The Southeast Asian Ministers of Education Organisation
Regional Centre for Education in Science and Mathematics
(SEAMEO RECSAM, Malaysia)

University of Tsukuba, Japan
https://www.criced.tsukuba.ac.jp/SEA-BES_CCRLS_and_Guidebooks.html

ISBN 978-967-930-057-4

Mathematics Challenges for Classroom Practices at the LOWER PRIMARY LEVEL

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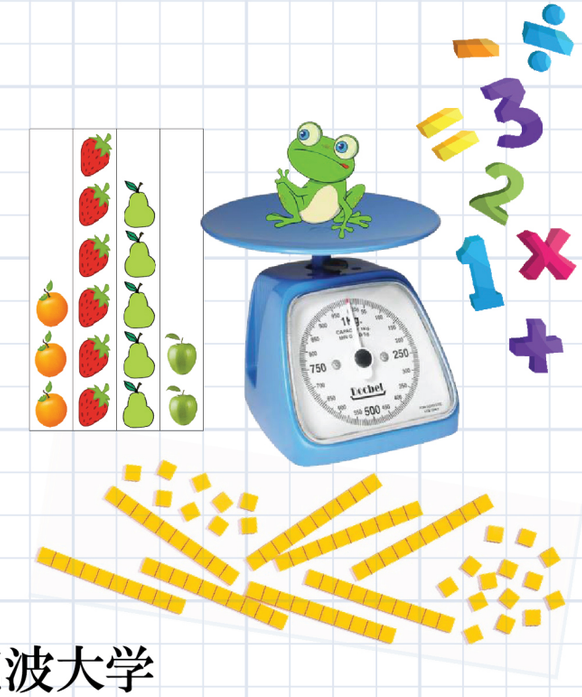
Based on SEAMEO Basic Education Standards:
Common Core Regional Learning Standards in Mathematics



Editors:

**TEH Kim Hong**
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With Support of:

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SEAMEO recsam 筑波大学 University of Tsukuba

*Published by*
SEAMEO RECSAM
Jalan Sultan Azlan Shah
11700 Gelugor
Penang, MALAYSIA

Cover design by Nur Syafiyah Binti Mohd Kassim

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ISBN: 978-967-930-057-4

## FOREWORD

Congratulations to CRICED, University of Tsukuba and SEAMEO RECSAM for another collaboration project in the publication of three series of guidebook entitled Mathematics Challenges for Classroom Practices based on the SEAMEO Basic Education Standards: Common Core Regional Learning Standards (SEA-BES CCRLS) in Mathematics. Two of the series are for the lower and upper primary level while the third series is for the lower secondary level. Generally, curriculum standards of subjects are not widely scrutinised by classroom practitioners and teacher educators compared to the curriculum specialists. The publication of these three new guidebooks anchor well and consolidate the role and importance of the SEA-BES Common Core Regional Standards document which was already introduced to all the 11 SEAMEO member countries and beyond since published in 2017.

The content of this guidebook series covers across Grade 1 to Grade 9 and consists of tasks written to understand the learning standards in Mathematics. The transfer of information from the SEA-BES CCRLS to the newly published book series will create an awareness among classroom teachers and teacher educators the importance and relevance of curriculum standards in formulating and designing learning specifications for students. The presentation of the book series emphasised on three aspects, namely highlighting the misconceptions, developed new ideas from the previously learned knowledge and explanation of new concepts in mathematics. The task-based approach will surely help readers to enhance their own mathematical understanding and ultimately provide better support for classroom teaching and learning.

I sincerely hope that the Minister of Education of SEAMEO members would provide support and promote the use of this guidebook series among educators and teachers in their respecting countries. This effort and spirit of cooperation among SEAMEO members and associate members can be realised to bring benefits for classroom practices, which will eventually benefit children of our future.

My sincere appreciation and congratulations to CRICED, University of Tsukuba as a project proponent and provided the financial support, SEAMEO RECSAM as the main collaborator, other collaborating partner institutions and individual educators and specialists for their expertise, commitment and contributions in this endeavour.

**Dr ETHEL AGNES P VALENZUELA**
**Director, SEAMEO Secretariat, Bangkok, Thailand**

## FOREWORD

On behalf of SEAMEO RECSAM, I would like to express my sincerest appreciations to the Centre for Research on International Cooperation in Educational Development (CRICED), University of Tsukuba for inviting the Centre as the main collaborator in the publication of the guidebook series, titled "Mathematics Challenges for Classroom Practices" for the i) Lower Primary Level, ii) Upper Primary Level and iii) Lower Secondary Level. Besides the involvement of SEAMEO RECSAM, many educators and specialists of other collaborating partners such as Khon Kaen University, Thailand; the Institute for the Promotion of Teaching Science and Technology (IPST), Thailand, SEAMEO QITEP in Mathematics, Indonesia; mathematics specialists in APEC economies, educators, local teacher educators, and curriculum specialists involved in the SEAMEO Basic Education Standards: Common Core Regional Learning Standards (SEA-BES CCRLS) in Mathematics had contributed their writings in this guidebook series.

SEA-BES CCRLS in Mathematics and Science was first published in 2017 by SEAMEO RECSAM but had limited and restricted usage despite being shared with all SEAMEO member countries and beyond. Today, SEA-BES CCRLS has been given a new life where the learning standards of the Mathematics component have been adopted and used as the main reference for this guidebook series. Having said that, I am grateful to the outstanding writing team who made this possible. I shall start by acknowledging the contribution of Professor Dr Masami Isoda, who initiated the idea and the project and had graciously invited SEAMEO RECSAM to produce this guidebook series; Ms Teh Kim Hong who coordinated the project with Mr Pedro Jr. Montecillo; Mr Gan Teck Hock who was later recruited to join the writing team and other mathematics specialists who were also invited to contribute their writings. Despite facing time constraints and changes of staff members, the writing team stayed intact with their contributions and commitment until this guidebook series is published. Besides the writing team, I also like to thank the panel reviewers of RECSAM who provided their constructive suggestions to improve the content of this guidebook series.

The guidebook series covers the mathematics content across Grade 1 to Grade 9 with the focus of utilising written tasks to understand the learning standards of SEA-BES CCRLS in Mathematics. The transfer of information from the later to the newly published guidebook series will create awareness among classroom teachers and teacher educators regarding the importance and relevance of curriculum standards in the planning of teaching and learning. The presentation of this guidebook series emphasised on highlighting misconceptions, contradictions, and developed new ideas from the previously learned knowledge to enhance learning and develop mathematical thinking. Such an approach of contradiction will foster deeper thinking among readers, thus enhancing mathematical understanding, translating into better support for classroom teaching and learning.

Without a doubt, much commitment and hard work had been invested to produce these guidebooks. I hope that this mathematics guidebook series will be used widely by teachers and educators of SEAMEO member countries for classroom practices. I sincerely hope SEAMEO Secretariat will also provide their support by promoting this guidebook series in the classrooms of educators and teachers in all SEAMEO member countries.

I am therefore proud to present this guidebook series as the contribution of SEAMEO RECSAM and CRICED, University of Tsukuba, for the promotion and development of mathematics education in this region. This would not have been possible without CRICED, University of Tsukuba's content expertise and financial support. I hope this valuable collaboration and cooperation will continue in other future projects to benefit education development in this region.

**Dr SHAH JAHAN BIN ASSANARKUTTY**
**Centre Director, SEAMEO RECSAM**

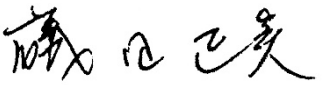
## FOREWORD

In addition to the Japanese Ministry of Education, Culture, Sports, Science Technology (MEXT), the University of Tsukuba has been playing the role of an affiliate member to collaborate with SEAMEO. As the Director of the Centre for Research on International Cooperation in Educational Development (CRICED), it is my pleasure to continue working with SEAMEO RECSAM on the SEAMEO Basic Education Standards for Mathematics (SEA-BES-M). This project was launched in 2014 as a reference book for curriculum reformers and teachers to develop the 21st century skills and OECD competency (2005) in education. For SEA-BES-M, I had collaborated with Pedro Montecillo Jr., Kim Hong Teh, and the late Mohd Sazali bin Hj. Khalid of RECSAM, along with the contribution from curriculum developers in SEAMEO countries, specialists in APEC economies and several internationally leading researchers. Even though it was published before OECD Education 2030, it is included the same structure as OECD Learning Compass.

Before SEA-BES-M was published, a comparative analysis of the mathematics curriculum documents of SEAMEO countries revealed that the higher order thinking presented as process standards is compartmentalised between every content description. Therefore, a major contribution of SEA-BES-M to the world, particularly the SEAMEO countries is its clear description of the meaning of higher order thinking for the mathematics curriculum standards to develop mathematical thinking. After the publication, we recognized that there are difficulties for readers to understand the intended meaning of every standard. This is mainly due to the fact that readers will interpret what is read based on their curriculum knowledge and experience teaching mathematics of their own countries. As such, it is crucial to develop a book series to be used as references for interpreting SEA-BES-M.

This book series is prepared, particularly for teacher educators, textbook authors, and curriculum developers as well as teachers to understand higher order thinking for developing mathematical thinking in their classrooms. In this book series, the authors had collaborated with leading researchers and educators in major teacher education institutions, CRME-IRDTP at KKU (Thailand), IPST (Thailand) and SEAMEO QITEP in Mathematics (Indonesia) with the support of the coordinators.

Furthermore, the authors had made some minor revisions to the SEA-BES-M to align with the needs of the Era of the 4th Industrial Revolutions to develop stakeholders and users of Artificial Intelligence and Big Data in business as well as establishing a successful life under the reality of humanity with technology. The minor revision was made based on the curriculum reform recommendation (2020) by APEC InMside Project with the purpose of promoting mathematical capitalism under mathematical-statistical-informational sciences on the demands of the Era. It is hoped that this book series is used in teacher education to develop new curriculum content knowledge for teaching in this Era. I would like to acknowledge SEAMEO secretariat and centres for their collaborations, especially Shah Jahan Bin Assanarkutty , the Director of RECSAM, who made possible this publication. Last but not least, I would like to convey my sincere appreciation to all contributing writers stated in the contributor list and Gakko Tosho (the Japanese Textbook Publisher) which provided us innovative ideas.



**ISODA MASAMI**
**Director of CRICED, University of Tsukuba, Japan**

## PREFACE

Realising reform in school curriculum beyond the 21st century and revitalising teacher education have been set as prioritised agendas in SEAMEO countries. On this demand, SEAMEO Basic Education Standards: Common Core Regional Learning Standards (SEA-BES:CCRLS) in Mathematic was published in 2017 for the main purpose of strengthening collaboration on curriculum standards and learning assessment across different educational systems in SEAMEO countries. In order for this document to be understood beyond the curriculum developers, supporting materials need to be developed for helping other users such as classroom teachers and teacher educators to acquire a deeper understanding of the standards. This book is an initiative to provide such support. With this support, it is anticipated that teachers and teacher educators will be able to innovate their classroom practices for developing competency and professional development aligning with the trends of the 4th Industrial Revolution.

**Mathematics Challenges for Classroom Practices at the Lower Secondary Level** consists of mathematical tasks for the following strands:

- Numbers and Operations
- Quantity and Measurement
- Shapes, Figures and Solids
- Pattern and Data Representations

These tasks are prepared to be used for pre-service and in-service mathematics teacher education. Its main purpose is to help readers develop mathematical knowledge for teaching (MKT) which consists of subject matter knowledge (SMK) and pedagogical content knowledge (PCK) (see, Ball, Thames, and Phelps, 2008)[1]. In developing the tasks, the English edtion of Japanese mathematics textbooks publised by Gakko Tosho[2] had been used as the main reference. These textbooks provided major guides for learning mathematics through problem solving approach to develope mathematical thinking. As such, the tasks in this book are focused on mathematical ideas and ways of thinking. Basically, the tasks are developed in three ways: (a) analysing misconception of ideas, (b) developing ideas from previously learnt knowledge, and (c) using inquiry-based investigation to learn new ideas. In designing the tasks, the importance of local contexts of the SEAMEO community had been considered. However, some essential elements of Japanese school mathematics were also incorporated into the tasks. It is hoped that these elements will set off a new breath of mathematics learning in the SEAMEO community, shaping our students to be critical and creative thinkers in the era of artificial intelligence and data science.

Each task is written based on a standard in SEA-BES: CCRLS in Mathematics and it serves to clarify (a) the curriculum knowledge of teaching in PCK, and (b) the mathematical ideas and ways of thinking on SMK related to the standard. Apart from that, SEA-BES CCRLS in Mathematics is used as the basic source for MKT. Since SEA-BES CCRLS in Mathematics was developed with curriculum specialists of SEAMEO countries, solving the tasks will also provide a bird's eye view of their national curriculum to the readers. Furthermore, it will broaden their perspective of mathematical ideas, ways of thinking and curriculum sequence with respect to their use of local textbooks.

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1. Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What makes it special? Journal of Teacher Education, 49(5), 389-407.
2. Hitotsumatsu, S. et al (2005). Study with your friends: mathematics for elementary school (G. 1-6). Tokyo: Gakko Tosho./ Isoda, M., Murata, A., Yap, A. (2011, 2015, 2020) Study with your friends: mathematics for elementary school (G. 1-6). Tokyo: Gakko Tosho/ Isoda, M., Tall, D. (2019). Junior High School Mathematics (G. 1-3). Tokyo: Gakko Tosho. For developing tasks in this book, authors are inspired by the tasks and task sequence of these mathematics textbooks.

Although the targeted readers of this book are teachers and teacher educators, most of the tasks can also be solved by students in classrooms as they are also aligned with school mathematics curriculum. In addition, teachers and teacher educators are expected to solve them without much difficulties. However, studying the tasks carefully will raise the awareness of the depth of SMK and PCK required to complete the tasks. This will triggered off a need to upskill their understanding of mathematical ideas and function effectively to develop students mathematical competency. Thus, solving tasks in this book will provide readers the opportunities to relearn the mathematics content for teaching. Furthermore, it will also help them to identify (a) the objectives of teaching the content, (b) the gap between students' prior knowledge and what is to be learnt, (c) what and how students reorganise the content knowledge of their learning, (d) students' difficulties in learning the content, and (e) what ideas will be developed through their new learning.

Readers may also choose to work with any task according to their interests. It is not necessary to work out all the tasks according to the sequence in the book. However, for a deeper understanding of the mathematical ideas embeded in a task, it is recommended that readers should solve the task in the following manner: (a) solve the task by themselves and read the related standards, (b) communicate their solutions with others to identify what is really new content for them, and (c) paraphrase and summarise the communication with others based on the perpective of mathematical ideas and ways of thinking that align with the framework of SEA-BES CCRLS as described in Chapter One.

This book is recommended for use in many ways and various contexts. Firstly, as all the tasks were designed based on school mathematics curriculum, so they can be used directly by students as learning tasks. In addition, teachers can also use the book as a quick guide to create similar mathematical tasks that incorporate mathematical thinking. Secondly, when the book is used in the context of in-service teacher education such as in lesson study, teachers can solve the tasks in this book as a step to gain a deep understanding of the mathematical ideas in order to prepare a unit of lesson plan based on the standard chosen. This may help to improve the effectiveness of lesson planning and anticipate responses of students to the tasks. Thirdly, in the context of pre-service teacher education, the tasks in this book can serve as a mean to acquire MKT which may be required for any teacher employment examination or entrace examination for an education graduate programme. Fourthly, in the context of mathematics education research, this book can be used as a reference for MKT. Last but not least, when the book is used in the context of curriculum reform and textbook revision, it could serve as a guide to formulate new objectives and tasks which are not existing in their current curriculum and textbooks.

**Mathematics Challenges for Classroom Practices at the Lower Primary Level** is the outcome of many contributions of educators and academia from different institutions. In order to ensure good quality of content and streamline the presentation of the writings, many rounds of editing and rewriting were unavoidable. It is our hope that the mathematical ideas and ways of thinking promoted through this book will enhance the teachers' capacities to develop their students' potentials in facing the challenging and demanding era ahead.

TEH Kim Hong
ISODA Masami
GAN Teck Hock

## ACKNOWLEDGEMENTS

The authors/editors would like to thank and express our gratitude to the following contributors, institutions and organisations in making this project and publication possible:

Dr Ethel Agnes Pascua-Valenzuela, Director of SEAMEO Secretariat, Bangkok, Thailand for providing CRICED the opportunities to collaborate with RECSAM and other SEAMEO centres;

Dr Shah Jahan Bin Assanarkutty, the Centre Director of SEAMEO RECSAM for his encouragement and support in realising the publication of the books;

Dr Johan Bin Zakaria, the ex-Centre Director of SEAMEO RECSAM for his confidence and generosity of granting permission to use the SEABES: CCRLS in mathematics to develop this guidebook series;

The University of Tsukuba generously supported the funding of this guidebook series project. The books will be disseminated to SEAMEO member countries and recommended for use by teachers, teacher educators for the benefit of the SEAMEO community;

Assoc. Professor Dr Maitree Imprasitha, the Vice President of Khon Kaen University(KKU) and Director of the Institute for Research and Development in Teaching Profession (IRDTP) for ASEAN, Thailand, for providing the support in organising a workshop to lead the KKU and other affiliated Universities lecturers in contributing writings to this guidebook series;

Dr Suppattra Pativisa, Senior Professional Expert of The Institute for the Promotion of Teaching Science and Technology (IPST), Thailand, in lending the support and led the lecturers in contributing writings to the guidebook series;

Dr Wahyudi, the former director of SEAMEO QITEP in Mathematics (SEAQIM) and currently the Deputy Director of SEAMEO Secretariat, together with Dr Sumardyono, the current Director of SEAQIM, provided the support to lead their mathematics lecturers in contributing the writings;

The specialists of APEC economies of the year September 2017 during the 13th APEC-Khon Kaen University Conference, and the year February 2018, in providing suggestions and contributions of resources in helping to conceptualise the content and outcome of the books;

All curriculum specialists and educators who attended the SEA-BES CCRLS in Mathematics workshop in March 2017, held in SEAMEO RECSAM, contributed ideas and suggestions on shaping the outcome of the guidebooks, as well as those who followed the sessions on how to write the tasks and submitted the writings for considerations;

Pedro Lucis Montecillo Jr, mathematics specialist of RECSAM (until May 2018) for assisting the coordination at the early stage of the project;

All staff of SEAMEO RECSAM who supported the production of these books by providing ideas, coordination and manage the distribution of books to other institutions.

This work was supported by The Japan Society for the Promotion of Science (JSPS) Grants-in-Aid for Scientific Research (KAKENHI) Grant Number 19H01662 and19K21743.

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# CHAPTER 1

## Guide to the SEA-BES CCRLS Framework in Mathematics

The Southeast Asia Basic Education: Common Core Regional Learning Standards (SEA-BES CCRLS) was developed and directed to create a harmonious SEAMEO Member community in the era of artificial intelligence and data science through mutual understanding. In this respect, the CCRLS framework in Mathematics (2017) outlines three basic components towards developing creative, competent and productive global citizens essential for achieving this aim. The comprehensive illustration of the framework is attached in Appendix A. The revised framework in Figure 1 shows the interconnection of the three components.

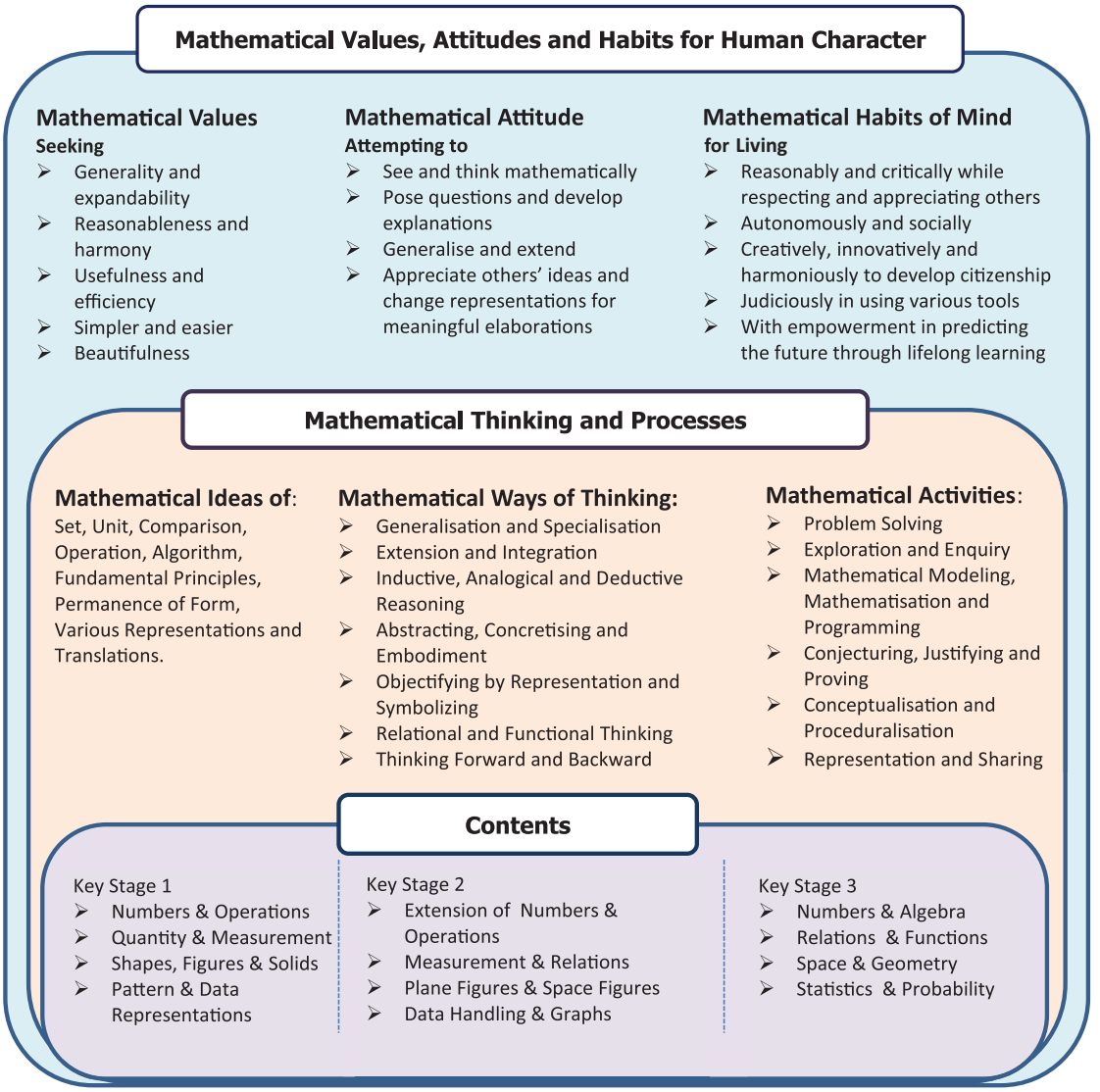



*Figure 1*. Revised CCRLS Framework in Mathematics

This book is written to guide readers acquire a better understanding of this framework particularly on mathematical thinking and processes which are embedded in all the tasks. The detailed explanation of mathematical ideas, mathematical ways of thinking and mathematical activities can

be found in Appendix B. The standards for the strand on Mathematical Processes-Humanity is attached in Appendix C to provide readers with challenging activities to promote metacognitive thinking at different level of arguments to make sense of mathematics. In order to understand the development and progression of learning from the primary level to the secondary level (Key Stage 3), the learning standards of Key Stage 2 and Key Stage 3 can be referred in Appendix D and Appendix E, respectively.

The interconnection of the three components is shown in Figure 2. The ultimate aim of the CCRLS framework is to develop mathematical values, attitudes and human characters which are the essence of a harmonious society. This component is closely related to the affective domain of human character traits which correspond to soft skills that can be developed through appreciation. In relation to this, acquisition of mathematics contents as hard skills and reflection on the thinking processes are needed to inculcate the capability of appreciation. The reflection is necessary for learners to recognise their cognitive skills derived through the contents. Even though contents appeared to be learned independently through acquisition, the mathematical thinking and process, and the appreciation of mathematical values, attitudes and habits for human character is possible to be developed through reflective experiences.

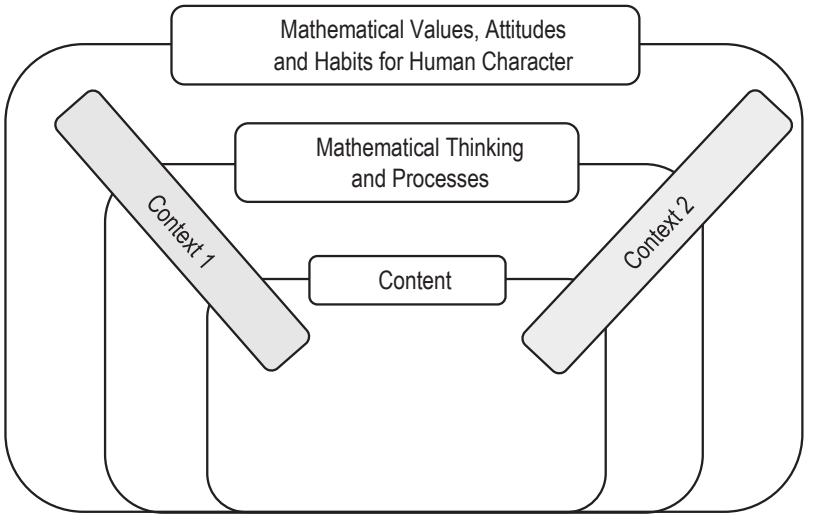



*Figure 2*. Interconnection of Components in CCRLS Framework in Mathematics

The three components will not be ideally operationalised without appropriate contexts. The tasks in this book provide the contexts for developing the mathematical thinking and processes, which are the key learning objectives. Completing the tasks correspond to gauging the readers' acquired mathematical knowledge for teaching. Thus, it is recommended that readers should constantly reflect on the appropriateness of their solutions to the tasks. Other than this, comparing solutions and discussion with others should always be done habitually in order to gain a deeper understanding of the mathematical processes. This may enable the readers to discover any hidden mathematical ideas and structures in the tasks with appreciation. The tasks are specifically designed to cater for this purpose.

In a nutshell, an important target of solving the tasks in this book is to enable readers to acquire a better insight of the learning standards. This insight will in turn help them to understand and appreciate their national mathematics curriculum from the perspective of SEABES CCRLS. Furthermore, since the learning standards are developed based on the framework which emphasised on the components of contents, thinking and processes, as well as values of mathematics, ultimately, readers will be able to acquire mathematical teaching knowledge with appreciation.

# CHAPTER 2

# Numbers and Operations

**Topic 1: Introducing Numbers Up to 120**

**Standard 1.1:**

Enjoying counting orally and manipulatively with number names, without symbolic numerals

i. Develop fluency of the order of number names and use it based on situations
ii. Set initial object for counting, direction of counting and recognise the last object with one-to-one correspondence
iii. Distinguish the original situation with concrete objects and the representation of counting chips, blocks or marbles

**Sample Tasks for Understanding the Standards**

**Task 1: Counting and Comparing Sets of Objects**

Diagram 1

i. Given the situation of both the birds and squirrels are looking for nests in Diagram 1.

- If one nest is only for one animal, can all the nests be occupied? Why or why not?
- If there is a condition that only the birds can occupy the nests, can all the nests be occupied? Why or why not?
- How is the idea of set embedded in this comparison task?

ii. Why is the condition of limiting the nests only for the birds significant in helping children learn the idea of one-to-one correspondence?

iii. When asked to count the number of squirrels in Diagram 1, some children counted 5 squirrels and some counted other numbers such as 6, or 7. In this task, 5 squirrels is a denomination number. Children consider the squirrel as a unit for counting.

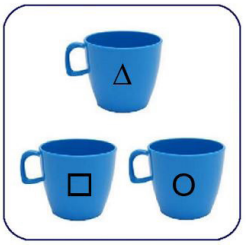
- Explain the meaning of denomination number.
- Based on Diagram 1, give some other examples of denomination numbers.
- Suggest a possible reason why some children's answers are more than 5.
- How can you help children avoid the common error of double counting?
- When asked "How many birds are there?", a student said 6, another said 6 birds. Which answer is more appropriate? Why?

Diagram 2

iv. A teacher used same coloured chips to represent the nests and birds.

- Can the chips for the birds and the nests be distinguished?
- What does the quantity of chips represent?
- Explain how the square grid in Diagram 2 can be used with the chips to help children make sense of one-to-one correspondence.
- What future learning can be followed after this task? Explain your answer.

## Task 2: Counting Objects



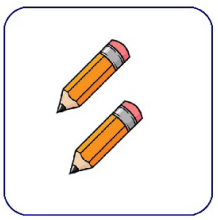



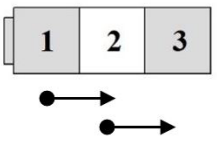
Diagram 3

Diagram 3 shows some picture cards used for children to learn counting the number "three".

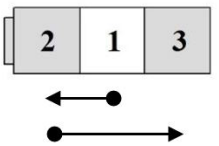
i. Why are numbers "one" and "two" included in the discussion of learning number "three"?

ii. Counting the cups can begin with any one of the three cups. As such, there are six counting sequences. One example is "Δ, □, O".

- What are the other five possible counting sequences?
- Will the choice of first cup affect the outcome of counting? Why or why not?
- How will the direction of counting the cups affect the outcome of counting?
- How the last cup counted is related to the total number of cups?




Direction ①




Direction ②

Diagram 4

iii. Diagram 4 shows two sets of three interlocking cubes. Two children counted the cubes starting with a different cube and in different directions as shown.

- Which direction of counting should be encouraged? Explain your reasons.
- Comparing the two situations, how will direction of counting affect the outcome of counting the cubes?

iv. When guiding children to count, why discussion of counting sequence could only start from number 3, not 1 or 2?

v. What are the requisite ideas for children to learn counting?

**Standards 1.2:**

Understanding and using the cardinality and ordinality of number with objects and numerals through activities of grouping, corresponding and ordering, and developing number sense and appreciate its beautifulness

i. Group objects for counting with conditions such as cups, flowers, and rabbits in situations and introduce the numerals
ii. Obtain fluency of counting concrete objects, understand counting on, and recognise the necessity of zero
iii. Compare different sets by one-to-one correspondence and recognise larger, smaller or equal with appreciation in drawing paths between objects
iv. Compose and decompose numbers for strengthening the number sense and cardinality
v. Understand the difference of ordinal and cardinal numbers and use them appropriately in situations and challenge the mixed sequence
vi. Acquire number sense and appreciate the beautifulness in ordering numerals with and without concrete objects

**Sample Tasks for Understanding the Standards**

**Task 1: Grouping Objects for Counting**

① How many tortoises?

② How many fish?

③ How many animals in the pond?

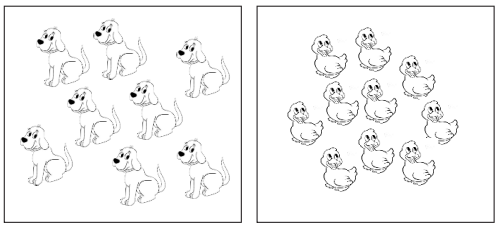
Diagram 1

i. Diagram 1 shows a task for children to learn "group objects for counting".

- What are the requisites for grouping objects to count?
- Why question ③ should not be asked before questions ① and ②?

ii. Develop a story based on Diagram 1.

- Explain how you will use the story to reinforce children's abilities to group objects for counting.



➢ Are there more puppies or ducklings?

Diagram 2

iii. Diagram 2 shows a task for children to learn comparing quantities of two sets of objects.

- Using one-to-one correspondence, explain how to find which set of animals is larger or smaller.
- What are the challenges that may be faced by children when completing this task?
- Suggest how you would help children to overcome the challenges.

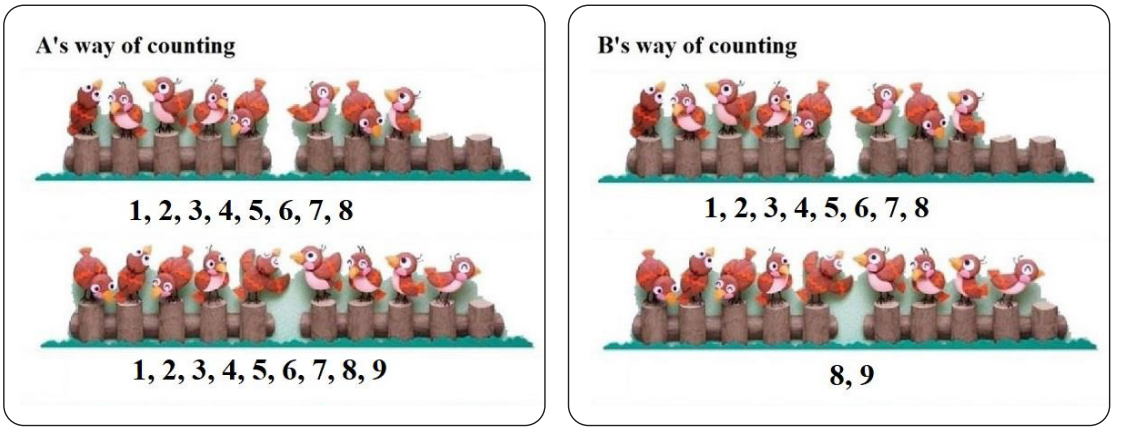



Diagram 3

iv. Diagram 3 shows two different counting strategies used by student A and B to count the number of birds. A used the count-all strategy while B used the count-on strategy.

- Explain briefly the two strategies of counting.
- Which strategy of counting is more efficient? Explain your answer.
- Explain how the more efficient way of counting can help children learn addition of numbers later.

## Task 2: Ordering Numbers

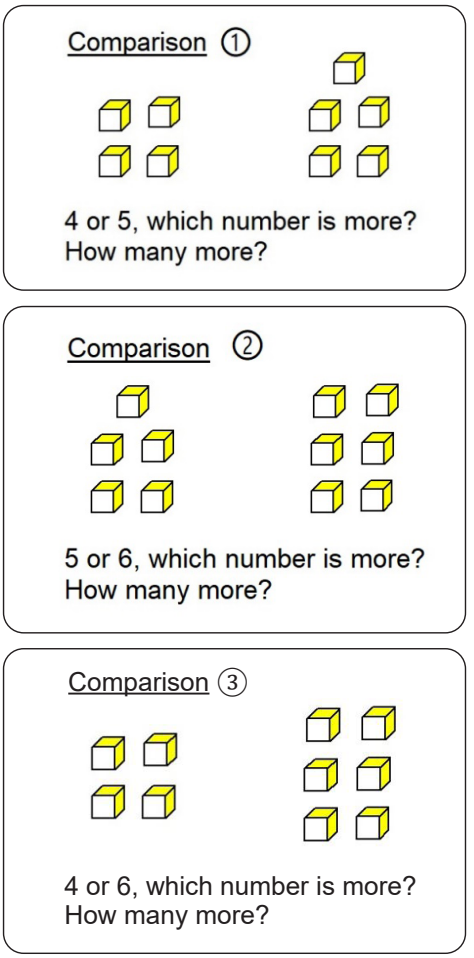



Diagram 4

i. Diagram 4 shows three comparisons of numbers using interlocking cubes.

- How will you arrange the interlocking cubes to help your students compare the two numbers?
- How will you sequence the three comparison tasks for your students?
  - ❖ Explain your reasons.
  - ❖ How does the transitive law affect your choice of sequencing the tasks?

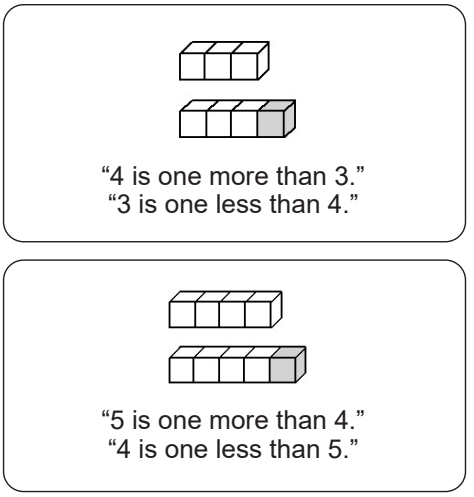



Diagram 5

ii. The interlocking cubes in Diagram 5 are used to show the numerical relationships between the numbers 3 & 4 and 4 & 5.

- Explain how you will use interlocking cubes to help your students determine the relationships between the numbers
  - ❖ 5, 3, 2, and 1.

Diagram 6

iii. Diagram 6 shows ordering of numbers from zero to 10.

- What number pattern do you observe in the ordering of these numbers?
- What questions should you ask children in order to raise their awareness on the number pattern?
- How would you use the diagram to help children learn counting forward and counting backward?
- Should zero be included in the diagram for children to learn ordering numbers up to 10? Explain your reasons.

iv. Explain how the use of base-ten blocks would help children to appreciate the beautifulness of ordering numbers up to 10?

## Task 3: Introducing Zero

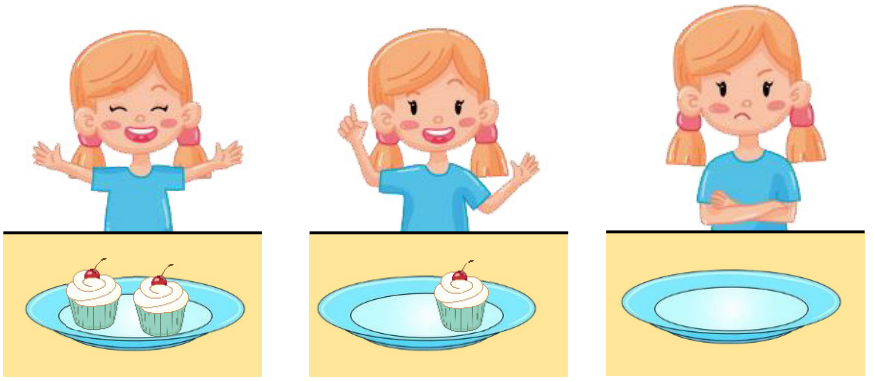


Diagram 7

i. Diagram 7 shows a situation involving a sequence of decreasing quantity to introduce zero.

- What type of counting is applied to introduce zero here?
- Design a series of questions to help children learn the number zero with the use of this diagram.
- Describe briefly how you would introduce the numeral “0” with the situation in this diagram.

Diagram 8

ii. Diagram 8 shows a situation involving 4 balls and a basket.

- Suggest an activity for children to explain the meaning of zero using the situation in Diagram 8.

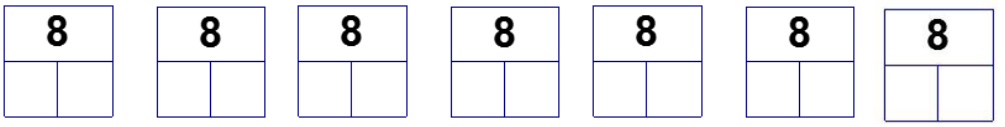
## Task 4: Composing and Decomposing Numbers

***Decomposing Numbers***


Diagram 9

i. Diagram 9 shows a set of 8 frogs at a pond.

- What are the subsets of frogs that can be formed from the 8 frogs?
- Explain how the subsets can be used to introduce the idea of number decomposition to your students.
- Decompose number 8 based on the subsets.



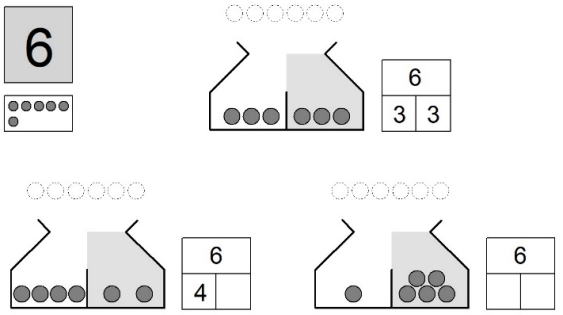




Diagram 10

ii. Decompose number 6 based on the situations in Diagram 10.

Diagram 11

iii. Diagram 11 shows 7 interlocking cubes. Explain how these interlocking cubes can be used to help children find all the decompositions of number 7.

iv. Find all the decompositions of number 9.

***Making Ten***

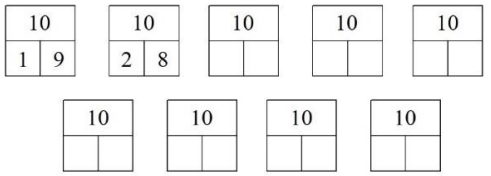



Diagram 12

i. Diagram 12 shows two decompositions of number 10.

- How many different decompositions are there for 10?
- What pattern is seen in all the decompositions of number 10?
- 1 & 9 and 9 & 1 are two similar pairs of decompositions of 10. What are other similar pairs?
- Explain how the ideas of decomposing 10 can help children learn number operations at a later stage.

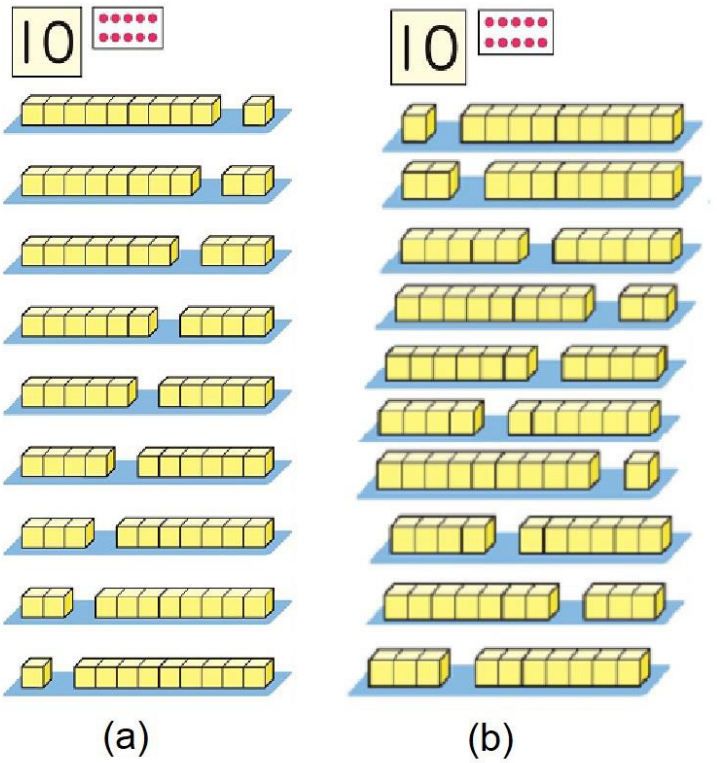



Diagram 13

ii. Diagram 13 shows two arrangements of interlocking cubes showing different compositions of 10.

- Which arrangement do you think is easier for children to find all the decompositions of 10 and enable them to appreciate the usefulness and beautifulness of ordering numbers?
  Explain your reasons.

## Task 5: Differentiating Cardinal, Ordinal and Nominal Numbers

**A convesation between two children:**

Rudy: I live at the **number 2 house** on the right side of the lift. The house number is **605**.

Rudy: I do not like this new house because I have to travel **six floors** down to come to the garden.

Din: Wao! You are rich. You live in **two houses**.

Rudy: I have only **one house**. There are **six houses** on my floor. I actually live in **the second** house.

Din: My friend, Raj, told me he lives in the **first floor**. Which button should I press to reach your house when I get into the lift?

Rudy: Don't worry. Just call this telephone number and I will come downstairs to meet you. My number is **04-68818288**

Diagram 14

i. With the help of examples, explain the differences between cardinal, ordinal and nominal numbers.

ii. Diagram 14 shows the conversation between Rudy and Din.

- How would you help children understand the differences in the meaning of numbers used in this conversation?

iii. In this context, what are the words you can use to describe order?

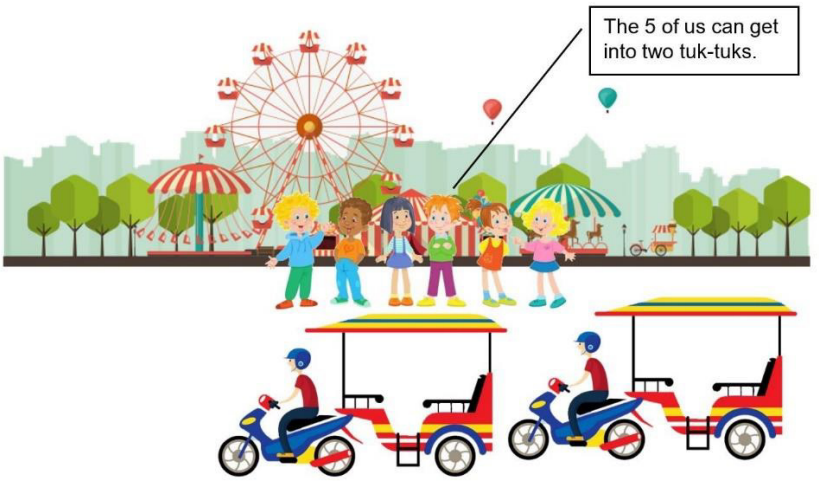



Diagram 15

iv. Diagram 15 shows Phirum and his friends after visiting a fun park. He was standing fourth from the front. He instructed the first 3 friends from the front to go into the first t*uk-tuk*, and another 2 friends behind him to go into the second *tuk-tuk*. He was happy and said 'The 5 of us can get into the 2 *tuk-tuks*'

From Phirum's last instruction, he seemed to have some confusion on the situation.

- What was not accurate in his last instruction?
- Why do you think he was confused?
- How to help him to overcome the confusion?

**Standards 1.3 :**

Introducing base-ten system with groupings of 10 and extend numbers up to 120

i. Extend numbers to more than 10 with base-ten manipulative representing numbers in ones and tens, and appreciate the base-ten numeration system.
ii. Extend the order sequence of numbers to more than 10 in relation to the size of ones and tens and compare numbers using numeral in every place value.
iii. Introduce number lines to represent the order of numbers and for comparison starting from zero and counting by ones, twos, fives, and tens.
iv. Enjoy various ways of the distribution of objects with counting such as playing cards, and explain it and enhance number sense
v. Draw diagram for representing the size of number with base-ten blocks such as a flat (square) for a unit of hundred and rectangular bar for a unit of ten

**Sample Tasks for Understanding the Standards**

**Task 1: Finding Easier Ways to Count Objects**

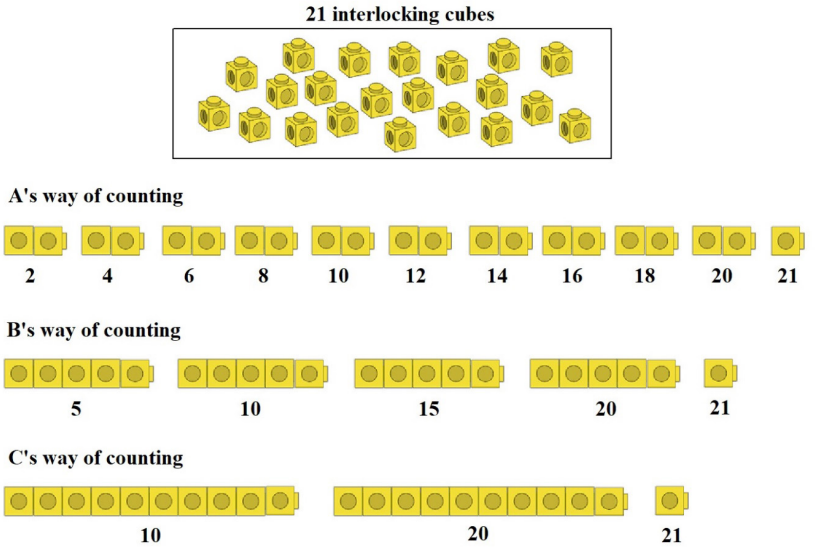



Diagram 1

i. Diagram 1 shows 21 interlocking cubes given to three children, A, B and C. The three children counted the number of cubes in different ways.

- Explain briefly the three ways of counting.
- What are the advantages of these ways of counting?
- The SEA-BES CCRLS framework in mathematics (refers Chapter 1) illustrates five mathematical values. Which of the values are linked to these ways of counting?
- Explain how these ways of counting can help children learn other mathematical ideas in their later stage of learning numbers and operations?

**Task 2: Place Value**

***Numeration System***

i. Explain the difference between number and numeral.

ii. The universally used Hindu-Arabic numerals are formed by ten basic symbols, known as digits, together with the idea of the base-ten place value.

- What are the ten digits?
- Why is the symbol for zero crucial in forming the numerals?
- Explain briefly the idea of the base-ten place value system.

iii. Other than the Hindu-Arabic numerals, another numeration system still in use is the Roman numerals. This system uses seven Latin alphabets as basic symbols to form numerals:

I – 1, V – 5, X – 10, L – 50, C – 100, D – 500, and M – 1000.

- Write in the Hindu-Arabic numerals, the numbers represented by each pair of the Roman numerals.
  (a) IV and VI (b) IX and XI (c) XIV and XVI
  (d) XL and LX (e) DC and CD (f) MMC and MCM
- Explain the major differences between the Roman numeration system and the Hindu-Arabic numeration system.

***Numbers Up to 20***

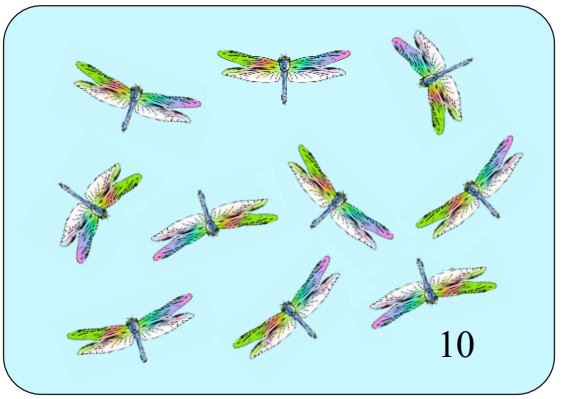



Diagram 2

i. Diagram 2 shows a numeral-picture card with 10 dragonflies. At this early experience with the numeral 10, children often see it as a representation of number 'ten', but do not understand the idea that the digit 1 represents 1 ten, and the digit 0 represents 0 ones.

- Explain why this is so.
- How would you help children to understand the idea.

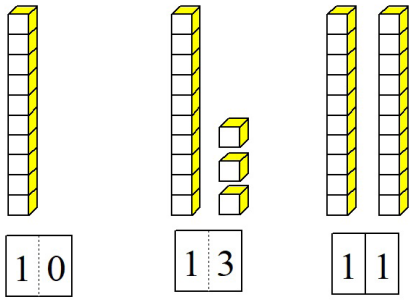



Diagram 3

ii. A teacher used some base-ten blocks to help children learn the idea of place value. Diagram 3 shows the work of a child.

- What is the misconception of the child?
- What would you suggest to the teacher in order to help the child correct the misconception?

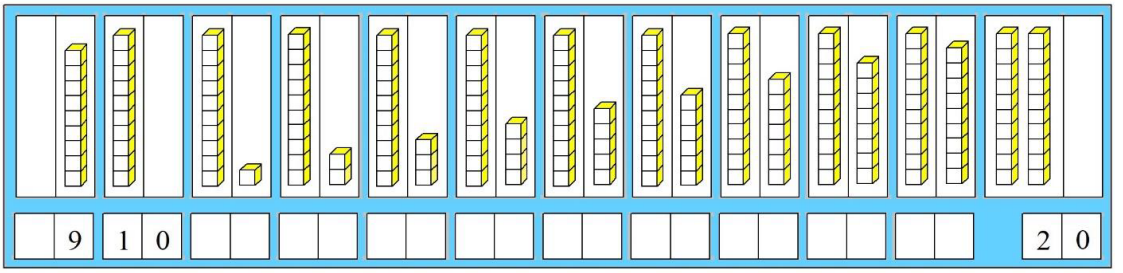



Diagram 4

iii. Diagram 4 shows how the numbers 9 to 20 are represented with base-ten blocks placed on a two-column chart.

- Why are the retangular bars placed on the left column of the chart?
- If the rectangular bars are placed on the right column, do they change the values of the numbers? Explain your reasons.
- Some children find it difficult to tell whether the numerals 16 or 19 is larger. How would you help the children overcome this difficulty?

iv. What key ideas are children expected to develop when using base-ten blocks to represent numbers?

***Numbers More than 20***

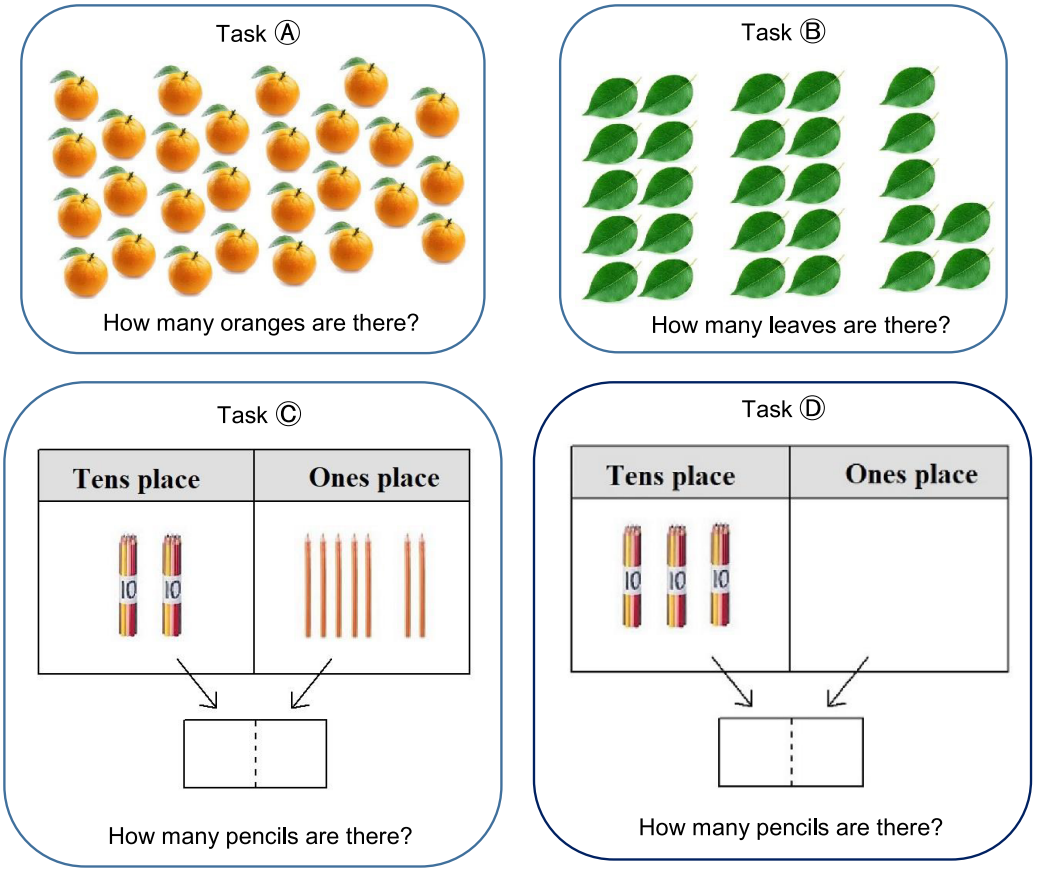



Diagram 5

i. Diagram 5 shows four tasks involving counting of objects for Grade 1 students.

- Explain why all four tasks are crucial in helping children to conceptualise base-ten place value.
- How will you sequence these four tasks?
- Explain your reasons.

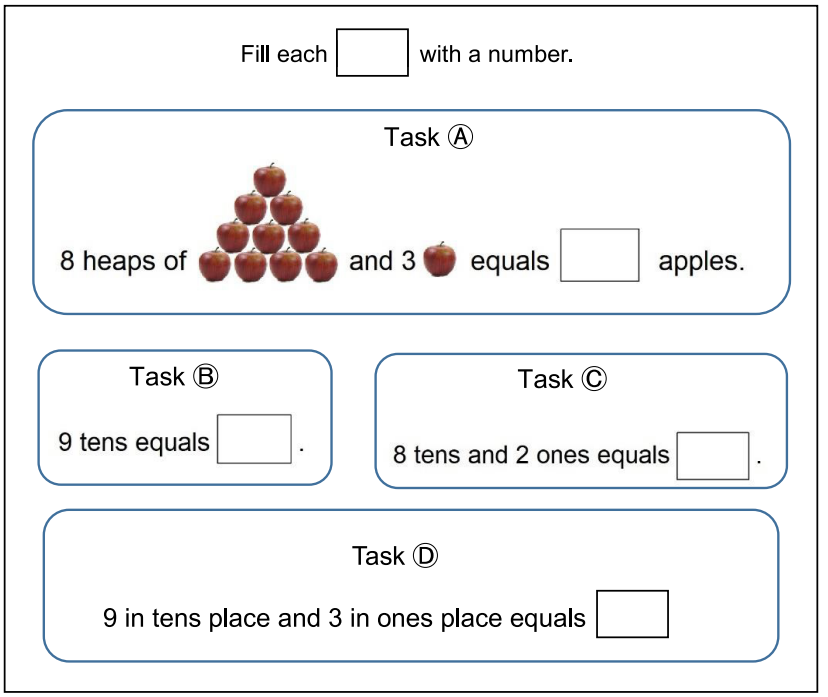



Diagram 6

ii. Diagram 6 shows another four tasks involving numbers up to 100.

- Will you ask your students to do all the four tasks? Explain your reasons.
- How will you sequence these four tasks? Explain your reasons.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | | 13 | 14 | 15 | 16 | 17 | 18 | | 20 |
| 21 | 22 | 23 | 24 | 25 | | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | | 35 | 36 | 37 | | | |
| 41 | 42 | | 44 | | 46 | | 48 | 49 | 50 |

Diagram 7

iii. Diagram 7 shows 'adding one' and 'adding 10' structure of number chart.

- Fill each ☐ with a number.
- Explain how this chart can be used to enhance children's understanding of place value.

## Task 3: Saying Numbers in Different Languages

Saying numbers in different languages can be a daunting task. Table 1 shows how 1 to 20 is read in different languages. Do enjoy reading them.

Table 1
Saying Numbers in Different Languages

| Language | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| English | one | two | three | four | five | six | seven | eight | nine | ten |
| | eleven | twelve | thirteen | fourteen | fifteen | sixteen | seven-teen | eighteen | nineteen | twenty |
| Malay | satu | dua | tiga | empat | lima | enam | tujuh | lapan | sembilan | sepuluh |
| | sebelas | dua belas | tiga belas | empat belas | lima belas | enam belas | tujuh belas | lapan belas | sembilan belas | dua puluh |
| Mandarin | yī | èr | sān | sì | wǔ | liù | qī | bā | jiǔ | shí |
| | shí yī | shí èr | shí sān | shí sì | shí wǔ | shí liù | shí qī | shí bā | shí jiǔ | èr shí |
| Japanese | ichi | ni | san | shi, yon | go | roku | shichi, nana | hachi | kyuu | jyu |
| | jyu ichi | jyu ni | jyu san | jyu yon | jyu go | jyu roku | jyu nana | jyu hachi | jyu kyuu | ni jyu |
| Tagalog | sa | dalawa | tatlo | apat | lima | anim | pito | walo | siyam | sampu |
| | labing-isa | labin dalawa | labin tatlo | labing-apat | labin lima | labing-anim | labim pito | labing walo | labing siyam | dalaw ampu |
| Thai | nùeng | sŏng | săm | sì | hâ | hòk | chèt | pàet | kâo | sìp |
| | sìp et | sìp sŏng | sìp săm | sìp sì | sìp hâ | sìp hòk | sìp chèt | sìp pàet | sìp kâo | yii sìp |
| Laos | neung | sawng | saam | sii | haa | hok | ched | bpaed | gao | sip |
| | sip-et | sip-song | sip-saam | sip-sii | sip-haa | sip-hok | sip-jet | sip-bpaet | sip-kao | sao |
| Khmer | mouy | pii | bei | buon | pram | pram muoy | pram pii | pram bei | pram buon | dap |
| | dap Mouy | dap pii | dap bei | dap buon | dap pram | dap pram muoy | dap pram pii | dap pram bei | dap pram buon | mpei |
| Vietnamese | một | hai | ba | bốn | năm | sáu | bảy | tám | chín | mười |
| | mười một | mười hai | mười ba | mười bốn | mười lăm | mười sáu | mười bảy | mười tám | mười chín | hai mươi |
| Burmese | tit | hnit | thone | lay | nga | chout | khoon-nit | shit | ko | ta-hseare |
| | seh-tit | seh-hni | seh-thoun | seh-lei | seh-nga | seh-chao | seh-kun-hni | seh-shit | seh-koe | hna-seh |
| Tamil | ondru | irandu | Moon dru | naangu | ainthu | aru | erlu | ettu | onpathu | pattu |
| | Pathin ondru | Pani rendu | Pathin moondru | Pathi naangu | Pathin ainthu | Pathin aru | Pathin Erlu | Pathin ettu | Path onpathu | Iru Pathu |

i. Study and compare how the numbers 11 to 20 are said in the different languages.

- Do you observe any interesting pattern in some of the languages? If yes, describe the pattern.
- Some of the patterns are more helpful for children to learn numbers 11 to 20. Identify the pattern and explain why it is helpful.

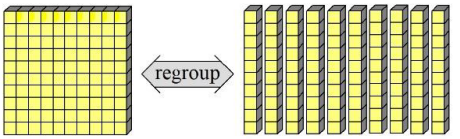
ii. The Khmer language uses an interesting pattern in saying numbers 6 to 9.

- Describe the pattern.
- Do you think the pattern makes it easier for Cambodian children to learn numbers? Explain your reasons.

**Task 4: Extending Numbers to 120**

Representation Ⓐ:
100 stamps

Representation Ⓑ:
10 sets of tens is 1 set of hundred.




Representation Ⓒ:
10 tens is 1 hundred.

| Hundreds | Tens | Ones |
|---|---|---|
| 0 | 10 | 0 |

regroup

| Hundreds | Tens | Ones |
|---|---|---|
| 1 | 0 | 0 |

Representation Ⓓ:
10 sets of ones is
1 set of ten.

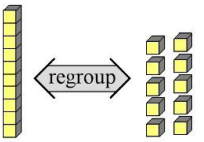



Diagram 8

i. Diagram 8 show four representations of regrouping involving ones, tens and hundreds.

- Explain how these representations can help children develop the concept of base-ten place value?
- How will you order the sequence of tasks using each of these representations? Explain your reasons.

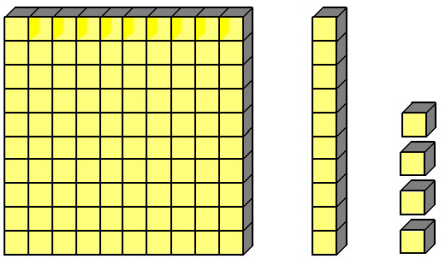


Diagram 9

ii. Looking at the base-ten blocks in Diagram 9, a student said: "*The number is 1 hundred and fourteen*".

But when asked to write the number, he wrote: 100104.

- Why such misconception happened?
- Explain how you can help the student to arrive at the correct answer and correct his misconception.

## Task 5: Introducing Number Lines

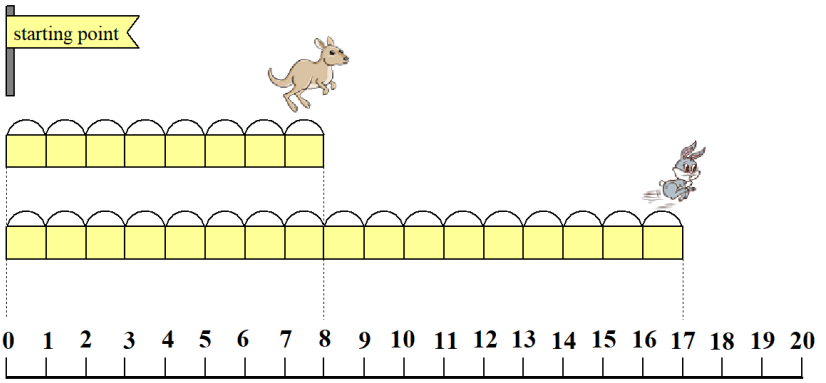



Diagram 10

i. Diagram 10 shows two situations on two scaled bars.

- Based on the scaled bars, a child says the kangaroo jumps 9 steps and the rabbit hops 18 steps. What is the child's misconception
- Without an initial reference point, can you distinguish an interval and a scale of measure? Explain your answer.
- A number line is then aligned to the scaled bars.
  - ❖ What is a number line?
  - ❖ What can a number line be used for?
  - ❖ Why is it important to have '0' at the begining of a number line?
- How does the number line help children to understand the order of numbers?
- How does the number line help children to determine the difference in the number of steps between the rabbit hop and the kangaroo jump?

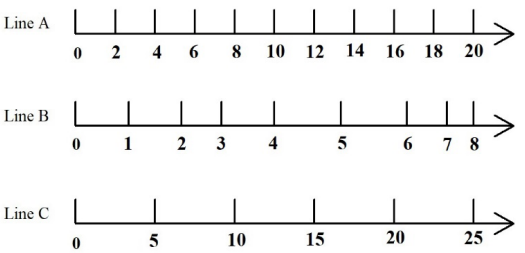



Diagram 11

ii. Between Line A, Line B and Line C in Diagram 11, which one **cannot** be considered as a number line? Explain your reasons.

iii. Must a number line begin with a zero point? Explain your reasons.

**Task 6: Different Types of Number Lines**

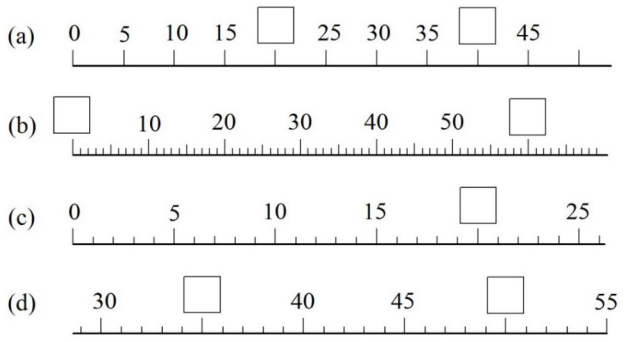



Diagram 12

i. Diagram 12 shows four number lines.

- Fill in each □ with a number for each of the number lines.
- What are the diffrences in number lines (a) (b), (c) and (d) ?
- How do you explain the differences and similarities between number lines (a) and (c)?

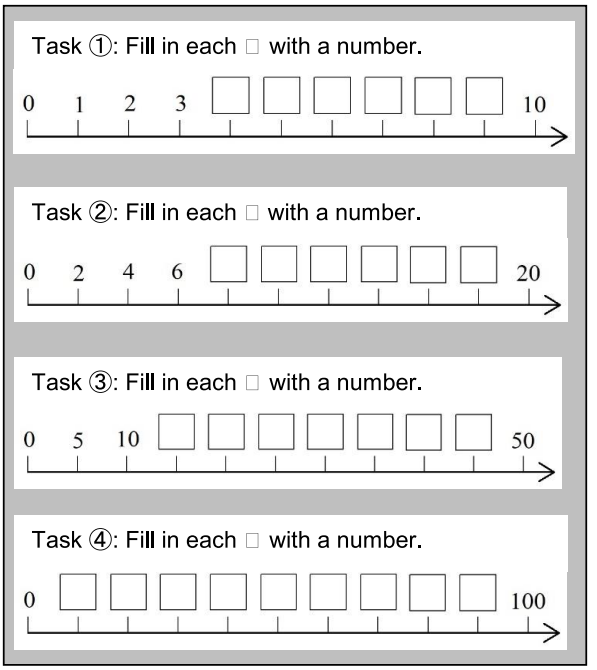



Diagram 13

ii. A teacher prepares four tasks involving number lines in the sequence as shown in Diagram 13. Explain how this sequence of tasks can facilitate students' understanding of drawing number lines for different scales.

## Topic 2: Introducing Addition and Subtraction

### Standards 2.1:

Understanding situations for addition up to 10 and obtaining fluency of using addition in situations

i. Introduce situations (together, increase and combine) for addition and explain it with manipulative and orally to define addition for operation.
ii. Develop fluency of addition expressions using composition of numbers for easier calculation with number sense for composition of numbers
iii. Apply addition fluency in learners' life

### Sample Tasks for Understanding the Standards

### Task 1: Introducing the Meanings of Addition

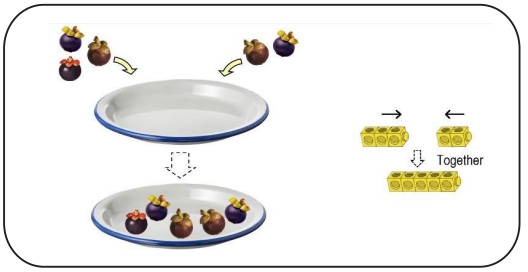



Diagram 1

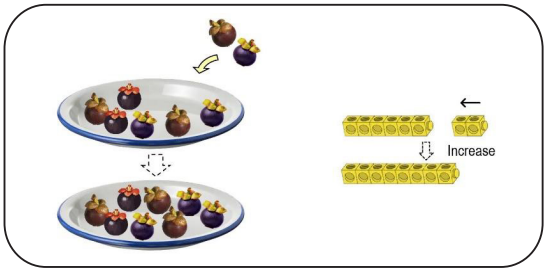



Diagram 2

Addition usually involves putting objects together, but it is not always so. Diagrams 1 and 2 show two addition situations inolving mangoesteens. The pictures of the blocks are used to illustrate the situations.

i. Study the situation in Diagram 1.
   - Describe the situation.
   - How to represent this situation with a number sentence involving addition?
   - Make an addition story similar to the situation in Diagram 1.

ii. Study the situation in Diagram 2.
   - Describe the situation.
   - How to represent this situation with a number sentence involving addition?
   - Make an addition story similar to the situation in Diagram 2.

iii. Compare the two addition situations in Diagrams 1 and 2.
   - Explain the different meanings of addition in the two situations.

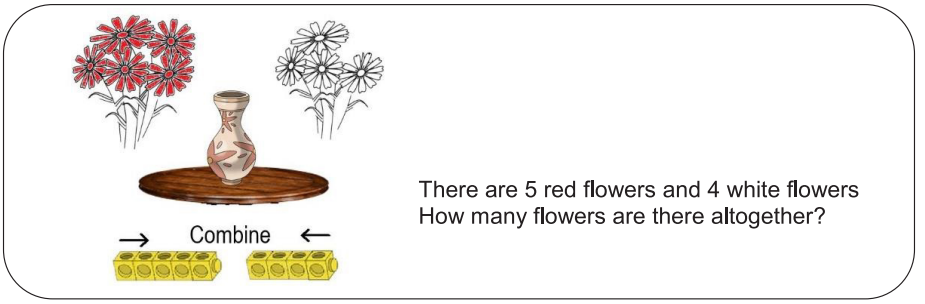



Diagram 3

iv. Diagram 3 shows another story of addition. Compare this story with Diagram 1.

- The story in Diagram 3 may be more difficult for children to perceive the idea of addtion as compare to the story in Diagram 1. Explain why.
- Make a similar addition story involving same category of objects, yet with different attributes.
- When learning addition, do you think it is necessary for children to do activity involving both situations in Diagram 1 and Diagram 3? Why or why not?

v. Consider the following task for children:

| Create a story for 3 + 4. |
|---|

- Explain how this task can help children conceptualise the meaning of addition.

**Task 2: Addition with Sum Less than 10**

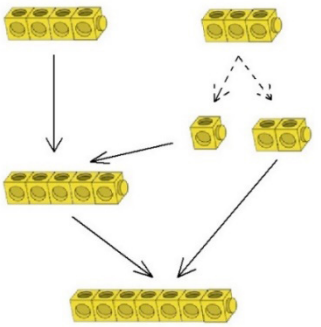


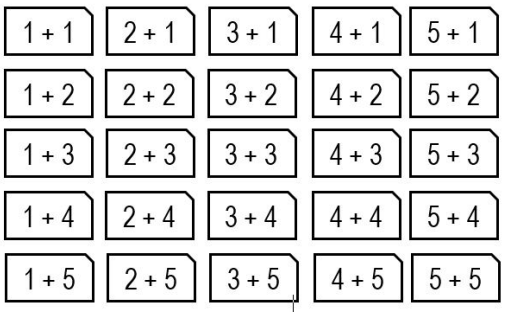
Diagram 4

i. At the early stage of learning addition, many children rely on counting to find the sum of two numbers. However, when the sum of two numbers is less than 10, such as 4 + 3, addition of the two numbers can also be obtained without counting if children have learnt composition and decomposition of numbers.

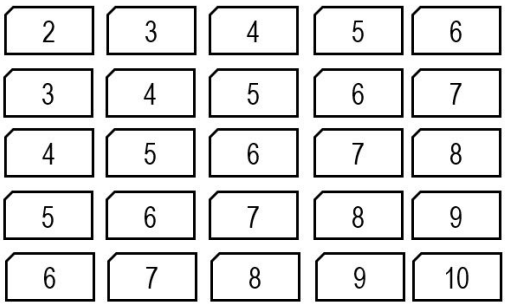
Diagram 4 shows how the addition is calculated with the help of interlocking cubes using the decompose-compose strategy.

- Explain the decompose-compose strategy.
- Without the interlocking cubes, how will you use a diagramatic representation to perform 4 + 3 with the strategy?
- Is this strategy suitable for calculating 7 + 2? Explain your reasons.
- What knowledge learnt previously is required to apply in this strategy?

**Task 3: Using Addition Cards to Develop Mathematical Thinking**




Addition Cards




Answers on Opposite Sides

Diagram 5

Diagram 5 shows an arrangement of 25 addition cards with the answer for each addition written on the corresponding opposite side of each card.

i. Describe any pattern that you observe in these cards.

ii. Suggest and explain how the pattern in these cards can be used to develop inductive reasoning among primary school students.

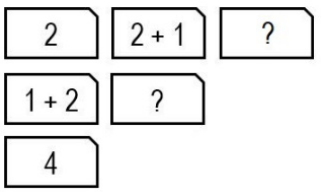



Diagram 6

iii. A teacher arrange 6 of the cards as shown in Diagram 6 and intend to use them to promote mathematical thinking among his primary school students.

- What are the possible addition expressions behind card “4”?
- Which addition expression should be in this card? Explain your reason.
- What expression should be in the two cards with “?”, respectively? Explain your reason.
- Explain how you will use these cards to develop a learning activity for promoting mathematical thinking.
- Explain what mathematical thinking is being promoted in your learning activity.

**Standards 2.2**:

Extending addition to more than 10 and obtaining fluency of using addition in situations

i. Extend addition situations and think about how to answer using the idea of making 10 with decomposition and composition of numbers
ii. Explain the idea of addition with place value using base-ten blocks
iii. Develop fluency of addition expressions to more than 10 for easier calculation
iv. Apply addition fluency in learners' life

**Sample Tasks for Understanding the Standards**

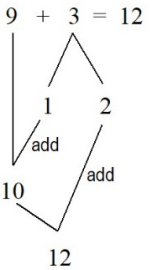
**Task 1: Addition With Sum More Than 10**

Diagram 1

i. Diagram 1 shows a situation involving some eggs in two trays.

- Why egg trays are used in this task?
- How many eggs are there altogether?
- Write an expression to represent the addition and explain how you find the answer.

ii. Without counting, addition with sum more than 10, such as 9 + 3, can be calculated using the decompose-and-compose stragety as follows:




- Describe the strategy briefly.
- Do you think this strategy is easier or more difficult for children in your class to learn addition? Explain your reasons.

Diagram 2

iii. Diagram 2 shows another addition situation involving 3 + 9.

- Calculate 3 + 9 using the decompose-and-compose stragety by drawing the diagram.
- Some teachers argue that it is easier for children to learn 9 + 3 than 3 + 9. Do you agree with this view? Explain your reasons.

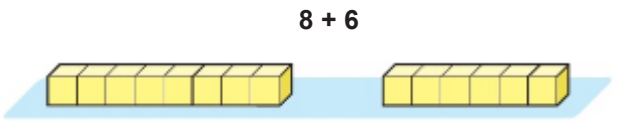



Diagram 3

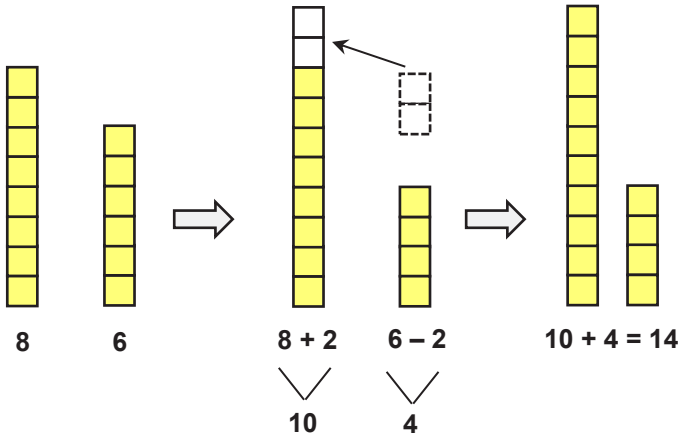



Diagram 4

iv. Diagram 3 shows a situation involving 8 + 6. Diagram 4 shows how David used interlocking cubes to calculate 8 + 6.

- What number did David decompose to make 10?
- What numbers did David compose to get the answer?
- How would you record David's strategies without drawing the interlocking cubes?

v. Calculate 8 + 6 by decomposing a different number from David to make 10.

vi. Between 8 and 6, which number would you decompose to do the addition? Justify your choice.

**Task 2: Using Addition Cards to Develop Fluency of Additon**

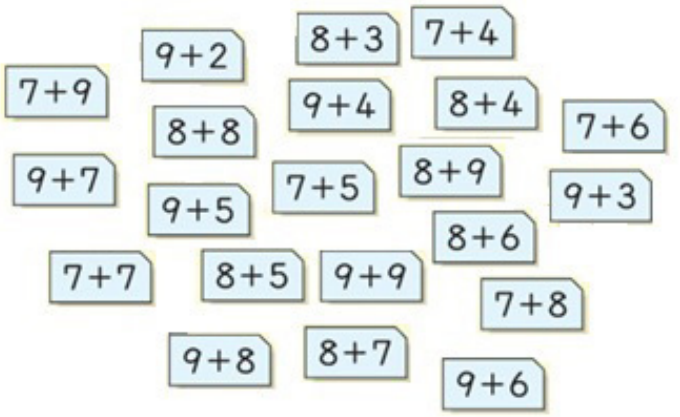



Diagram 5

i. Diagram 5 shows 21 cards for basic facts of addition in disarray.

- Rearrange the cards so that all cards with the same sum will be in the same horizontal row.
- What patterns do you notice in the horizontal rows and the vertical columns?

ii. There are another 15 cards for basic facts of addition other than those shown in Diagram 5.

- Find all the missing 15 cards.

iii. From the complete set of arranged cards, describe any pattern that can be used to help children develop fluency in addition.

iv. Explain how the discovery of patterns in the arrangement of cards can help children appreciate the beautifulness of numbers.

**Standards 2.3:**

Understanding situations for subtraction up to 10 and obtaining fluency of using subtraction in situations

i. Introduce subtraction situations (remaining, complement and difference) and explain orally with manipulative to define subtraction for operation
ii. Develop fluency of subtraction expressions using decomposition of numbers for easier calculation
iii. Apply subtraction fluency in learners' life

**Sample Tasks for Understanding the Standards**

**Task 1: Introducing the Meanings of Subtraction**

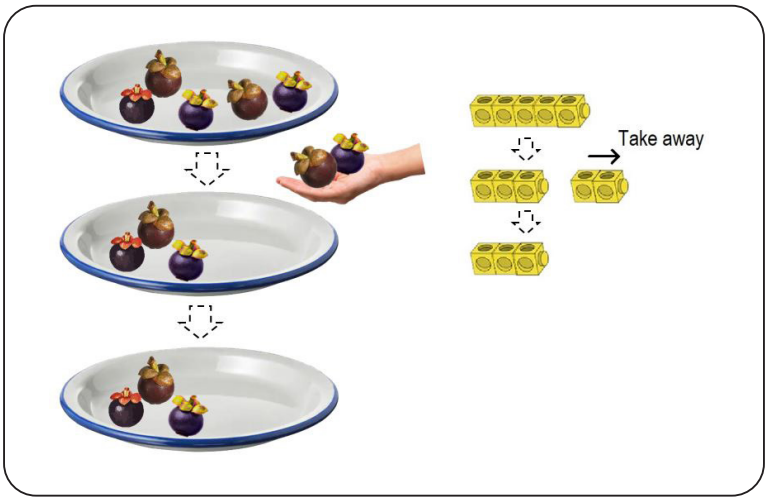



Diagram 1

i. Diagram 1 shows a situation involving subtraction. The picture of the interlocking cubes are used to illustrate the situation.

- How is the terminology "left" related to this situation?
- How to represent this situation with a number sentence involving subtraction?
  - ❖ Explain the meaning of subtraction in this situation.
- Two commonly used expressions in subtraction are "5 minus 2" and "take 2 away from 5". Explain the different contexts in using the two expressions.
- When looking at the picture in Diagram 1, Amin explained the situation as "3 mangosteens plus 2 mangosteens is 5 mangosteens altogether".
  - ➢ Give a possible reason for Amin's explanation.
  - ➢ How will you guide Amin to extend his idea of addition to subtraction?

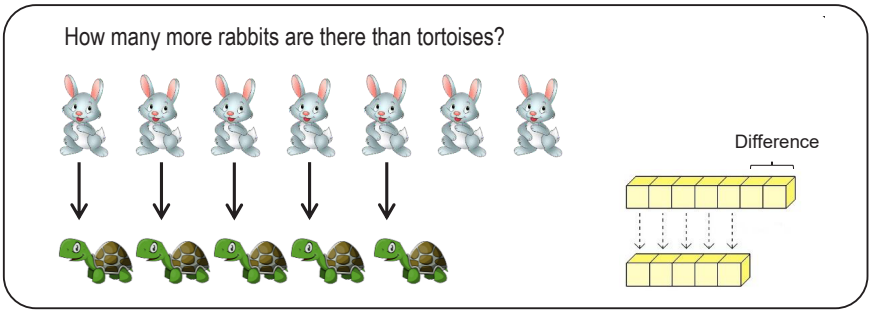



Diagram 2

ii. Diagram 2 shows another subtraction situation involving 7 rabbits and 5 tortoises.

- How is the terminology “difference” related to this situation?
- How to represent this situation with a number sentence involving subtraction.
- Explain the meaning of subtraction in this situation.


Diagram 3

iii. Diagram 3 shows a third situation involving 8 – 3 = 5.

- Explain the differences between this subtraction situation from situations in Diagrams 1 and 2.
- Why is it **not** appropriate to read ‘8 – 3 = 5’ as ‘Take 3 away from 8 equals 5’ for this situation?
- Explain the meaning of subtraction in this situation.

iv. Compare the three subtraction situations in Diagrams 1, 2 and 3.

- Which of these situations would you use as a first example to teach subtration to children? Explain your reasons.
- Why is it **not** appropriate to teach subraction by using the ‘take away’ situation **only**?

v. Consider the following task for children:

| Create a story for 9 – 4. |
|---|

- Explain how this task can help children conceptualise the meaning of subtraction.

**Task 2: Subtraction Less than 10**

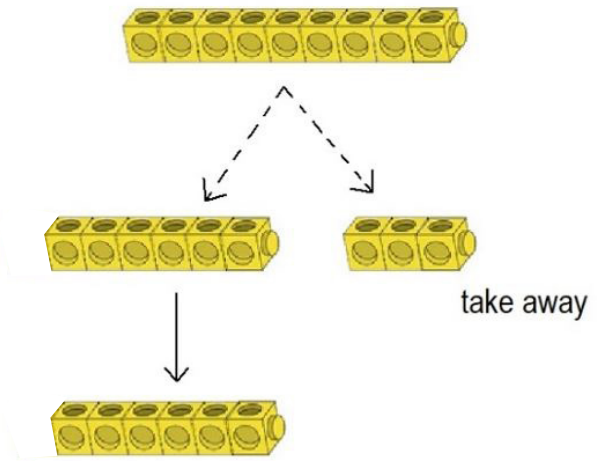



Diagram 4

i. Diagram 4 shows how 9 – 3 is calculated with the help of interlocking cubes.

- Explain the strategy of subtraction.
- Without the interlocking cubes, how will you use a diagramatic representation to perform 9 – 3 with the strategy?
- Use this strategy to calculate:
  - ❖ 5 – 2
  - ❖ 7 – 3

ii. Do you think it will be easier for the children in your class to learn subtraction by this strategy? Why or why not?

**Standards 2.4:**

Extending subtraction to more than 10 and obtaining fluency of using subtraction in situations

i. Extend subtraction with situations and think about how to answer using the idea of 10 with addition and subtraction of numbers (composition and decomposition of numbers)
ii. Explain the idea of subtraction in place value using base-ten blocks
iii. Develop fluency of subtraction expressions to more than 10 for easier calculation
iv. Apply subtraction fluency in learners' daily life

**Sample Tasks for Understanding the Standards**

**Task 1: Extending Subtraction to More Than 10**

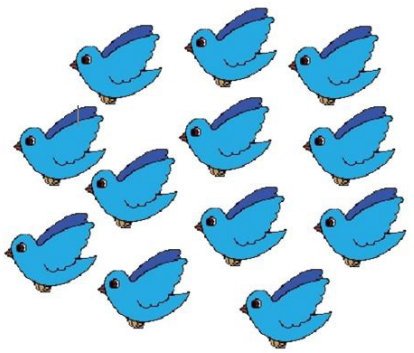


Diagram 1

i. Diagram 1 shows 12 bird stickers to be used for teaching subtraction. The problem is as follows:

> There are 12 stickers. 9 stickers are given away. How many stickers are left?

A child calculated the answer as follows:

$$\begin{array}{r} 12 \\ -\ 9 \\ \hline 7 \end{array}$$

- What is the misconception of the child?

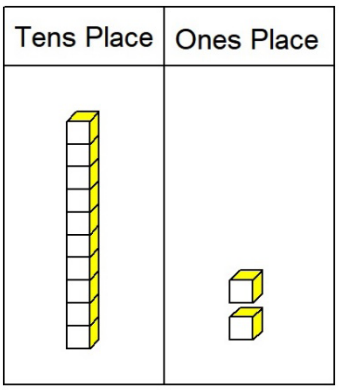
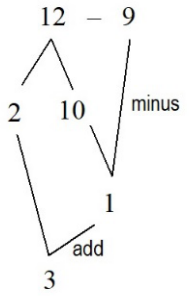



Diagram 2

ii. Diagram 2 shows some base-10 blocks used with a place-value chart to represent number 12.

- Explain with the idea of place value, how you will use the base-10 blocks to help the child in (i) to overcome his misconception?

Child A's strategy:




Child B's strategy:

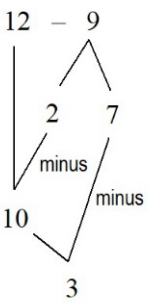



Diagram 3

iii. Two children, A and B, used the decompose-and-compose strategy in different ways to calculate 12 – 9 as shown in Diagram 3.

- What are the differences in the two ways?
- Which way do you think is easier for the children in your class? Explain your reasons.

iv. Consider the following task for children:

> Use the decompose-and-compose strategy to calculate:
> ◊ 17 – 9
> ◊ 15 – 6
> ◊ 11 – 2

- Explain how this task of proceduralisation can enhance the computational fluency of subtraction.

v. A teacher argue that the task “12 – 9” should be discussed before the task “12 – 3”.

Do you agree? Justify your reasons based on the idea of decomposition and composition of numbers.

**Task 2: Using Subtraction Cards to Develop Fluency of Subtraction**

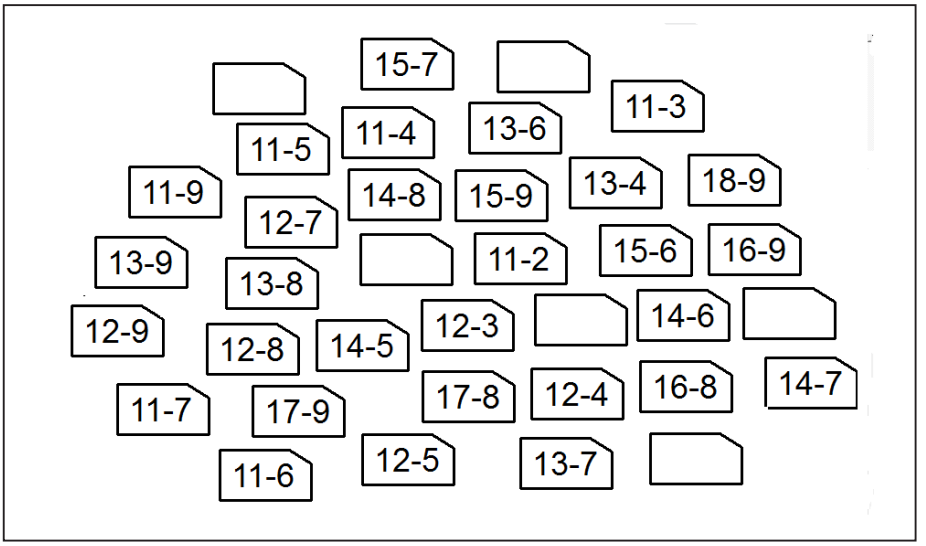



Diagram 4

i. Diagram 4 shows some subtraction basic fact cards and a few empty cards.

- Rearrange the cards so that all cards with the same answers will be in a same horizontal row.
- What patterns do you notice in the horizontal rows and the vertical columns?
- Explain why each pattern is formed in such a way.
- Based on the patterns identified, write an expression on each of the blank cards to complete the pattern.

ii. Explain how these patterns can be used to help the children in your class to develop fluency in subtraction.

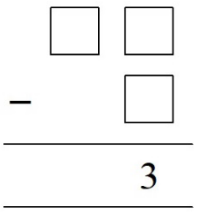


Diagram 5

iii. Diagram 5 shows a subtraction problem involving a 2-digit number with a 1-digit number.

- Find all possible solutions for the problem.
- How many solutions are there altogether?
- What strategy have you used in solving this problem?

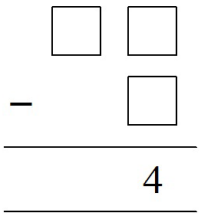


Diagram 6

iv. Diagram 6 shows another subtraction problem.

- Find all possible solutions for the problem.
- How many solutions are there altogether?

v. How many solutions are there altogether for the problem □□ – □ = 2?

vi. Given another problem □□ – □ = 7.

- Make a conjecture on the number of all possible solutions for the problem.
- Test your conjecture.

vii. Given another problem □□ – □ = 9.

- How many solutions are there altogether?
- How confident are you that your answer is correct? Explain your reasons.
- What about □□ – □ = 8?
- What about □□ – □ = 10?

viii. What are the advantages of posing this kind of tasks for your students?

**Topic 3: Utilising Addition and Subtraction**

**Standards 3.1:**

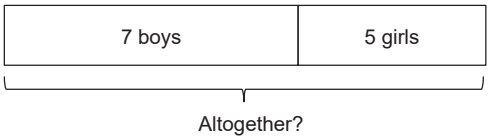
Utilising addition and subtraction in various situations and understanding their relationships

i. Understand the difference between addition and subtraction situations with tape diagrams
ii. Explain subtraction as an inverse of addition situations with tape diagrams
iii. Understand addition with three numbers, subtraction with three numbers and combination of addition and subtraction situations
iv. Apply addition and subtraction in various situations such as in ordering numbers

**Sample Tasks for understanding the standards**

**Task 1: Addition and Subtraction with Tape Diagram**

Teacher A's Diagram:




Teacher B's Diagram:

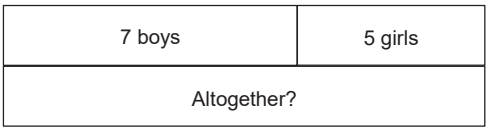



Diagram 1

i. Two teachers, A and B, drew two different tape diagrams as shown in Diagram 1 to help their students solve the following addition problem:

> There are 7 boys and 5 girls in a group.
> How many children are there altogether?

- In what way do you think tape diagram is useful in helping your students understand number operations?
- Which of the two ways of drawing tape diagram do you think is easier for your students to understand addition? Explain your reasons.

> 8 children are playing in the park.
> 3 of them go back.
> How many children are left?

> 6 children are playing in the park.
> 3 more children came to join them.
> How many children are there?

Diagram 2

ii. Diagram 2 shows two word problems.

- Draw a tape diagram for each of the problems to help your students understand the problem situation.
- Base on each tape diagram, write a mathematics sentence to represent the problem situation.

There are 9 black marbles and 3 white marbles in a box.

How many more black marbles are there than white marbles?

There are some black and white marbles in a box. Black marbles are 3 less than white marbles. 9 of the marbles are black.

How many white marbles are there?

Diagram 3

Diagram 4

iii. Diagram 3 shows another two word problems.

- Explain why many children find these two problems difficult to understand.
- Draw a tape diagram for each of the problems to help your students understand the problem situation.
- Base on each tape diagram, write a mathematics sentence to represent the problem situation.

iv. Diagram 4 shows a situation involving some monkeys in a zoo.

- Base on the diagram, create an addition story and represent it with a mathematics sentence.

v. Subtraction is the inverse operation of addition.

- Create a subtraction story to show the inverse property of addition.
- Represent the story with a mathematics sentence.

vi. Explain with tape diagrams, how you will help chidren understand the inverse relationship between addition and subtraction.

**Task 2: Situations of Combined Operations**

$$7 + 5 - 8$$

Diagram 5

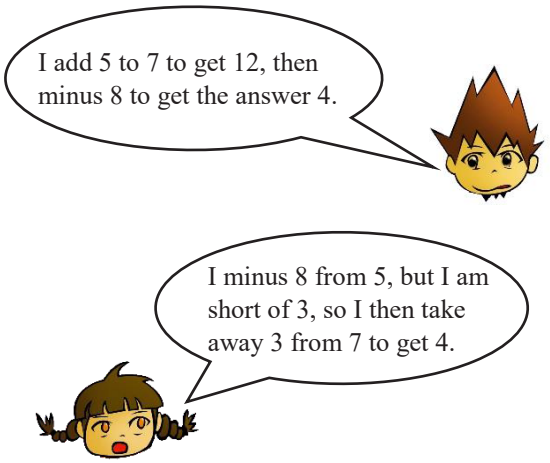



Diagram 6

A combined operation in Diagram 5 may be difficult for children to understand.

i. Two students, Daniel and Pearly explained the methods they used to get their answers in Diagram 6.

- Explain if both the methods are correct.
- Create a mathematics story to help children understand 7 + 5 – 8.

Daniel's solution:

$$9 - 3 + 2$$
$$= 9 - 5$$
$$= 4$$

Pearly's solution:

$$9 - 3 + 2$$
$$= 6 + 2$$
$$= 8$$

Diagram 7

ii. Diagram 7 shows how Daniel and Pearly solved the combined operation 9 – 3 + 2.

- Which method is correct? Explain your reasons.
- Create a mathematics story to help children understand 9 – 3 + 2.

iii. In what way is the process of thinking in creating stories can enhance the conceptual understanding in combined operations of your students.

**Task 3: Making Addition and Subtraction Easier**

$$8 + 6 + 2$$

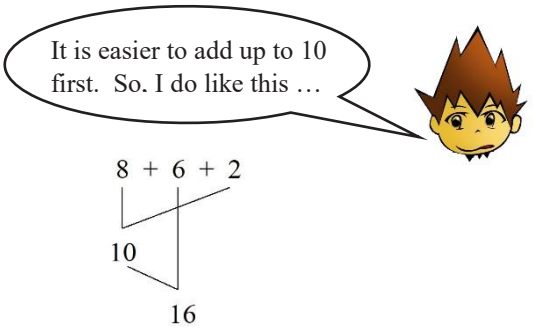






Diagram 8

i. Diagram 8 shows an addition involving three numbers. Daniel explained the method he used to find the answer.

- Explain Daniel's method.
- What property of addition did Daniel use in his method?

ii. Daniel used a method to make addition of three numbers easier as follows:

$$\begin{aligned}&(628 + 41) + 59\\&= 628 + (41 + 59)\\&= 628 + 100\\&= 728\end{aligned}$$

- Explain the method.
- What property of addition was used?

iii. There are also easier ways to do subtraction such as the following examples:

$$56 - 16 - 8 = 60 - 20 - 8$$
$$56 - 18 - 7 = 60 - 20 - 9$$
$$56 - 19 - 6 = 60 - 20 - 9$$

- Explain why these subtractions are correct?
- In what way can these help to make the subtraction easier?

iv. What is the significance of these methods in developing computational fluency?

**Task 4: Applying Addition and subtraction**

Diagram 9

Diagram 9 shows 9 birds perching on a telephone line. The rounded bird is at the middle of the birds.

i. What is the position of the rounded bird from the left side?

ii. The third bird from the left side flew away, follow with the last two birds from the right.

- What is the new position of the rounded bird?
- How many birds are left on the telephone line? Write a mathematics expression to find the answer.

iii. Children are lining up in the school field. Adriel is at the 4th place from the front.

- Babyrina is the 3rd child behind Adriel. What is the position of Babyrina from the front? Write a mathematics expression to find the answer.
- There are 8 children behind Babyrina. How many children are there altogether?
- Explain how a number line can be used to help children solve this type of problem.

iv. Why situations or contexts are crucial in helping children learn addition and subtraction involving three numbers?

## Topic 4: Extending Numbers with Base-ten System Up To 1 000 000 Gradually

### Standards 4.1:

Extending numbers using base-ten system up to 1 000

i. Experience counting of 1 000 by using various units and appreciate the necessity of the base-ten system
ii. Extend the order of numbers to introduce more than 1 000 in relation to the size of ones, tens and hundreds
iii. Use a partial number line to compare size of numbers through translation of the size of every digit and select appropriate scale
iv. Represent appropriate diagram to show the size of numbers without counting such as three of hundreds mean 30 of tens and visualise the relative size of numbers
v. Represent larger or smaller numbers by symbol of inequality

### Sample Tasks for Understanding the Standards

### Task 1: Counting Up to 1000

***Grouping by 10s***

i. Grouping by tens is a crucial process in the base-ten numeration system. Children need to understand this process in order to understand and appreciate the system.

- Explain the grouping process in the base-ten numeration system.

ii. Various materials have been used to model the grouping-by-10 process. Diagram 1 shows a comparison of two such materials, pencils and blocks.

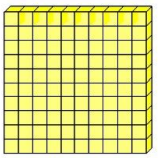
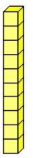
| Material | 100 | 10 | 1 |
|---|---|---|---|
| Pencils |   |   |  |
| Blocks |  Hundred-flat |  Ten-bar |  One-cube |

Diagram 1

- What is a crucial difference between the nature of these two materials?
- What are the advantages and disadvantages of these two materials in helping children learn the idea of regrouping?

- Do you think these two materials are equally effective in helping your students learn the idea of grouping by 10s? If yes, why? If no, which of these two materials do you think is more effective? Explain your reasons.

- How can each of these materials be used to represent one thousand?

iii. Diagram 2 shows the number 230 represented by bundles of pencils. 230 is the sum of 2 sets of 100 and 3 sets of 10.

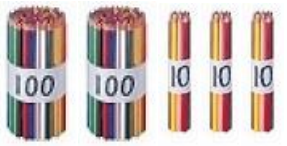



Diagram 2

- Write a mathematics sentence to represent the sum.

- How many sets of 10 are there in 230?

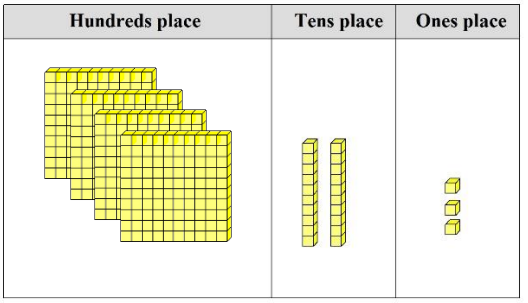



Diagram 3

iv. Diagram 3 shows some base-ten blocks placed on a place-value chart to represent the number 423.

- Explain the development of place value by using base-ten blocks that consist of one-cubes, 10-bars and 100-flats to represent 423.

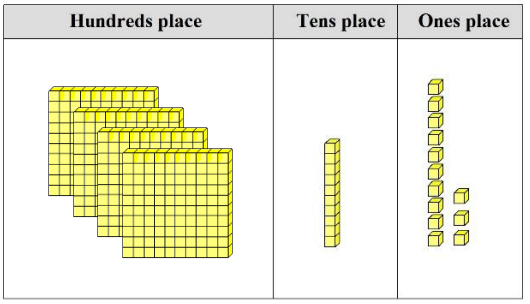



Diagram 4

v. Diagram 3 also shows 423 grouped as a sum of 4 hundreds 2 tens and 3 ones, which can also be written as 400 + 20 + 3. Diagram 4 shows 423 regrouped as 4 hundreds 1 tens and 13 ones.

- Does the regrouping change the written numeral for 423? Explain your answer.

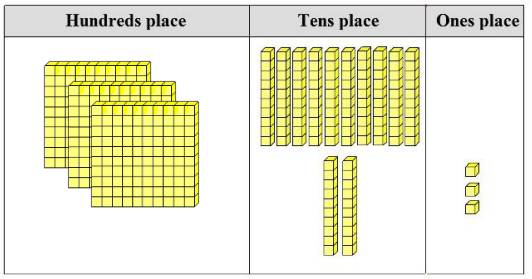



Diagram 5

vi. Diagram 5 shows 423 regrouped as 3 hundreds 12 tens and 3 ones.

- Does the regrouping change the value of 423? Explain your answer.

vii. What changes and what remains unchanged after a number is regrouped?

***Counting Numbers More Than 100***


Diagram 6

i. Diagram 6 shows 422 pencils in bundles of 100, 10 and 1. Base on this example, how to explain to children the extension of counting by 10s to counting by 100s?

- Explain why this example is important in helping your students to appreciate the extension from counting by 10s to counting by 100s.

ii. Diagram 7 shows five lines of numbers arranged in some sequences. Fill in the missing numbers.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Line ① | 117 | 118 | 119 | | 121 | | |
| Line ② | 780 | 790 | | | 820 | | 840 |
| Line ③ | 200 | | 400 | | 600 | 700 | |
| Line ④ | 650 | | 750 | | | | 950 |
| Line ⑤ | 854 | 852 | | | 846 | | |

Diagram 7

- In what ways is this task important in helping children understand larger numbers?
- Create another similar task for your students.

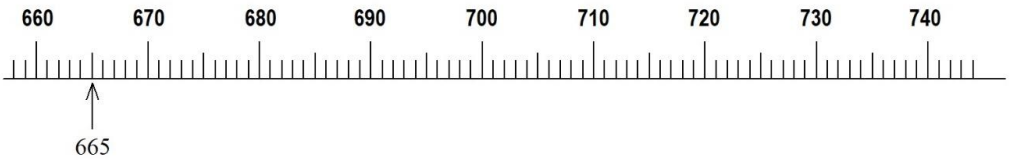



Diagram 8

iii. Diagram 8 shows part of a number line with scale. The number 665 is marked with an arrow.

- Mark each of the following numbers on the number line with an arrow:
  - ❖ 658, 676 and 699
  - ❖ 704, 725, 737 and 743
- What benefits will children acquire by doing this kind of task?

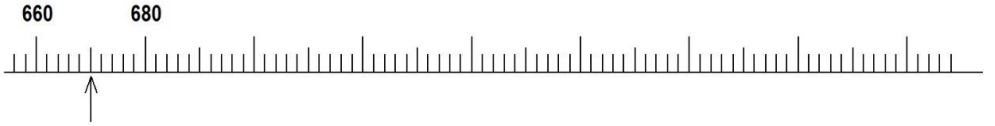



Diagram 9

iv. Using the same number line, but 670 is replaced with 680 as shown in Diagram 9.

- What is the new number indicated by the arrow?
- Complete the new scale of this number line.
- What will be the new number indicated by each of the arrows in (iii)?

## Task 2: Comparing Numbers

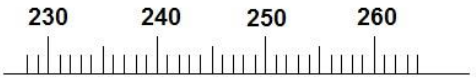



Diagram 10

i. Children often confuse with the values of numbers such as in 236 and 263.

- Mark each number with an arrow on the number line in Diagram 10.
- Explain why a number line is helpful in helping children overcome their confusion in recognising values of numbers.

| Hundreds | Tens | Ones |
|---|---|---|
| 2 | 3 | 6 |
| 2 | 6 | 3 |

Diagram 11

ii. Diagram 11 shows the numbers 236 and 263 written on a place-value chart.

- Between a number line and a place value chart, which do you think is easier to compare the values of 236 and 263? Explain your reasons.
- What about comparing the values of 253 and 510? Explain your answer with suitable diagrams.

| > | or | < |
|---|---|---|
| larger than | | smaller than |

Diagram 12

iii. Diagram 12 shows two symbols used to represent inequality.

- Write the relationship between the pair of numbers 236 and 263 by using the symbols ‘larger than’ or ‘smaller than’.

  (Note. When learning positive and negative numbers in upper grades, the terms “greater than” and “less than” are used.)

| Hundreds | Tens | Ones |
|---|---|---|
| 4 | 8 | 3 |
| 7 | 1 | 2 |

Diagram 13

iv. Diagram 13 shows two numbers written on a place-value chart.

- Use the symbols > or < to represent the numerical relationship for the pair of numbers 483 and 712.
- Sketch the positions of the two numbers on a suitable number line to show their numerical relationship.
- When a place-value chart is used to compare 483 and 712, which place value should we look at? Why?

  - ❖ What about 527 and 561?
  - ❖ What about 309 and 604?
  - ❖ What about 382 and 942?

**Standards 4.2:**

Extending numbers using base-ten system up to 10 000

i. Visualise the 10 000 by using thousands, hundreds, ten and one as units
ii. Extend the order sequence of numbers to introduce more than 10 000 in relation to the size of ones, tens, hundreds and thousands
iii. Use number line with appropriate scale to show size of numbers and relative size of numbers while focusing on the scale

**Sample Tasks for Understanding the Standards**

**Task 1: Counting Large Numbers**

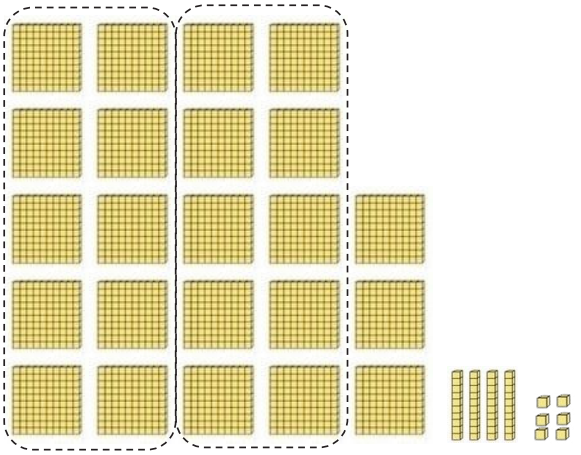


Diagram 1

i. Diagram 1 shows some base-ten blocks used to represent a number. Two students, Roy and Jess describe the number as follows:

Roy: "*There are 23 sets of hundreds, 4 sets of tens, and 6 ones*."

Jess: "*There are 2 sets of thousands, 3 sets of hundreds, 4 sets of tens and 6 ones*."

- Are both the answers correct? Justify your explanation.
- What is a better way of representing 23 sets of hundreds? Why is it better?

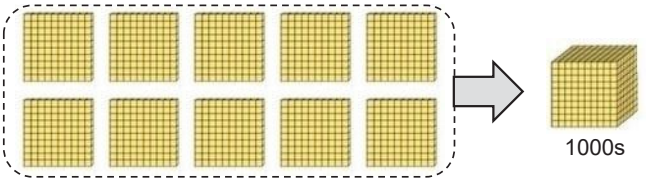



Diagram 2

ii. Diagram 2 shows 10 sets of hundreds being changed to 1 set of thousands. Why is this change important?

iii. How do you explain the new place value of 1000s using the place-value chart?

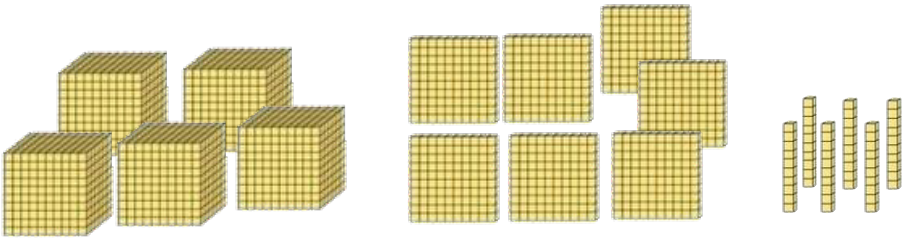


Diagram 3

iv. Diagram 3 shows a number represented by some base-ten blocks. A student counted the number of unit cubes as follows:

*"One thousand, two thousand, three thousand, four thousand, five thousand ... five thousand one hundred, five thousand two hundred, five thousand three hundred, five thousand four hundred, five thousand five hundred, five thousand six hundred, five thousand seven hundred ... five thousand seven hundred and ten, five thousand seven hundred and twenty, five thousand seven hundred and thirty, five thousand seven hundred and forty, five thousand seven hundred and fifty, five thousand seven hundred and sixty!"*

- Suggest a more efficient way of counting the number of one-cubes.

**Task 2: Writing Large Numbers**

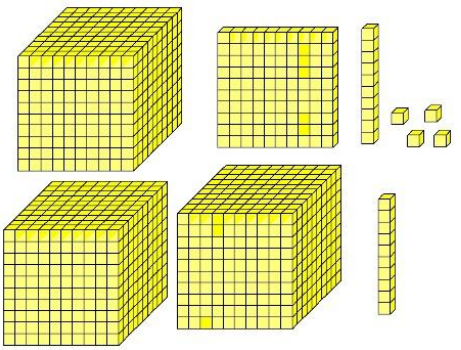


Diagram 4

i. Diagram 4 shows a number represented by some base-ten blocks. A student wrote the number as 3000124.

- What was the error in the number written by the student?
- What were the possible causes of the error?
- Explain how a place-value chart can help the student to correct the error.

*13 sets of hundreds,*
*4 sets of tens and*
*6 sets of ones.*

Diagram 5

ii. When given a task to write the number represented by the expressions in Diagram 5, a student wrote it as 1300406.

- What was the error of the number written by the student?
- What were the possible causes of the error?
- Explain how the error can be corrected with a place-value chart.
- What skills need to be developed in order for students to acquire the fluency of reading and writing large numbers?

**Task 3: Using Number Line to Show Large Number**

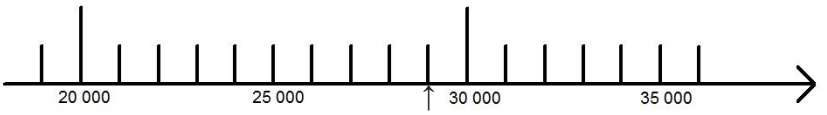



Diagram 6

i. Diagram 6 shows a number line with large numbers

- Describe the scale of the number line.
- Write down the number marked by the arrow.

ii. Can 23 460 be marked accurately on the number line? Why or why not?

iii. Explain how this number line can be use to compare the numbers 23 460 and 34 520.

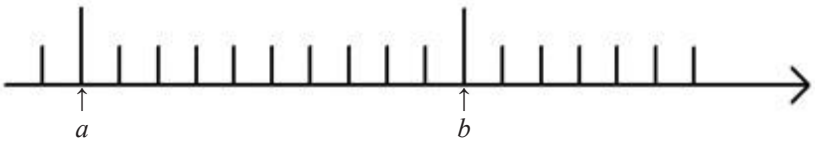



Diagram 7

iv. Diagram 7 shows another number line without a scale.

- What should numbers $a$ and $b$ be so that 23 460 can be marked accurately on the number line?
- Mark the accurate location of 23 460 on this number line.

v. Sketch a number line with appropriate scale to compare the values of 23 460 and 97 680.

- Explain how the comparison can be done.
- Is there a neccesity to compare large numbers with detailed scales such as ones? Explain your reasons.

vi. How to draw a line of numbers with appropriate scale if 76 586 need to be marked accurately to show its relative position with 76 705?

vii. What are the important elements you need to consider when drawing and reading values of numbers on a number line?

**Standards 4.3:**

Extending numbers using based-ten system up to 1 000 000

i. Extend numbers up to 1 000 000 and learn the representation of the place value for grouping every 3-digit numeral system up to million
ii. Write large numbers using grouping of 3-digit numeral system such as thousand as a unit and compare numbers in relation to it
iii. Develop number sense such as larger and smaller based on comparison of place values through visualisation of relative size of numbers

**Sample Tasks for Understanding the Standards**

**Task 1: Beyond the Thousands Place Value**

***The 3-Digit Numeral System***

| ③ | ② | ① | Thousands | Hundreds | Tens | Ones |
|---|---|---|---|---|---|---|
| | | | | | | |

Diagram 1

i. Diagram 1 shows a base-ten place value chart up to thousands.

- How many ones are regrouped to form tens?
- How many tens are regrouped to form hundreds?
- How many hundreds are regrouped to form thousands?
- What should be the place values ① , ② and ③ , respectively? Explain your reasons.

ii. Write the number “two hundred fifty two thousand three hundred and sixty seven” on the place-value chart in Diagram 1.

- Which place values do you see a repeat of ones, tens and hundreds?
- This repeatation of ones, tens and hundreds enable us to form a 3-digit numeral system as shown by the grouped place-value chart in Diagram 2.

  ❖ Explain how this chart can be used to help children read large numbers in the English language.

| Group of Millions | | | Group of Thousands | | | Group of Ones | | |
|---|---|---|---|---|---|---|---|---|
| ② | ① | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| | | | | | | | | |

Diagram 2

iii. Write the number 756 238 on the grouped place-value chart in Diagram 2.

- How would you guide children to determine the value of each digit?
- Read and write the number in words.

iv. What should be the place values ① and ② for the group of millions? Explain your reasons.

v. Write and read the following numbers in words.

- 63 502 000
- 438 167 064

vi. Given two large numbers: 43 816 764 and 438 167 064.

- Which number is larger?
- What different methods can you used to compare the numbers?
- Explain the different methods.
- Which method do you think is most efficient in comparing large numbers? Justify your answer

***The Structure of Base-Ten Place Value System***

| Name | Place Value | Pattern | Power of 10 |
|---|---|---|---|
| Hundred millions | 100 000 000 | | $10^{?}$ |
| Ten millions | 10 000 000 | | $10^{?}$ |
| Millions | 1 000 000 | | $10^{?}$ |
| Hundred thousands | 100 000 | | $10^{?}$ |
| Ten thousands | 10 000 | | $10^{?}$ |
| Thousands | 1 000 | | $10^{?}$ |
| Hundreds | 100 | X10 | $10^{2}$ |
| Tens | 10 | X10 | $10^{1}$ |
| Ones | 1 | | $10^{?}$ |

Diagram 3

i. Diagram 3 shows the based-ten place values from ones to hundred millions which explain the comprehensive structure of the base-ten numeration system. Study the structure carefully.

- What pattern do you notice in the place values?
- Complete the missing power of 10 indicated by each question mark.

ii. What do you think a base-five structure for place values should be?

- Explain the structure.

## Topic 5: Producing Vertical Form Addition and Subtraction and Acquiring Fluency of Standard Algorithms

### Standards 5.1:

Thinking about the easier ways for addition and subtraction and producing vertical form algorithms

i. Think about easier ways of addition or subtraction situations by using models with base-ten blocks meaningfully for representing base-ten system
ii. Produce and elaborate efficient ways and identify the standard algorithms in relation to base-ten system with appreciation
iii. Explain the algorithms of borrowing and carrying with regrouping of base-ten models
iv. Acquire fluency in addition and subtraction algorithms

### Sample Tasks for Understanding the Standards

### Task 1: *Addition in Vertical Form*

***Addition Without Regrouping***

Candy Problem: Rizal has 13 candies, and Ricky has 24 candies.
How many candies are there altogether?

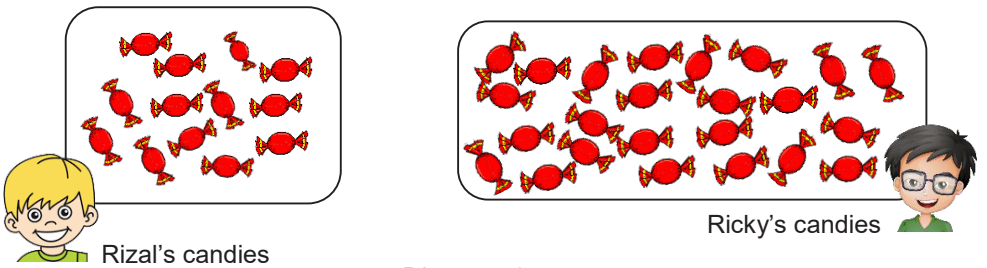



Diagram 1

i. Diagram 1 shows a candy problem. Diagrams 2(a) to 2(c) show three stages of developing an algorithm, not arranged in order, for solving the Candy Problem.

Stage ①
Grouping tens and ones in vertical form.

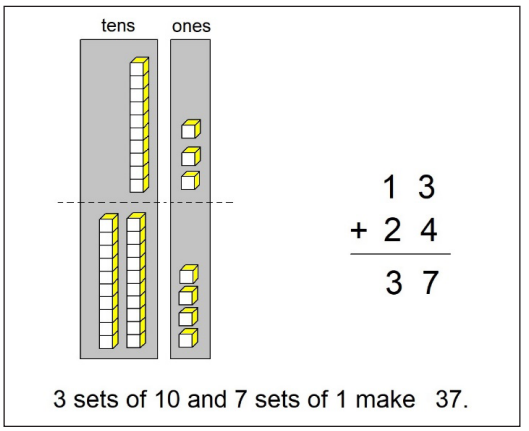



Diagram 2(a)

Stage ②
Grouping tens and ones in horizontal form.

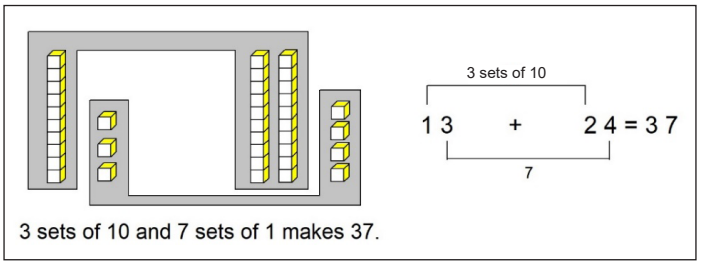



Diagram 2(b)

Stage ③
Grouping by tens, then add in sets of tens and sets of ones.

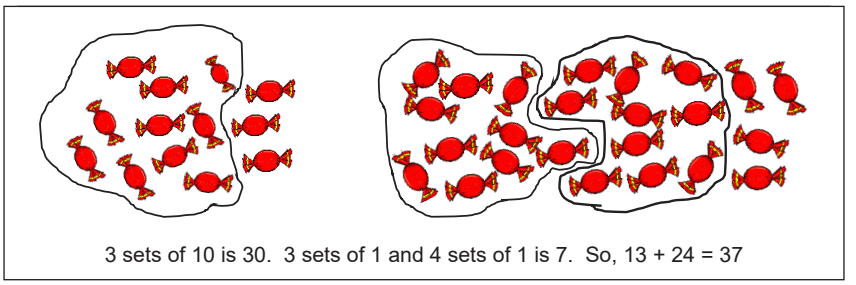



Diagram 2(c)

- How will you sequance the stages to help your students learn the vertical form of addition? Explain your reasons.

| Student A: | Student B: |
|---|---|
| 4<br>+ 25<br>65 | 4<br>+ 25<br>29 |

Diagram 3

ii. Diagram 3 shows how two students, A and B, adding two numbers in the vertical form. After comparing their answers, Student B told Student A that the answer 65 was wrong. He then tried to explain his reason that could be seen in the following conversation.

Student B: "*You cannot write 4 on top of 2.*"
Student A: "*Why not*?"
Student B: "*Because the two numbers on the right side must be on the same line, not the left side*."

- What does Student B mean?
- Why must the numbers be aligned on the right side?
- How can base-ten blocks be used to help Student A correct his mistake

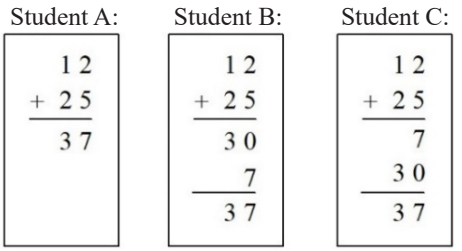



Diagram 4

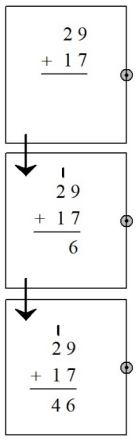
iii. Three students, A, B and C, calculated 12 + 25 in different ways as shown in Diagram 4.

- Which of the ways do you think will be most difficult for your students? Explain your reasons.
- Which of the ways do you think can help your student understand vertical form addition better?
- Student A explained how he got the answer as follows:
  "*2 plus 5 is 7 and then 1 plus 2 is 3. So, the answer is 37*".
  - ❖ Did Student A represent each digit of 25 and 12 with the correct value in his explanation? Explain your reasons.
- Student B used some based-ten blocks to explain how he got the answer as follows:
  *"I use 1 bar of 10s and 2 units of 1s, then another 2 bars of 10s and 5 units of 1s. Altogether, I have 3 bars of 10s and 7 units of 1s which is 37."*
  - ❖ Draw a diagram of base-ten blocks to show the process explained by Student B.
  - ❖ How does the arrangement of the base-ten blocks help to explain the algorithmn in vertical form?
- Compare Student B's way and Student C's way of adding the two numbers.
  - ❖ What do you think is the difference in the two students' thinking?
  - ❖ Is the difference significant? Explain your reasons.

iv. Teachers in different part of the world practise slightly different ways of writing the vertical form of addition. For examples, while there are teachers who use an answer bar as in Diagram 3, there are also teachers who do not use the answer bar as in Diagram 4.

- Which of these practices do you prefer? Explain your reasons.

***Addition in Vertical Form with Regrouping***



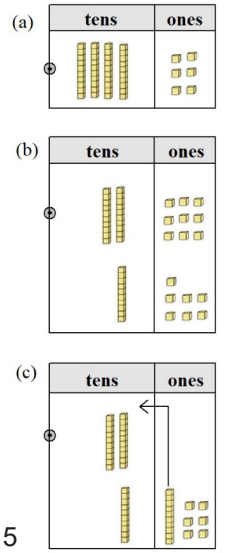




Diagram 5

i. Diagram 5 shows the sequencial steps in adding two numbers with regrouping in vertical form and pictures of base-ten blocks illustrating each of the steps. The pictures of base-ten blocks are not arranged in the correct sequence.

- Draw a straight line to match each step with the correct picture of base-ten blocks.
- Explain how the regrouping of 10 ones as 1 tens is represented in the written vertical form.
- In what way can the use of base-ten blocks help children understand the regrouping process when using addition in the vertical form?

[*Note*. The term "carrying" is also used to mean regrouping involving addition in some countries.]

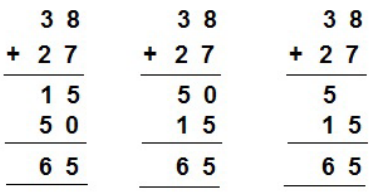



Diagram 6

ii. A teacher discovered several ways her students performed addition in vertical form as shown in Diagram 6.

- What are the advantages of using these vertical forms to help children understand place values and addition?

iii. Another student showed an alternative way to perform 28 + 27 as shown in Diagram 7.

- Comapring this way with those in Diagram 6, do you consider it easier and more efficient? Explain your reasons.

[*Note*. The line below the final answer is used to indicate the end of the calculation. Depending on a country's common practices, the line may not be used.]

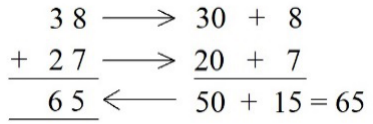



Diagram 7

(a)

$$\begin{array}{r} {\scriptstyle 1\phantom{0}} \\ 38 \\ +\ 27 \\ \hline 65 \end{array}$$

(b)

$$\begin{array}{r} 38 \\ +\ 27 \\ {\scriptstyle 1\phantom{0}} \\ \hline 65 \end{array}$$

(c)

$$\begin{array}{r} 38 \\ +\ 27 \\ {\scriptstyle 1\phantom{0}} \\ \hline 65 \end{array}$$

Diagram 8

iv. As in the writing of the answer bar, teachers from different part of the world also practice different ways to represent the 'regrouped number' when using the vertical form of addition. Diagram 8 shows three such practices.

- Which of these ways do you practice in your classroom?
- Mathematically, is there any difference in value between the three ways of marking the regrouped '1'? Explain your reasons.
- Can you say it is wrong if the method do not conform to your local practice? Why or why not?

## Task 2: Subtraction in Vertical Form

***Subtraction Without Regrouping***

i. Diagram 9 and Diagram 10 show how two teachers, A and B, used base-ten blocks to perform 48 – 13.

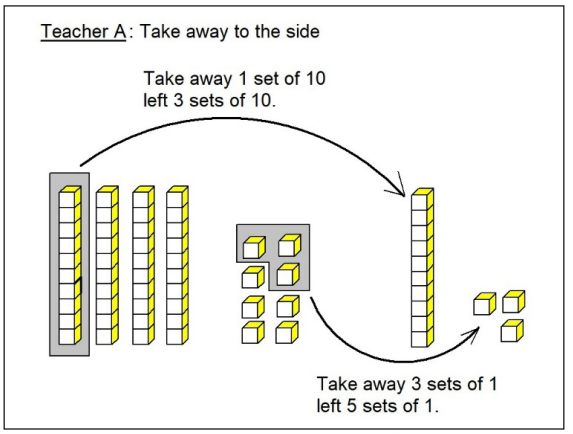



Diagram 9

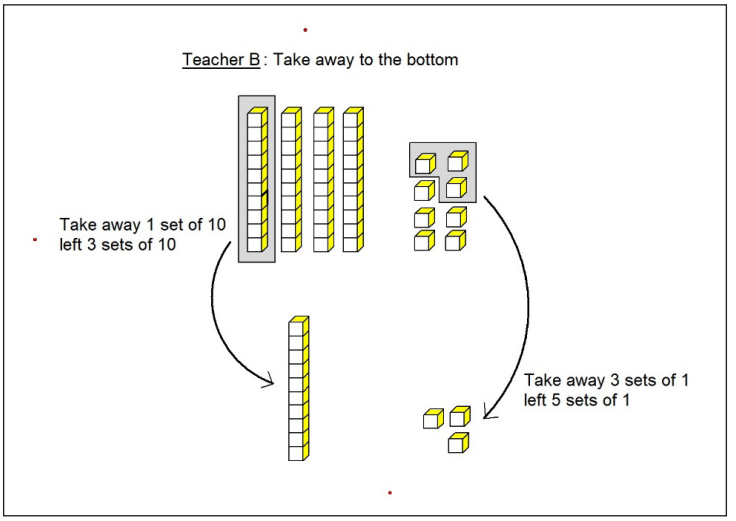



Diagram 10

- Compare the two ways of take away used by the teachers. Are there any conceptual difference in the meaning of subtraction as shown by the two ways? Why or why not?

$$\begin{array}{r} 48 \\ -\ 13 \\ \hline 35 \end{array}$$

Diagram 11

- Diagram 11 shows 48 – 13 in the vertical form.
  - ❖ Which way of take away will you follow if you want to introduce this vertical form subtraction to your students? Explain your reasons.

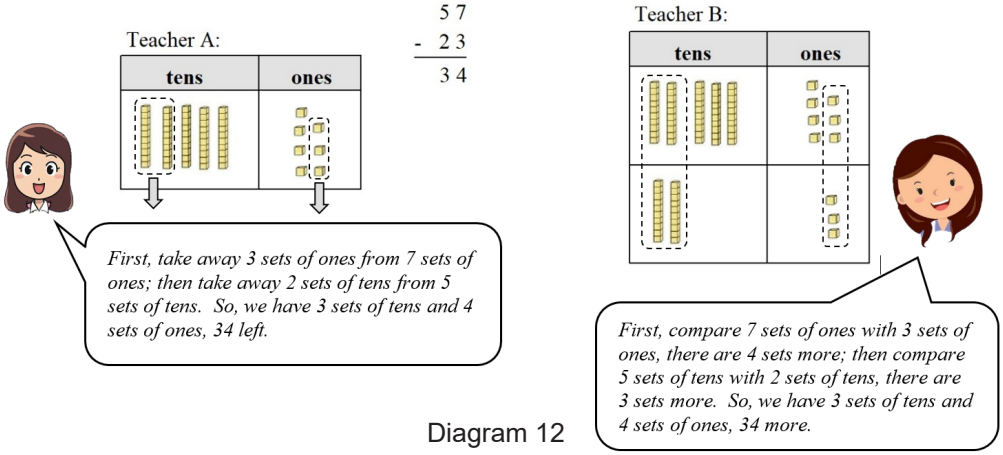



Diagram 12

ii. Two teachers, A and B, used base-10 blocks to show vertical form of subtraction 57 – 23 in two different ways as shown in Diagram 12.

- What meaning of subtraction does each of the ways show?
- Discuss the practicality of each way in learning the procedure of subtraction?
- Which of the ways is more efficient? Explain your reasons.

***Subtraction With Regrouping***

$$\begin{array}{r} 4\ 5 \\ -\ 2\ 8 \\ \hline \end{array}$$

Diagram 13

i. Diagram 13 shows an example of subtraction in vertical form that requires regrouping.

- Use base-ten blocks to perform the subtraction.
  - ❖ Explain how the regrouping is done using base-ten blocks.
- Think and show different algorithms in vertical form to represent the subtraction.

ii. Explain why algorithm is neccesary to help children do calculation?

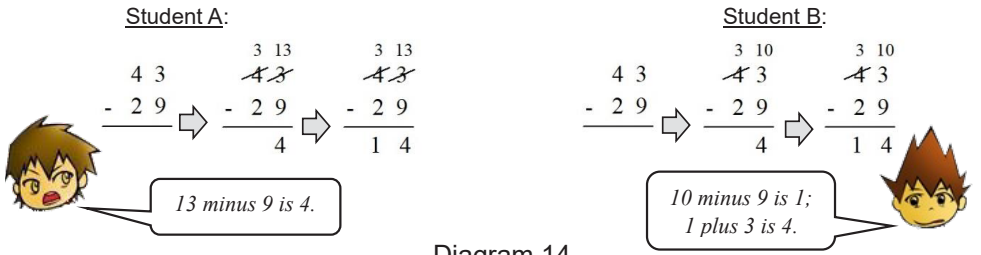



Diagram 14

iii. Diagram 14 shows two students' ways of doing subtraction in vertical form with regrouping.

- Explain the different ways used by the students.
- Which way do you think is easier for your students? Explain your reasons.

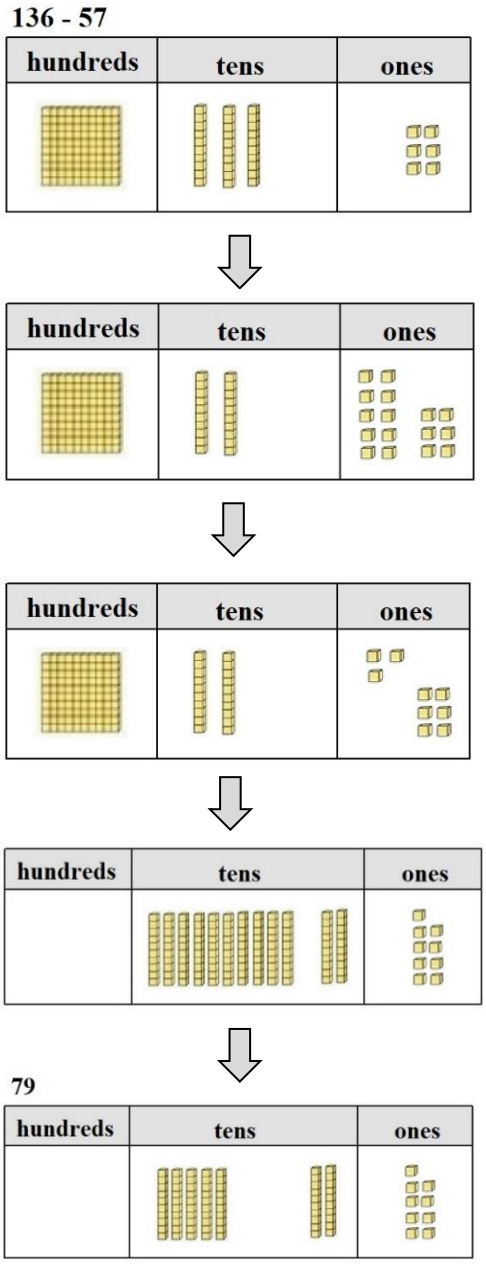



Diagram 15

iv. Diagram 15 shows how the subtraction 136 – 57 is done by using base-ten blocks in the vertical form.

- Explain each step of the subtraction with appropriate written vertical form of subtraction.
- What should you emphasise when using these concrete blocks to teach written vertical form to your students?

v. Children could easily get confused about written algorithm with abstract symbols. The vertical form of subtraction is an abstract representation as compared to the concrete representation using base-ten blocks. Hence, special effort is needed to help children connect the two representations especially for subtraction involving regrouping.

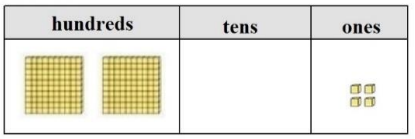
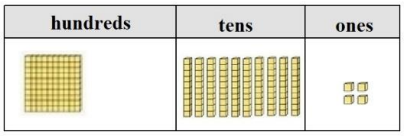
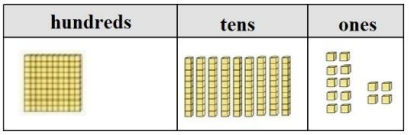
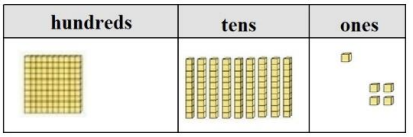
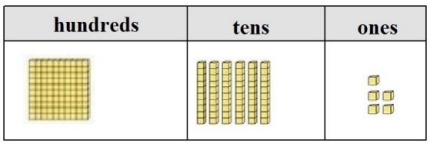
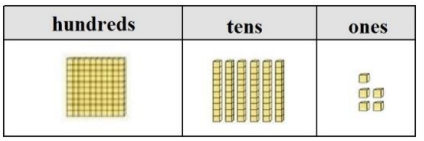
| Concrete Representation (in pictorial form) | | Abstract Representation |
|---|---|---|
| Step ①<br>  | ⟺ | 2 0 4<br>- 3 9 |
| Step ②<br>  | ⟺ | 1 10<br>~~2~~ 0 4<br>- 3 9 |
| Step ③<br>  | ⟺ | |
| Step ④<br>  | ⟺ | |
| Step ⑤<br>  | ⟺ | |
| Step ⑥<br>  | ⟺ | |

Diagram 16

Diagram 16 shows 6 steps in calculating 204 – 39 using the base-ten blocks. The vertical form of the first two steps had been written for you.

- Complete the vertical form for each of the remaining steps.
- When making connection between concrete and abstract representations, what do you think are the important points to consider?

**Task 3: Common Errors of Addition and Subtraction**

Student A:

| | |
|---|---|
| 28 + 65 | 2 8 |
| | + 6 5 |
| | 8 3 |

Student B:

| | |
|---|---|
| 28 + 65 | 2 8 |
| | + 6 5 |
| | 8 1 3 |

Student C:

| | |
|---|---|
| 43 - 18 | 4 3 |
| | - 1 8 |
| | 3 5 |

Student D:

| | |
|---|---|
| 84 - 7 | 8 4 |
| | - 7 |
| | 1 4 |

Diagram 17

i. Diagram 17 shows the work of four students with errors.

- Identify the error in each case.
- Explain the possible causes that children make such errors.

ii. Explain how the error in each case can be corrected.

iii. Collect a list of common errors of your students when doing addition and subtraction in the vertical form. Analyse the errors.

- Identify the difficulties faced by your students.
- Suggest ways to help your students overcome the difficulties.

**Standards 5.2:**

Acquiring fluency of standard algorithms for addition and subtraction and extend it up to 4-digit numbers

i. Extend the vertical form addition and subtraction through the extension of numbers and appreciate the explanation using base-ten block model
ii. Develop fluency of every extension up to 3-digit numbers and simple case for 4- digit numbers

**Sample Tasks for Understanding the Standards**

**Task 1: Vertical Form Addition for 3-Digit Numbers**

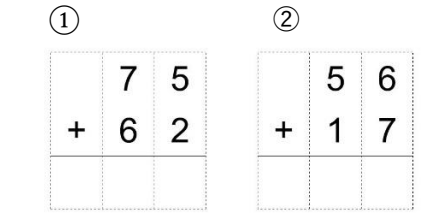



Diagram 1

i. Diagram 1 shows two additions of 2-digit numbers which the students have learnt before addition of 3-digit numbers.

- Which of the task is most suitable to be used to extend addition of 2-digit number to addition of 3-digit numbers?

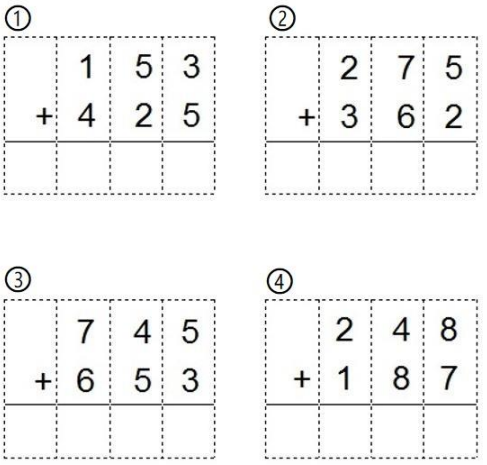



Diagram 2

ii. Diagram 2 shows five addition tasks involving 3-digit numbers.

- How do you structure the sequence of these five tasks to enhance the acquisition of computational fluency? Justify your decision.

- Which of these tasks is most suitable to be used to extend addition of 3-digit numbers to 4-digit numbers? Justify your decision.

**Task 2: Vertical Form Subtraction for 3-Digit Numbers**

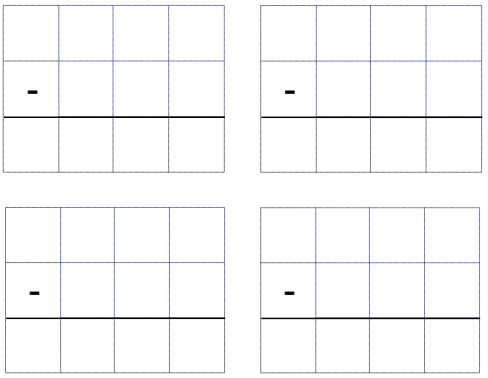



Diagram 3

i. Diagram 3 shows four sets of grid boxes for subtraction. Create a subtraction task involving two 3-digit numbers for each of the following conditions:

- One time regrouping from the hundreds place.
- One time regrouping from the tens place.
- The 3 digits in minuend are larger than the subtrahend.
- Two times regrouping from the tens and hundreds place.

ii. How do you sequence these tasks to develop computational fluency?

## Task 3: Representing a Subtracton Using Base-Ten Blocks

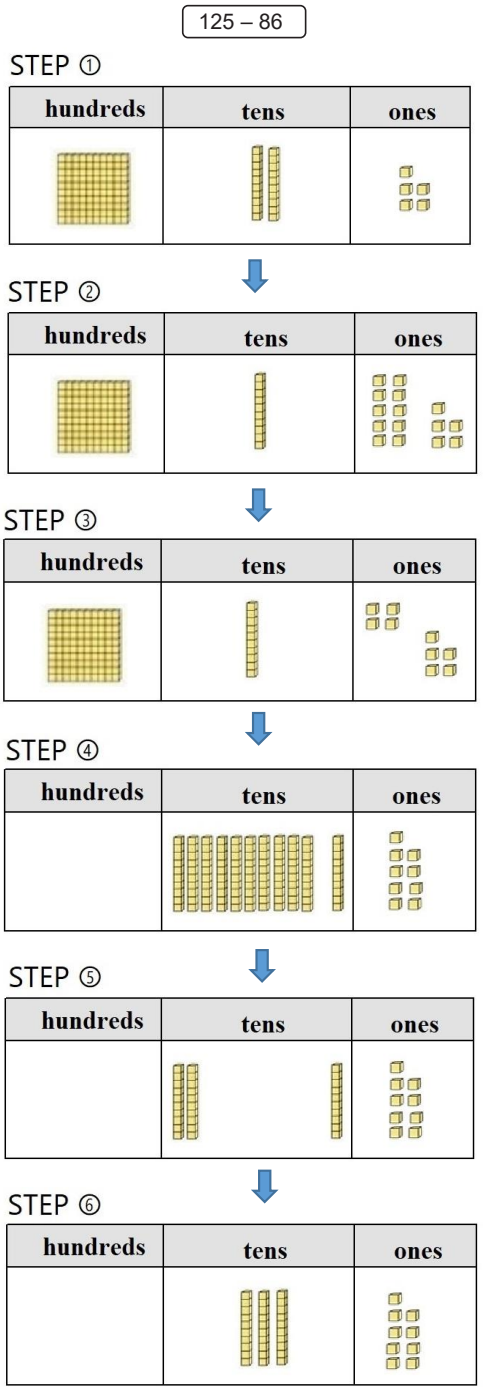



Diagram 4

i. Diagram 4 shows six steps in using base-ten blocks to calculate 125 – 86.

- At which step did the regrouping process carry out for the first time? Explain the regrouping.
- At which step did the regrouping process carry out for the second time? Explain the regrouping.

ii. Write the corresponding vertical form subtraction of each step.

iii. Why is it important for your students to write the corresponding vertical form at each step?

[*Note*. The term “borrowing” is also used to mean regrouping involving subtraction in some countries.]

**Standards 5.3:**

Developing number sense for estimation and using calculator judiciously for addition and subtraction

i. Develop number sense for mental arithmetic with estimation for addition or subtraction of numbers
ii. Identify necessary situations to use calculators judiciously in real life
iii. Appreciate the use of calculator in the case of large numbers for finding the total and the difference

**Sample Tasks for Understanding the Standards**

**Task 1: Making Sense of Estimation in Addition and Subtraction of Numbers**

16 + 19 + 18 =

Diagram 1

i. In real life, computational estimation can produce answers that are close enough for making good decisions without finding the exact calculation.

- What background knowledge is neccesary for children to make estimation for the addition task in Diagram 1?
- A child estimated the addition sum to be larger than 30, but less than 60.
  - ❖ How did the child come up with the answer?

① 398 + 140

② 387 + 56 + 43

③ 52 + 365 + 48

④ 91 – 28

⑤ 256 – 137

Diagram 2

ii. Diagram 2 shows five expressions involving addition and subtraction of numbers

- A child rounded up the two numbers in expression ① and estimated the sum to be 500. Explain how the child did the estimation.
- Without using the vertical form, find an estimated value for each of the remaining epressions ② to ⑤.
  - ❖ Explain your strategy used in the estimation of each task in Diagram 2?

ⓐ 3096 + 5218

ⓑ 6274 + 2726

ⓒ 5128 – 2835

ⓓ 5000 – 1782

ⓔ 966 – 28

Diagram 3

iii. Diagram 3 shows some expressions of large numbers involving addition and subtraction.

- Which of these expressions will you encourage young children to use mental arithmatic even though the numbers appear large? Explain your reasons.

**Task 2: Using Calculators at Right Situations**

i. Calculation using algorithm can be challenging for some young children.

- What types of situations will you recommend the use of calculators to help young children learn addition and subtraction? Give examples to illustrate your answers.

ii. Diagram 4 shows some addition and subtraction problems in vertical form. Some of the digits in each problem are stained with spilled ink.

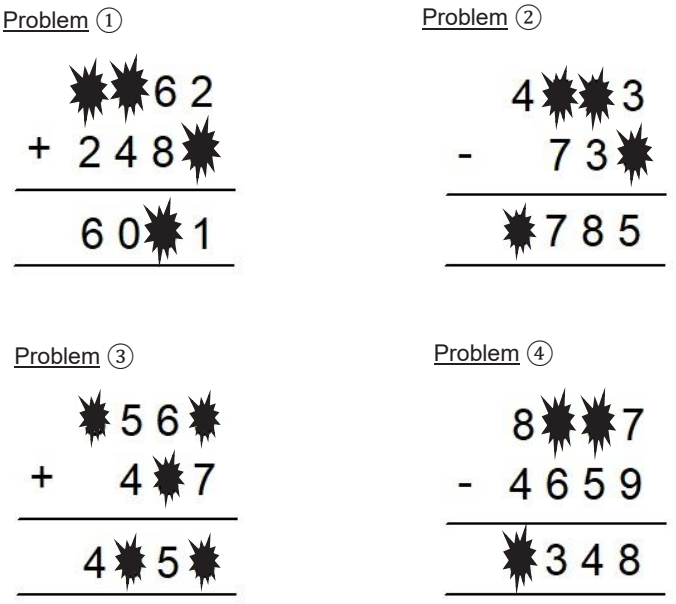



Diagram 4

- Find the missing digits in each problem.
- If calculator is allowed to be used, do you think children will be able to solve these problems by pressing the calculator keys without thinking carefully?
- In what ways can children benefit from the use of calculator when solving these problems?

iii. Diagram 5 shows some mixed addition and subtraction problems in vertical form with some missing numbers as indicated by each ➢.

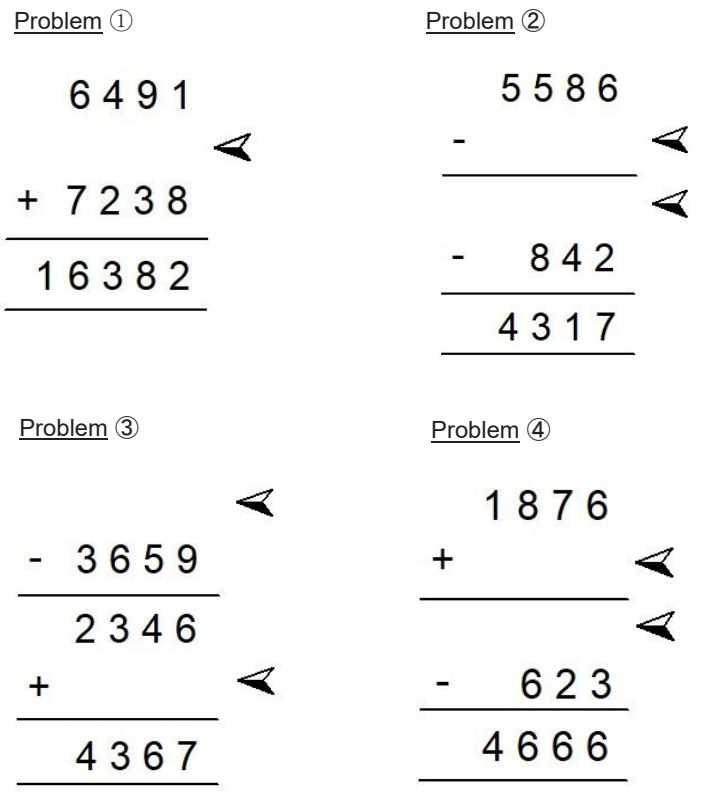



Diagram 5

- Find the missing numbers in each problem.
- Will you allow your students to use calculator when solving these problems?

  Explain your reasons.

## Topic 6: Introducing Multiplication and Producing Multiplication Algorithm

### Standards 6.1:

Introducing multiplication and mastering multiplication table

i. Understand the meaning of multiplication situations with models using the idea of addition and distinguish from the common addition to find the total number
ii. Produce multiplication table in the case of counting by 2 and 5 with array diagrams, pictures or block models and extend it until 9 and 1 with appreciation of patterns
iii. Develop sense for multiplication through mental calculation with fluency
iv. Use multiplication in daily life, differentiating the situations for multiplication in various situations with understanding that any number can be a unit for counting in multiplication

### Sample Tasks for Understanding the Standards

### Task 1: Situations of Multiplication

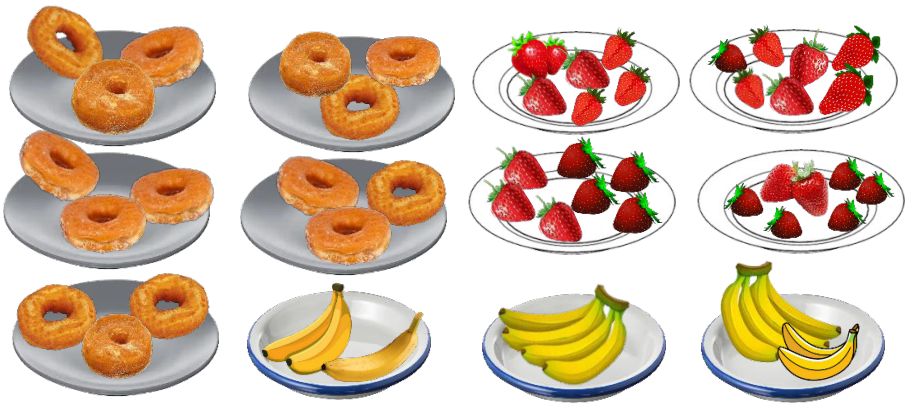


Diagram 1

i. Diagram 1 shows some plates of bananas, strawberries and doughnuts.

- Consider answering the following three sets of questions:

  Set ①
  - o How many plates of bananas are there?
  - o How many bananas are there altogether?

  Set ②
  - o How many plates of strawberries are there?
  - o How many strawberries are there altogether?

  Set ③
  - o How many plates of doughnuts are there?
  - o How many doughnuts are there altogether?

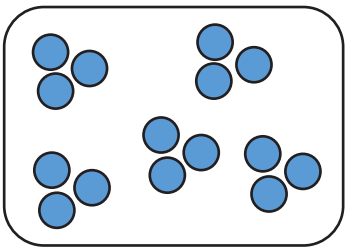
- Which of these sets of questions cannot be answered using a multiplication sentence?

- Why is the set of questions important in developing the concept of multiplication?

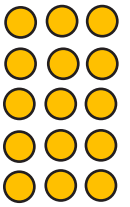
- Based on this task, how will you help your students to relate multiplication to addition?
- Base on the situations, identify two ideas you consider most important in developing the meaning of multiplication as repeated addition.
- How does this task cultivate the value of seeking simpler ways to find answer in mathematics among your students?

ii. When asked to find the total number of doughnuts in Diagram 1, four students, A, B, C and D, used four different methods to do so as shown in Diagram 2

Student A's Method



Student B's Method



Student C's Method

3 + 3 + 3 + 3 + 3 = 15

Student D's Method

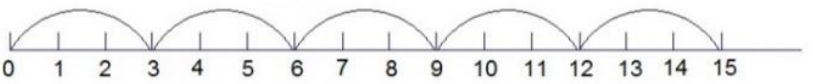



Diagram 2

- Explain how each student get the answer of 15.
- How does skip counting connect to multiplication?

iii. All the situations in Diagram 2 represent 3 doughnuts in each plate, for 5 plates to make 15 doughnuts. Different countries may follow different conventions in writing the multiplication sentence.

- In your country, are the situations in Diagram 2 written as 5 x 3 = 15 or 3 x 5 = 15? Explain your answer.
- How do you address this issue of different convention in writing the multiplication sentence in your teaching?

  [Note. In this book, 5 x 3 means 5 sets of 3 unless it is stated otherwise. However, the method of expressing multiplication sentence should be based on your national curriculum.]

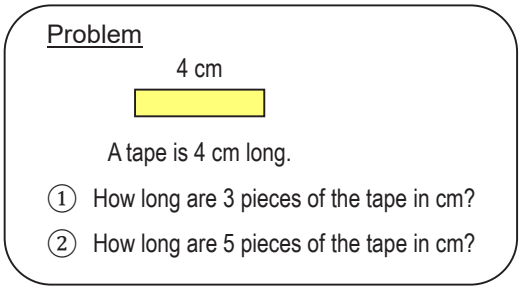
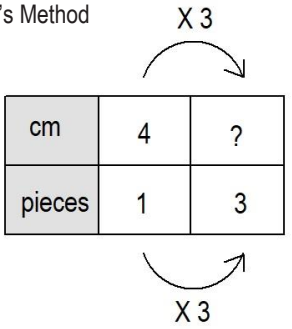



Diagram 3

Rizal's Method




Ricky's Method

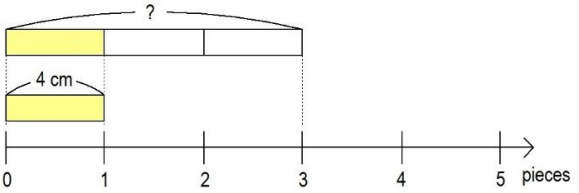



Diagram 4

iv. Diagram 3 shows a problem involving lengths of several pieces of tapes. Two students, Rizal and Ricky, solved question ① with different methods as shown in Diagram 4.

- Explain each method.
- Solve question ② using each method.
- Represent the situation in question ② with a multiplication sentence.
  - ❖ Using your multiplication sentence as example, explain the meanings of
    (a) multiplier,
    (b) multiplicand, and
    (c) product.
- Which method do you prefer for your students to learn?
  Justify your choice.
- Use each method to solve the following problem.

  "John has 18 cm of green ribbon and 6 cm of yellow ribbon. How many times the length of the yellow ribbon is equal to the length of the green ribbon?"

  - ❖ What future learning of number operation is this problem leading your students to? Explain your answer.

**Task 2: Producing Multiplication Tables of 2 and 5**

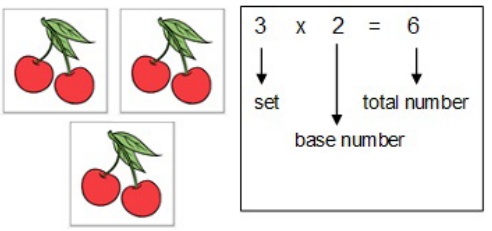



Diagram 5

i. Diagram 5 shows three sets of cherries, each set with two cherries. Students are taught to read the multiplicaton sentence as '3 times 2 is 6'.

- "3 times 2 is 6" (3x2=6) can also be read as "2 multiply by 3 equals 6". In this case, multiplicand, multiplier, and product are three terms related to the multiplication sentence.
  - ❖ Identify which number is the multiplicand, the multiplier and the product.
  - ❖ Explain the meaning of each term.
- Concrete materials such as interlocking cubes can be used to help children understand multiplication.
  - ❖ How do you use interlocking cubes to represent 3 x 2 = 6?
  - ❖ Draw the interlocking cubes that show 3 x 2 = 6.

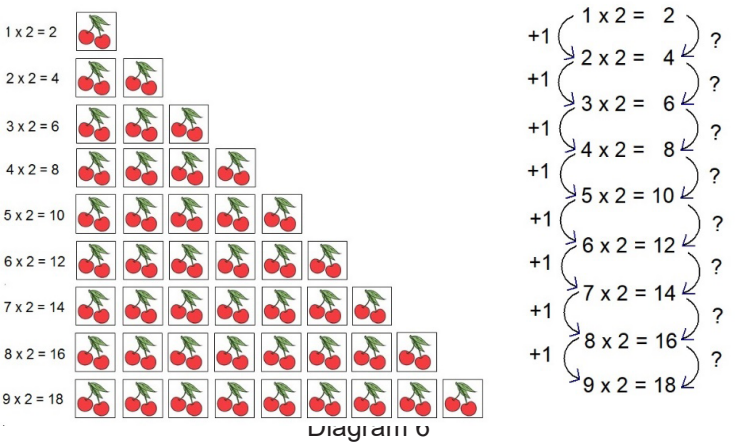



Diagram 6

ii. In Diagram 6, the cherry sets are arranged starting from one set, and added a set to the next row until the 9th row. The corresponding multiplication table 2 is also shown by the side of the arrangement of cherry.

- As the number of sets increase by 1, what pattern do you notice in the total number of cherries?

- Explain how the pattern can help children find the answers for 10x2, 11x2 and 12x2?
- What is the relationship of increasing the number of sets and the total number of cherries?

Convention ①

| |
|---|
| 1 x 5 = 5 |
| 2 x 5 = 10 |
| 3 x 5 = 15 |
| 4 x 5 = 20 |
| 5 x 5 = 25 |
| 6 x 5 = 30 |
| 7 x 5 = 35 |
| 8 x 5 = 40 |
| 9 x 5 = 45 |

Convention ②

| |
|---|
| 5 x 1 = 5 |
| 5 x 2 = 10 |
| 5 x 3 = 15 |
| 5 x 4 = 20 |
| 5 x 5 = 25 |
| 5 x 6 = 30 |
| 5 x 7 = 35 |
| 5 x 8 = 40 |
| 5 x 9 = 45 |

Diagram 7

iii. Diagram 7 shows two different conventions in writing the multiplication table of 5 which are practised in different countries.

- Which convention does your country's national curriculum follow?
- Will this different conventions cause a confusion to your practice in the classroom? Explain your answer.

iv. Why are base numbers 2 and 5 usually used first to introduce multiplication before other numbers?

v. A multiplication table is usually built from 1 to 9. Why is table 10 not included?

**Task 3: Multiplication Beyond Tables 2 and 5**

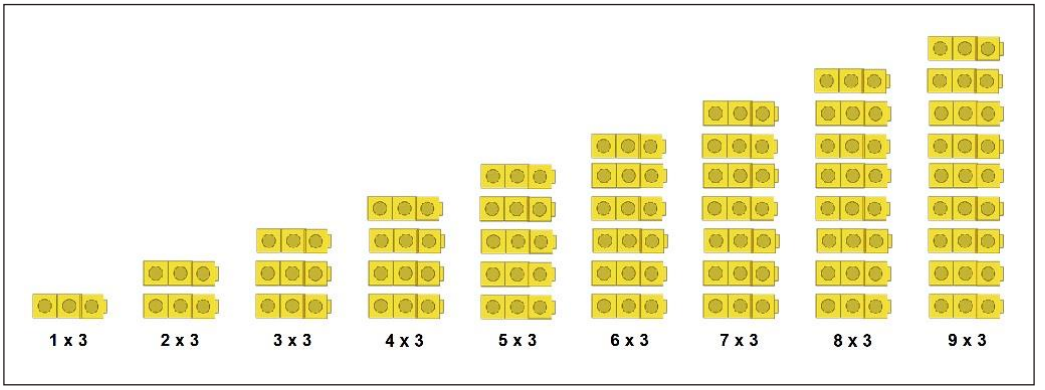



Diagram 8

i. Diagram 8 shows multiplication table 3 represented by interlocking cubes. Compare this diagram with the arrangement of cherries in diagram 6.

- In what way is the arrangement of interlocking cubes different from the arrangement of cherries?
- Explain how this arrangement of interlocking cubes can be used to show another multiplication situation, which is knows as a multiplication array.

  [Note. In this book, 5 x 3 means 5 rows by 3 columns unless it is stated otherwise. Once again, the method of expressing a multiplication array should be based on your national curriculum.]

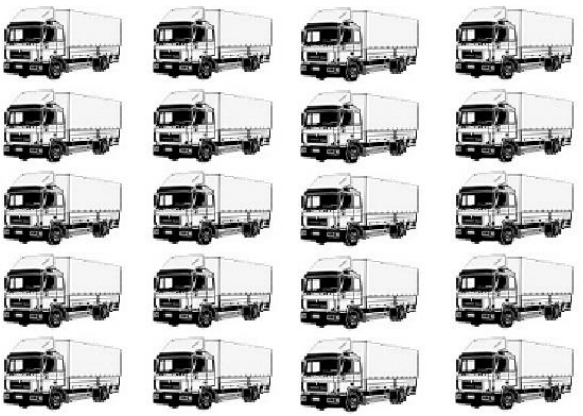


Diagram 9

ii. Diagram 9 shows 5 rows or 4 columns of lorries.

- Write a multiplication expression to represent the array.

iv. A teacher used an array of 9 by 4 lorries as shown in Diagram 10 and a rectangular card to help his students developed the multiplication table 4.

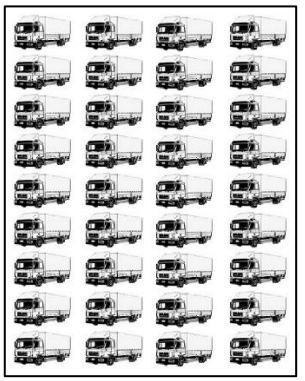
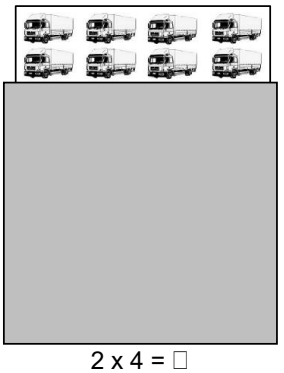
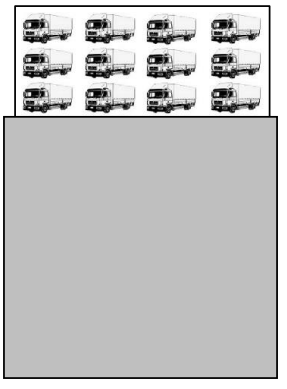


Diagram 10

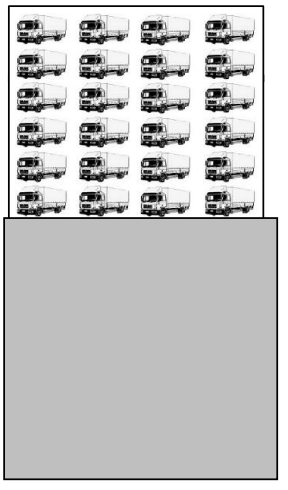
He moved the rectangular card up and down and used appropriate questions to guide his students determine the basic facts for multiplication table 4. Diagram 11 shows some examples of such facts.



2 x 4 = □



3 x 4 = □

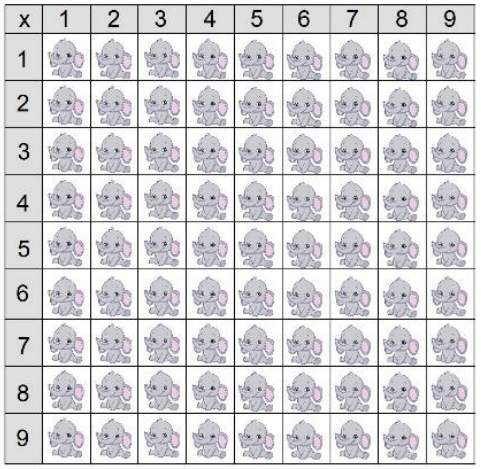


6 x 4 = □

Diagram 11

- What questions will you ask not only to help your students determine the basic facts, but also to develop their sense for multiplication?
- Justify your choice of questions.
- Explain how this task can help your students learn the idea of area in future grades.

v. Diagram 12 shows a 9 row by 9 column arrays, a L-shaped card and an empty 9 by 9 multiplication table.




| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | | | | | | | | | |
| 4 | | | | | | | | | |
| 5 | | | | | | | | | |
| 6 | | | | | | | | | |
| 7 | | | | | | | | | |
| 8 | | | | | | | | | |
| 9 | | | | | | | | | |

Diagram 12

- A teacher used the array together with the card to help children fill in the 9 by 9 multiplication table. Explain how this can be done.
- What are the advantages and disadvantages of this method in helping children develop the multiplication table?

## Task 4: Fluency in Multiplication Basic Facts

***Patterns in Multiplication Tables***

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | | | | | | | | |
| 2 | | 4 | | | | | | 16 | |
| 3 | | | 9 | | | | | | |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 5 | | | | 20 | 25 | | | | 45 |
| 6 | | | | | | 36 | | | |
| 7 | | | | | | | 49 | | |
| 8 | | | | | | | | 64 | |
| 9 | | | | | | | | | 81 |

Diagram 13

Diagram 13 shows a chart with multiplication tables 1 to 9, leaving some products as blanks.

i. Complete the table.

ii. Explore number patterns in the chart.

- What different patterns do you notice among the numbers?
- Explain how and why each of the patterns is formed?
- Is any of the patterns related to the commutative property of multiplication? Explain your answer with examples.
- Suggest how these patterns could help your students learn the basic facts of multiplication.

iii. Look at the numbers along the diagonal from the top left to the bottom right and also the diagonal from the top right to the bottom left. Explore these two lines of diagonal numbers.

- What interesting patterns do you find in these two lines of numbers?
- What about the difference between consequtive numbers?
- Explain why each pattern exists.

iv. Why is fluency with mental calculation important in developing sense for multiplication among your students?

***Pattern in Rows and Columns***

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | ← Row 1 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | ← Row 2 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | ← Row 3 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | |

Diagram 14

Diagram 14 shows a complete chart with multiplication tables 1 to 9.

Study the numbers in rows 1, 2 and 3.

i. What pattern do you observe about these three rows of numbers?

ii. Make a conjecture about the relationship between any three rows of numbers.

- Is your conjecture always true?
- Justify your conjecture using a property of multiplication.

iii. Suggest how the relationship can help your students master basic facts of multiplication.

***Sum of Products in Multiplication Tables***

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | ← Row 5 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | |

Diagram 15

Diagram 15 shows a horizontal box with three consecutive products 8, 10 and 12 in it. Another vertical box has three consecutive products 20, 24 and 28 in it.

i. After studying the horizontal box carefully, a student makes the following conjecture:

*"The sum of any row of three consecutive products is equal to the middle product multiply by 3".*

- Is the conjecture always true?
- How could you justify the conjecture without actually adding up the products?
- Is the conjecture extendable to the sum of any row of five consecutive products, respectively? Explain your reasons.
- Is the conjecture extendable to the sum of any products in a vertical box such as 20, 24, and 28? Explain your reasons.

***Pattern in Rows and Columns***

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |

↑ Column 5

Diagram 16

ii. Diagram 16 shows Column 5 of the products in the chart. Another student makes a conjecture as follows:

*"The sum of all products in Column 5 is 9* x *25."*

- Is the student's conjecture true?
- Other than adding up all the products in Column 5, how could you justify the student's conjecture?
- Can the conjecture be extended to other columns? Explain your reasons.

iii. A third student makes another conjecture as follows:

*"The sum of all the 81 products is 25* x *81."*

- Is the conjecture true?
- Justify the conjecture, without adding up all the products.

iv. With reference to the Common Core Regional Learning Standards in Mathematics, in what way can these tasks inculcate mathematical values among your students?

**Task 5: Multiplication in Daily Life**



Diagram 17

i. Many things in grocery stores are packed in sets. Diagram 17 shows some grocery items that come in packages.

- The unit of counting for the package of oranges is 5.
  - ❖ What is the unit of counting for each of the remaining two items?
- For each of the items, create a daily-life problem involving the item in a multiplication situation.
- Find other suitable items with different unit of counting. Create a dailly-life situation involving the items.

**Standards 6.2:**

Producing multiplication in vertical form and obtaining fluency

i. Think about easier ways of multiplication in the case of numbers greater than 10 using array diagrams and block models
ii. Develop multiplication in vertical form using multiplication table, array, model, and base ten system with appreciation
iii. Extend multiplication algorithm to 3-digit times 2-digit numbers
iv. Obtain fluency for standard algorithm for multiplication
v. Use estimation with multiplication of tens or hundreds in life
vi. Compare the multiplication expressions which is larger, smaller or equivalent
vii. Appreciate the use of calculator sensibly in life in the case of large numbers

**Sample Tasks for Understanding the Standards**

**Task 1: Multiplication Involving Numbers Greater Than 10**

i. Multiplication becomes difficult for many children when it involves numbers greater than 10, such as 15 x 8.

- Suggest and explain some easier ways to calculate 15 x 8.

ii. Diagram 1 shows 14 sets of interlocking cubes.

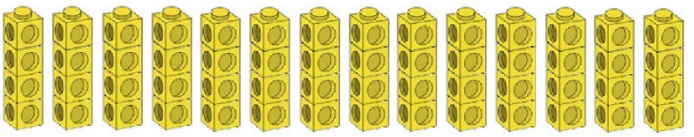


Diagram 1

Three students, A, B and C, used different strategies to find the total number of the cubes as shown in Diagram 2.

Student A's Strategy:

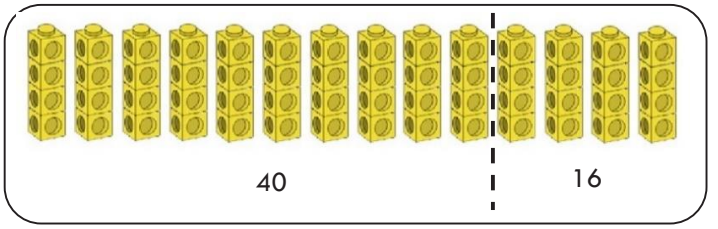
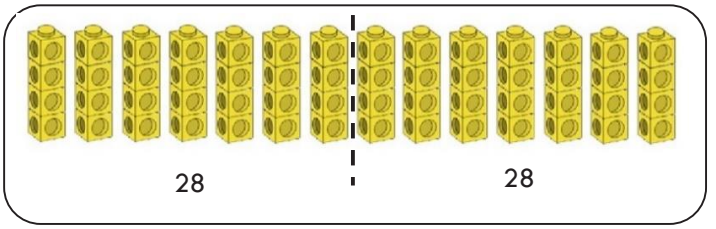



Diagram 2

Student B's Strategy:




Student C's Strategy:

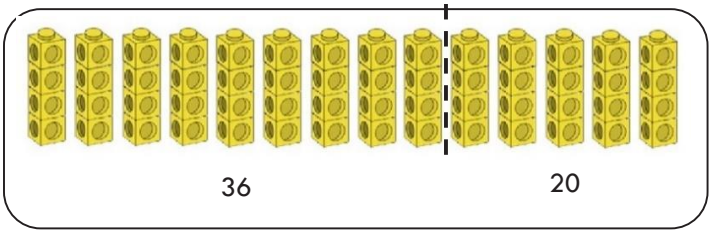



Diagram 2 (Contd.)

- Explain the strategy of each student.
- Write a mathematics sentence to represent each strategy.
- What property of multiplication was used in the students' strategies?
- Explain how the property can be used to simplify the multiplication 45 x 12 in future grades.
- Which strategy will you encourage your students to use? Justify your answer.

**Task 2: Multiplication in Vertical Form**

Diagram 3 shows the multiplication 3 x 23 done in the vertical form.

```
  2 3
x   3
-------
    9   <-- 3 x 3
  6 0   <-- 3 x 20
-------
  6 9
-------
```

Diagram 3

i. Explain how the distributive property of multiplication is used to perform the multiplication.

ii. Teacher A used the array situation as shown in Diagram 4 to explain the vertical form of multiplication, whereas another teacher, B, explained it using the repeated addition situation as shown in Diagram 5.

Teacher A's explanation:

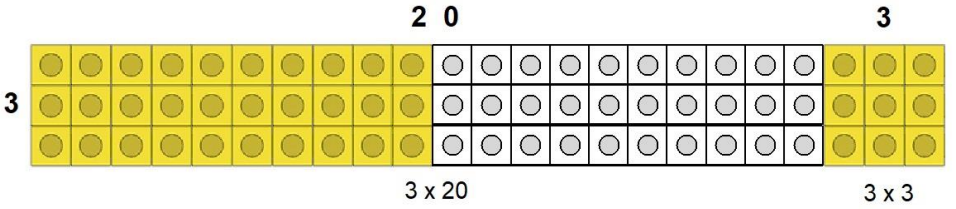



Diagram 4

Teacher B's explanation:

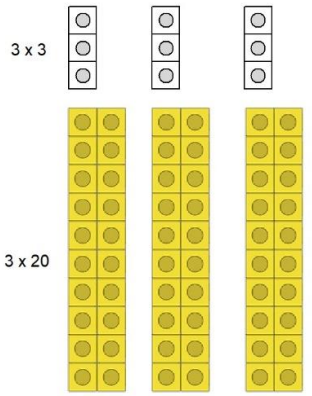



Diagram 5

- Which situation do you think is easier to explain the vertical form of multiplication to your students? Explain your reasons.

**Task 3: Multiplication of 2-Digit Number by 2-Digit Number and Beyond**

i. Three teachers, A, B and C, taught their students three different methods to perform multiplication in vertical form as shown in Diagram 6.

Teacher A's method :

$$\begin{array}{r} {}^{1}2\,4 \\ \times\ 1\,3 \\ \hline 7\,2 \\ 2\,4 \\ \hline 3\,1\,2 \\ \hline \end{array}$$

Teacher B's method :

$$\begin{array}{r} {}^{1}2\,4 \\ \times\ 1\,3 \\ \hline 7\,2 \\ 2\,4\,0 \\ \hline 3\,1\,2 \\ \hline \end{array}$$

Teacher C's method :

$$\begin{array}{r} 2\,4 \\ \times\ 1\,3 \\ \hline 1\,2 \\ 6\,0 \\ 4\,0 \\ 2\,0\,0 \\ \hline 3\,1\,2 \\ \hline \end{array}$$

Diagram 6

- Which method do you think is easier for your students to learn? Explain your reasons.

ii. A student calculated 13 x 24 using a method that was different from his teacher's method. The student's method is shown in Diagram 7.

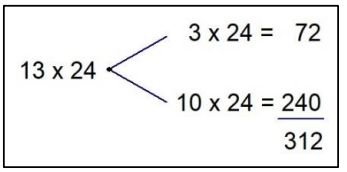



Diagram 7

- Explain the student's method.
- How can the student's method help to explain each of the teachers' methods in Diagram 6?
- Why methods in diagram 6 are usually taught to students instead of this method?
- What are the similarities and differences between these methods?

iii. Another student calculated 13 x 24 using base-ten blocks as shown in Diagram 8.

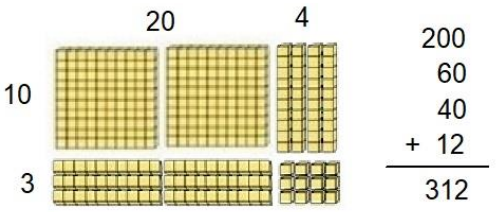



Diagram 8

- Explain the method.
- How can these base-ten blocks help to explain each of the teachers' methods in Diagram 6?

iv. Diagrams 9 shows three different methods to calculate 25 x 34.

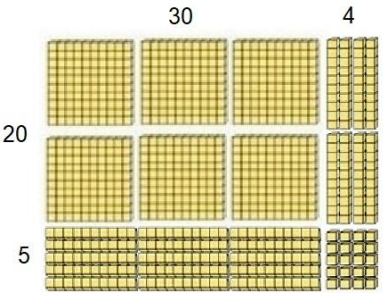
Method ① : Using Table

**25 x 34**

| x | 30 | 4 |
|---|---|---|
| 20 | 600 | 80 |
| 5 | 150 | 20 |

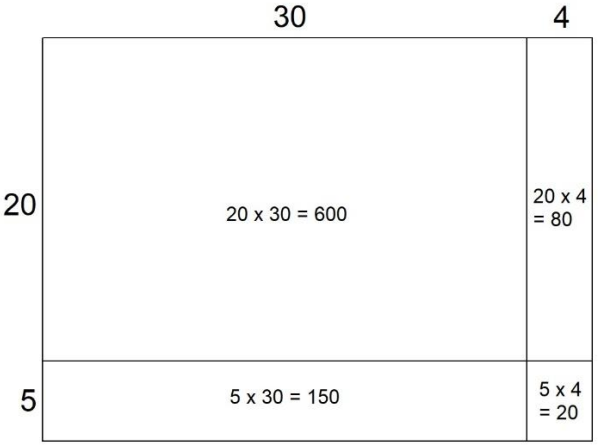
600
150
80
+ 20
850

Method ② : Using Base-10 Blocks




600
150
80
+ 20
850

Method ③ : Using Block Diagram




600
150
80
+ 20
850

Diagram 9

- If you are to introduce these three methods to your students, how will you sequence the order of teaching? Justify your decision.

v. Three students calculated 48 x 235 by three different methods as shown in Diagram 10.

Student A's Method:

**48 x 235**

| **x** | **200** | **30** | **5** |
|---|---|---|---|
| **40** | 8000 | 1200 | 200 |
| **8** | 1600 | 240 | 40 |

8000
1200
1600
200
240
\+ 40
Total 11280

Student B's Method:

```
  235
x  48
 1880
 940
11280
```

Student C's Method:

```
  48
x235
 240
1440
9600
11280
```

Diagram 10

- What are the advantages and disadvantages of each of the methods?
- How can Student A's method be used to explain Student B's and Student C's methods?

**Task 4: Problem Solving With Multiplication**

i. Diagram 11 shows a problem involving multiplying a 3-digit number with a 2-digit number.

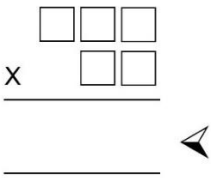


Diagram 11

- The product of a 3-digit number with a 2-digit number is at least a 4-digit number.
  - ❖ Do you agree with the statement? Why or why not?
- What 3-digit and 2-digit numbers, when multiplied together will give a product of 5-digit numbers? Justify your answers.

ii. Diagram 12 shows two multiplication problems in vertical form with some missing numbers as indicated by each ◅.

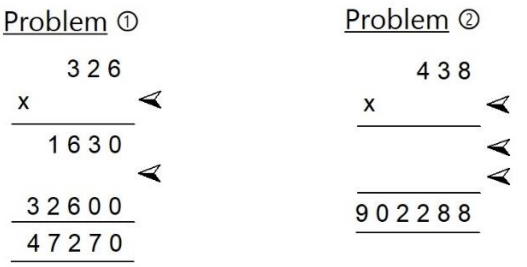



Diagram 12

- Find the missing numbers in each problem. Use calculator to help you solve the problems.
- Does the use of calculator inhibit or promote your thinking processes when solving the problems? Explain your reasons.

iii. Ben was given four digit cards:

| 2 | 3 | 4 | 6 |
|---|---|---|---|

He used the four cards to make two multiplications by interchanging the digits as follows:

32 x 46 and 23 x 64

- Work out the answers for the two multiplications.
- What interesting results do you get?
  - ❖ Try with other combinations of the four cards to see if you can find similar results.

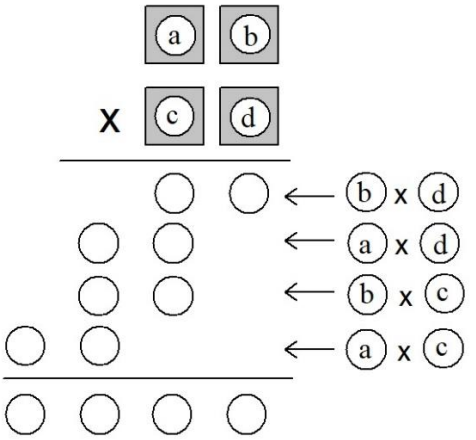



Diagram 13

- Use the structure of multiplication in the vertical form in Diagram 13 to help you analyse the problem.
- What is the rule to explain the interesting result you found in the two multiplications?
  - ❖ Using the rule, make other problems with four other different digits that can produce the same interesting result.

## Topic 7: Introducing Division and Extending It to Remainder

### Standards 7.1:

Introducing division with two different situations and find the answers by multiplication

i. Understand division with quotative and partitive division for distribution situations
ii. Think about how to find the answer of division situations by distribution using diagrams, repeated subtractions and multiplication
iii. Obtain fluency to identify answers of division through inverse operation of multiplication
iv. Appreciate the use of multiplication table for acquiring mental division

### Sample Tasks for Understanding the Standards

### Task1: Quotative and Partitive Division

***Meanings of Division***

Diagram 1

Diagram 1 show 12 tomatoes. Consider two division situations as follows:

Partitive Situation: Pack the tomatoes equally into 2 boxes.
How many tomatoes are there in a box?

Quotative Situation: Pack 2 tomatoes into a box.
How many boxes of tomatoes are there?

i. Diagram 2 shows the two situations graphically, both representing 12 ÷ 2 = 6.

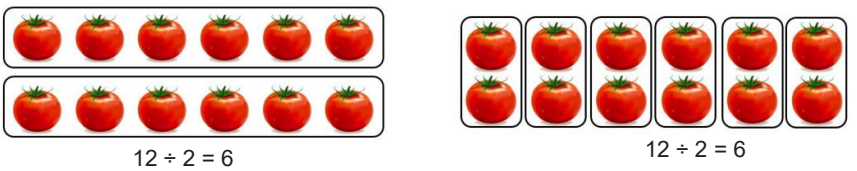



Diagram 2

- Match each of the situations with the correct picture in diagram 2.
- Explain the key difference between the two situations.

ii. Illustrate the key difference between the two division situations with appropriate tape diagrams.

iii. Diagram 3 shows two number lines.

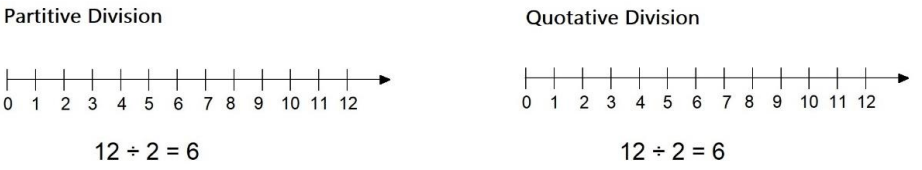



Diagram 3

- Draw on the number lines to show the key difference between the two division situations.

iv. Diagram 4 shows the dividend, divisor and quotient in a division sentence.

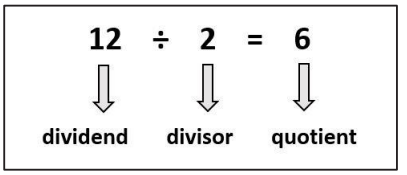



Diagram 4

- What does the divisor '2' mean in each of the division situations?
- What does the quotient '6' mean in each of the division situations?

v. Division is the inverse of multiplication. Depending on the contexts, different division situations may relate to different multiplication situations.

- Which division situation is related to 6 x 2 = 12? Explain your answer.
- Which division situation is related to 2 x 6 =12? Explain your answer.

vi. Which division situation shows repeated subtraction? Explain your answer.

***Basic Facts of Division***

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| **1** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| **2** | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| **3** | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| **4** | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| **5** | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| **6** | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| **7** | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| **8** | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| **9** | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |

Diagram 5

i. Diagram 5 show the 9 by 9 table of multiplication.

- Explain with specific examples, how the inverse relationship between division and multiplication can be used to find basic facts of division from his table.

**Standards 7.2:**

Extending division into the case of remainders and use division for distribution in daily situations

i. Extend division situations with remainders and understand division as a repeated subtraction with remainders
ii. Obtain fluency for division and apply it in daily situations
iii. Understand simple case of division algorithm

**Sample Tasks for Understanding the Standards**

**Task 1: Division with Remainder**

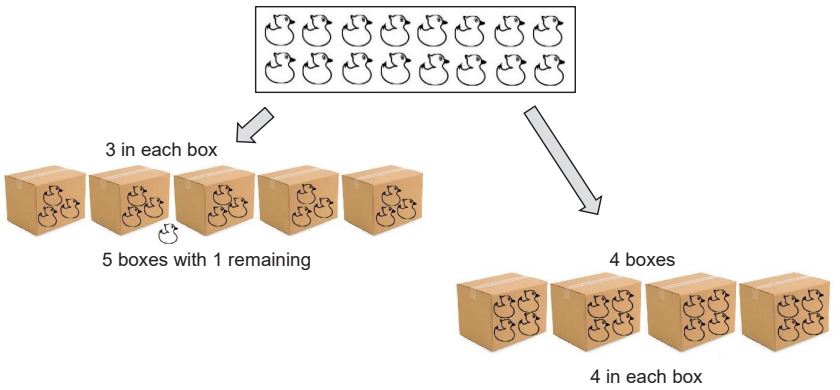



Diagram 1

i. Diagram 1 shows 16 toy ducks. A student packed 3 toy ducks into each box. There are 5 boxes of toy ducks and a remainder of 1 toy ducks. Another student packed the toy ducks equally into 4 boxes. There are 4 toy ducks in a box with no remainder.

- Explain the similarities and differences between the two division situations.
- It is said that 16 is divisible by 4, but not divisible by 3. Explain the meaning of divisible.

| | |
|---|---|
| 24 ÷ 4 = 6 | |
| 23 ÷ 4 = 5 | remainder 3 |
| 22 ÷ 4 = 5 | remainder 2 |
| 21 ÷ 4 = 5 | remainder 1 |
| 20 ÷ 4 = 5 | |
| 19 ÷ 4 = 4 | remainder 3 |
| 18 ÷ 4 = 4 | remainder ? |
| 17 ÷ 4 = 4 | remainder 1 |
| 16 ÷ 4 = 4 | |
| 15 ÷ 4 = 3 | remainder ? |
| 14 ÷ 4 = 3 | remainder ? |
| 13 ÷ 4 = 3 | remainder 1 |
| 12 ÷ 4 = 3 | |

Diagram 2

ii. A student carried out an investigation on division with remainders. The result of investigation is shown in Diagram 2.

- What questions will you ask to enable your students to determine whether a number is divisible or not divisible by 4?
- Explain the pattern of the remainders in Diagram 2.
- One student said that 23 ÷ 4 = 4 remainder 7.
  - ❖ Will you accept this answer?
  - ❖ How will you fix the matter?

- Why a remainder is always smaller than the divisor?
- If a divisor of a division expression is 7, What are the possible remainders?

**Task 2: Calculate Division in Vertical Form**

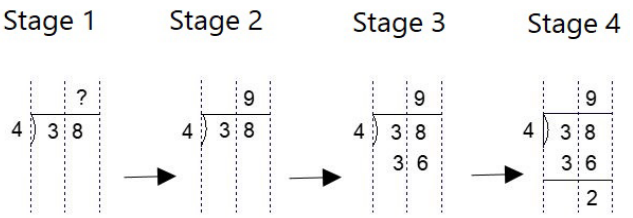



Diagram 3

i. Diagram 3 shows four stages of calculating 38 ÷ 4 in the vertical form.

- What questions will you ask at each stage to guide students perform the division?

ii. What other operations are also involved in calculating division in vertical form?

- Why and how is each of the identified operations involved with division?


Diagram 4

iii. Diagram 4 shows how the calculations 16 x 42 and 672 ÷ 16 are done in the vertical form.

- Why is addition involved in the multiplication algorithm?
- Why is subtraction involved in the division algorithm?

iv. Three students, A, B and C, used three different methods to calculate division in the vertical form as shown in Diagram 5.

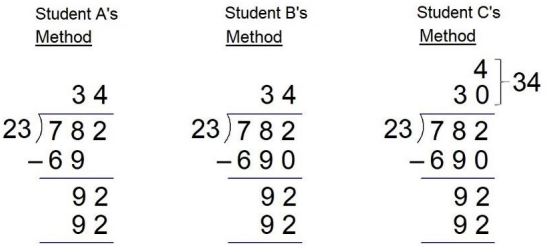



Diagram 5

- Which of these methods will you prefer to teach your students?
- Justify your choice.

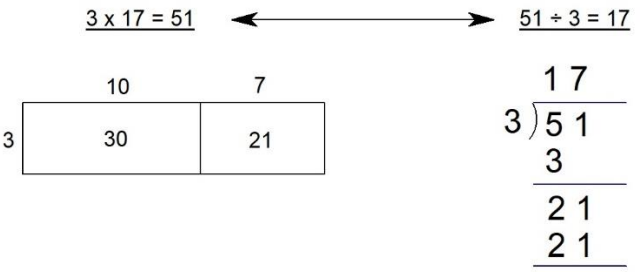



Diagram 6

v. Diagram 6 shows the connection between the vertical form of division and multiplication performed in block diagram.

- Explain the connection between the two forms.
- How will this idea of connection between the two forms help you in guiding your students explain the process of vertical form of division?

## Topic 8: Introducing Fractions and Extending to Addition and Subtraction of Similar Fractions

### Standards 8.1:

Introducing simple fractions such as halves, quarters and so on using paper folding and drawing diagrams

i Introduce simple fractions using paper folding and drawing diagrams in the context of part whole relationship
ii Use "*a half of*" and *"a quarter of*" in a daily context such as half a slice of bread
iii Count a quarter for representing one quarter, two quarters, three quarters, and four quarters
iv Compare and explain simple fractions in the case where the whole is the same

### Sample Tasks for Understanding the Standards

### Task 1: Representing Simple Fractions

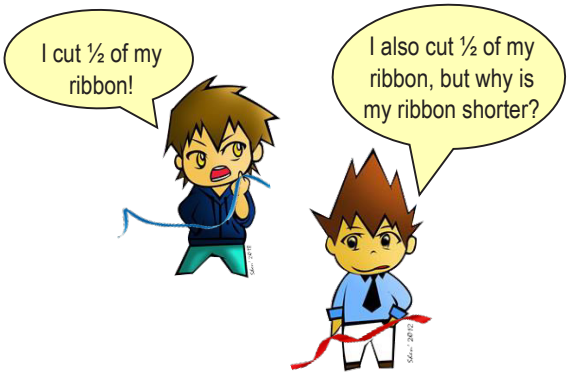



Diagram 1

i. John and Jason found a ribbon each at their homes. They cut ½ of each ribbon and brought to the school the next day. In school, they decided to exchange the ribbons.

When they compared the ribbons, they got confused and Diagram 1 shows the conversation between them.

- Why are ½ of each ribbon not the same length?
- What questions will you ask to adress the confusion?

ii. Diagram 2 shows three shapes used by a teacher to introduce ¼ to children.

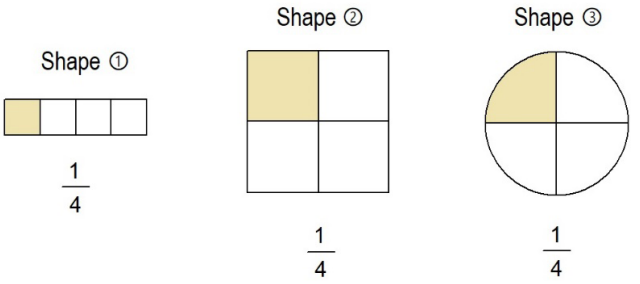



Diagram 2

- How will you convince your students that all four parts of each shape are equal in size?
- Explain how paper folding can be used to make the three shapes?

iii. Both the tasks in (i) and (ii) could be used to introduce simple fraction. What are the similarities and differences between the two tasks?

iv. Shapes with shaded parts are commonly used to introduce simple fraction to children. Diagram 3 shows four examples of shapes with shaded parts.

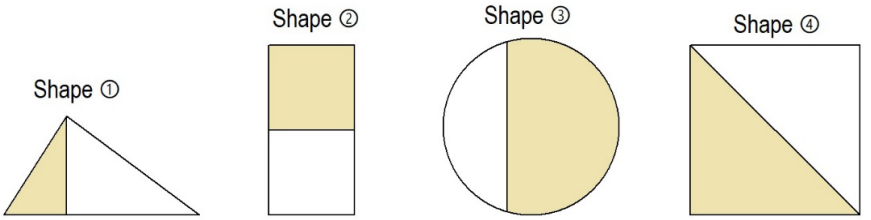
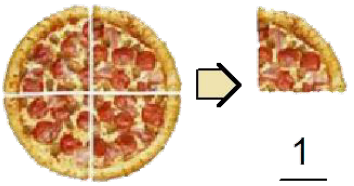



Diagram 3

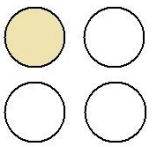
- Which shapes are inappropriate for introducing ½?
- Explain your answer based on the following perspective:
  - ❖ Students' previously learnt knowledge
  - ❖ Sizes of the parts
  - ❖ The shape of the 'whole'

Representation ①



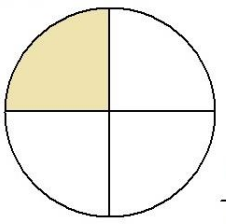
$\frac{1}{4}$

Representation ②



$\frac{1}{4}$

Representation ③



$\frac{1}{4}$

v. Diagram 4 shows three representations of ¼.

- If you want to discuss the concept of fraction with your students, how will you order the sequence of discussion?
- Justify your decision.

Diagram 4

vi. Various representations of fractions are presented to children in order to help them learn simple fraction at different complexity. Diagram 5 shows some representations of ¼.

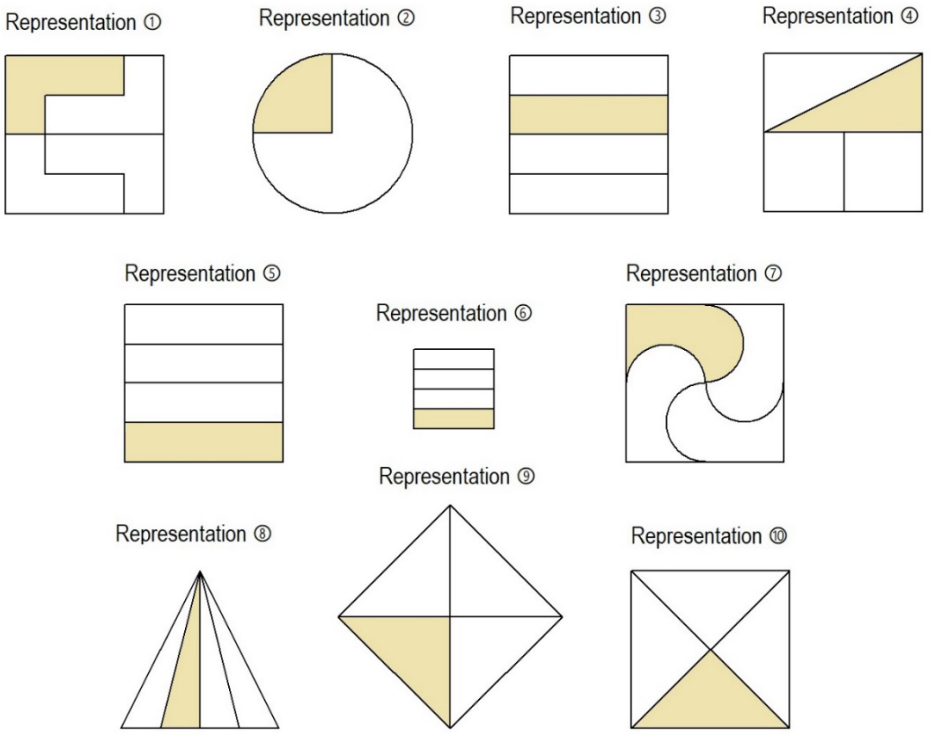



Diagram 5

- Which representations will you use for each of the following purposes?
  - ❖ Introducing simple fraction to your students.
  - ❖ Learning simple fraction by paper folding.
  - ❖ Promoting mathematical reasoning among your students.
- For each of the purposes, explain how you will use the representations to achieve it.

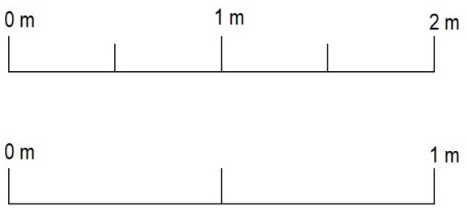



Diagram 6

vii. Diagram 6 shows two number lines with different scales. One number line represent a length of 2 metres, whereas the other number line represents a length of 1 metre.

- Use ↑ to mark the position of $\frac{1}{4}$ m on each of the two number lines.
- How do you explain the different position of the ↑?
- How will you use paper folding to help your students solve this problem?

## Task 2: Comparing Simple Fractions

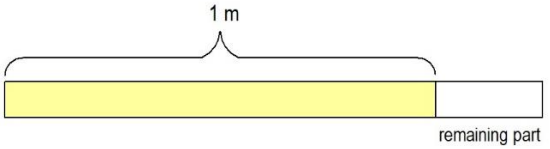



Diagram 7

i. A student used a tape to measure the length of a table. The measurement was 1 m and a remaining part as shown in Diagram 7.

In order to answer the question in Diagram 8, the student cut off the remaining part and used it as a unit to measure the length of 1 m. The result is shown in Diagram 8.

- How many metre is one remaining part?
- How many metre is two remaining parts?
- How many metre is three remaining parts?
- How could this task be used to compare fractions?
- Design another similar task for ½.

Question

"How many pieces of the remaining part are equal to 1 m?"

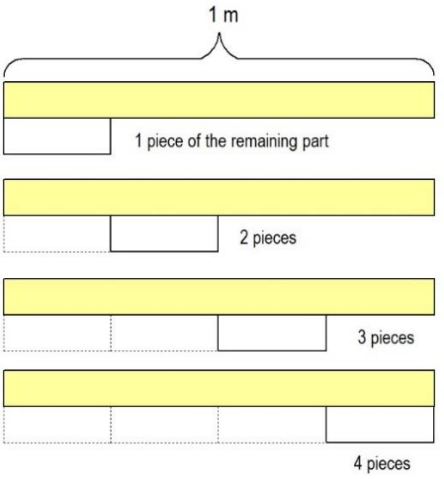



Diagram 8

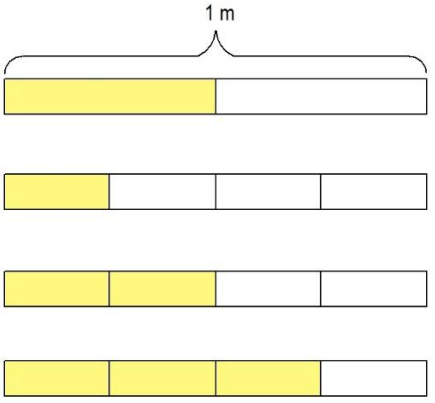



Diagram 9

ii. Diagram 9 shows four identical strips of paper. Each strip of paper is 1 m long. One strip is divided into two equal parts while the other three strips are divided into four equal parts. The strips of papers are then shaded as shown in Diagram 9.

- Explain how you would use these strips of paper to make the following comparisons.
  - ❖ Which is longer, $\frac{1}{2}$ m or $\frac{1}{4}$ m?
  - ❖ Which is longer, $\frac{2}{4}$ m or $\frac{3}{4}$ m?

How many sets of $\frac{1}{4}$ m are there in 1 m?

How many sets of $\frac{1}{4}$ m are there in $\frac{2}{4}$ m?

How many sets of $\frac{1}{4}$ are there in $\frac{3}{4}$ m?

So, which is longer, $\frac{2}{4}$ m or $\frac{3}{4}$ m?

Why?

Diagram 10

iii. Unit fraction is a fraction with numerator 1. Two examples of unit fraction are $\frac{1}{2}$ and $\frac{1}{4}$ which are commonly called one half and one quarter, respectively.

Diagram 10 shows a sequence of questions involving unit fractions designed by a teacher to help his students compare simple fractions.

- How will the use of unit fraction promote your students' ability to compare $\frac{1}{4}, \frac{2}{4}$ and $\frac{3}{4}$ ?

iv. Another teacher argues that it is not important to use the unit metre in asking the questions. It will be easier for students to answer the questions in terms of $\frac{1}{4}, \frac{2}{4}$ and $\frac{3}{4}$ directly.

- Do you agree with the teacher? Why or why not?

**Standards 8.2:**

Extending fractions using tape diagram and number line to one, and think about how to add or subtract similar fractions for producing simple algorithm

i Extend fractions to more than one unit quantity for representing the remaining part (unit fraction) such as measuring the length of tape, recognising the remaining parts as a unit measure of length, and understand proper and improper fractions

ii Appreciate fractions with quantities in two ways; firstly, whole is a unit of quantity and secondly, based on the number of unit fraction

iii Compare fractions in the case where the whole is the same and explain it with tape diagram or number line, and develop fraction number sense such as $\frac{1}{10}$ with quantities and so on

iv Think about how to add or subtract similar fractions with tape diagram or number line and produce simple algorithm with fluency

**Sample Tasks for Understanding the Standards**

**Task 1: Proper Fractions**

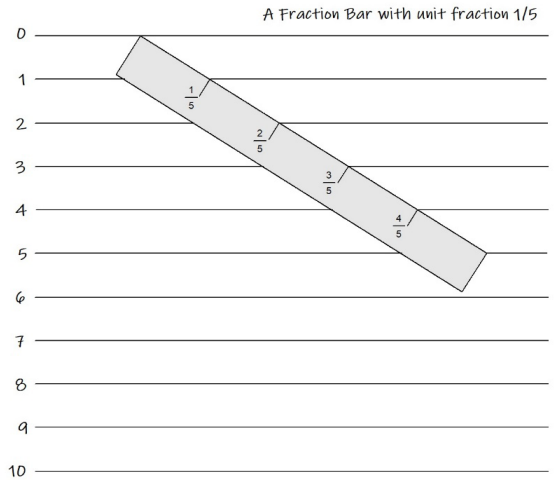



Diagram 1

| | | | | | |
|---|---|---|---|---|---|
| $\frac{1}{6}$ | | | | | |

Diagram 2

i. Paper folding can be easily used to represent ½ and ¼.

- What other unit fractions can also be represented easily using the same method?

ii. Unit fractions such as $\frac{1}{3}$ and $\frac{1}{5}$ are not easily represented by paper folding. Diagram 1 shows a method to represent $\frac{1}{5}$ on a fraction bar using parallel lines.

- Describe how the parallel lines in Material Sheet 1 at the end of this topic can be used to represent $\frac{1}{3}$, $\frac{1}{8}$, and $\frac{1}{10}$, respectively on a fraction bar.

iii. Diagram 2 shows a $\frac{1}{6}$ fraction bar.

- List all proper fractions that can be represented using this bar.
- Explain the meaning of proper fraction.

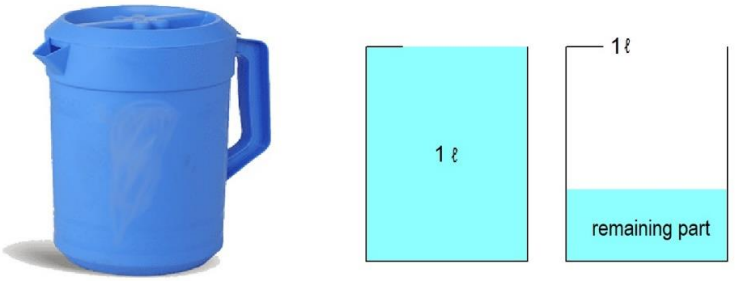



Diagram 3

iv. John poured a jug of water into two identical 1-litre containers. The amount of water is 1 ℓ and a remaining part as shown in Diagram 3.

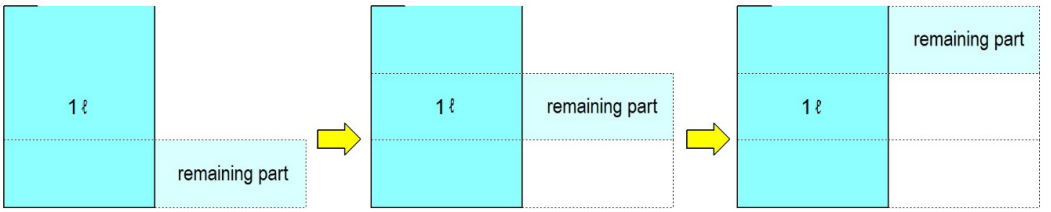



Diagram 4

v. He then compared the remaining part of water with the 1-litre container as shown in Diagram 4.

- How many remaining parts are there in 1 ℓ of water?
- How many litre is one remaining part?

vi. Fractions with quantities can be understood in two ways as shown in the following questions and answers:

Question ①: How many litre are two remaining parts?
Answer ①: 2 remaining parts are $\frac{2}{3}$ ℓ .

Question ②: How many litre are 2 sets of $\frac{1}{3}$ ℓ ?
Answer ②: 2 sets of $\frac{1}{3}$ ℓ are $\frac{2}{3}$ ℓ ".

- What are the similarities and differences between the two ways?
- Which way do you think is easier for your students to understand fractions?

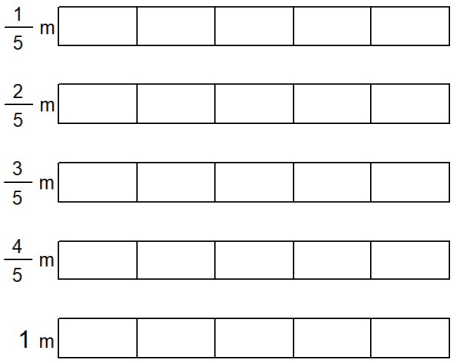



Diagram 5

vi. Diagram 5 shows five 1-metre tapes divided into 5 equal parts.

- How many $\frac{1}{5}$ m are there in
  - ❖ 1 m?
  - ❖ $\frac{3}{5}$ m?
- Compare the following fractions using Diagram 5.
  - ❖ Which is longer, $\frac{2}{5}$ m or $\frac{3}{5}$ m?
  - ❖ Which is longer, $\frac{4}{5}$ m or 1 m?
- How will you compare the fractions using unit fraction?

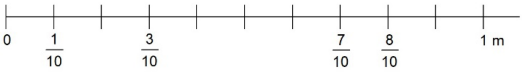



Diagram 6

vii. Diagram 6 shows a fraction number line.

- How many $\frac{1}{10}$ m are there in
  - ❖ 1 m?
  - ❖ $\frac{4}{10}$ m?
- Compare the following fractions using Diagram 6.
  - ❖ Which is more, $\frac{5}{10}$ m or $\frac{6}{10}$ m?
  - ❖ Which is more, $\frac{9}{10}$ m or $\frac{2}{10}$ m?
  - ❖ Which is more, $\frac{3}{10}$ m or $\frac{7}{10}$ m?

viii. What rules on comparing fractions do you expect your students to conclude from these tasks?

**Task 2: Introducing Improper Fractions**

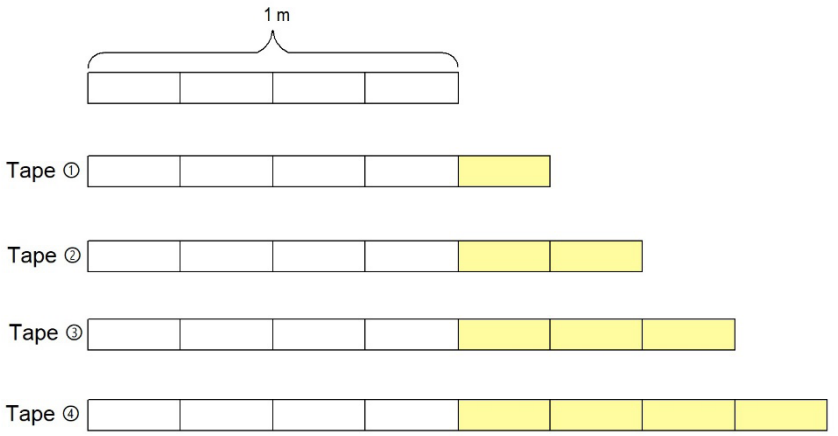



Diagram 7

i. Diagram 7 shows a 1-metre tape divided into 4 equal parts and four other tapes.

- How many metre is 1 part?
- How many metre is tapes ①, ②, ③ and ④, respectively?
  - ❖ Express each length as a fraction in metres.
- What is the difference between proper and improper fractions?
- Is $\frac{4}{4}$ proper or improper fraction? Explain your answer.

## Task 3: Addition and Subtraction of Similar Fractions

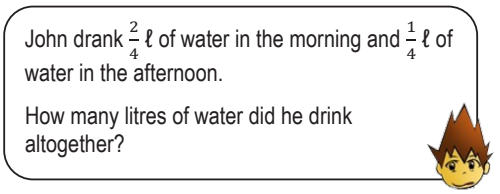



Diagram 8

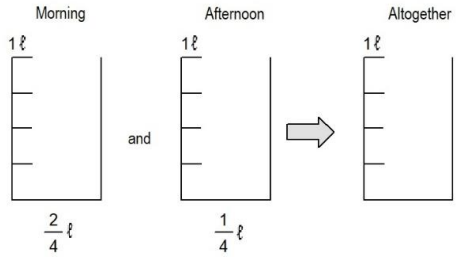



Diagram 9

Problem

A 1-metre tape was divided into 7 equal parts. John took 2 pieces and his brother took 3 pieces of those parts. How many metres were taken by John and his brother?

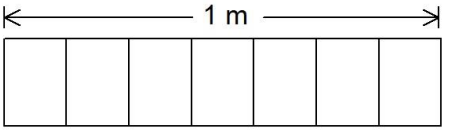



Diagram 10

Problem

A tumbler bottle has $\frac{7}{8}$ ℓ of water. John drinks $\frac{3}{8}$ ℓ of the water. How many litre of water is left in the bottle?

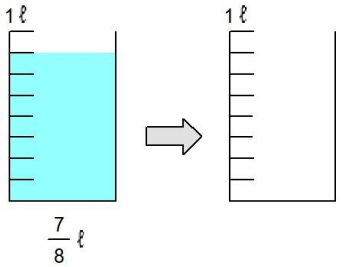



Diagram 11

i. Diagram 8 shows a problem involving amount of water drank by John.

Diagram 9 shows three identical 1-litre containers, each divided into four equal parts.

- Write an addition sentence to represent the problem situation in Diagram 8.
- Show how you will use Diagram 9 to solve the problem.
- A teacher used the following questions to help students solve the problem.

  "How many sets of $\frac{1}{4}$ ℓ are there in $\frac{2}{4}$ ℓ ?"

  "How many sets of $\frac{1}{4}$ ℓ are there altogether?"

  ❖ To what extend will these questions help your students understand addition of fractions?

ii. Diagram 10 shows a problem and a 1-metre tape divided into 7 equal parts.

- Write an addition sentence to represent the problem situation.
- Show how you will use Diagram 10 to solve the problem.

iii. Diagram 11 shows a problem and two identical 1-litre containers, each divided into 8 equal parts. One container is filled with $\frac{7}{8}$ ℓ of water.

- Show how you will use Diagram 11 to solve the problem.
- Write a subtraction sentence to represent the situation.
- Design a sequence of questions involving $\frac{1}{8}$ ℓ to help your students solve the problem.

Problem

From a $\frac{5}{6}$ m tape, $\frac{3}{6}$ m was cut off. How many metres are left?

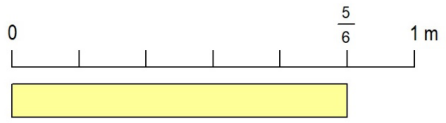
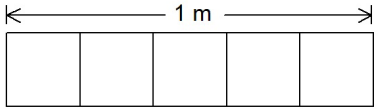



Diagram 12



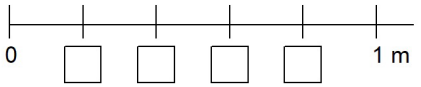




Diagram 13

i. Diagram 12 shows a problem and a $\frac{5}{6}$ m tape.

- Write a subtraction sentence to represent the problem situation.
- Show how you will use Diagram 12 to solve the problem.

ii. Diagram 13 shows a 1-metre tape divided into 5 equal parts and a number line.

- How many metre is one part of the tape?
- Fill in the missing fraction in each ☐ of the number line.
- Use the tape diagram or the number line to find
  - ❖ $\frac{1}{5}$ m + $\frac{2}{5}$ m
  - ❖ $\frac{4}{5}$ m − $\frac{3}{5}$ m
  - ❖ Between tape diagram and number line, which do you choose to do the calculations? Justify your choice.

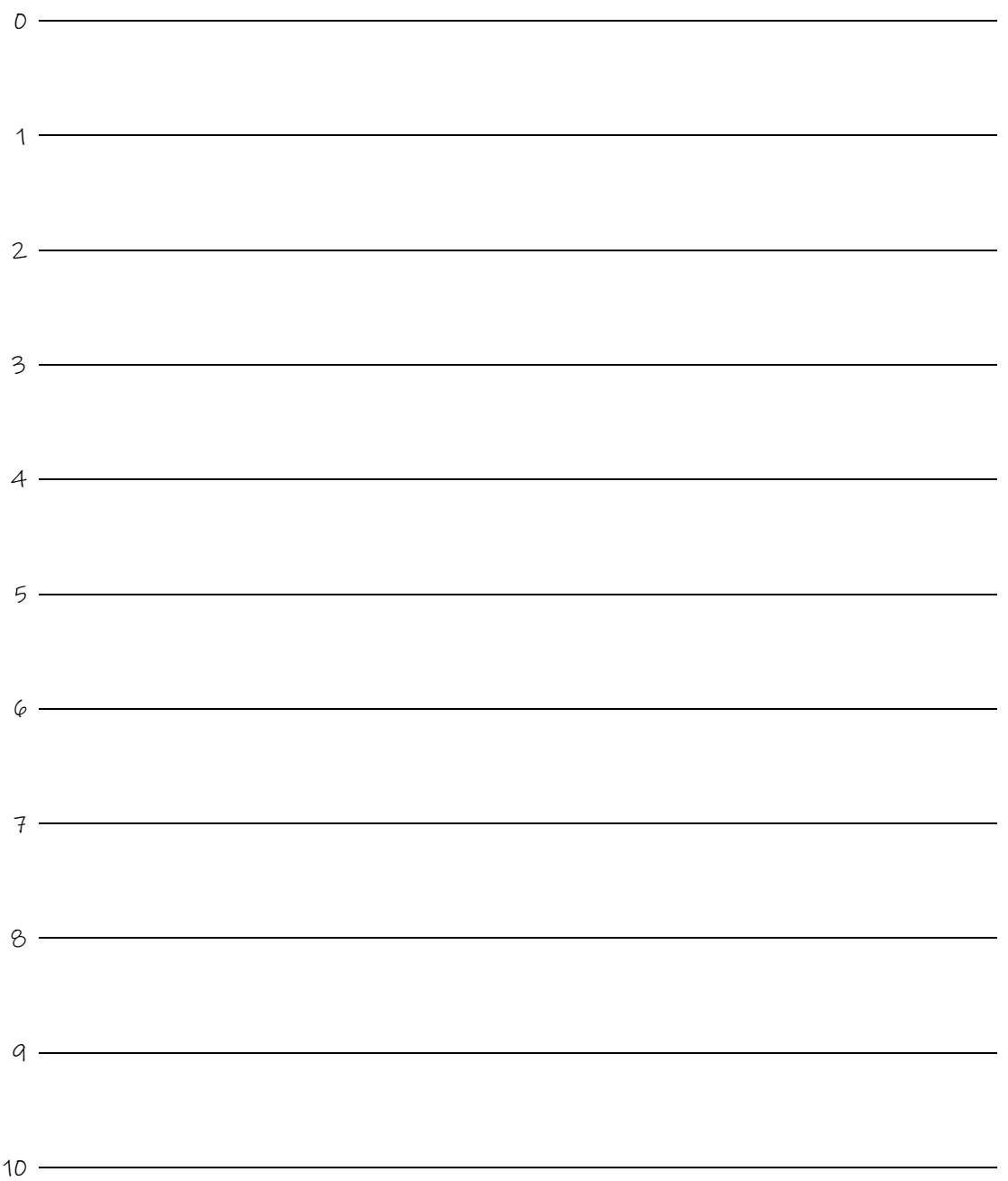
iii. Calculate

- $\frac{2}{5}+\frac{3}{5}$
- $\frac{5}{7}-\frac{2}{7}$
- How will you guide your students to do these calculations without using tape diagram or number line?

iv. Find as many solutions as possible for each of the following problems.

- $\frac{\square}{10}+\frac{\square}{10}=\frac{8}{10}$
- $\frac{\square}{9}-\frac{\square}{9}=\frac{3}{9}$
- $\frac{\square}{8}+\frac{2}{8}=\frac{\square}{8}$
- $\frac{5}{7}-\frac{\square}{7}=\frac{\square}{7}$
- How will these problems help your students to develop fluency in algorithm for addition and subtraction of fractions?

## Material Sheet 1: Equidistant Parallel Lines for Constructing Fraction Scales



## Topic 9: Introducing Decimals and Extending to Addition and Subtraction

### Standards 9.1:

Introducing decimals to tenths, and extend addition and subtraction into decimals

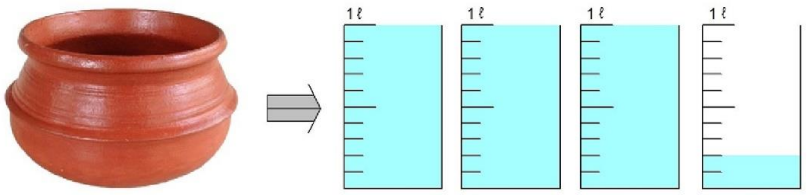
i. Introduce simple decimals to tenths by remaining part such as using tape diagram with appreciation
ii. Compare size of decimal numbers on a number line with the idea of place value
iii. Extend addition and subtraction of decimals utilising the place value system in vertical form until up to tenths
iv. Think about appropriate place value for applying addition and subtraction in life

### Sample Tasks for Understanding the Standards

### Task 1: Decimal Numbers

i. A task is used to help children learn the concept of 0.1 using measurement of water as shown in Diagram 1.

The amount of water in a clay jar is measured with 1-litre containers. Each container is divided into 10 equal parts. One equal part of the container will contain $\frac{1}{10}$ ℓ or 0.1 ℓ of water.




There are exactly 3 containers and a remaining part of water as shown in the diagram above.

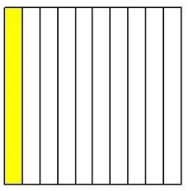
◊ How many sets of 0.1 ℓ of water are there in 1 ℓ of water?

◊ How many sets of 0.1 ℓ of water are there in the remaining part of water?

◊ What is the total amount of water in ℓ ?

Diagram 1

- In what way does the use of clay jar and measurement of water help children to concretise the concept of 0.1?



◊ What fraction of the square is shaded?
◊ How is the fraction related to 0.1?

Diagram 2

ii. Diagram 2 shows another task, also used to help children understand the meaning of 0.1. Compare this task with the task in Diagram 1.

- Which task do you think is more effective in helping your students understand the concept of 0.1?
- Justify your decision.

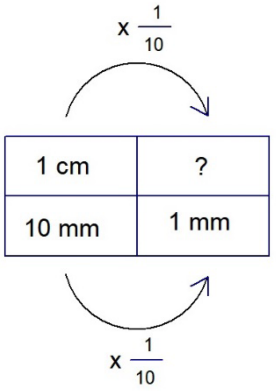



Diagram 3

iii. A teacher intends to introduce decimal numbers by using Diagram 3 to help students understand the relationship 1 mm = $\frac{1}{10}$ cm = 0.1 cm.

- How could the teacher explain the relationship?
- How could the discussion be extended to other decimal numbers such as 0.2 and 0.3.

iv. Another teacher intends to extend the learning of decimal numbers using a 1-metre tape. The 1-metre tape is divided into 10 equal parts as shown in Diagram 4.

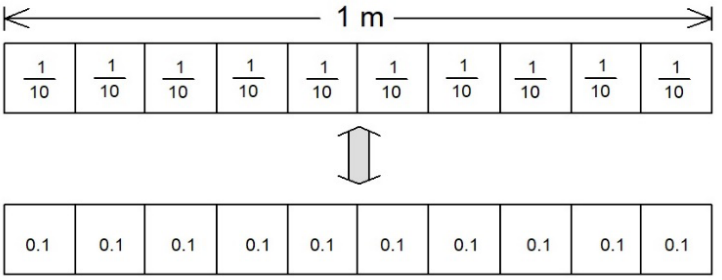



Diagram 4

- What numerical relationship is expected to be learnt through the diagram?
- To what extend do you think the diagram is effective in helping your students understand the relationship?
- Based on the diagram, a teacher asks the following questions:

  "How many m is one of the 10 parts?"
  "How many sets of 0.1 m are there in 1 m?"

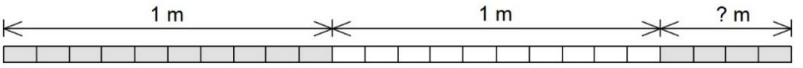
  ❖ What is the main purpose of asking these questions?

  ❖ In what way is the use of the terminology "set" useful in helping students conceptualise the relationship?

v. Diagram 5 shows two tasks involving decimal numbers.

- How will you order the sequence of these tasks?
- Justify your decision.

Task ①



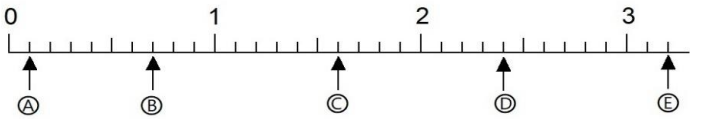

The diagram shows a tape of length more than 2 m.

Each metre of tape is divided into 10 equal parts.

◊ How many sets of 0.1 m are there altogether?

◊ What is the length of the tape in m?

Task ②




The diagram shows a decimal number line.

◊ Write down the decimal number that each ↑ is pointing at.

◊ How many sets of 0.1 are there in each of the decimal numbers expressed in Ⓐ to Ⓔ?

Diagram 5

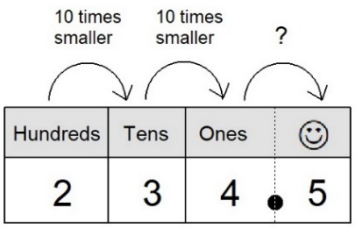



Diagram 6

vi. Diagram 6 shows a decimal number 234.5 written on a place-value chart.

- Explain how the place-value concept is extended from whole numbers to decimal numbers.
- Explain why the place value of ☺ is $\frac{1}{10}$.
- How will you guide your students to predict the next place value to the right of ☺?

vii. How will you sequence the following tasks based on difficulty levels?

Task ① : 234.5 is the sum of ____ sets of hundreds ____ sets of tens ____ sets of ones and ____ sets of tenths.

Task ② : 234.5 = 200 + ____ + 4 + ____

Task ③ : 2.8 is the sum of ____ sets of 0.1.

Task ④ : The sum of 12 sets of 0.1 is ____.

**Task 2: Addition and Subtraction of Decimal Numbers**

```
  25.3
+  14
------
  26.7
```

Diagram 7

i. Diagram 7 shows the work of a student when adding 25.3 and 14.

- What is the student's error?
- How will you help this student correct the error?

ii. Diagram 8 shows two 1-litre containers each divided into 10 equal parts. The two containers have 0.8 ℓ and 0.5 ℓ of water, respectively as shown in Diagram 7.

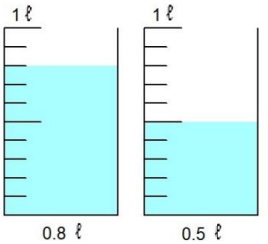



Diagram 8

- Explain how you will use Diagram 8 to find the answers for
  - ❖ 0.8 ℓ + 0.5 ℓ
  - ❖ 0.8 ℓ – 0.5 ℓ

iii. When teaching 0.8 + 0.5 in the vertical form, two teachers, A and B, used different questions to guide their students. Diagram 9 shows the two sets of questions.

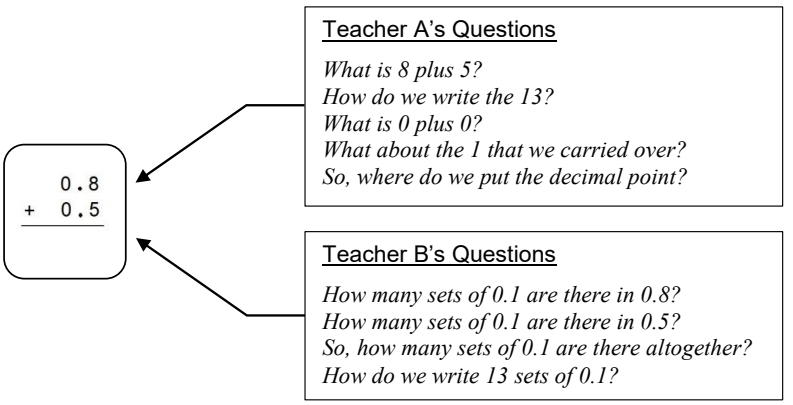



Diagram 9

- Which set of questions do you think will help your students develop a better understanding on addition of decimal numbers? Explain your reasons.
- Design a set of questions that you will use when guiding your students to do 0.8 – 0.5 in the vertical form.

iv. Diagram 10 shows a measuring tape and two tapes of length 0.8 m and 1.5 m, respectively.

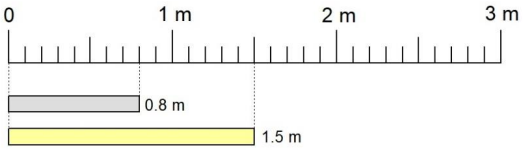



Diagram 10

- Show and explain how Diagram 10 can be used to find the answers for

❖ 1.5 m + 0.8 m

❖ 1.5 m – 0.8 m

v. A teacher was guiding a student to add 1.5 and 0.8. The following is the conversation between the teacher and the student.

| | | |
|---|---|---|
| Teacher | : | *"Try to think about how many sets of 0.1 are there. First, how many sets of 0.1 are there in 1.5?"* |
| Student | : | *"15 sets."* |
| Teacher | : | *"Then, how many sets of 0.1 are there in 0.8?"* |
| Student | : | *"8 sets."* |
| Teacher | : | *"So, how many sets of 0.1 are there altogether?"* |
| Student | : | *"Emm .. 15 sets and 8 sets ... 23 sets altogether?"* |
| Teacher | : | *"So, what number is 23 sets of 0.1?"* |
| Student | : | *"Oh! I got it ... 23 sets of 0.1 is 2.3. So, 1.5 plus 0.8 is 2.3!"* |

- Explain the student's thinking processes triggered by the teacher's questioning.

vi. Diagram 11 shows a set of questions designed by a teacher to guide students do 1.5 + 0.8 in vertical form. The questions are not arranged in the correct sequence.

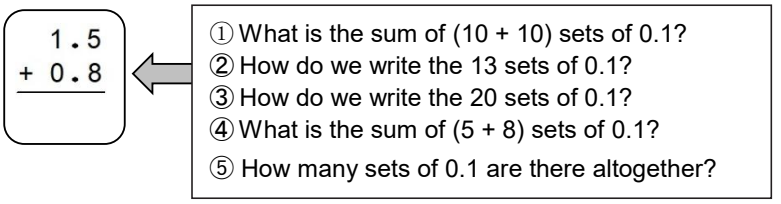



Diagram 11

- How will you sequence the questions?
- Design a set of questions to guide your students to do 1.5 – 0.8 in vertical form.

vii. Fractions and decimal numbers are different forms to represent parts of a whole. Two examples of such representations are $3\frac{1}{2} = 3.5$ and $2\frac{4}{5} = 2.8$.

- Perform and compare the following addition and subtraction.
  - ❖ $3\frac{1}{2} + 2\frac{4}{5}$ and $3.5 + 2.8$
  - ❖ $3\frac{1}{2} - 2\frac{4}{5}$ and $3.5 - 2.8$
- What is the advantage of decimal number as compare to simple fractions in doing computation?

# CHAPTER 3

# Quantity and Measurement

**Topic 1: Comparing Size Directly and Indirectly Using Appropriate Attributes and Non-standard Units**

**Standards 1.1:**

Comparing and describing quantity using appropriate expression

i. Compare two objects directly by attributes instead of stating in length and amount of water such as longer or shorter and less or more
ii. Compare two objects indirectly using non-standard units to appreciate the unification of units
iii. Use appropriate denomination of quantity (such as number of cups) for counting and appreciate the usage of units for quantity for suitable context

**Sample Tasks for Understanding the Standards**

**Task 1: Introduce Measurement Using Attributes**

***Direct Comparison***

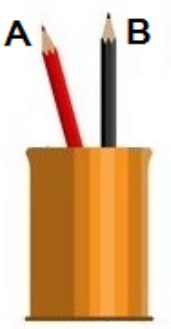



Diagram 1

i. A teacher showed pencil A and pencil B in a container as in Diagram 1 to her class. One student claimed that pencil B is longer than pencil A, but another student disagreed.

- What made them disagree?
- What should you do to help these two students settle their disagreemnet over the length of the pencils?


Diagram 2

ii. Two threads P and Q are placed side by side as in Diagram 2 and students are asked to compare the two lengths.

- What must the students first do in order to compare the two lengths?

  Explain your answer.

- Length can be stated in different forms such as height. State and explain other forms of length.

***Indirect Comparison***

Diagram 3

i. Diagram 3 shows a situation of comparing the height of a metal table and the breadth of a photo frame which is fixed on the wall.

- Why is direct comparison not suitable in this case?
- Without using a ruler or a measuring tape, how indirect comparison can be done in this case?

Diagram 4

ii. Diagram 4 shows a pen and a pencils of different lengths. Two students, Zaw Zaw and Khin, used two different methods to compare these two lengths as shown in Diagrams 5 and 6.

- Explain the similarities and differences of the two methods.
- Which of these methods will you recommend for your students? Explain your reasons.

Zaw's Method

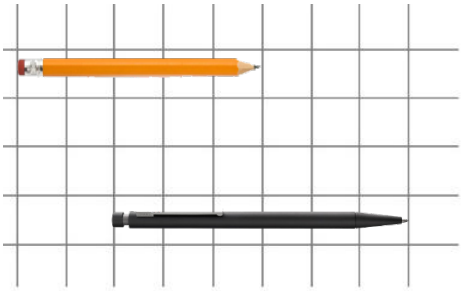


Diagram 5

iii. The paper clips used by Khin is an example of non-standard unit for length.

- Using these paper clips as units for measurement, what is the length of the pencil and the pen, respectively?
- Suggest two other non-standard units for length that you can use.
- What measuring aspects should you consider when choosing the non-standard unit?

Khin's Method

Diagram 6

Diagram 7

iv. The pen in Diagram 4 is measured using different paper clips as shown in Diagram 7.

- What is the length of the pen?
- How can this task help students appreciate the importance of standard units in measurement?

Diagram 8

v. Diagram 8 shows two cups with different shapes, but can hold same amount of water.

- What could you ask your students to do in order to convince them that both the cups hold the same amount of water?
- What conclusion can your students make from this task?


Diagram 9

vi. Diagram 9 shows two glasses with a same amount of orange juice and a scoop. However, most children will choose glass A thinking that it has more juice since its level of juice is higher.

- Design a task using the scoop to compare the amount of juice in the two glasses.
- Explain how the idea of denomination of quantity for counting is used in your task for comparing volume of the two glasses.

## Topic 2: Introducing Quantity of Length and Expanding It to Distance

### Standards 2.1:

Introducing centimetre for length and extend to millimetre and metre

i. Compare length of different objects and introduce centimetre with calibrated tape of one centimetre
ii. Demonstrate equivalent length with addition and subtraction such as part-part whole
iii. Extend centimetre to millimetre to represent remaining parts with ideas of equally dividing and idea of making tens
iv. Extend centimetre to metre to measure using metre stick
v. Estimate length of objects and select appropriate tools or measuring unit for measurement with fluency
vi. Convert mixed and common units of length for comparison
vii. Convert mixed and common units of length when adding or subtracting in acquiring the sense for quantity

### Sample Tasks for Understanding the Standards

### Task 1: Representing Length Using Various Units

***Comparing Length***

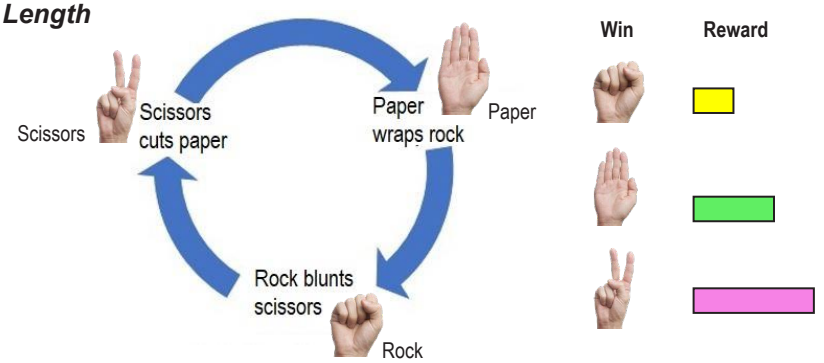



Diagram 1

i. Rock-paper-scissors is a game for two players. Diagram 1 shows the rules of the game. A win with the rock, the paper and the scissors will be awarded a one-unit, two-unit, and three-unit card respectively. The cards won by each player are arranged in front of each player on the opposite side of a same table. At the end of the game, the player with the longer card arrangement is the winner.

Ben and Maria played the game and the results are shown in Diagram 2.


Diagram 2

- Without moving the cards, explain how Ben and Maria could use the one-unit card to determine the winner.
- How will you relate the unit cards to the learning of centimetre?

***Using Calibrated Tape to Introduce Centimetre and Milimetre***

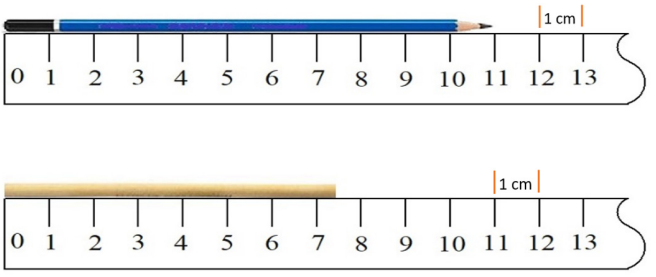



Diagram 3

i. Diagram 3 shows the scales on calibrated tapes which are marked 1 centimetre apart.

- Why is cm a better unit than other arbitrary units for measurement?
- The pencil is 11 cm in length, but the stick is a little longer than 7 cm. How can you use this situation to introduce millimetre as a remaining part of one centimetre?

ii. A teacher placed a ruler callibrated with mm beside the stick as shown in Diagram 4.

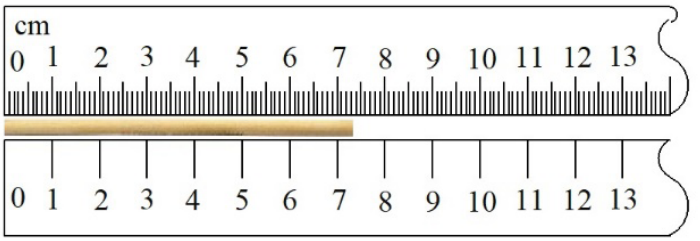



Diagram 4

- How many mm are there in 1 cm?
- The length of the stick is 7 cm and a remaining part. How many mm is the remaining part?
- What is the length of the stick in cm and mm?

***Extending Centimetre to Metre***

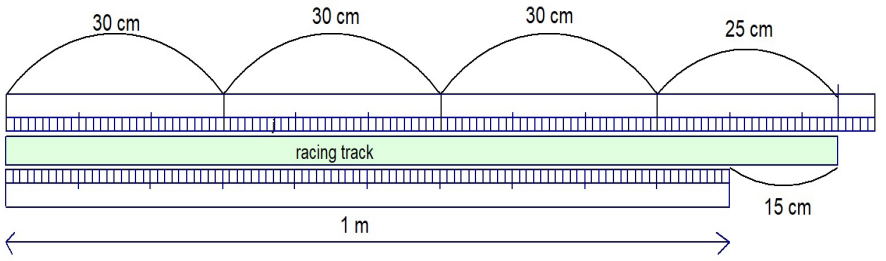



Diagram 5

i. Diagram 5 shows the length of a toy car racing track measured using a 30 cm ruler. The total length of the track can be expressed as:

30 cm + 30cm + 30 cm + 25 cm = 115 cm

- Why is a metre stick with a length 100 cm more appropriate to measure the track?
- How to explain the relationship between metre and centimetre?

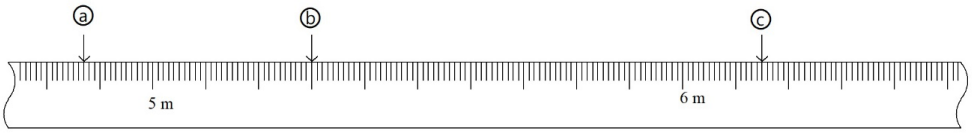



Diagram 6

ii. Diagram 6 shows part of a measuring tape. Three students gave their readings as follows:

| **Dave** | Mary | **Alan** |
|---|---|---|
| ⓐ is 4 m | ⓐ is 3 m 7 cm | ⓐ is 4 m 87 cm |
| ⓑ is 8 m | ⓑ is 8 m | ⓑ is 5 m 30 cm |
| ⓒ is 7 m | ⓒ is 7 m 5 cm | ⓒ is 6 m 15 cm |

- Which sets of readings are wrong?
- What are the possible difficulties faced by the students who made the wrong readings?
- How would you help the students overcome the difficulties in reading of measuring scale?

Diagram 7

iii. Why is a metre stick not suitable for measuring the size of the tree trunk in Diagram 7?

- Explain briefly how you could measure the size of the trunk with a suitable measuring tool.
- What units will you use? Justify your choice.
- Which of the following lengths could be acceptable estimates for the size of the tree trunk?

  □ 30 to 50 mm
  □ 30 to 50 cm
  □ 2 to 3 m
  □ 2 to 3 km

Justify your answers.

**Task 2: Conversion of Units for Length**

***Adding and Subtracting Length***

Diagram 8

i. Diagram 8 shows a white board with measurement given by two students, Ary and Mimi where the length is 325 cm and the width is 1 m 40 cm.

Ary and Mimi found different answers when adding the total length of the four sides of the board as shown in Diagram 9.

- Explain the differences between the two ways of calculating the answers.
- Which answer is correct?
- How will you help the student with the wrong answer?

| **Ary** | **Mimi** |
|---|---|
| 325 cm | 325 cm |
| 325 cm | 325 cm |
| 1 m 40 cm | 140 cm |
| + 1 m 40 cm | + 140 cm |
| 652 m 80 cm | 930 cm |

Diagram 9

ii. How will you guide students in the following calculations involving m, cm and mm?

- 7 cm 2 mm + 5 cm
- 3 m 20 cm + 2 m
- 23 cm 8 mm – 8 cm
- 8 m 25 cm – 5 m 20 cm

**Standards 2.2:**

Introducing distance for the extension of length

i. Introduce kilometre to measure distance travelled using various tools and appreciate the experiences of measuring skills
ii. Distinguish distance travelled and the distance of two places on the map
iii. Compare mixed units of length with appropriate scale on number line

**Sample Tasks for Understanding the Standards**

**Task 1: Visualising a Distance of 1 Kilometre**

***Measuring Distance with km***

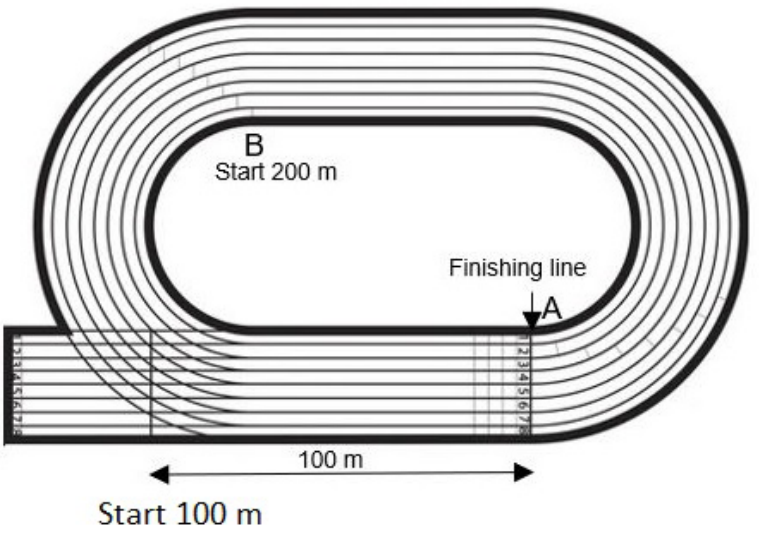



Diagram 1

i. Diagram 1 shows a standard 400-metre with 8 running tracks in a school field.

- Which track will exactly give a distance of 400 m in a complete round?
- How do you use the running track to help students visualise and understand a distance of 1 km? Explain your answer.

ii. A group of children is going for a 1000-metre race.

- How many rounds should they run?
- Which track the children need to take for their advantage? Explain your answer.
- To what extend can this task help children understand the relationship between km and m?

**Task 2: Distance Travelled and Distance Between Two Places on the Map**

***Adding and Subtracting Length***

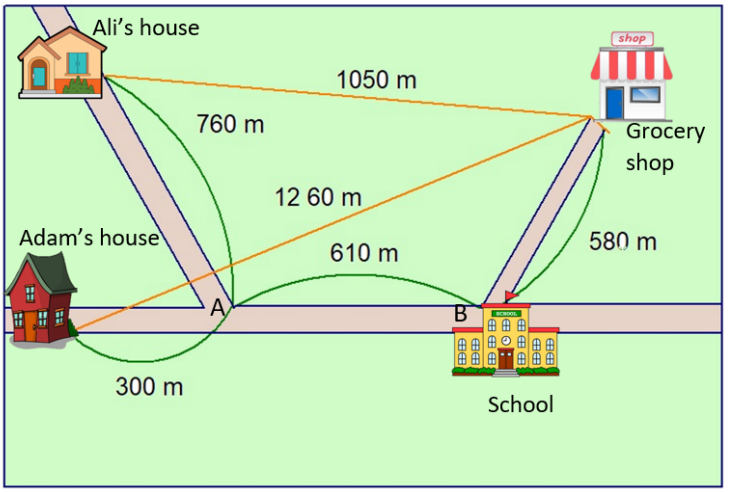



Diagram 2

i. Diagram 2 shows a location where Ali and Adam live.

Ali has to travel by car through junction A, junction B and to reach the grocery shop.

- What is the distance travelled from Ali's house to the grocery shop?
- If Ali walks through a straight path from his house to the grocery shop, what is the distance he walked?

ii. What is the distance travelled by Adam from his house to the school?

- If Adam walks through a straight path from his house to the school, what is the distance he walked?

iii. How can you use this task to help your students learn the relationship between kilometre and metre?

**Task 3: Compare Mixed Units of Length**

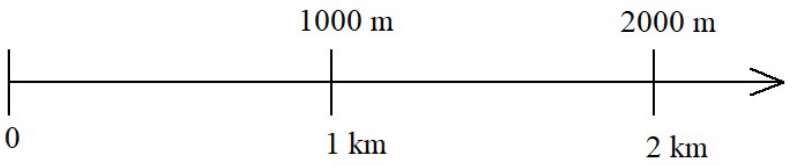



Diagram 3

i. Diagram 3 shows a number line marked with km and m. Locate and mark the following distance on the diagram

- 865 m
- 1280 m
- 1 km 50 m
- 1 km 500 m

ii. Draw a number line to compare the following pairs of distance.

- Which is longer, 3 km 60 m or 3080 m?
- Which is longer, 8 km 6 m or 7 km 920 m?
- Which is longer, 2580 m or 2 km 58 m?

## Topic 3: Introducing Quantity of Mass for Its Measurement and Operation

### Standards 3.1:

Introducing gram for mass and extend to kilogram and tons

i. Compare mass of different objects directly using balance and introduce gram
ii. Demonstrate equivalent mass with addition and subtraction such as part-part whole
iii. Extend gram to kilogram, measure with weighing scale
iv. Extend kilogram to metric ton through relative measure (such as 25 children, each weigh 40 kilogram)
v. Estimate mass of objects and select appropriate tools or measuring unit for measurement with fluency
vi. Convert mixed and common units of mass for comparison
vii. Convert mixed and common units of mass for addition and subtraction in acquiring the sense for quantity

### Sample Tasks for Understanding the Standards

### Task 1: Comparing Mass of Objects

***Which object is the heaviest?***

Diagram 1

i. Teacher Lim displayed a whiteboard eraser, a small piece of rock, a paper puncher and a sponge block on a table as shown in Diagram 1. He than asked his students to guess which was the heaviest object. Some students said the sponge block was the heaviest.

- Explain why the answer could be wrong.
- Without using any balance, what will you do to help the students correct their wrong perception, if any?

ii. Diagram 2 show a simple balance used for comparing mass. At any one time, the balance can compare the mass of two objects.

Diagram 2

- What is the minimum number of times you need to compare the four objects for ordering them from the heaviest to the lightest object?

  Explain your answer with diagrams.

Diagram 3

iii. Diagram 3 shows some stacks of Malaysian coin.

- How could you use these coins with a simple balance to order the four objects in Diagram 1?
- What need to be considered when choosing the types of coins in the comparisons?
- How will you use this situation to help your students learn measuring mass with non-standard units?

iv. Diagram 4 show two faces of a Japanese 1-yen coin. Interestingly, one piece of 1-yen coin weighs 1 gram (g).

Diagram 4

Diagram 5

- As shown in Diagram 5, the eraser is balanced by 24 pieces of 1-yen coin.
  - ❖ How many g is the whiteboard eraser? Explain your answer.
  - ❖ If the number of coins is increase to 48 pieces, predict what will happen to the left pan of the balance?
  - ❖ What will happen to the balance if another eraser is added to the left pan?
- Find any small item that weighs 1 g.
  - ❖ Design a task for your students to measure mass of objects in g using the item.

**Task 2: Addition and Subtraction Involving Mass**


Diagram 6

i. Diagram 6 show two situations with a balance scale of 1 kilogramme mass capacity.

- Some students recorded the mass of oranges as 850 g.
  - ❖ What is the error?
- How will you help the students to correct the error?
- A 90 g apple is added to the basket with oranges.
  - ❖ What is the new reading on the balance scale?

## Task 3: Extend Units of Mass to Larger Units

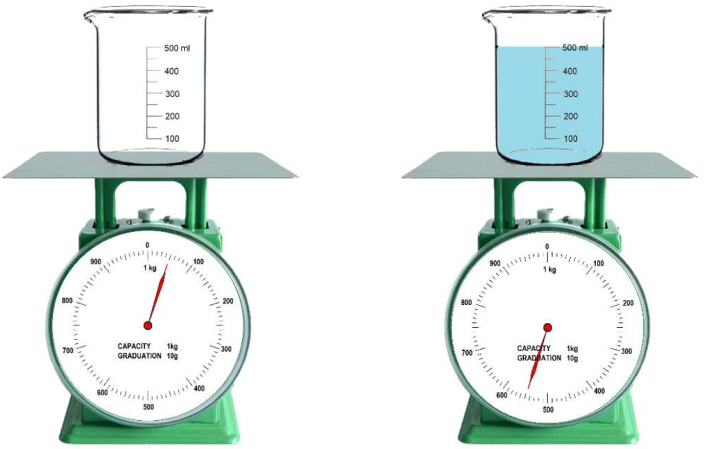



Diagram 7

i. Diagram 7 shows two situations involving a beaker on a 1-kg weighing scale.

- What is the mass of 500 ml of water?
- What is the mass of 1000 ml of water?
- How will you use this situation to introduce kilogramme to your students?

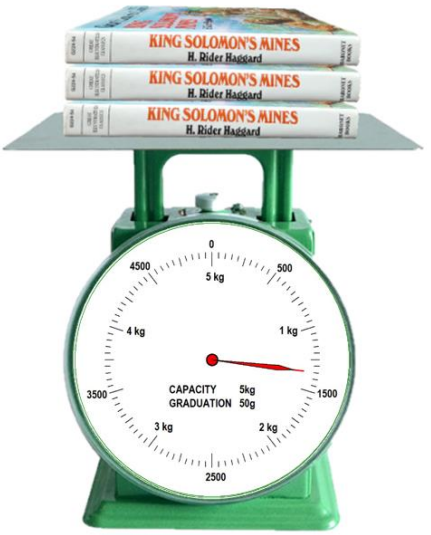



Diagram 8

ii. Diagram 8 shows the reading for the mass of three identical books.

- What is the mass of each book?

iii. A teacher asked his students to find the mass of their respective school bag using a weighing scale. However, the students are asked to estimate the mass of their bags before the actual weighting.

- How does making an estimation before actual weighing help in developing a sense of quantity among your students?

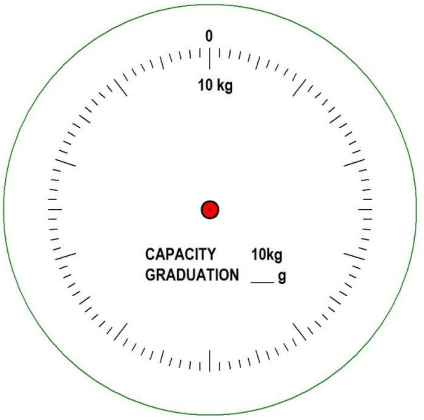



Diagram 9

iv. Diagram 9 shows another 10-kg weight scale. Label the appropriate graduation marks on the scale.

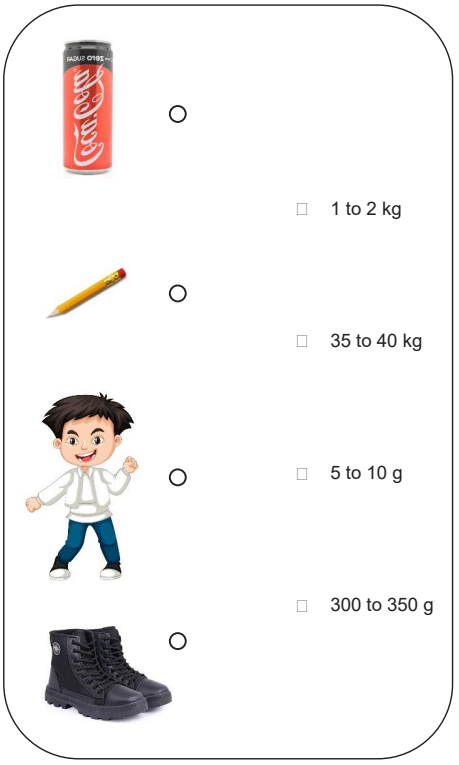



Diagram 10

v. Diagram 10 shows a pencil, a can of drink, a pair of boots, a boy and a list of mass.

- Match each item with its acceptable estimated mass.
- How does this task help in developing a sense of kg among your students?

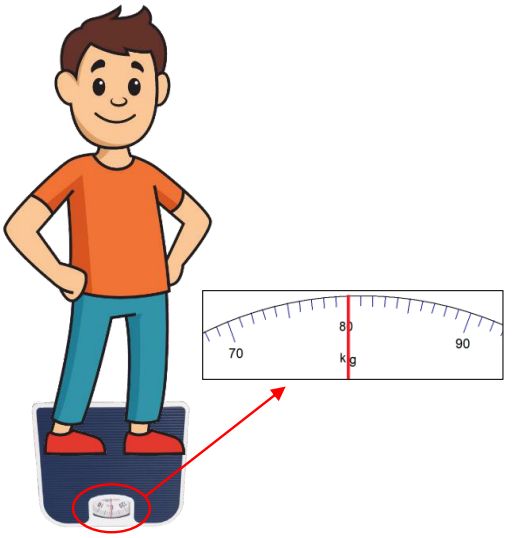



Diagram 11

vi. Diagram 11 shows a man standing on a bathroom scale. The reading shows 80 kg.

- What is the mass of 10 men?
- Approximately, how many men will weigh 1000 kg?
- A bus can take 42 passengers. Approximately, what is the maximum load of the bus in kg?
- How will you use these situations to introduce metric tonne?
- How does this task help in developing a sense of metric tonne among your students?


Diagram 12

vii. In Malaysia, the capacity of a lorry is measured based on the goods it carries and expressed in metric tonne.

- What does it mean to say “a 2 -metric tonne lorry” ?
- Diagram 12 shows a lorry with the following information:

| |
|---|
| BDM 2500KG<br>(Permisible Laden Weight)<br>BTM 1810KG<br>(Curb weight=actual weight of lorry) |

❖ What is the actual mass of the lorry?

❖ How many kg of goods can the lorry carry?

**Task 4: Conversion of Units**

***Comparing Mass***

i. Which object is heavier? Explain your answers.

- One kilogram of cotton or one kilogram of iron?
- 1 kg 100 g of iron or 1200 g of cotton?
- 1 kg 20 g iron or 1100 g cotton?


Diagram 13

ii. Diagram 13 shows a number line marked with kg and g. Locate and mark the following mass.

- 736 g
- 2480 g
- 1 kg 60 g
- 2 kg 600 g

iii. Draw a number line to compare the following pairs of mass.

- Which is lighter, 5 kg 20 g or 4580 g?
- Which is heavier, 9 kg 15 g or 8 kg 120 g?

***Addition and Subtraction Involving Mass***

- 350 kg 80 g + 85 kg
- 2 tonne 245 kg – 786 kg

Diagram 14

Diagram 14 shows two calculations involving units of mass.

i. What are the common errors of your students?

ii. How will you help your students to correct the errors?

## Topic 4: Introducing Quantity of Liquid Capacity for its Measurement and Operation

### Standards 4.1:

Introducing litre for capacity of liquid and extend to millilitre

i. Compare amount of water in different containers and introduce litre with measuring cups of 1 litre
ii. Demonstrate equivalent capacity with addition and subtraction such as part-part whole
iii. Extend litre by decilitre/100 millilitre cup for representing remaining parts with ideas of equally dividing and making 10, and extend until millilitre
iv. Estimate capacity of containers and select appropriate measuring unit
v. Convert mixed and common units of capacity for comparison
vi. Convert mixed and common units of capacity for addition and subtraction in acquiring the sense for quantity

### Sample Tasks for Understanding the Standards

### Task 1: Measuring Amount of Liquid in Litre and Decilitre


Diagram 1

i. Diagram 1 shows Jason and John, each with a bottle of orange juice. Over the phone, Jason and John are discussing which of their bottle can hold more juice.

- Can Jason claims that his bottle can hold more juice? Why or why not?
- How can you compare the amount of juice accurately?


Diagram 2



Diagram 3

ii. Diagram 2 shows a big container that can hold 1 litre of water. The water in one big container can fill 10 small containers. One small container is 1 decilitre.

- What is the relationship between litre and decilitres?

iii. Jason and John use these containers to measure the amount of juice in their respective bottle. The results are shown in Diagram 3.

- What is the amount of orange juice in Jason's bottle?
- What is the amount of orange juice in John's bottle?

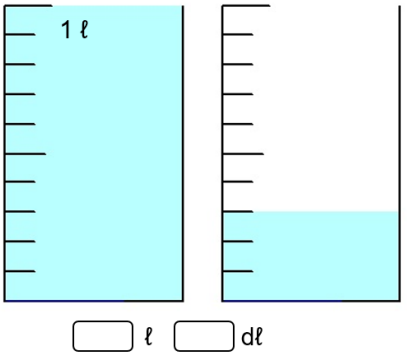



Diagram 4

iv. Diagram 4 shows a task given as an exercise for students.

- What is the purpose of the task?

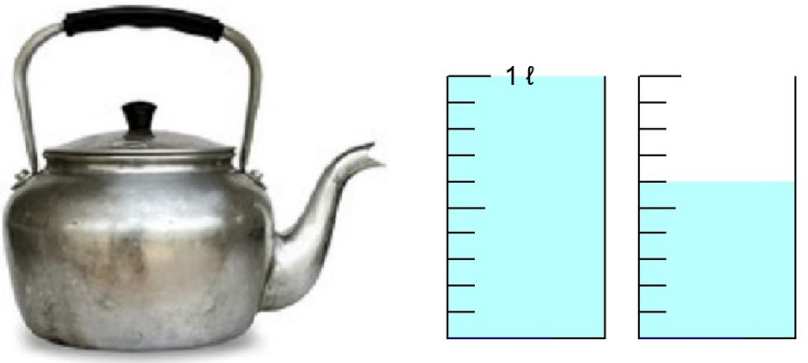



Diagram 5

v. Diagram 5 shows the amount of water in a kettle measured using 1-litre cups.

- What is the amount of water in the kettle?

**Task 2: Measuring Amount of Water in Millilitre**

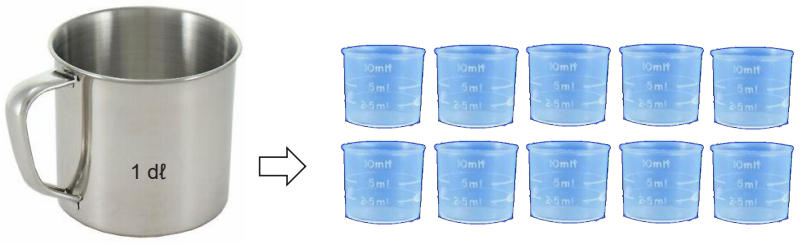



Diagram 6

i. Diagram 6 shows a one-decilitre container and a ten-millilitre medicine cup. Water in the one-decilitre container can fill 10 ten-millilitre medicine cups.

- What is the relationship between decilitre and millilitre?
- What is the relationship between litre and millilitre?


Diagram 7

ii. Water from a jug is poured into these containers. The result is shown in Diagram 7.

- What is the amount of water in the jug?

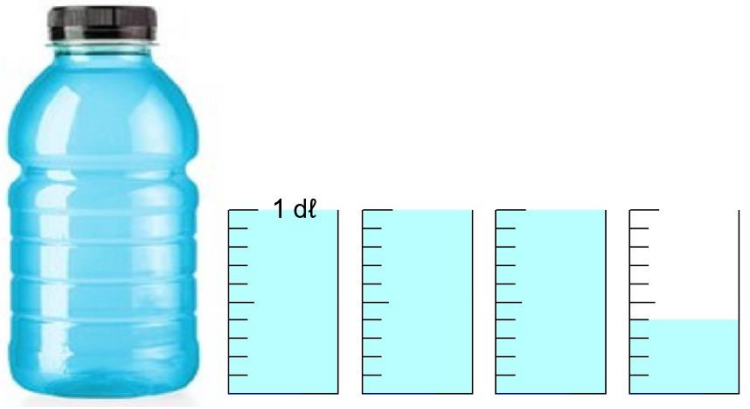



Diagram 8

iii. Diagram 8 shows the amount of water in a bottle measured using 1-decilitre cups.

- What is the amount of water in the bottle?


500 mℓ 200 mℓ

Diagram 9

iv. Many types of drinks are packed and measured in mililitre. For examples, a bottle of mineral water is 500 mℓ and a package of orange juice is 200 mℓ as shown in Diagram 9.

- How will you use these items to help students estimate volume of some common containers such as
- a drinking cup,
- a bowl, and
- a bottle of fruit juice?

Diagram 10

v. Diagram 10 shows a teaspoon and a tablespoon.

- Why are these spoons usually used to measure liquid cooking ingredients in daily life?
- Is there a standard capacity for each of these spoons?

**Task 3: Conversion of Units**

***Comparing Liquid Capacity***

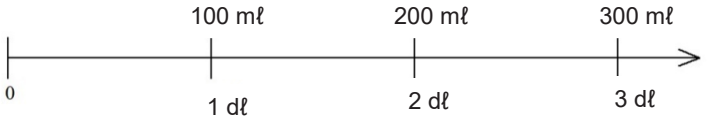



Diagram 11

i. Diagram 11 shows a number line marked with decilitre and millilitre. Locate and mark the following liquid capacity.

- 35 mℓ
- 280 mℓ
- 1 dℓ 50 mℓ
- 2 dℓ 8 mℓ

ii. Draw a number line to compare the following pairs of liquid capacity.

- Which is more, 7 ℓ or 72 dℓ?
- Which is more, 5 dℓ 28 mℓ or 485 mℓ?
- Which is more, 2 ℓ 40 mℓ or 380 mℓ?

iii. A centilitre, cℓ, is $\frac{1}{100}$ of a litre.

- How many mℓ is 1 cℓ?
- How many cℓ is 1 dℓ?

***Addition and Subtraction Involving Liquid Capacity***

There are 7 ℓ of water in a bottle.
2 *dℓ* has been drank.
How much is left?

Diagram 12

i. Diagram 12 shows a problem involving liquid capacity. A student gave the answer as 5 ℓ.

- Why is there an error?
- How will you help the student overcome the error?

ii. Based on your explanation in (i), how will you help students who give 8 dℓ as the answer for (7 dℓ + 1 ℓ)?

iii. In what ways are measuring of length, mass and liquid capacity the same?

## Topic 5: Introducing Time and Duration and Its Operation

### Standards 5.1:

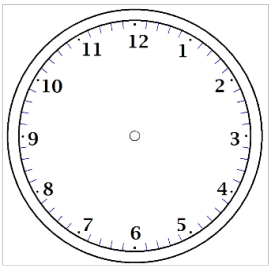
Introducing analogue time and extend it to duration

i. Tell and write analogue time of the day corresponding with different activities in daily life such as morning, noon, afternoon, day and night
ii. Show time by using clock face with hour hand and minute hand
iii. Understand the relative movement of clock hands

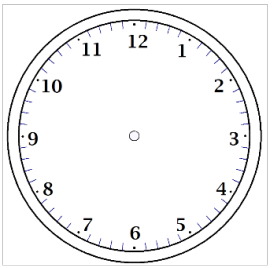
### Sample Tasks for Understanding the Standards

### Task 1: Telling and Writing Time

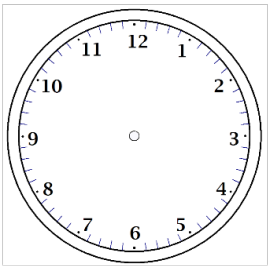

① Going to school



② Going home from school



③ Dinner time



④ Bedtime

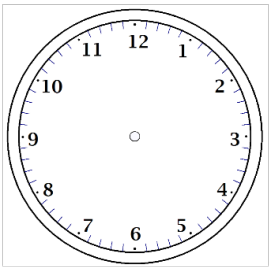


Diagram 1

i. Diagram 1 shows four activities in a day.

- Draw the hour and minute hands to show your time to do each activity.
- How does this task help your students to understand time?
- Discuss the common difficulties faced by students in showing and telling time.

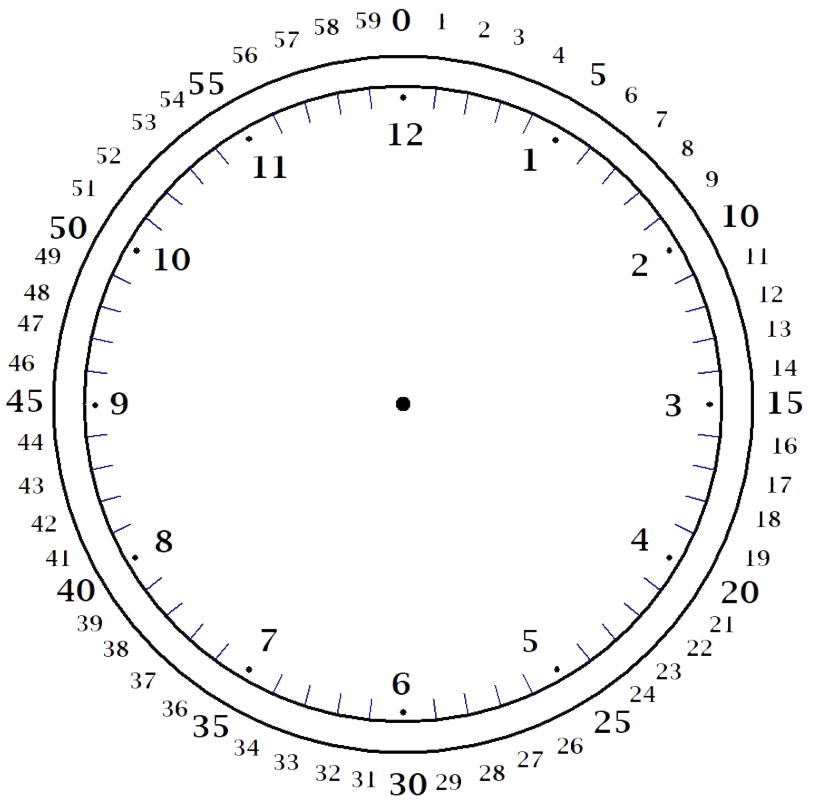



Diagram 2

ii. Diagram 2 shows a clock face marked with hours and minutes.

- How will you use this clock face to help your students understand and tell time?
- How do you explain the relative movement of the minute hand and the hour hand?
- How many rounds will the hour hand travel if the minute hand travels one complete round?
- How many rounds will the minute hand travel if the hour hand travel one complete round?

**Standards 5.2:**

Extending clock time to duration of one day

i. Introduce duration in hours and minutes based on beginning time and end time of activities
ii. Express time and duration on time line, and understand duration as difference of two distinguished times
iii. Addition and subtraction of duration and time
iv. Extend time and duration to seconds
v. Convert mixed and common units of duration for comparison
vi. Estimate duration of time and select appropriate measuring unit for measurement with fluency and appreciate the significance of time and duration in life
vii. Appreciate the difference in time depending on the area (time zone) and the seasons

**Sample Tasks for Understanding the Standards**

**Task 1: A Sense of Hour and Minute**

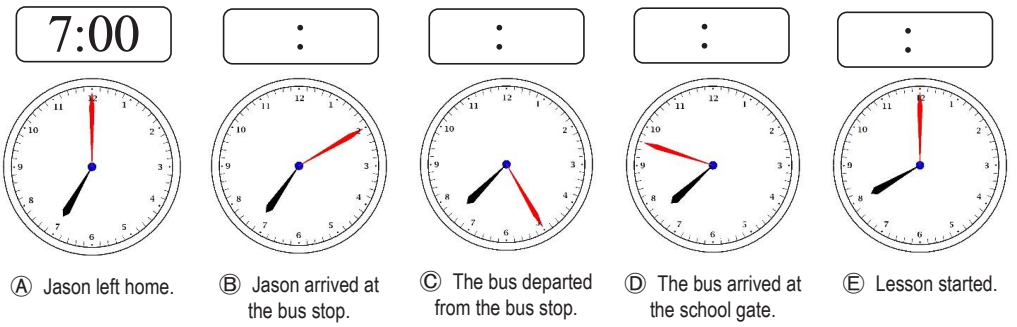



Diagram 1

i. Diagram 1 shows the times for Jason to travel from home to school.

- Read and write the time for each activity.
- How many minutes was the bus journey?

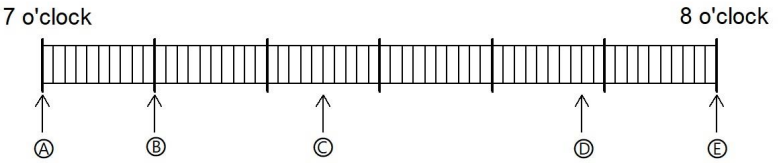



Diagram 2

ii. Diagram 2 shows a time scale between 7 o'clock to 8 o'clock divided into 60 intervals.

- What is the duration for 1 interval?
- Mark the duration of the bus journey on Diagram 2.
- Some students faced difficulties in finding the duration of the bus journey. How will you use Diagram 2 to help these students overcome the difficulties?

**Task 2: One Day in Hours**

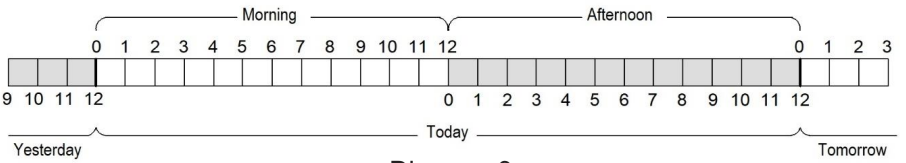



Diagram 3

i. Diagram 3 shows time band where 1 day is divided into 24 intervals.

- What is the duration for 1 interval?
- John went for a field trip. The trip started at 7 o'clock in the morning and ended at 3 o'clock in the afternoon. Mark the duration of the trip on Diagram 3.
- What is the duration of the trip in hours?

ii. How many complete rounds will each of the clock hands moves in a day?

iii. Ali says he will visit his best friend, Ahmad at 8 o'clock tomorrow.

- Why is the stated time confusing?
- How to get rid of the confusion?

**Task 3: Time Duration in Seconds**

i. Stand on one foot with your eyes closed.

- How long can you balance yourself?

ii. Hold your breath.

- How long can you hold?

iii. Some examples of units for measuring time duration are day, hour, minute and second.

- What unit is appropriate to measure the duration in (i) and (ii)? Explain your answer.

Table 1
Running Time

| Student | Time taken to run 400 m |
|---|---|
| Mira | 1 minute 50 seconds |
| Joe | 2 minutes |
| Judy | 65 seconds |

iv. Three students competed in a 400 m race. The result is recorded in Table 1.

- Who ran the fastest?
- What is the difference in time between the fastest and the slowest student?
- Another student reached the finishing line 37 seconds after Judy.
  - ❖ What is the time taken by this student to complete the run?

**Task 4: Estimation Involving Time**

Camping ○

Sweeping the floor ○

Gardening ○


Running a 200 m ○

☐ 40 s

☐ 1 hour

☐ 5 minutes

☐ 2 days

Diagram 4

i. Diagram 4 shows four sets of activities and time durations.

- Match each activity with the appropriate duration.
- Determine an appropriate tool for measuring the time for each activity.

**Task 5: Additon and Subtraction Involving Duration of Time**

i. Adam's father sent him and his friends to the bus station to take a bus to the Youth Park. The bus left the station at 7:30 in the morning and it took 80 minutes to reach the Youth Park.

- What time did Adam and his friends arrive at the Youth Park?
- If Adam's father drove 20 minutes to the bus station from his home and arrived 5 minutes before the bus departed, what time did they start from home?

ii. A group of students left the school at 9:25 in the morning and arrived at the National Monument at 3:10 in the afternoon. Three students recorded the duration of the journey as follows:

Student A: 12 hours 35 minutes

Student B: 6 hours 15 minutes

Student C: 5 hours 45 minutes

- Which student recorded the correct answer?
- How many different ways can be used to solve the problem?
- Which of the ways do you think is easier for your students to learn? Why?

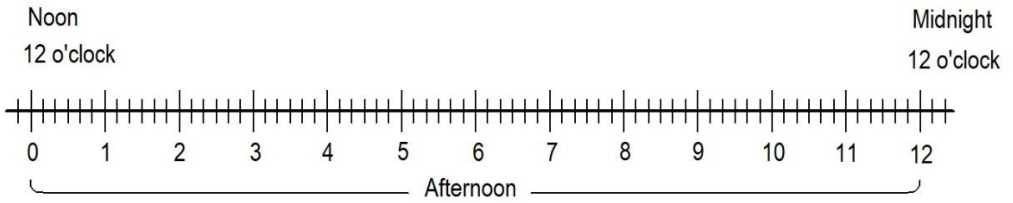



Diagram 5

iii. Diagram 5 shows a time line from 12:00 noon to 12:00 midnight.

- Why is each hour on the time line divided into 6 intervals?
- Mark the following times with ↓ on the time line.
  - ❖ 9 o'clock in the evening
  - ❖ 10 minutes past 2 in the afternoon
  - ❖ 20 minutes to 6 in the evening
  - ❖ Half past 10 in the evening

- Use the time line to help you solve the following problem.

  Over the phone, Kapono told his best friend what he did on new year day.

  "*After lunch, I went to the cinema for a show. Two hours later, I returned home. While at home, I had a shower for 10 minutes. Then I did my school work. It took me 80 minutes to complete the work. Then I chat with my father and sisters for 40 minutes. After that, I spent another 1 hour to read my favourite story book. Then, I played chess with my younger brother for 50 minutes. At that time, my mother called us for dinner, I looked at my watch, and the time was exactly 7 o'clock in the evening*."

  ❖ What time did Kapono start doing his school work?

  ❖ What time did Kapono start playing chess?

  ❖ What time did Kapono finish his lunch?

**Task 6: Different Time Zone**

i. Sirichai lives in Bangkok and his friend Kenji lives in Tokyo. However, Sirichai does not understand why Kenji is always in bed when he calls at 10.00 o'clock at night.

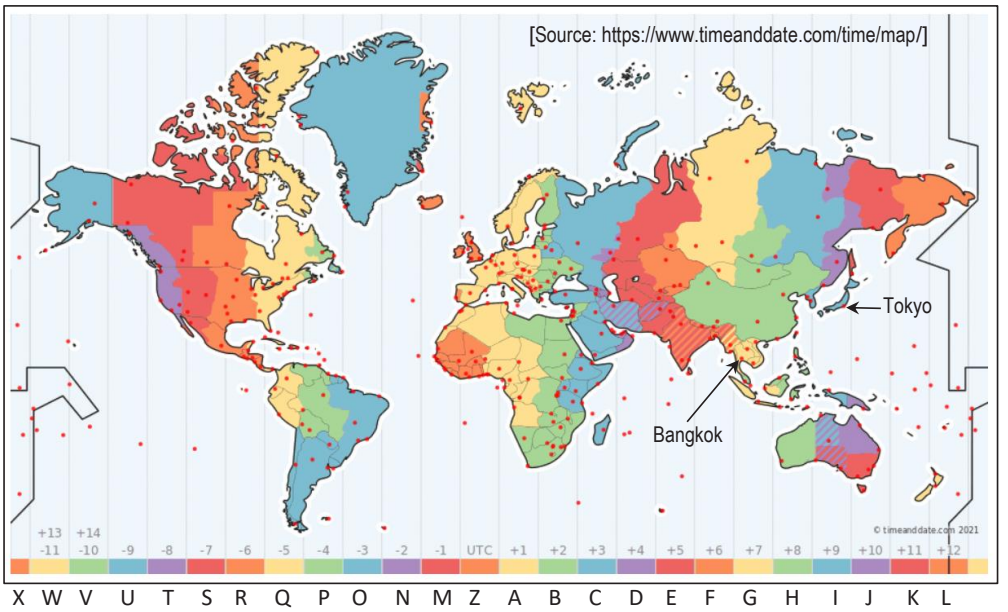



Diagram 6

Diagram 6 shows the locations of Bangkok and Tokyo in different time zones.

- How do you use Diagram 6 to help Sirichai understand the situation?
- What is the difference in hours between consecutive time zones?
- Why are there different time zones in the world?
- Bangkok is in zone G and Tokyo is in zone I. What is the time in Tokyo when Sirichai calls at 10.00 o'clock at night?
- What is the best range of time Sirichai should call Kenji if he wishes to talk to Kenji at night?

ii. To what extend can this task cultivate your students' appreciation of time zone towards the usefulness of mathematics in daily life?

## Topic 6: Introducing Money as Quantity

### Standards 6.1:

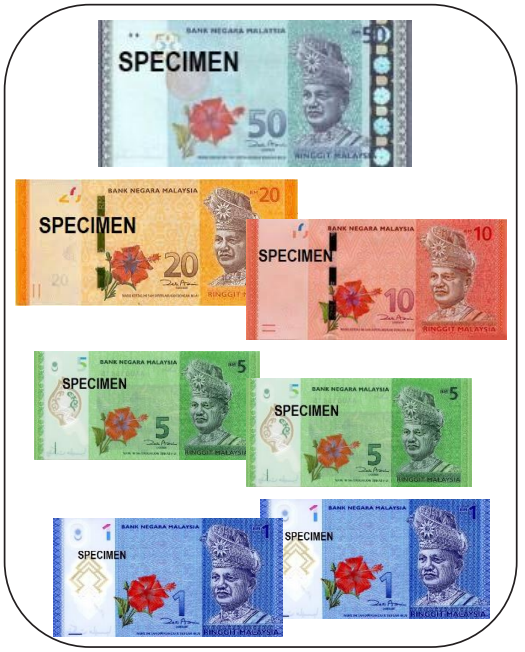
Introducing money as quantity and use it as the model of base ten system

i. Introduce unit of money using notes and coins and determine the correct amount of money
ii. Use counting by fives and so on for base-10 system
iii. Appreciate the fluency for calculation of money with all the four operations
iv. Appreciate number sense for conversion and transaction of money in life

### Sample Tasks for Understanding the Standards

### Task 1: Learning about Money in SEAMEO Countries

Set ① : Malaysian Ringgit




RM 92

Set ② : Thai Baht


Set ③ : Singaporean Cent


Set ④ : Philippines Piso


Diagram 1

i. Determine the amount of money in different currencies as shown in Diagram 1

Brunei Dollar



Indonesian Rupiah



Vietnamese Dong



Lao's Kip



Diagram 2

ii. Diagram 2 shows a few sets of currencies of some SEAMEO countries.

- Match the sets of currencies with the same value.

iii. Different systems of currency use different units of money to denote the values of coins and notes. Table 1 shows the units of money used in some SEAMEO countries.

Table 1
Money Units of Some SEAMEO Countries

| Country | Unit of Money | Symbol |
|---|---|---|
| Malaysia | Ringgit Malaysia | RM |
| Singapore | Singapore Dollar | S$ |
| Brunei | | |
| Cambodia | | |
| Indonesia | | |
| Lao | | |
| Myanmar | | |
| Philippines | | |
| Thailand | | |
| Timor Leste | | |
| Vietnam | | |

- Complete Table 1.

iv. Table 2 shows the different denominations of coins of Malaysia and Singapore.

Table 2
Malaysian and Singaporean Coins

| Country | Coin Denomination |
|---|---|
| Malaysia |  5 sen, 10 sen, 20 sen, 50 sen |
| Singapore |  5 cents, 10 cents, 20 cents, 50 cents, S$1 |

- Why is there no denomination for 30 cents?
- What will happen if there is no denomination for 20 cents?

v. Table 3 summarises the denomination of Indonesian currency notes.

Table 3
Denomination of Indonesian Rupiah

| Currency Unit | Denomination | | | | | | |
|---|---|---|---|---|---|---|---|
| Rupiah | 1 000 | 2 000 | 5 000 | 10 000 | 20 000 | 50 000 | 100 000 |

- Prepare a summary of the denomination of currency notes in your country and compare it with the denomination of rupiah.
  - ❖ What are the similarities and differences between the two currency notes?
- Explain how the currency notes are aligned to the base-10 system.
- Why denominations involving the numerals 1, 2 and 5 are commonly found in currency?

**Task 2: Operations Involving Money**

i. Calculate.
- S$8 + S$5
- RM23.50 – 65 sen
- 3 x ₭2500
- ₫45000 ÷ 5

ii. In a particular day, 1 piece of S$1 can exchange for 3 pieces of RM1 as shown in Diagram 3.


Diagram 3

- How many pieces of RM1 can you get for 5 pieces of S$1?
- How many pieces of S$1 can you get for 75 pieces of RM1?


Diagram 4

iii. Diagram 4 shows the prices of three types of fruits. Mr Wahyudi bought 2 bunches of bananas and a box of mangoes.
- How much did he pay for the fruits?
- If he return the box of mangoes in exchange for 2 packets of apples, how much balance will he get from the seller?

# CHAPTER 4

# Shapes, Figures and Solids

### Topic 1: Exploring Shapes of Objects

**Standard 1.1:**
Exploring shapes[1] of objects for finding their attributes

i. Roll, fold, stack, arrange, cut, draw, and trace objects (blocks such as boxes, cans and so on) for knowing their attributes
ii. Use attributes of blocks for drawing pictures by tracing shapes on the paper and explain how to draw it with the shapes
iii. Create patterns of shapes (trees, rockets and so on) by using the attributes and recognise the characteristics of shapes
iv. Appreciate the functions of shapes of objects in life
v. Appreciate the names of shapes in daily life by using mother-tongue

### Sample Tasks for Understanding the Standards

### Task 1: Exploring Attributes of Shapes

Diagram 1

Diagram 2

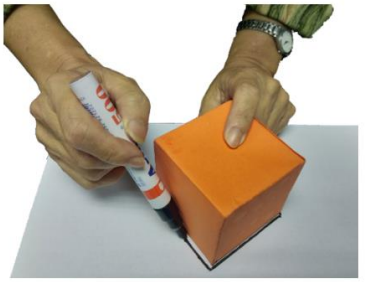
i. Diagram 1 shows two Grade 1 students doing a task of stacking objects given by their teacher, Mr Botum. The challenge of the task is to stack the objects to the tallest possible height.

Another teacher, Mr Udin gives a runway and some objects to his Grade 1 student as shown in Diagram 2. The task of the student is to identify and group together objects that could roll down the runway smoothly.

- What attributes of objects does each of the teachers expect the students to identify through the activity?
- What questions will you ask the students in order to guide them understand the attributes of objects?
- What questions will you ask the students in order to promote their mathematical thinking?

[1] Shape is not referred as figure.


Trace



Draw

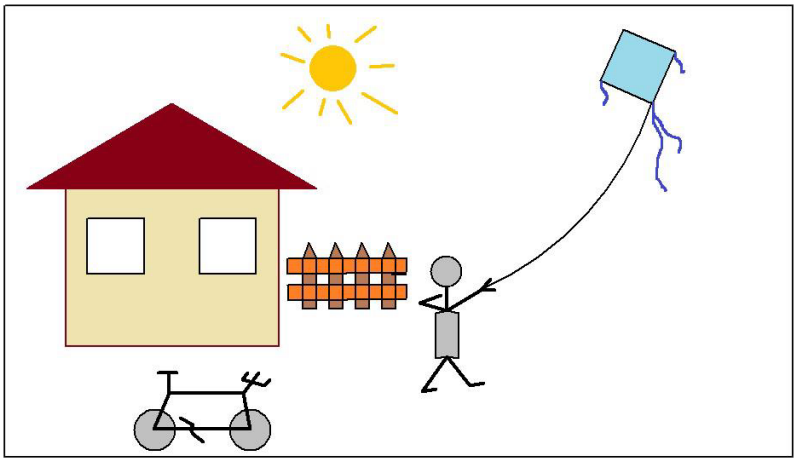


Diagram 3

ii. Diagram 3 shows the work of a Grade 1 student tracing shapes using daily objects to create a picture. The windows and the kite in the picture were traced using the same squared box. The following is the conversation between the student and his teacher.

Teacher : "*What shapes are your windows?*"

Student : "*Squared face of the box.*"

Teacher : "*What other shapes do you have in your drawing?*"

Student : "*Round shapes, triangular face of the chocolate box, rectangular face of the tissue box, and … mmm … so many!"*

Teacher : *Do you use any other shape that is the same as the window?*"

Student : "*Mmm … no*!"

Teacher : "*What about the flying kite*?"

Student : "*Mmm … no, it is not the same as the window.*"

Teacher : "*Why not?*"

Student : "*Because they don't look the same. It is just a kite*."

- Why does the student see the window and the kite as different shapes?

iii. A student traced and cut out many pieces of triangular faces with different sizes. He then pasted the cut-out pieces to create a sailing boat as shown in Diagram 4.

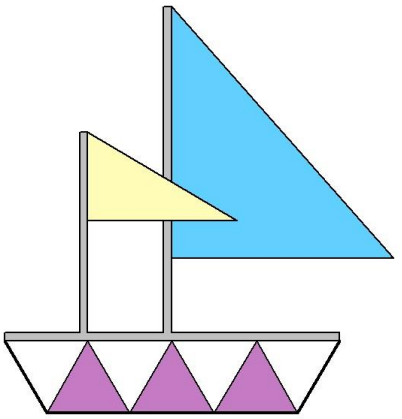


Diagram 4

- Describe the shapes used in drawing the boat.
- How can this task be extended for students' future learning?
- How can this activity develop a sense of appreciation towards mathematics among primary children?

## Topic 2: Characterising Shapes for Figures and Solids

### Standard 2.1:

Describing figures with characters of shapes

i. Use characteristics of shapes for understanding figures (quadrilaterals, square, rectangle and triangle, right angle, same length)
ii. Introduce line and right angle with relations to activities such as paper folding and use it for describing figures with simple properties (such as triangle has 3 sides)
iii. Classify triangles by specific components, such as side, vertex and angle (right-angled triangle, equilateral, isosceles) and then know the properties of each classification
iv. Reorganising rectangular shape and squared shape as figures by using right angle and length of sides

### Sample Tasks for Understanding the Standards

### Task 1: Exploring Shapes

i. A sheet of paper is folded to form a right angle, $a$ as shown in Diagram 1. This paper-folding activity can be used to introduce right angle to primary school students.

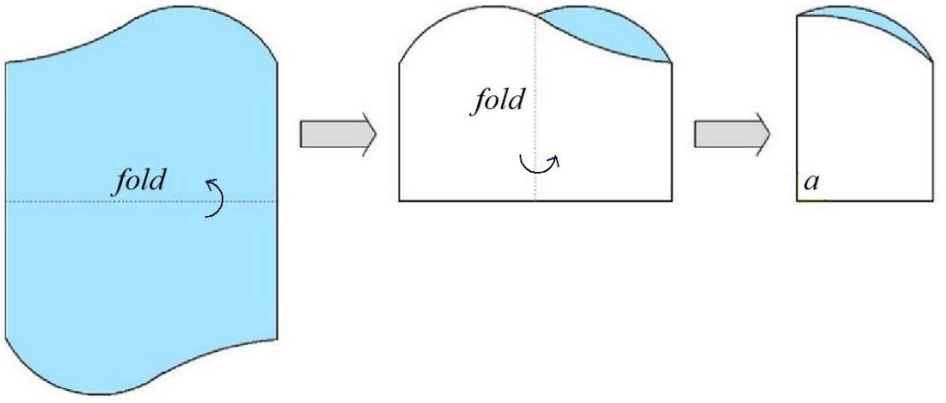



Diagram 1

- How will you extend this paper-folding activity to reinforce your students' understanding of right angles?
- Draw a shape that has 2 right angles only.
- Is it possible to have a shape with 3 right angles only? If yes, draw the shape. If no, why not?
- A shape has 4 right angles only, but it is not a rectangle neither a square. Draw the shape.
- A closed shape has 4 equal sides, but it is not a square. What shape can it be?

ii. A piece of square paper is cut along its two diagonals to get four right-angled triangles as shown in Diagram 2.

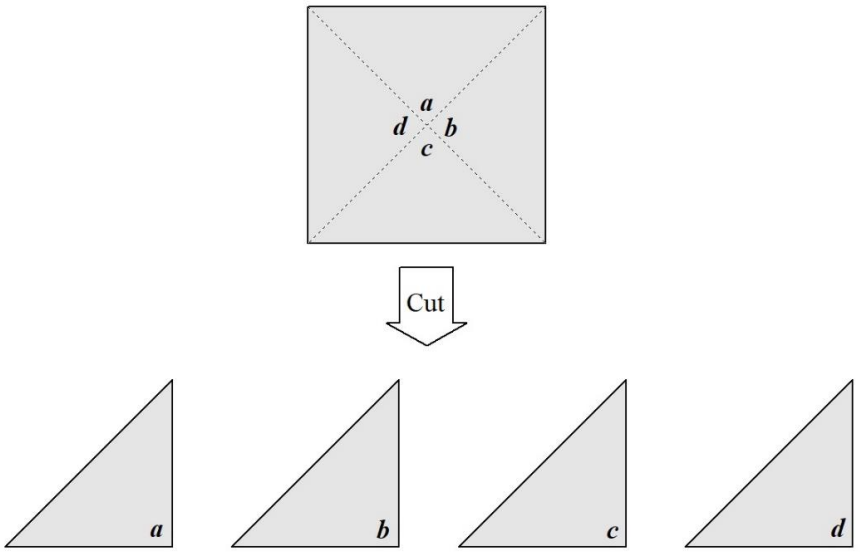



Diagram 2

- How will you convince your students that the four triangles are identical?
- How will you guide your students to determine whether $a$, $b$, $c$ and $d$ are right angles?

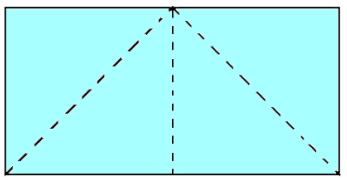


Diagram 3

- The four pieces of triangles can be rearranged to form a rectangle as shown in Diagram 3.
  - ❖ Arrange the four pieces of triangles in another way to form a rectangle.
  - ❖ Arrange the four pieces of triangles to form figure Ⓐ and figure Ⓑ in Diagram 4, respectively.

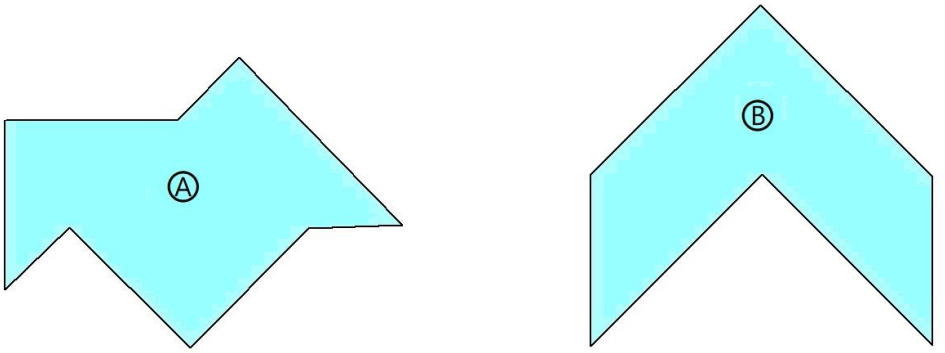
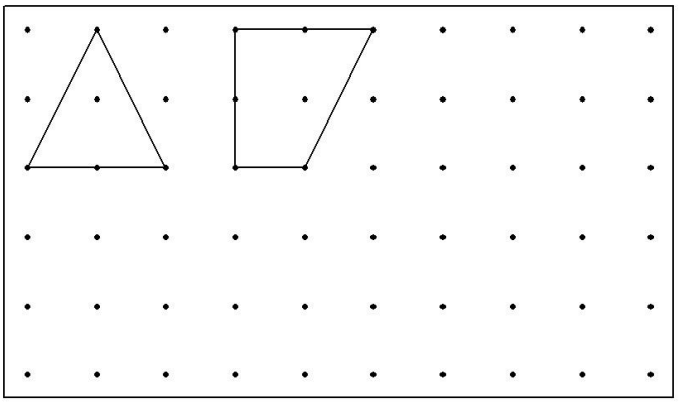



Diagram 4

- What mental ability about shapes and figures can students develop from this activity? Explain your answer.

iii. Diagram 5 shows two tasks designed by a teacher for his primary school students to learn triangles and quadrilaterals.

Task ① : Connect points with straight lines to draw triangles and quadrilaterals.



Task ② : Draw a straight line connecting each pair of points with the same numbers. Colour the triangles and quadrilaterals formed by the straight lines.


Diagram 5

- What is the difference between figure and plane figure?
- Compare Task Ⓐ and Task Ⓑ.
  - ❖ What are the similarities and differences in the ideas of triangles and quadrilaterals learnt through these two tasks?

iv. Diagram 6 shows some figures.

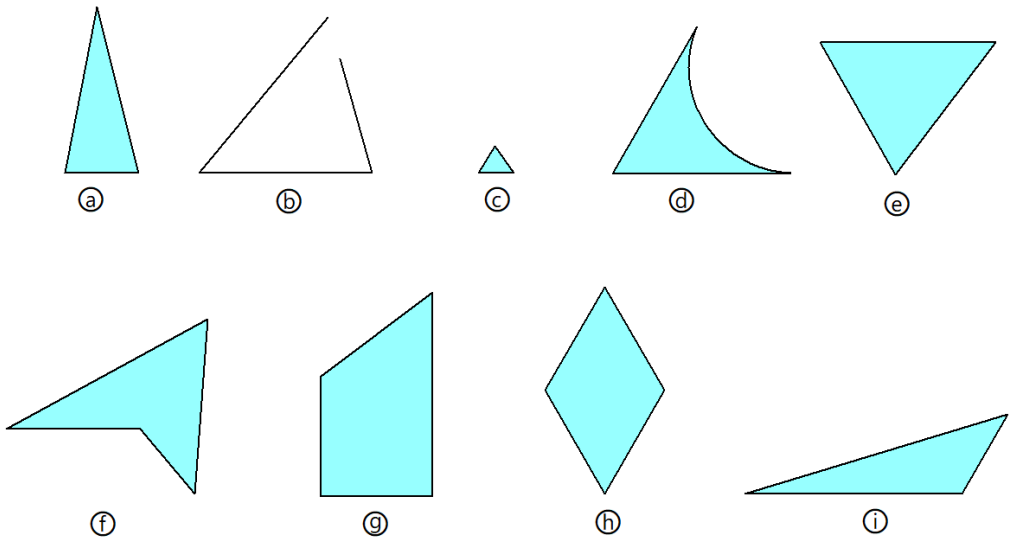



Diagram 6

- Determine which of these figures are not triangles.
- For each figure that is not a triangle, explain why it is **not** a triangle.
- What is a triangle?

v. Identify and explain briefly the characteristics of the following figures.

- Rectangle
- Square
- Determine whether each of the following statements is true or false. Explain your reasons.
  - ❖ All rectangles are not squares.
  - ❖ All squares are rectangles.
  - ❖ Some rectangles are squares.

## Task 2: Classifying Triangles

***Constructing Triangles with Straws***

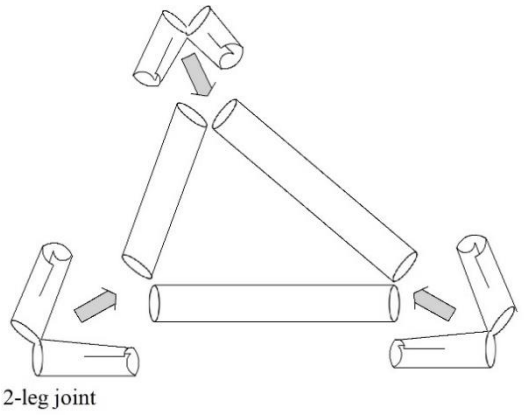



Diagram 7

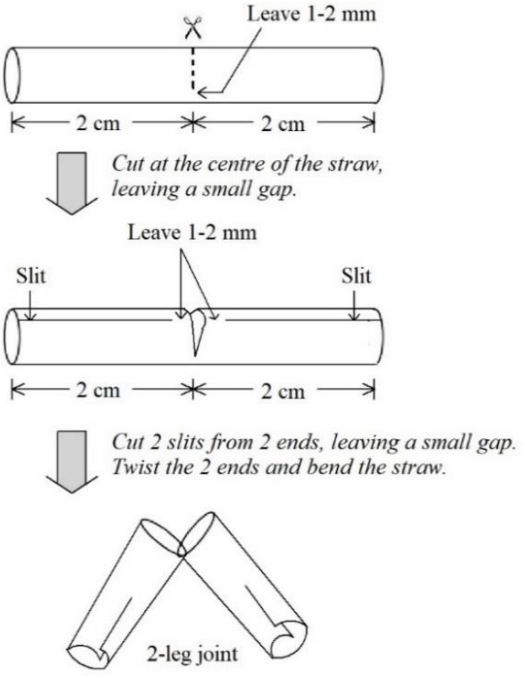
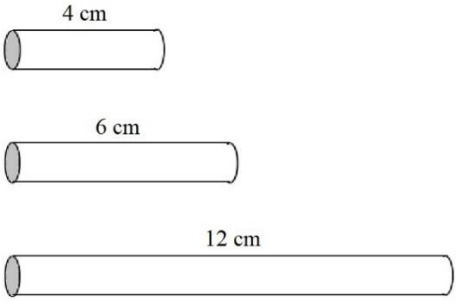



Diagram 8

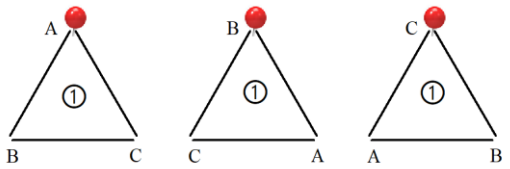
Diagram 7 shows how a triangle can be constructed using three straws and three 2-leg joints. Diagram 8 shows the construction of a 2-leg joint.

i. The following is a set of 3 straws.



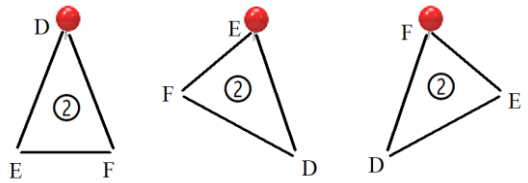

- Explain why this set of straws cannot make a complete triangle?
- What is the condition, in term of lengths, for three straws to form a complete triangle?

Triangles Type ①



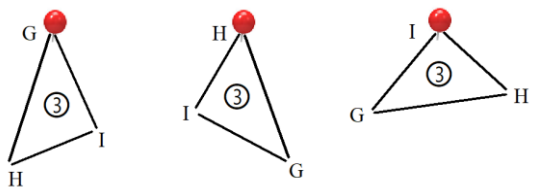

- *Triangle ① hanging at vertices A, B and C.*
- *All horizontal bases.*

Triangles Type ②




- *Triangle ② hanging at vertices D, E and F.*
- *Only one horizontal base.*

Triangles Type ③




- *Triangle ③ hanging at vertices G, H and I.*
- *All slanted base.*

Diagram 9

ii. A teacher, Ms Malee, asked her students to construct triangles using straws of different lengths. All the triangles constructed were pin on the class bulletin board as a display of the students' work. From the display, she discovered that there are 3 types of triangles as shown in Diagram 9.

Type ① are triangles that always have a horizontal base at any point of hanging.

Type ② are triangles that have a horizontal base at only one point of hanging.

Type ③ are triangles that always have a slanted base at any point of hanging.

- Explain how the three types of triangles are related to classification of triangles by lengths of the sides.
- Explain how you will use this construction of triangles from straws to help your students learn equilateral, isosceles and scalene triangles.

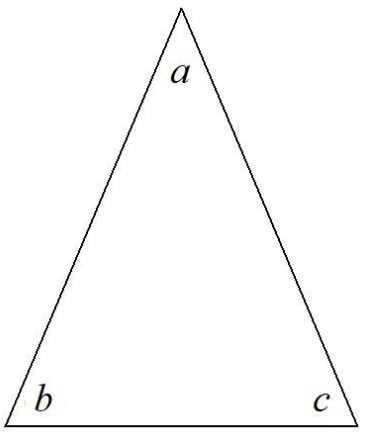



Diagram 10

iii. Diagram 10 shows an isosceles triangle cut out from a piece of paper.

- Explain how you can fold the paper triangle to show that:
  - ❖ Angle $b$ is same size as angle $c$
  - ❖ Angle $a$ is not the same size as angle $b$

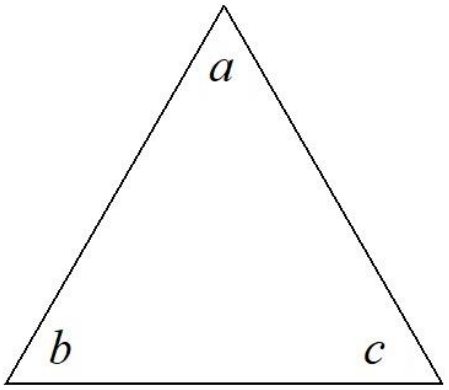
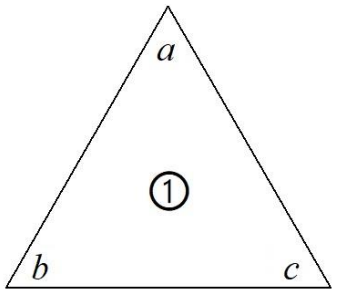



Diagram 11

iv. Diagram 11 shows an equilateral triangle cut out from a piece of paper.

- Explain how you can fold the paper triangle to show that:
  - ❖ Angle $a$ is the same size as angle $b$.
  - ❖ Angle $b$ is same size as angle $c$.
  - ❖ Without folding the triangle, how do you reason that angle $a$ and angle $c$ are the same?



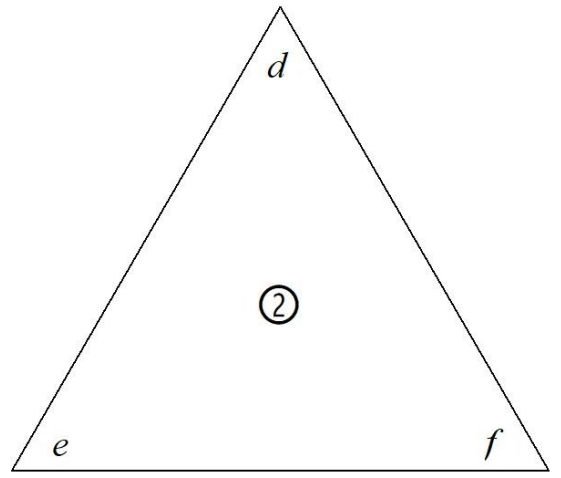




Diagram 12

v. Diagram 12 shows two equilateral triangles of different sizes. A student claimed that the sizes of angles $a$, $b$, $c$ in triangle ① are smaller than the sizes of angles $d$, $e$, $f$ in triangle ②.

- How will you use paper cutting to convince your students that angles $a$, $b$, $c$, $d$, $e$ and $f$ are the same?

**Standards 2.2:**

Describing solids with characteristics of shapes

i. Use the characteristics of shapes to understand solids such as boxes can be developed by six rectangular parts with simple properties
ii. Develop boxes with the properties
iii. Appreciate the solids around daily life through considering the function of the solids

**Sample Tasks for Understanding the Standards**

**Task 1: Properties of a Box**


Diagram 1

i. Diagram 1 shows a rectangular box with 6 faces. Faces ① , ② , and ③ are shown in the diagram, whereas the other three faces are hidden.

- How many edges and vertices does the box have?

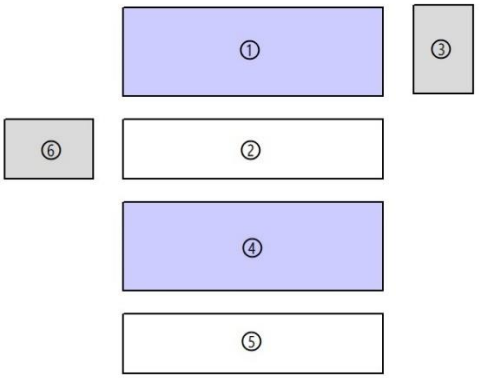



Diagram 2

ii. Each face of the box is traced on paper as shown in Diagram 2.

- What shape is each face?

iii. Faces ④ , ⑤ , and ⑥ are opposite to faces ① , ② , and ③ , correspondingly. Each face is joined with 4 other faces and hence forming 4 adjoining edges. For example, face ① is joined with faces ② , ③ , ⑤ and ⑥ .

- Explain the condition in terms of opposite faces and adjoining edges for the 6 faces to form the box.

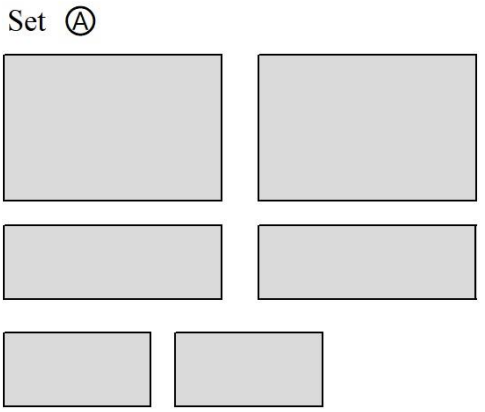



Diagram 3

iv. Diagram 3 shows a set of faces. If the faces are cut and joined by their edges.

- Can the set of faces form a proper box? Why or why not?

Set Ⓑ

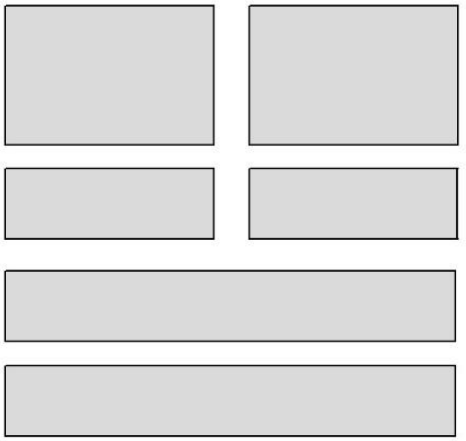


Diagram 4

v. Diagram 4 shows another set of faces.

- Can the set of faces form a proper box? Why or why not?

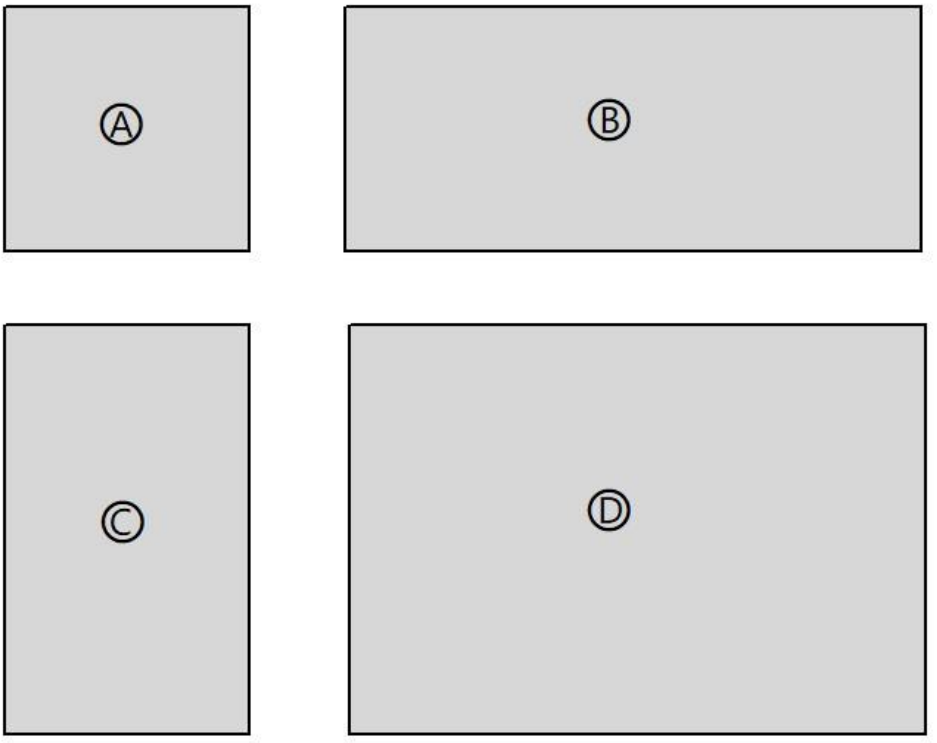



Diagram 5

vi. Diagram 5 shows four types of cards with different measurements. A proper box can be constructed using 6 pieces of these cards. For examples, 6 pieces of card Ⓐ can make a cube, and 2 pieces of card Ⓐ with 4 pieces of card Ⓑ can make a cuboid.

- How many different boxes are you able to construct using these cards?
- Photocopy Diagram 5 and construct all the boxes.

**Standards 2.3:**

Drawing circle and recognising sphere based on circle

i. Think about how to draw a circle and find the centre and radius
ii. Draw a circle with instrument such as compass
iii. Enjoy drawing pictures using circles such as Spirograph
iv. Find the largest circle of the sphere with diameter and identify the sphere by its centre and radius
v. Appreciate circles and spheres in daily life such as manhole, and the difference between soccer ball and rugby ball

**Sample Tasks for Understanding the Standards**

**Task 1: Properties of a Circle**

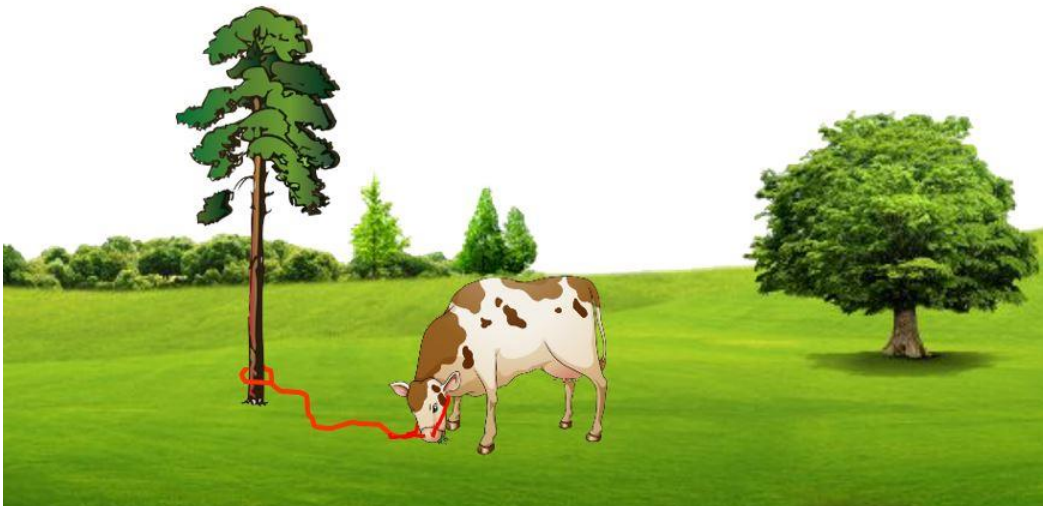


Diagram 1

i. A cow is tied to a tree with a 2-metre rope in a grass field.

- What is the furthest distance the cow could move away from the tree?

  [Note. Taking the mouth of the cow as the reference point.]

- After a day, the cow has eaten all the grasses within its' reach, left with a patch of bare land. What is the shape of the patch of bare land?

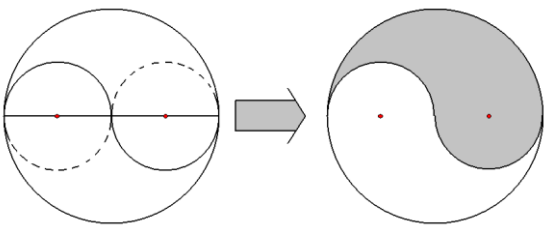


Diagram 2

ii. A compass is a simple instrument used to draw a circle. Diagram 2 shows how the *taizhi* symbol can be constructed from 3 circles.

- Use a compass to construct the *taizhi* symbol.

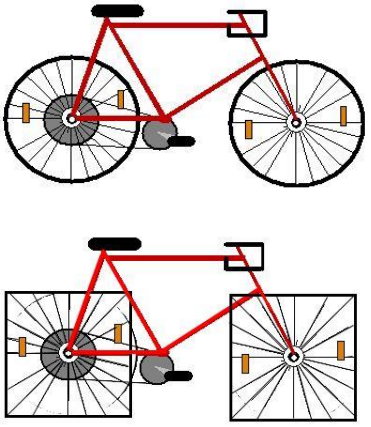


Diagram 3

iii. Diagram 3 shows a bicycle with circular wheels, and another bicycle with squared wheels.

- Why is a circular wheel better than a squared wheel?
- What properties of a circle make it a better choice than a square?

Diagram 4

iv. Diagram 4 show a manhole.

- Why is the manhole made in the shape of a circle and not any other shape such as square?

Diagram 5

v. Diagram 5 shows a broken circular plate.

- Photocopy the diagram and explain how to find the centre of the circle.
- Explain how you can determine the diameter of the circular plate.

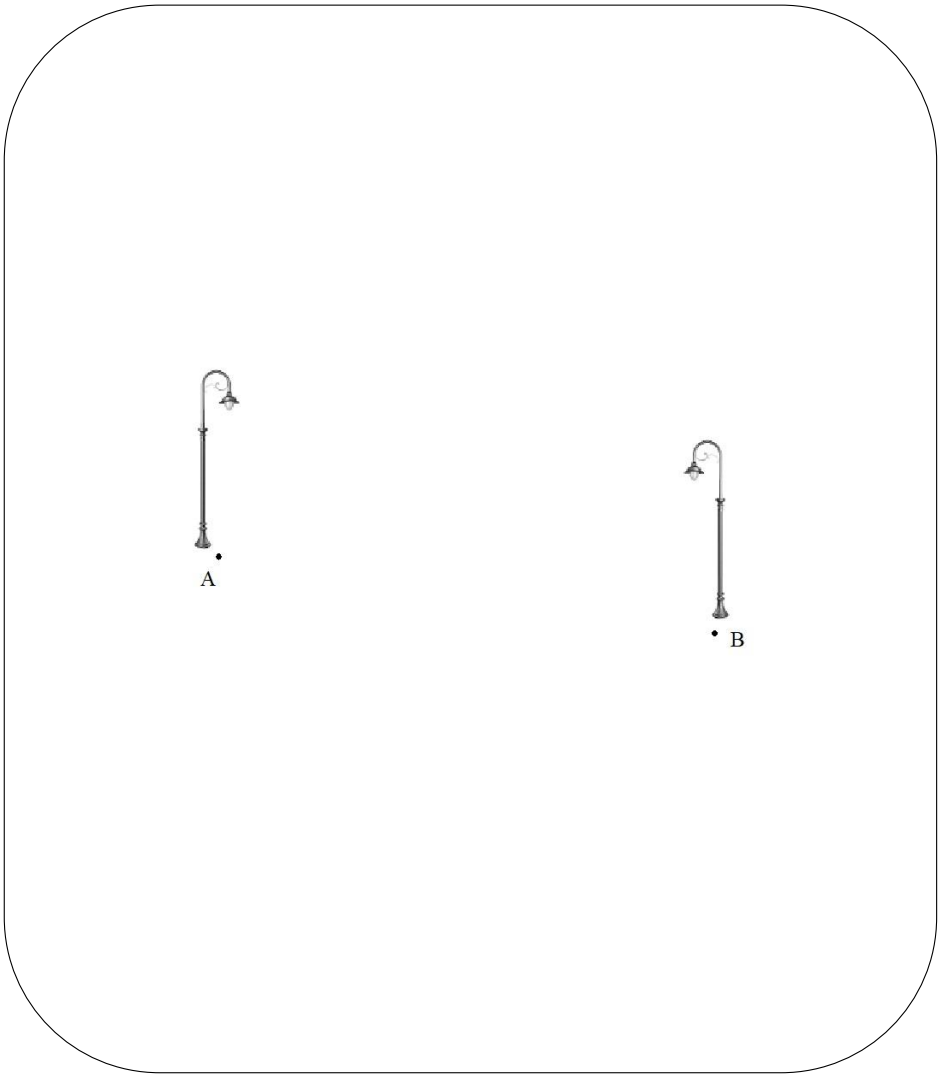



Diagram 6

vi. Diagram 6 shows a picture of two lamp poles, A and B. There is a treasure hidden somewhere in the diagram.

- Given that the treasure is 8 cm from point A.
  - ❖ Is the information enough to locate the treasure? Explain your reasons.
- Also given, the treasure is 6 cm from point B.
  - ❖ Is the information enough to locate the treasure now?
  - ❖ How certain are you now on the location of the treasure?
  - ❖ Use a compass to determine and mark the possible locations of the treasure on Diagram 5.

**Task 2: Properties of a Sphere**

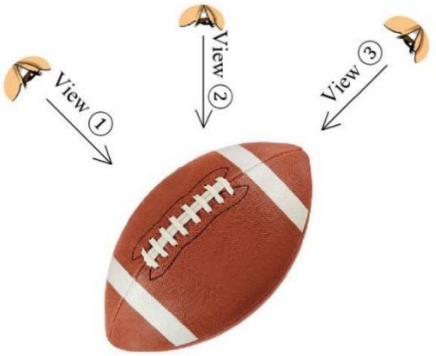



Diagram 7

i. Diagram 7 shows a rugby ball and three directions of viewing at it.

- Sketch the shape you see from each of the directions of viewing.

ii. Diagram 8 shows a football and three directions of viewing at it.

- What shape will you see from each of the directions of viewing?
- What can you conclude about the shape of a sphere when view from different directions?

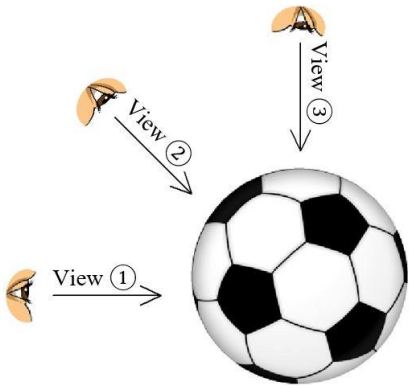



Diagram 8

iii. Explain why the football can roll smoother than the rugby ball.

iv. Explain how you guide young children to find the diameter of the football.

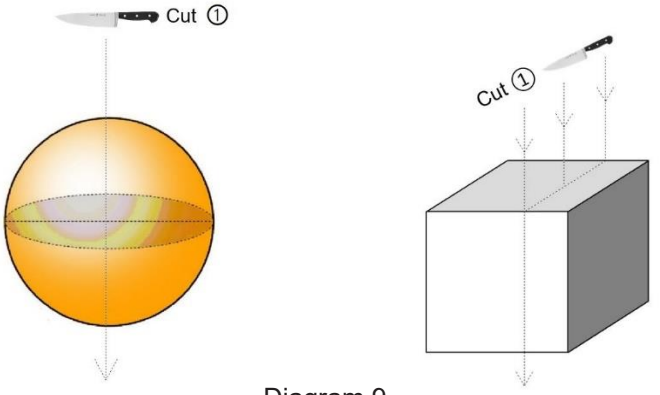



Diagram 9

v. Diagram 9 shows two solids, a sphere and a cube, cut in a vertical direction.

- Sketch the shape of the cross section for each of the solids.

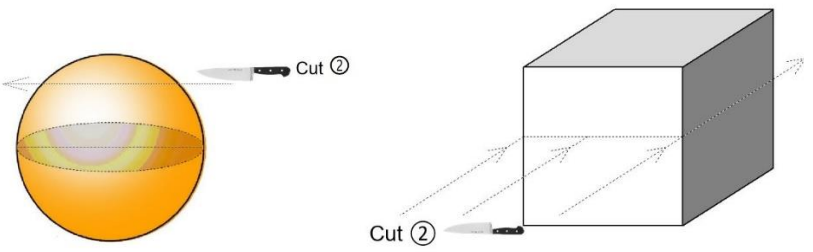



Diagram 10

vi. Diagram 10 shows the same sphere and cube, but cut in a horizontal direction.

- Sketch the shape of the cross section for each of the solids.

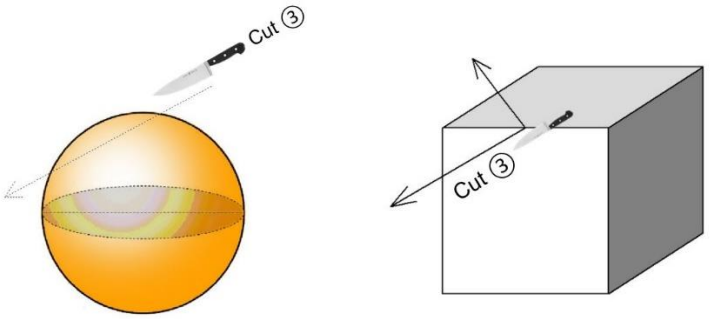



Diagram 11

vii. Diagram 11 shows the same two solids cut in a slanting direction.

- Sketch the shape of the cross section for each of the solids.

viii. What will you conclude on the cross-sectional shape of a sphere?

- When will you get the largest cross-sectional circle from a sphere?

## Topic 3: Explaining Positions and Directions

### Standards 3.1:

Exploring how to explain a position and direction

i. Identify simple positions and directions of an object accurately using various ways such as in my perspective, in your perspective in the classroom, and the left, right, front, back, west, east, north, south and with measurement
ii. Draw the map around the classroom with consideration of the location
iii. Design a game to appreciate the changing of positions and directions in a classroom

### Sample Tasks for Understanding the Standards

### Task 1: Position and Direction of an Object

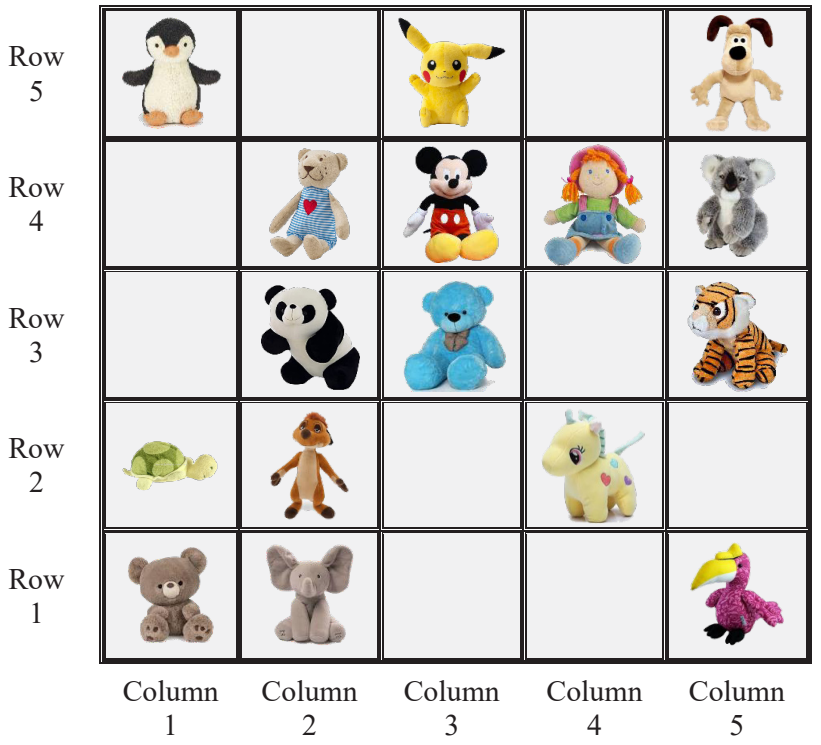



Diagram 1

Diagram 1 shows a wall shelf with 25 compartments labelled with its rows and columns. Soft toys are placed in some of the compartments. A class of children are given the opportunity to choose a favourite toy from the shelf.

i. Alfonso says: "My favourite toy is in the fourth row from the bottom."

- Is "fourth row from the bottom" the same as "fourth row from the top"?
- Is the information enough for you to locate the boy's favourite toy? Why or why not?

ii. Dylan says: “I like the toy in Column 5.”

- Can you locate the toy? Why or why not?

iii. What toy is located at the following compartments?

- First column from the left and fifth row from the bottom.
- Fourth column from the right and second row from the top.
- Column 2 and row 3
- Column 5 and row 4
- What minimum information is needed to locate a toy in the wall shelf?

iv. Design a method to identify a location in the wall shelf by using only numbers.

- Explain your method.
- How will your method differentiate the location of Pikachu and Tiger?

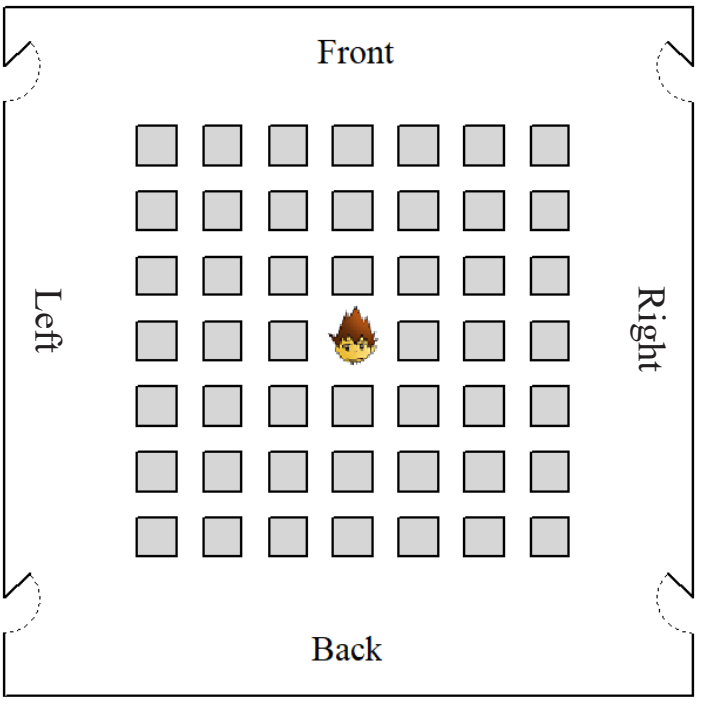



Diagram 2

v. Diagram 2 shows a game board which is a plan of table arrangement in a classroom. The rules of the game are shown in Diagram 3.

Game Rules:

① The game board is displayed in the front of the class.

② The class is divided into two teams.

③ Members from each team will take turn to choose a table on the game board by giving instruction to the teacher (taking the boy as reference point). Examples of instruction are:

"3 tables to the front."

"3 tables to the front and 4 tables to the left."

④ If an instruction is given correctly, the corresponding table will be claimed by the team and the teacher will mark the table for the team on the game board.

⑤ The first team that claims 4 tables consecutively in a horizontal, vertical or diagonal line, is the winner.

Diagram 3

- Diagram 4 shows the instructions given by members from two team, A and B, during a game session.

| Table Marked | Team A | Team B |
|---|---|---|
| 1st | ▪ 2 tables to the right | ▪ 1 table to the front and 2 tables to the left |
| 2nd | ▪ 1 table to the right and 1 table to the back | ▪ 3 tables to the left and 1 table to the front |
| 3rd | ▪ 2 table to the back | |

Diagram 4

❖ Mark the tables according to the instructions given, using different marks for different teams.

❖ After marking their 3rd table, the members of Team A shouted: "Yeah! We win! We win!" Do you agree with them? Explain your reasons.

- To what extend can this game help to inculcate appreciation towards the ideas of positions and directions among your students?

vi. Compare the two systems of determining locations on a plane in Diagram 1 and Diagram 2.

- What are the similarities and differences between the two systems?
- Which system will you discuss first with your students? Explain your reasons.

# CHAPTER 5

# Patterns and Data Representations

**Topic 1: Using Patterns Under the Number Sequence**

**Standard 1.1:**
Arranging objects for beautiful patterns under the number sequence

i. Know the beautifulness of patterns in cases of arranging objects based on number sequence
ii. Arrange objects according to number sequence to find simple patterns
iii. Arrange expressions such as addition and subtraction to find simple patterns
iv. Express the representation of patterns using placeholders (empty box)
v. Enjoy the arrangement of objects based on number sequence in daily life
vi. Find patterns on number tables such as in calendars

**Sample Tasks for Understanding the Standards**

**Task 1: Patterns from Number Sequence**

i. Diagram 1 shows 10 coloured chips in a row.

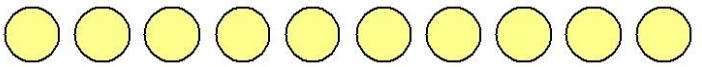
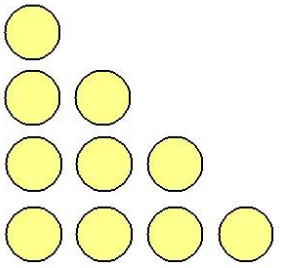


Diagram 1

Diagram 2 shows the 10 chips rearranged into a pattern that represents a number sequence.



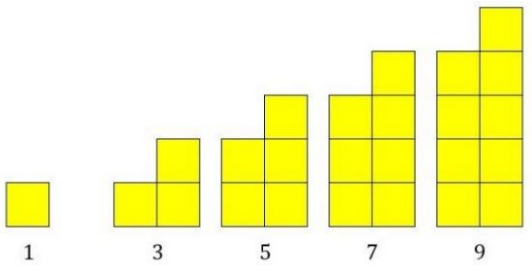
**4, 3, 2, 1**

Diagram 2

- Rearrange the chips to form more possible patterns and write the corresponding number sequence.
- How does the formation of patterns promote children's learning of number sequence?
- How does the number sequence lead to learning of addition?
- What number patterns can lead to multiplication? Explain your answer with examples.

ii. Two students, A and B, used multi-linked cubes to represent the first five odd numbers as shown in Diagram 3.

Student A's Representation




Student B's Representation

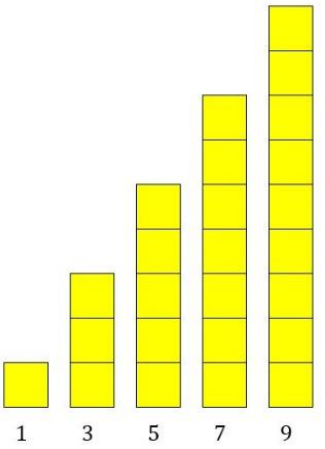



Diagram 3

- Which representation do your think is better in helping students learn the property of odd numbers? Justify your choice.
- How will you use multi-linked cubes to represent the first five even numbers?

iii. In what way can this task develop primary students' appreciation toward the beautifulness of patterns in number sequence?

**Task 2: Patterns from Addition and Subtraction Expressions**

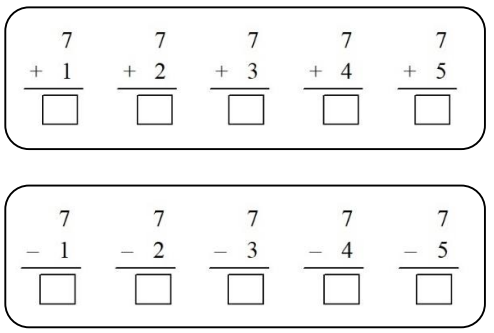



Diagram 4

i. Diagram 4 shows two sets of expressions involving addition and subtraction.

- Describe the pattern in each set of expressions.

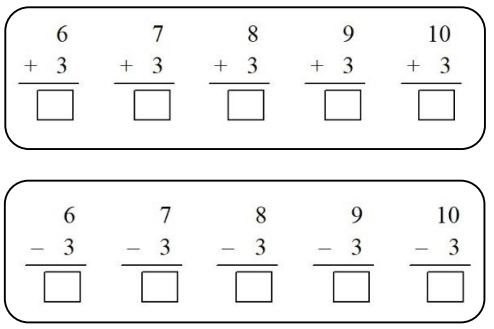



Diagram 5

ii. Diagram 5 shows another two sets of expressions.

- Describe the patterns in these two sets of expressions.

iii. How will these patterns help your students learn addition and subtraction of numbers?

iv. Diagram 6 shows a pattern of expressions.

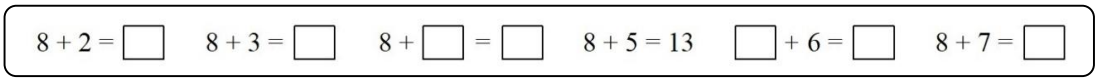



Diagram 6

- Fill in all missing numbers in the pattern. Justify each of your answers.
- In what way can this task promote mathematical thinking among primary students?

**Task 3: Patterns in Daily Life**

***Tablet Counter***

Diagram 7

Diagram 7 shows a triangular tablet counter used by pharmacists to automatically calculate the total number of tablets according to the number of rows filled printed on the counter itself.

i. What is the number of tablets for each of the following rows?

- Row 3
- Row 5
- Row 10

ii. Table 1 shows the total number of tablets for the first 3 rows.

Table 1
Total Number of Tablets for Each Row

| Row | Total |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 3 | 6 |
| 4 | |
| 5 | |
| 6 | |

Fill in the missing totals in Table 1.

- Describe the number pattern in the table.
- What mathematical attitude can be developed through this task? Explain your answer.

***Calendar Patterns***

**January 2021**

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|---|---|---|---|---|---|---|
| | | | | | 1 | 2 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | | | | | | |

Diagram 8

Diagram 8 shows the calendar for January 2021.

i. Choose any horizontal row of numbers from the calendar such as 3, 4, 5, 6, 7, 8, and 9.

- Describe the number pattern in each row.
- Choose three consecutive numbers from any of the rows. What pattern can you find from the three numbers? Explain why.

ii. Choose any vertical column of numbers from the calendar such as 5, 12, 19, and 26.

- Describe the number pattern in each column.
- Choose three consecutive numbers from any of the columns. What pattern can you find from the three numbers? Explain why.

iii. Choose any square box of four numbers such as

| | |
|---|---|
| 1 | 2 |
| 8 | 9 |

- What pattern can you find from the four numbers? Explain why.

## Topic 2: Producing Harmony of Shapes Using Patterns

### Standard 2.1:

Arranging tiles of different or similar shapes in creating harmony

i. Know the beautifulness of patterns in cases of arranging the objects based on shapes, colours and sizes
ii. Arrange objects according to shapes, colours and sizes to show patterns
iii. Arrange boxes according to shapes, colour and sizes to create structure
iv. Arrange circles and spheres for designing
v. Enjoy the creation based on different shapes, colour and sizes in daily life

### Sample Tasks for Understanding the Standards

### Task 1: Tiling Pattern

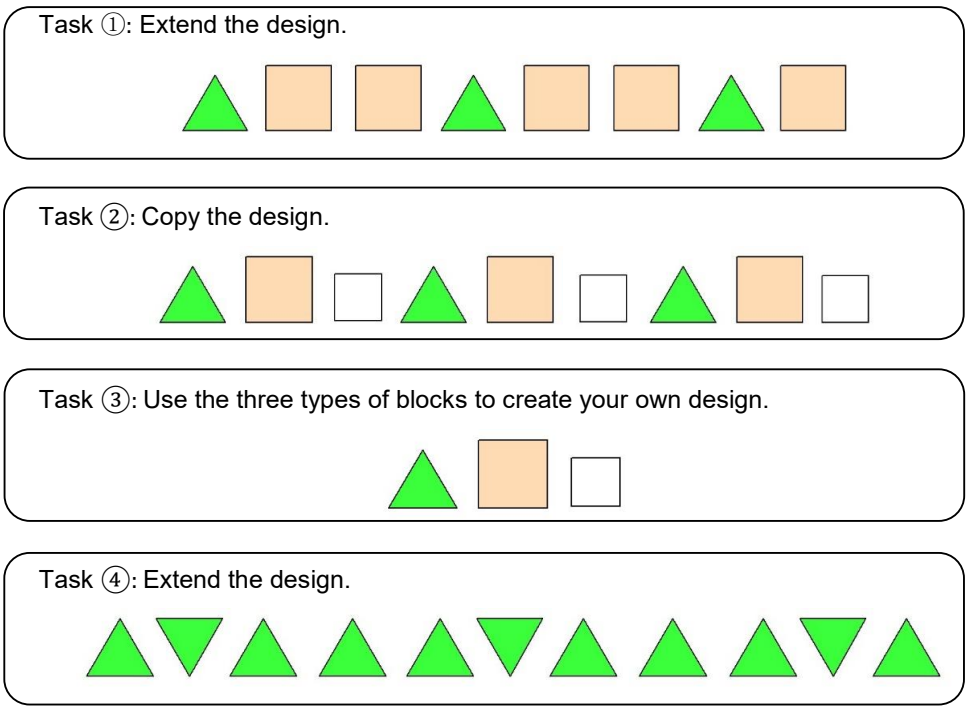



Diagram 1

i. Diagram 1 shows four tasks on designing patterns using geometrical tiles for primary children.

- How will you sequence the tasks for your students?
- Explain your reasons.

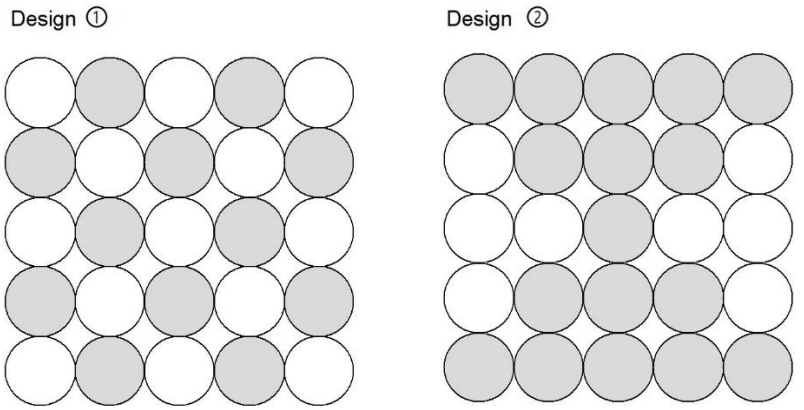



Diagram 2

ii. Diagram 2 shows two designs created by two primary students using circular tiles.

- Based on each of the designs, pose a task involving number pattern for primary mathematics.
- How could you help your students relate the designs to number pattern?

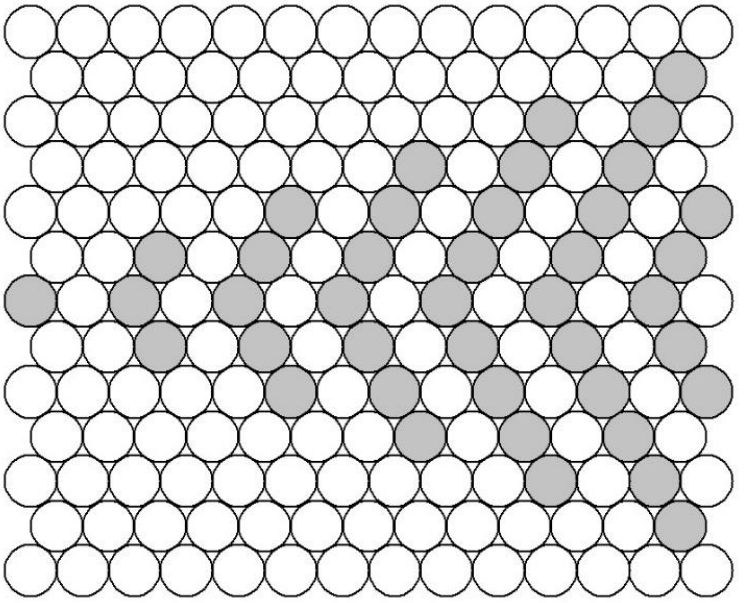


Diagram 3

iii. Diagram 3 show a part of a floor tiling design which consists of six complete and one incomplete arrangements of dark tiles.

- How many dark tiles will you predict to complete the $7^{th}$ arrangement?
- What mathematical thinking and processes could be promoted through this task?

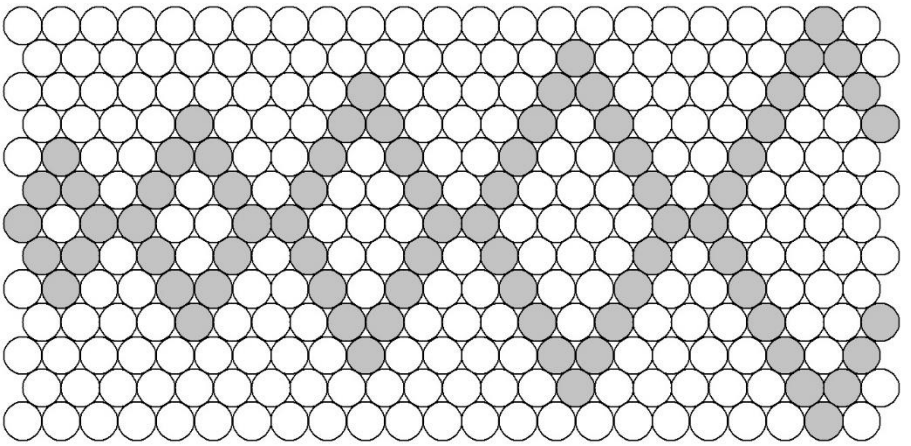


Diagram 4

iv. Diagram 4 shows another part of a floor tiling design which consists of four complete and one incomplete rhombus.

- How many white tiles are inside each of the complete rhombus?
- How many white tiles will you predict to complete the fifth rhombus?
- What mathematical ideas in future grade are embedded in this task?

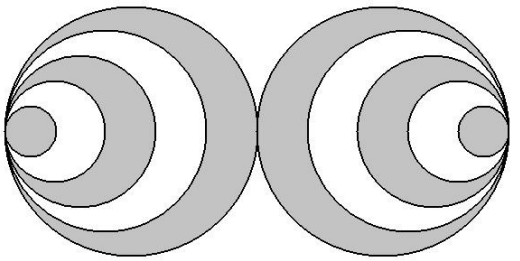


Diagram 5

v. Diagram 5 shows a design formed by 10 circles.

- Draw the design with the help of a compass.
- Based on the design, what embedded mathematical ideas can be discussed with your students?
- How will you use the design to prepare your students for learning the embedded ideas in higher grades?

**Task 2: Creative Design in Daily Life**

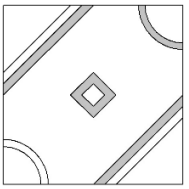


Diagram 5

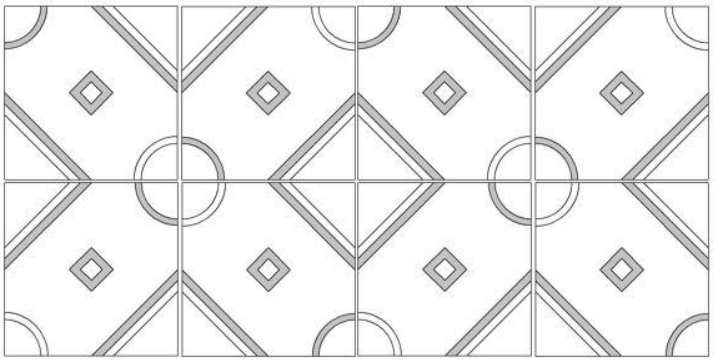


Diagram 6

i. Diagram 5 shows a basic piece of flooring tile. These basic pieces of tiles can be arranged to form various creative designs. Diagram 6 shows an example of design created using 8 pieces of the tiles.

- Based on this design, what embedded mathematical ideas can be discussed with your primary students?

ii. Use the basic flooring tiles in Material Sheet 2 at the end of this topic to create another flooring design which is different from Diagram 6.

- In what ways can this task cultivate mathematical habits of mind?
- In what ways can this task help to develop appreciation towards mathematics?

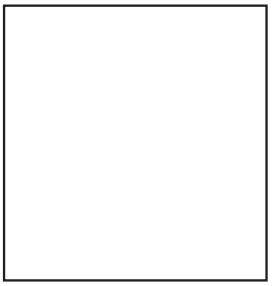


Diagram 7

iii. Diagram 7 shows an empty squared tile.

- Create a basic flooring design on this squared tile.
- What do you need to consider when creating the basic flooring tile in order to make it attractive?

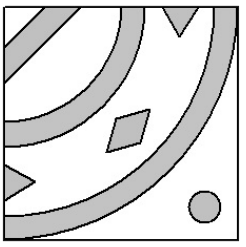
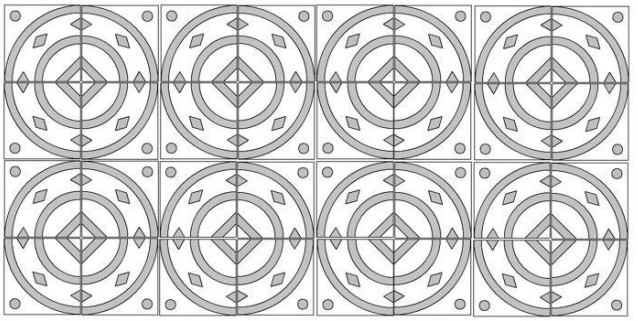


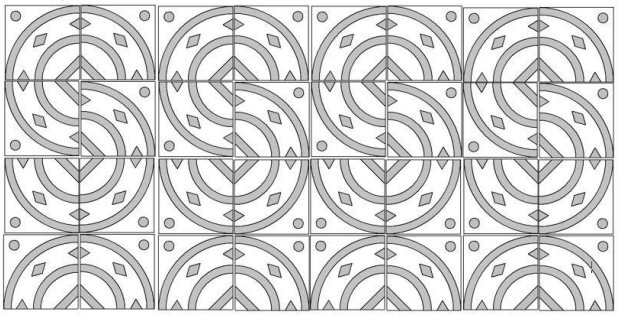
Diagram 8

iv. Diagram 8 shows another basic tile. Three floor tiling designs were formed by arranging 32 pieces of this basic tile as shown in Diagram 9.

Arrangement ①



Arrangement ②



Arrangement ③

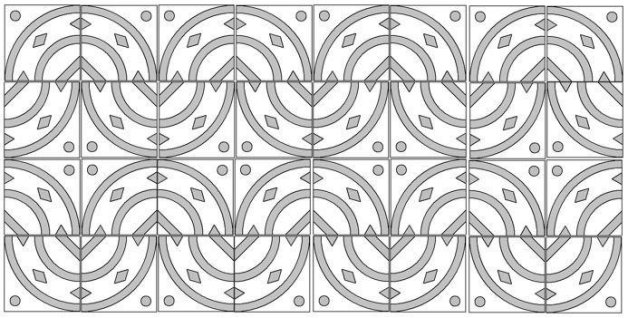
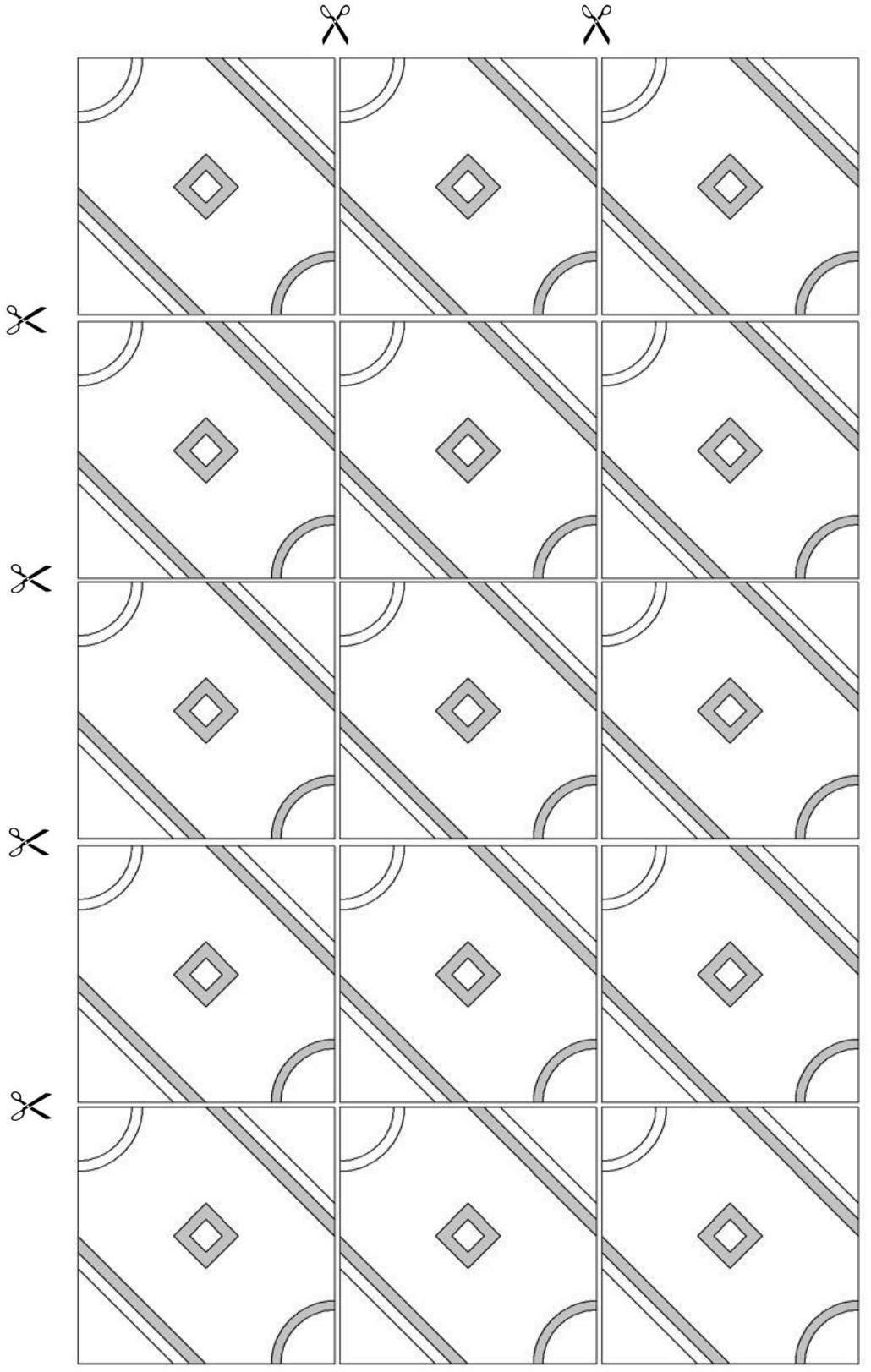


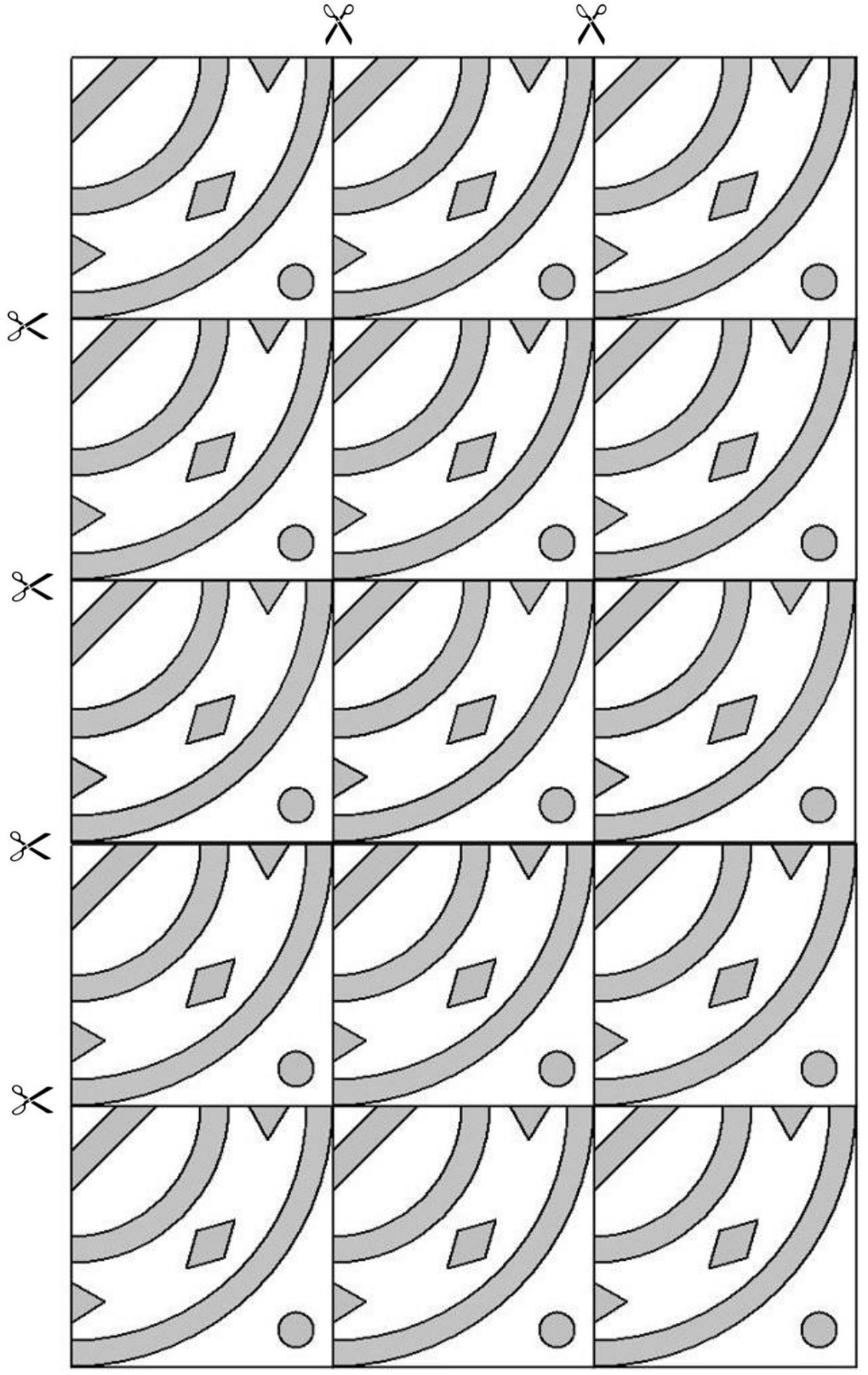
Diagram 9

- Use the basic flooring tile in Material Sheet 3 at the end of this topic to duplicate the three designs.
- Study arrangements ①, ② and ③ carefully.
  - ❖ Describe how arrangement ① is changed to arrangement ②?
  - ❖ Describe how arrangement ① is changed to arrangement ③?
- What mathematical ideas in future grade are embedded in this task?
- Interpretation in art refers to attribution of meaning to an art work. What is your personal interpretation on each of the geometrical art in Diagram 9?
- How could interpretations of these designs help your students to appreciate the harmony of geometrical art?

**Material Sheet 2: Basic Flooring Tiles – Design I**



**Material Sheet 3: Basic Flooring Tiles – Design II**



**Topic 3: Collecting Data and Represent the Structure**

**Standard 3.1:**

Collecting data through categorisation for getting information

i. Explore the purpose of why data is being collected.
ii. Grouped data by creating similar attributes on the denomination of categories and count them (check mark and count)
iii. Think about what information is obtained from the tables with categories and how to use it

**Sample Tasks for Understanding the Standards**

**Task 1: Collecting Data**

i. Every student from a class was asked to choose a picture card to represent his/her favourite fruit. Then, everyone placed their favourite fruit on the board as shown in Diagram 1.

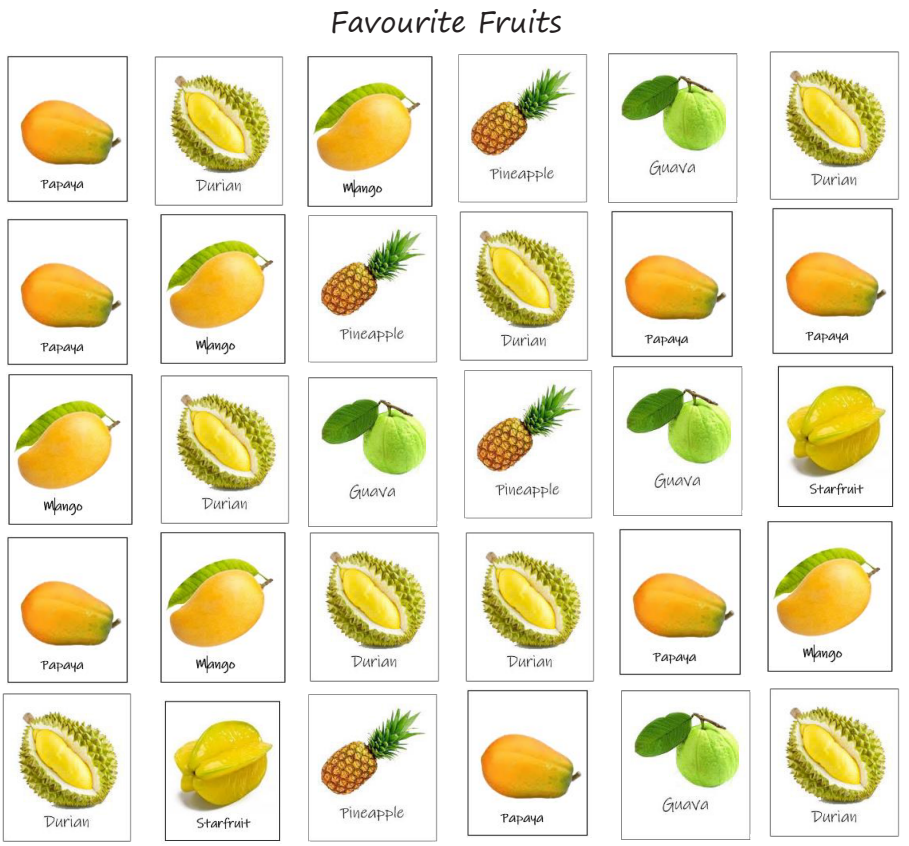



Diagram 1

- What is the most popular fruit?
- How many students choose pineapple as their favourite fruit?
- What can we do with the data collected in order to make it easier to answer these questions?

ii. With the purpose of answering the questions in (i), three students, A, B and C, counted the number of students for each category of fruits using different methods as shown in Diagram 2.

Student A's Method (Using Check Marks)

| Durian | Papaya | Pineapple | Mango | Starfruit | Guava |
|---|---|---|---|---|---|
| ✓✓✓✓✓✓✓✓ | ✓✓✓✓✓✓✓ | ✓✓✓✓ | ✓✓✓✓✓ | ✓✓ | ✓✓✓✓ |
| 8 | 7 | 4 | 5 | 2 | 4 |

Student B's Method (Using Tally Marks)

| Durian | Papaya | Pineapple | Mango | Starfruit | Guava |
|---|---|---|---|---|---|
| 𝍸 \|\|\| | 𝍸 \|\| | \|\|\|\| | 𝍸 | \|\| | \|\|\|\| |
| 8 | 7 | 4 | 5 | 2 | 4 |

Student C's Method (Line up the Cards)

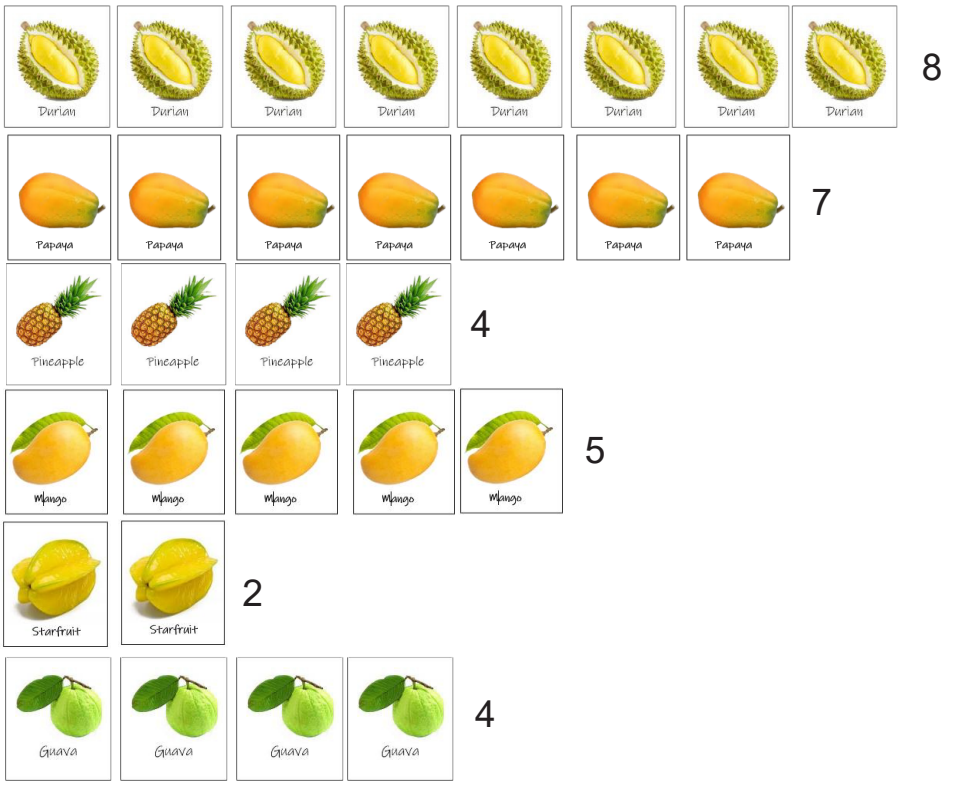



Diagram 2

- Compare the three methods of counting. Which method will be easiest for your students? Explain your reasons.
- Why is tally mark more efficient than check mark in counting the number of students in each category?

**Standards 3.2:**

Organising the data collected and represent using pictogram for easy visualisation
i. Produce the table and pictograms from collected data under each categories
ii. Interpretation of tables and pictograms as a simple conclusion about the data being presented.
iii. Appreciate pictograms through collecting data and adding data in daily activities in learners' life

**Sample Tasks for Understanding the Standards**

**Task 1: Representing Data**

i. Table 1 shows the data collected on students' favourite fruits.

Table 1
Students' Favourite Fruits

| Favourite Fruits | Durian | Papaya | Pineapple | Mango | Starfruit | Guava |
|---|---|---|---|---|---|---|
| Number of Students | 8 | 7 | 4 | 5 | 2 | 4 |

A student drew a pictogram as shown in Diagram 1 to represent the data in Table 1.

*Students' Favourite Fruits*

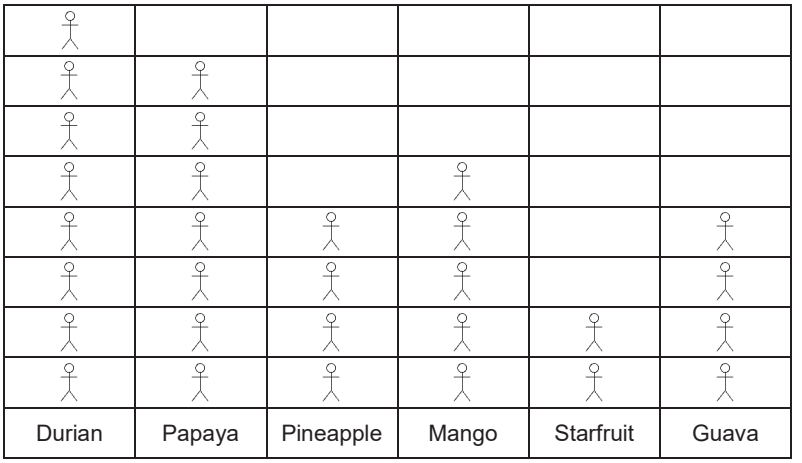



Diagram 1

- What is the most popular fruit?
- How many more students like durian than starfruit?
- Is it better to show the data in the table or in the pictogram? Explain your reasons.
- What could be done to the graph in order to make it easier to read?
- Three stores next to the school, each selling durian, papaya and mango respectively to the students. Another store sells other fruits. If each student is buying one favourite fruit from these stores, which store will have the highest number of fruits sold?
    - ❖ What is the consequence of changing the categories of this set of data?

ii. A class of 28 students is investigating their birthday in 2021. Each of them is given a smiley card as shown in Diagram 2 to represent himself/herself.

Diagram 2

Then, the students were asked to tell which day of the week their birthdays in 2021 will be by placing their smiley cards on a square grid drawn on a big piece of cardboard as shown in Diagram 3.

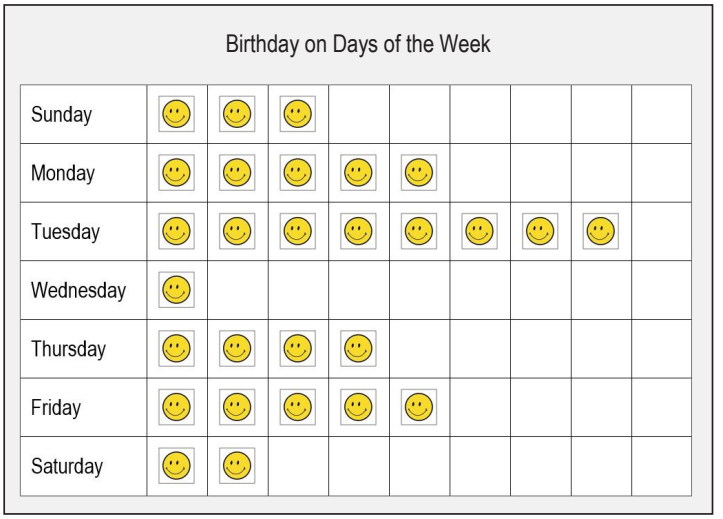



Diagram 3

- What does each  represent?
- How many more students have birthday on Tuesday than Friday?
- Is it necessary to order the categories of the data when drawing a pictogram? Explain your answer.

iii. Determine whether each of the following statements about pictogram is true or false.

_____ (a) A pictogram may not have a title.

_____ (b) A picture can only be used to represent 1 unit.

_____ (c) The pictures used must be of same size.

_____ (d) The pictures used must be equally spaced.

_____ (e) A key must be provided to determine the value of each picture.

_____ (f) A pictogram can be presented both horizontally and vertically.

iv. The pictogram in Diagram 4 shows the number of children using different means of transportation to school.

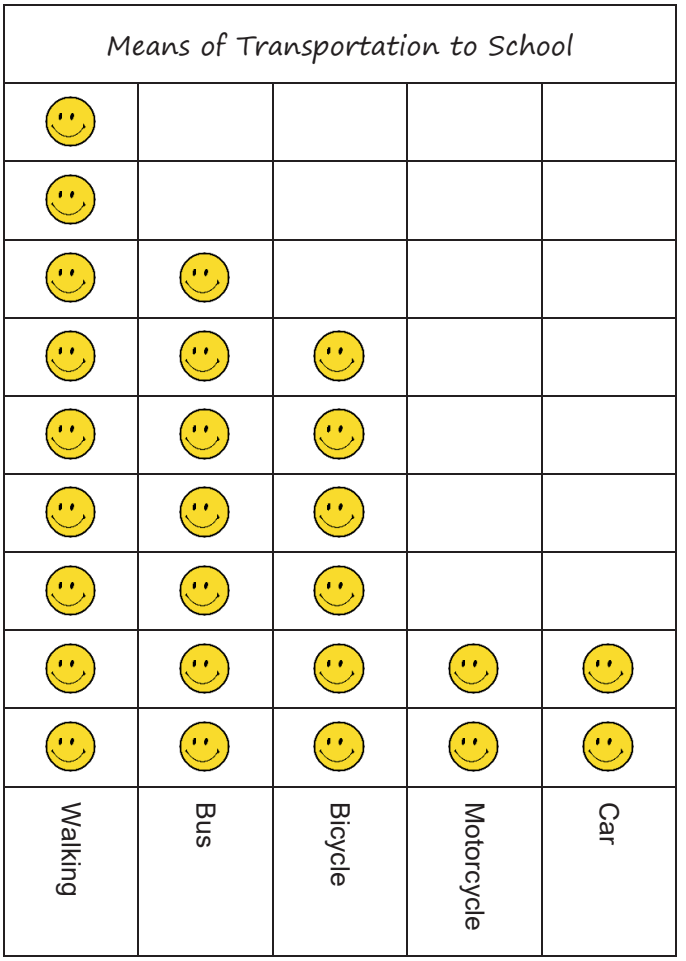



Diagram 4

- Looking at the pictogram, a student said there are 9 children who walked to school.
  - ❖ What is the misconception of the student?
  - ❖ How will you help the student overcome the misconception?

**Standards 3.3:**

Representing a data structure by using bar graph to predict the future of communities

i. Understand how to draw bar graph from table using data categories and sort the graph for showing its structure
ii. Appreciate ways of presenting data such as using tables, pictograms and bar graphs with sorting for predicting their future communities
iii. Appreciate the using of data for making decision

**Sample Tasks for Understanding the Standards**

**Task 1: Bar Graphs**

Table 1 shows the number of children in each grade who visited the school library during recess time for a week. Due to unforeseen circumstances, the data for Grade 6 is missing.

Table 1
Number of Children Who Visited the School Library for Grade 1 to Grade 6

| Grade | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of Children | 6 | 10 | 15 | 31 | 36 | ? |

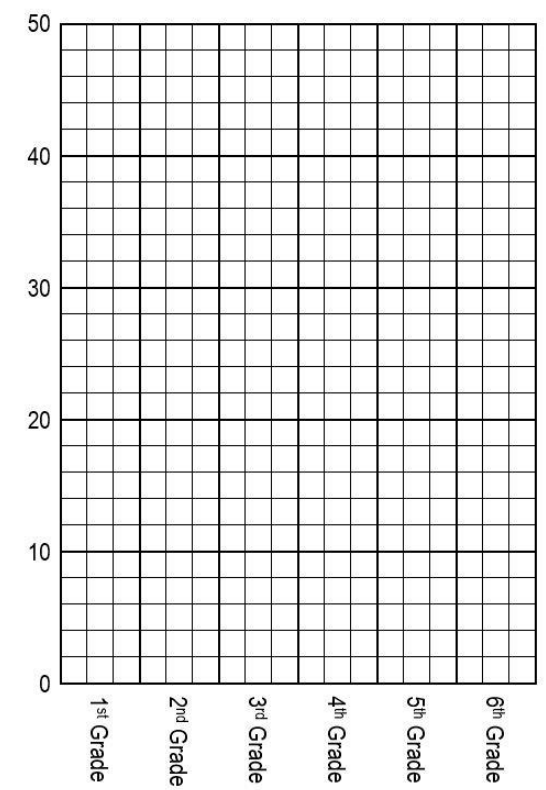



Diagram 1

i. Draw a bar graph on Diagram 1 to represent the data in Table 1.
   - What is the scale used on the vertical axis?
   - How do you draw the bars for Grades 3 and 4?

ii. Based on the bar graph, predict the missing number of children for Grade 6.
   - Explain the reasons for your prediction.
   - How confident are you with your prediction? Explain your answer.

iii. In what way can bar graphs help to develop appreciation towards mathematics among your students?

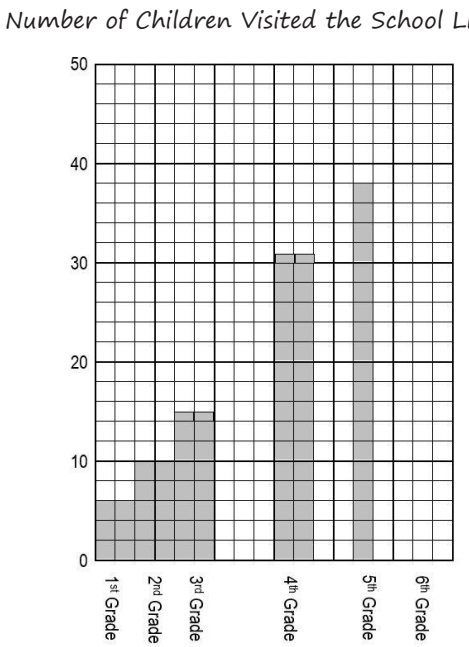



Diagram 2

iv. Diagram 2 shows a bar graph drawn by a student. There are two major errors in the bar graph.

- Identify and explain the major errors.
- How will you help your students to avoid these errors?

v. Diagram 3 shows a bar graph representing the number of teachers for each favourite colour.

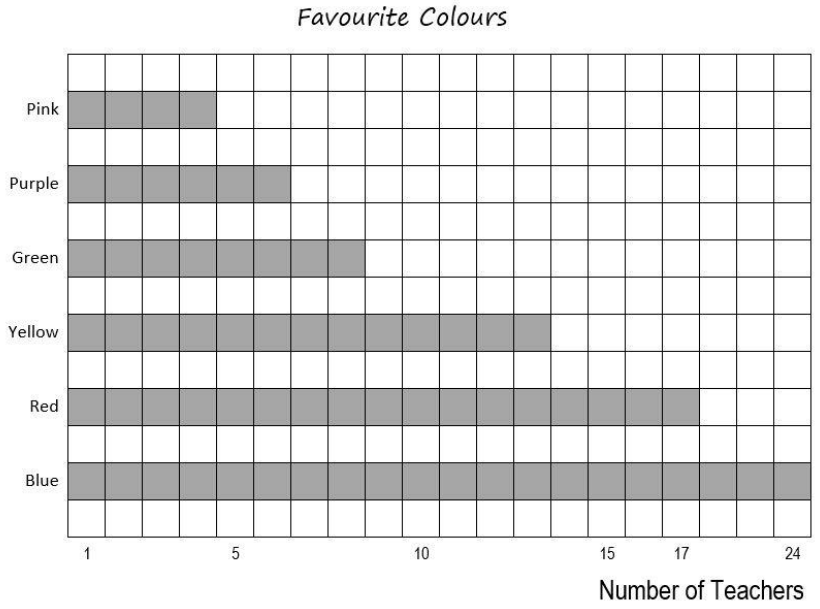



Diagram 3

There are two major errors in the scale used in this bar graph.

- Identify and explain the major errors.
- How will you help your students to avoid these error?

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# APPENDIX A

## Framework for CCRLS in Mathematics: Edited Edition

### Nature of Mathematics

Mathematics has been recognized as a necessary literacy for citizenship and not only living economically but also to establish a society with fruitful arguments and creations for better living. It has been taught as a basic language for all academic subjects using visual and logical-symbolic representations. In this information society, mathematics has increased its role to establish 21st century skills through reviewing mathematics as the science of patterns for future prediction and designing with big data which produces innovation not only for technology advancement but also for business model.

Mathematics is an essential subject to establish common reasoning for sustainable development of society through viable argument in understanding each other and develop critical reasoning as the habits of mind. Mathematics should be learned as basis for all subjects. For clarifying the framework in CCRLS on mathematics and by knowing the role of mathematics education, the humanistic and philosophical natures of mathematics are confirmed as follows:

The Humanistic nature of mathematics is explained by the attitudes of competitiveness and understanding of others by challenging mathematicians such as Blaise Pascal, Rene Descartes, Isaac Newton and Gottfried Wilhelm Leibniz. For example, if you read the letter from Pascal to Pierre de Fermat, you recognise the competitive attitude of Blaise Pascal to Fermat's intelligence and seek the way to be understood on the excellence of his finding on Pascal's Triangles. If we read Pascal's **Pensées,** you recognise how Pascal denied Descartes geometry using algebra from the aspect of ancient Greek geometry. On the other hand, Descartes tried to overcome the difficulties of ancient geometry by algebra. If you read the letter from Descartes to Elisabeth, you recognise how Descartes appreciated and felt happy the Royal Highness Elisabeth used his ideas of algebra in geometry. Despite being a princess, Elisabeth had been continuously learning mathematics in her life.

There were discussions on who developed calculus between Britain and Continent. On that context, Johann Bernoulli, a continental mathematician, posed a question on the journal about the Brachistochrone problem, locus of the point on circumference of the circle when it rotates on the line. There was no response and Bernoulli extended the deadline of the answer and asked Newton to reply. Newton answered it within a day. Finally, six contributions of the appropriate answer including Newton and other Continental mathematicians were accepted. All those stories show that mathematics embraces the humanistic nature of proficiency for competitiveness and understand others for sharing ideas.

The Philosophical nature of mathematics can be explained from ontological and epistemological perspectives. From the ontological perspective, mathematics can be seen as a subject for universal understanding and common scientific language. Plato and Aristotle are usually compared from this perspective. Plato believes that the existence of the world of "idea" and mathematics existed in the world of "idea" on Platonism. In this context, mathematical creation is usually explained by the word "discover" which means taking out the cover from which it has already existed. At the moment of discovery, reasonable, harmony and beautifulness of mathematical system is usually felt. Aristotle tried to explain about reaching an idea from the "material" to the "form". This explains that abstract mathematics can be understood with concrete materials using terms such as "modelling", "instruments", "embodiment", "metaphor" and "change representation". On this context, "invention" is also used for development of mathematical instruments, representation. and so on. From both ontological perspectives, mathematics can be understood and acquired by anyone and if acquired, it serves as a common scientific language that is used to express in any subject. Once representing the ideas using the shared common language, the world can be perceived in the same view autonomously.

From the epistemological perspective, mathematics can be developed through processes that are necessary to acquire mathematical values and ways of thinking. From this perspective, idealism and materialism are compared. In the context of Hegel, a member of German idealism, Imre Lakatos explained the development of mathematics through proof and refutation by using counter example. In another words, beyond the contradiction is the nature of mathematical activity and it provides the opportunity to think mathematically for overcoming. On this context, mathematics is not fixed but an expandable system that can be restructured through a process

of dialectic in constructing viable arguments. Plato also used dialectic for reaching ideas with examples of mathematics. The origin of dialectic is known as the origin of indirect proof. In education today, dialectic is a part of critical thinking for creation. Parallel perspectives for mathematical developments are given by George Polya and Hans Freudenthal. For the discovery of mathematics, Polya explained mathematical problem-solving processes with mathematical ideas and mathematical ways of thinking in general. Freudenthal enhanced the activity to reorganize mathematics by the term mathematisation on the principle of reinvention.

Genetic epistemologist Jean Piaget established his theory for operations based on various theories, including the discussion of Freudenthal and explained mathematical development of operations by the term reflective abstraction. Reflection is also a necessary activity for mathematisation by Freudenthal. On materialism, under the Vygotskyian perspective, intermediate tools such as language become the basis for reasoning in the mind. Under his theory, high-quality mathematical thinking can be developed depending on the high-quality communication in mathematics classrooms. Dialectical-critical discussion should be enhanced in the mathematics class. From both the epistemological perspectives, mathematics can be developed through the processes of communication, problem solving and mathematisation which include reorganization of mathematics. Those processes are necessary to acquire mathematical values and ways of thinking through reflection.

**Aims of Mathematics in CCRLS**

The aims of mathematics in CCRLS for developing basic human characters, creative human capital, and well qualified citizens in ASEAN for a harmonious society are as follows:

- Develop mathematical values, attitudes and habits of mind for human character,
- Develop mathematical thinking and able to engage in appropriate processes,
- Acquire proficiency in mathematics contents and apply mathematics in appropriate situations.

Framework for CCRLS in Mathematics as shown in Figure 3 is developed based on the three components with discussions of the humanistic and philosophical nature of mathematics. This framework also depicts the concrete ideas of mathematics learning of the above aims.

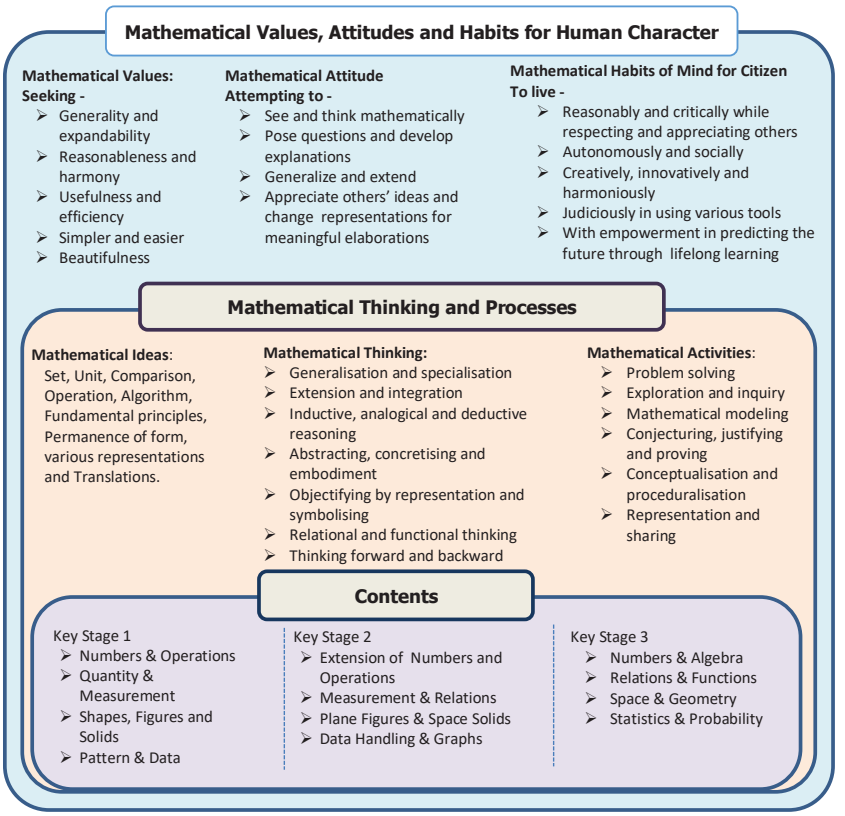


*Figure 3.* CCRLS Framework for Mathematics and Aims of Mathematics Learning: Old edition

### Mathematical Values, Attitude and Habits for Human Character

For cultivating basic human characters, values, attitudes and habits of mind are essentials to be developed through mathematics. Values are basis for setting objectives and making decisions for future directions. Attitudes are mindsets for attempting to pursue undertakings. Habits of mind are necessary for soft skills to live harmoniously in the society. Mathematical values, mathematical attitudes and mathematical habits of mind are simultaneously developed and inculcated through the learning of the content knowledge.
Essential examples on values, attitudes and habits of mind are given in Figure 3. On mathematical values, generalisable and expandable ideas are usually recognised as strong ideas. Explaining why a proving is necessary in mathematics is a way of seeking reasonableness. Harmony and beautifulness are described not only in relation to mathematical arts, but also in the science of patterns and system of mathematics. Usefulness and simplicity are used in selection of mathematical ideas and procedures.

On mathematical attitudes, "seeing and thinking mathematically" means attempting to use the mathematics learned for seeing and thinking about the objects. Posing questions and providing explanation such as the "why" and the "when" are ordinary sequence for thinking mathematically. Changing representation to other ways such as modelling can overcome the running out of ideas in problem solving. The mindset for trying to understand others is the basis to explain one's own ideas that is understandable by the rest with appreciation. Producing a concept with definition operationally is a manner of mathematics.

On mathematical habits of mind for citizens to live, mathematical attitudes and values are necessary for reasoning critically and reasonably. Appreciating and respecting other ideas is also necessary. Mathematics is developed independently for those who appreciate life creatively, innovatively and harmoniously. Seeking the easier and effective manner of selecting appropriate tools is necessary. Mathematics is a subject to challenge and experience competitiveness, appreciation with others, develop the mindset for lifelong learning, personal development and social mobility.

### Mathematical Thinking and Processes

For developing creative human capital, mathematical ideas, mathematical ways of thinking and mathematical activities are essential. Mathematical ideas are process skills involving mathematical concepts. Mathematical thinking is mathematical way of reasoning in general which does not depend on specific concepts. Mathematical activities are various types of activities such as problem solving, exploration and inquiry. Mathematical processes which include these components are necessary skills to use mathematics in our life, such as innovation in this society (e.g. Internet of Things (IOT)). In the context of education, competency referring to mathematical processes is the basis for STEM and STEAM[1] education as well as basis for social science and economy education.

Mathematical ideas serve as the basis of content knowledge related to promoting and developing mathematical thinking. Some key ideas of mathematics are used as special process. The fundamental ideas of set and unit lead to a more hierarchical and simple structural relationship. The ability to compare, operate, and perform algorithm of related functions enables efficient ways of learning mathematics and solving problem in learners' life with mathematics.

In the case of set, set is a mathematical ideas is related with conditions and elements. It is related with activity in grouping and distinguishing with other groups by conditions. Example, 3 red flowers and 4 white flowers become 7 flowers, if we change the condition of the set by not considering the colours. "A" and non A" is a simple manner to distinguish sets with logical reasoning. For categorizing, we use intervals such as $x > 0$, $x < 0$, $x = 0$. This situation can be seen in the hyperbolic graph where $y=1/x$.

In the case of unit, it is a mathematical idea that is related with the process to produce and apply the unit with operations. On some cases, trying to find the common denominator is the way to find the unit of two given quantity. Tentative unit such as arbitrary units can be set and applied locally whereas standard units are used globally. In the combination of different quantities, it produces new measurement quantity such as distance with respect to time produce speed. Square unit such as square centimetres is a unit for area.

[1] *STEM refers to Science, Technology, Engineering and Mathematics. STEAM refers to Science, Technology, Engineering, Arts and Mathematics or Applied Mathematics.*

Mathematical thinking is well discussed by George Polya. Inductive, analogical and deductive reasoning are major logical reasoning at school. However, deductive reasoning is enhanced in relation to formal logic and inductive and analogical reasoning are not well recognized. Polya enlightens the importance of those reasoning in mathematics. On the process of mathematisation by Hans Freudenthal, objectifying of the method is necessary. David Toll mentioned it by the term thinkable concept on the process of conceptual development. Polya mentioned thinking forward and backward in relation to ancient Greek term analysis.

Mathematical activities are ways to represent mathematical process. Problem solving process was analysed by Polya. He influenced problem solving with various strategies. Technology enhances the activities of conjecturing and visualizing for inquiries. Conceptualization is done based on procedures such as the procedure 3+3+3+3=12, become the basis for 4x3. The proceduralisation of multiplication is done through developing the multiplication table, idea of distribution and memorizing.

## Content

For cultivating well qualified citizens, content knowledge of mathematics is essential. Content of mathematics is usually divided by the set of mathematics. However, for developing human characters and creative human capitals, it should be developed through the mathematical processes. Values, attitudes and habits of mind are driving force for engagement in mathematical processes. Thus, without involving human character formations with mathematical process skills, content knowledge of mathematics cannot be realised. The content is divided into three stages in CCRLS and every stage has four strands. Between the stages, the names of the strands are directly connected and those on the standard level are well connected too. The names of strands for every key stage are as follows:

| Key Stages | Strands |
|---|---|
| Key Stage 1 | Numbers and Operations<br>Quantity and Measurement<br>Shapes, Figures and Solids<br>Pattern and Data Representations |
| Key Stage 2 | Extension of Numbers and Operations<br>Measurement and Relations<br>Plane Figures & Space Figures<br>Data Handling and Graphs |
| Key Stage 3 | Numbers and Algebra<br>Relations and Functions<br>Space and Geometry<br>Statistics and Probability |

In every stage, four content strands are mutually related[2]. Between the key stages, all strands in different key stages are mutually related. The same content strand names are not used to indicate development and reorganisation beyond each stage. For example, “Numbers and Operations” in Key Stage 1, “Extension of Numbers and Operation” in Key Stage 2, and “Numbers and Algebra” in Key Stage 3 are well connected. These names of the content strands show the extension and integration of contents. For example, even and odd numbers can be taught at any stage with the different definition. At Key Stage 1 even numbers can be introduced as “counting by two” which does not include zero. In Key Stage 2, it can be re-defined by a number divisible by two. Finally, in Key Stage 3 it can be re-defined as a multiple of two in integers which includes zero. Although we use the same name as even number, they are conceptually different. The definition in Key Stage 1 is based on counting, Key Stage 2 is based on division while Key Stage 3 is based on algebraic notation[3].

[2] *Strands used to explain mutual relation of content (Jeremy Kilpatrick, Jane Swafford, Bradford Findell. “Adding it up”, National Academies Press. 2001). The term domain is sometimes used for compartmentalization through categorisation of content.*

[3] *In algebraic notation of numbers, addition and multiplication are major operations. Subtraction can be represented by addition of negative numbers and division can be represented by reciprocal or multiplicative inverse property.*

Expressing such theoretical differences requires name of strands for content be distinguished. In the case of measurement, there is no strand name of measurement in Key Stage 3. Key Stage 1 relates with quantity and setting the units. In Key Stage 2, it extends to non-additive quantity beyond dimension. In Key Stage 3 the idea of unit and measurement is embedded in every strand. For example, square root in Numbers and Algebra strand is an irrational number which means unmeasurable, Pythagorean Theorem in Space and Geometry strand is used for measuring, proportional function in Relations and Functions strand is used for counting the number of nails by weight, and in Statistics and Probability strand, new measurement units are expressed such as quartile for boxplot.

### Context to Link the Three Components

Three components in Figure 3 should be embedded in every key stage as standards for the content of teaching. “Mathematical values, attitudes, habits for human character” component and “Mathematical thinking and processes” component cannot exist without “Content” component. The first two components can be taught through teaching with the content. For teaching those three components at the same time, context is introduced as shown in Figure 4. For example, context 1 is considered as problem solving situation in the real world situation and context 2 is considered as mathematical situation under the mathematical task sequence to develop further mathematics itself.

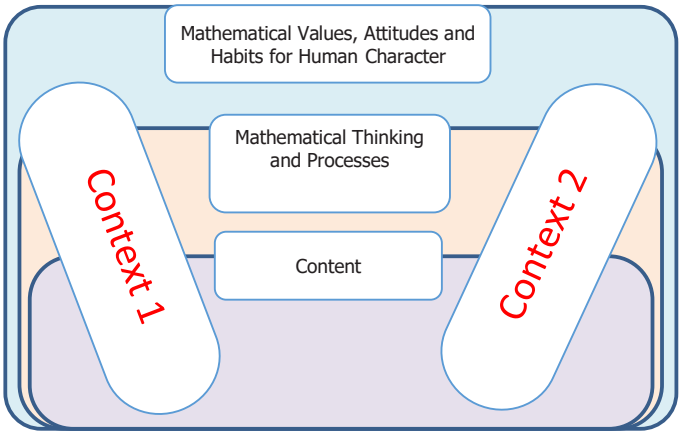


*Figure 4.* Interlinking of the three components with the context

On a given context, three components are well connected. For well connecting, classroom activities for developing competencies should be designed to link all of them. The following contexts are samples:

- Explore a problem with curiosity in a situation and attempting to formulate mathematical problems
- Apply the mathematics learned, listen to other’s ideas and appreciate the usefulness, power and beauty of mathematics
- Enjoy classroom communications on mathematical ideas in solving problems with patience and develop perseverance
- Feel the excitement of “Eureka” with enthusiasm for the solutions and explanation of unknown problems
- Think about ways of explanation using understandable representations such as language, symbols, diagrams and notation of mathematics
- Discuss the differences in seeing situations before and after learning mathematics
- Explain, understand others and conclude mathematical ideas
- Explore ideas through inductive and deductive reasoning when solving problems to foster mathematical curiosity
- Explore ideas with examples and counter examples
- Feel confident in using mathematics to analyse and solve contextual problems both in school and in real-life situations
- Promote knowledge, skills and attitudes necessary to pursue further learning in mathematics
- Enhance communication skills with the language of mathematics
- Promote abstract, logical, critical and metacognitive thinking to assess one’s own and other’s work
- Foster critical reasoning for appreciating other’s perspectives

- Promote critical appreciation on the use of information and communication technology in mathematics
- Appreciate the universality of mathematics and its multicultural and historical perspectives

Those contexts are chosen for illustrating the interwoven links of the two components with contents. It looks like methods of teaching, however all the three components are the subjects of teaching on the contexts.

Extracted and edited from:

SEAMEO Basic Education Standards (SEA-BES): Common Core Regional Learning Standards (CCRLS) in Mathematics and Science. SEAMEO RECSAM. (Mangao, Ahmad, & Isoda, 2017, p. 2-11)

# APPENDIX B

## Terminologies Explained

### Mathematical Thinking and Processes

Higher order thinking is the terminology for curriculum but it is not specified in mathematics. Here, it is explained generally as acceptable terms from the perspective of mathematics in education. On the Mathematical thinking and processes, the following terms are the sample which can be seen on SEA-BES: CCRLS for making clear descriptions of the objective of teaching. If you can use these terminologies for writing the objectives of teaching, you would be able to consider how you teach them in the process. Mathematical Thinking can be explained through Mathematical Ideas, Mathematical Ways of Thinking, and Mathematical Attitude. Mathematical attitude is a component of the Value, Attitude and Human Character Formation in Appendix A. Mathematical Ideas, Mathematical Ways of Thinking can be developed through the reflection of the process and Value and Attitude can be developed through appreciation.

#### Mathematical Ideas

Even through every mathematics content embedded some ideas, there are essential mathematical ideas which are used in various occasions. Mathematical ideas are not exclusive but functions as complementary. The followings are samples of essential mathematical ideas.

| Terminology | Explanation |
|---|---|
| **Set** | A set is a collection of elements based on certain conditions. When the condition of the set changes, result of reasoning related to the set may change too. Sets are compared by one-to-one correspondence. Basically, the idea of set is reflected through activities that require us to think about membership (elements and conditions) of a set. In addition, activities involving subsets, cardinality and power are extended ideas of set. Number of elements called cardinal or cardinal number (or set number). Ordinal number does not imply number of elements. Other ideas include operations of sets such as union, intersection, complement, the ordered pair/combination of elements such as Cartesian products and dimension mapping. A number system is a set with structures that has the structures of equality, order (greater, less than), and operations, which is developed and extended throughout the curriculum from natural number to complex number: the set of complex numbers does not have the structure of order. In Mathematics, a set is the bases to set algebraic structure and the bases for logic such as universal proposition and existential proposition. |
| **Unit** | Unit is necessary for counting, measurement, number line, operations and transformation. It is represented as “denomination’ for discrete quantity, such as 1 “apple’ for situations involving counting, or continuous quantity, such as 1 gram for situations involving measurement. Mathematically, unit is used to indicate a number by mapping it with the quantity in a situation.<br><br>In a situation, it can be fixed based on the context of comparison, which can either be direct or indirect comparison. In this context, a remainder or a difference from a comparison can be used for fixing a new arbitrary unit for measurement which is a fraction of the original unit. This process of determining a new unit is the application of Euclidean algorithm for finding the greatest common divisor.<br><br>For the base-10 place value number system, every column is defined by the units such as ones, tens, hundreds and so on. However, in other place value number system such as the binary system, every column is defined by the units such as |

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| | ones, twos, fours and so on. Therefore, in a place value number system, the unit is not always a multiple of the power of ten.<br><br>In addition, various other number systems are made up of different units. For the calendar system, the lunar calendar is based on 30 (29.5) days, while the solar calendar is based on 365 (365.25) days. On the other hand, the imperial and U.S. customary measurements include units in the base-12 and base-16 systems. In the ancient Chinese and Japanese systems, there were units in the base-4, thus also included the base-16. On the other hand, the units used in different currency systems are dependent on differing culture and countries. However, many countries had lost the unit of 1/100 on their currency systems, which originated from "per centos" that means percent. Even though the based-10 place value system is used to represent the value of money, many currency systems are using the units for 2, 5, and 25 in their denominations instead.<br><br>Unit for a new quantity can be derived from ratio of different quantities. For example, the unit for speed (km/h) is the ratio of distance (km) over time (h) which cannot be added directly. A car moves at 30km/h and then moves at 20km/h does not mean the car moves at 50km/h.<br><br>Identity element for multiplication is one but additive identity is zero. Identity for multiplication is the base for multiplicative and proportional reasoning. Inverse element for multiplication is defined by using one. |
| **Comparison** | Comparison of concrete objects can be done directly or indirectly without measurement unit. As mentioned at the Unit, direct comparison can be used to fix a new unit of measurement, whereas indirect comparison can be used to promote logic for transitivity which includes syllogism.<br><br>Comparison of multiple denominated numbers with different unit quantities on the same magnitude such as 5.2 m and 5 m 12 cm can be done if they are represented by a single denominated number by the unified unit quantity, such as 520 cm and 512 cm. Furthermore, comparison of expressions which has the same answers on the same operation such as 2+4, 3+3 and 4+2 can be used to find rules and patterns. For example, 2+4 = 4+2 can show commutative rule for addition whereas for 2+4 = 3+3 can be used to show pattern when 1 is added to 2 and subtracted from 4, the sum is still the same.<br><br>Comparison of fractions is an activity to find the unit fraction. For comparison of fractions such ad $\frac{1}{2}$ and $\frac{1}{3}$, we have to find the unit fraction $\frac{1}{6}$ which can measure $\frac{1}{2}$ and $\frac{1}{3}$. $\frac{1}{6}$ is the common denominator for $\frac{1}{2}$ and $\frac{1}{3}$. The algorithm to find the unit fraction as the common denominator is called 'reduction of fraction'.<br><br>On numbers, the relationship of two numbers can be equal, greater or less. The number set up to real number is a total/linear order set, thus up to real number set, two numbers can be compared. However, complex number as an extension of real number cannot be compared directly because it has two dimensions.<br><br>On the number line as real number, the size of number (distance) is defined by the difference, 1 = 2-1 = 3-2 = 4-3 =.... Here the difference is the value of subtraction as binary operation and can be seen as the equivalence class. On the idea of equivalence class, the value of operations can be compared. On the plane such as complex plane, even though the number is not simply ordered, the size of |

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| | number (distance) is defined by Pythagorean theorem. By using this definition, $I1I= I(\frac{\sqrt{2}}{2})(1+ i)I = IiI = I(\frac{\sqrt{2}}{2})(1- i)I=...$ the theorem produce the distance on the plane and the distance can be compared.<br><br>Explained at the Unit, on measurement, the magnitude is given by defining the unit of magnitude. One of the ways to produce the unit magnitude is a direct comparison which provides the difference and Euclidean Algorithm produce the unit of measurement as the greatest common devisor. Another way to produce the unit is by using the ratio and so on. Such a newly produced magnitude usually lost the linearity. In Physics, 'db' is the size of volume which is produced by the common logarithm of sound pressure. 'db' is fitting well for human impression of the size of sounds on its linearity. It is known as Weber-Fechner's law that human senses are proportional to the logarithm of stimulus. On science, logarithmic scale is used for Semi-log graph and Log-log graph for demonstrating the linearity even it is an exponential phenomenon. Logarithm produce the scale to illustrate multiplicative phenomena as an additive phenomenon.<br><br>In any mathematical investigation, particularly in the mathematics classroom, problem solving approach, comparisons of various ideas, representations and solutions are key activities for discussion and appreciation. This comparison is a nature of mathematical activity. |
| **Operations** | Addition, Subtraction, multiplication and division are four basic arithmetic operations. Mathematically, these are binary operations involving any two numbers with symbols of operations, +, –, × and ÷. Polynomial expressions can be seen as a combination of binary operations. Mental arithmetic may be used in column method with the base-10 place value system. An operation is not just a rule but can be demonstrated by using various representations. For example, an operation can be represented by the manipulation of concrete objects as well as expressions. However, the manipulating process is not the same as the operation process because it cannot be recorded without using a diagram.<br><br>From Key Stage 3 up to the field theory at the university levels, arithmetic operations are expressed as addition and multiplication. On the axiom of vector space, negative vector is represented by using the minus symbol. Mathematically, operations and set are used for to establish mathematical structure. |
| **Algorithm** | Algorithm is a set of sequential activity at the special situation to produce a particular solution to a task. The column method is based on the base-10 place value system. When using this method, 200 + 300 is done by just 2 + 3 on the hundreds place value. This algorithm is adapted for mental calculation. Representation of column method is not universal like an expression as algebraic representation. Algorithm for the column method is fixed as a formal form based on culture, however it can be created from the manipulation of the base ten blocks.<br><br>A formula also functions like an algorithm. It can be applied without understanding its meaning. However, it cannot be created without understanding and recognising its underlying structure or meaning. If we understand the structure or meaning such as ratio and proportionality, it is not necessary to memorise it. In Informatics, algorithm is usually defined by programing language through analysing procedure and setting several functions by the program are necessary. |

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| **Fundamental Principles** | Fundamental principles are the rules which is related with mathematical structures and forms in general.<br><br>Commutativity, Associativity and Distributivity are three fundamental principles for arithmetic operations. Commutativity does not work on subtraction and division. On the discussion of Distributivity, if *a*, *b*, and *c*, are positive numbers, then the expressions $a(b + c)$, $(b + c)a$, $a(b – c)$, and $(b – c)a$ are different. However, if *a*, *b*, *c* is both positive and negative numbers, then the four expressions can be seen as the same.<br><br>There are also other fundamental principles for arithmetic operations at the elementary level such as the followings:<br><br>$1 + 9 = 10$ $\quad\quad$ $2 \times 3 = 6$<br>$\downarrow +1 \quad \downarrow -1$ $\quad\quad$ $\downarrow \times 10 \quad\quad \downarrow \times 10$<br>$2 + 8 = 10$ $\quad\quad$ $20 \times 3 = 60$<br><br>$8.1 \div 9 = 0.9$ $\quad\quad$ $8.1 \div 9 = 0.9$<br>$\downarrow \times 10 \quad\quad \uparrow \div 10$ $\quad\quad$ $\downarrow \times 10 \quad \downarrow \div 10$<br>$81 \div 9 = 9$ $\quad\quad$ $8.1 \div 90 = 0.09$<br><br>Principles can be identified through comparison. They are necessary for the explanation of algorithms and thinking about how to calculate by using models and other representations. On the extension of numbers and operations, principles are used for the discussion of the permanence of form (see the permanence of form).<br><br>Not only algebraic participles, there are other forms that can be extended. In geometry, the extendable nature of a line changes its functions in curriculum. For example, shape is extended to figure; edge, which may include the inner part of a shape, is extended to side, which may not include the inner part of a figure. Then, the side is extended to a line which enable the discussion on the possibility of escribed circles. In addition, parallel lines are necessary to derive the area formula for triangles with various heights. |
| **Permanence of Form** | Permanence of the equivalence of form, Hankel's Principle, is known as Commutativity, Associativity, and Distributivity for algebra for the field theory.<br><br>Permanence of form had appeared in history of mathematics in 16th century and functioned to shift from arithmetic algebra to symbolic algebra. On Peacock's Permanence of Form, it is not only the limited three rules like Hankel's, but it is applied to algebraic symbolic form for overcoming arithmetic algebra which had limited the positive number and 0.<br><br>In Education, the form is not a limited expression but includes the patterns and the permanence of form can be used in various occasions. Especially, it is used for the extension of numbers and operations from elementary level to secondary level education like the followings:<br><br>$(+3) + (+2) = +5$ $\quad\quad$ $(+3) - (+2) = (+1)$<br>$\downarrow -1$ $\quad\quad$ $\downarrow +1$<br>$(+3) + (+1) = +4$ $\quad\quad$ $(+3) - (+1) = (+2)$<br>$\downarrow -1$ $\quad\quad$ $\downarrow +1$<br>$(+3) + 0 = +3$ $\quad\quad$ $(+3) - 0 = (+3)$<br>$\downarrow$ $\quad\quad$ $\downarrow$<br>$(+3) + (-1) = ?$ $\quad\quad$ $(+3) - (-1) = ?$ |

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| | The '?' are unknown, not yet learned. However, people could imagine the '?' by analogical reasoning with the idea of the permanence of the patterns. Here, the permanence of patterns is used as hypothesis and it makes possible to apply it to the unknown cases.<br><br>Permanence of form is used for initiation of number in Key Stage 1 and later throughout all other key stages. For examples, it is used to explain the necessity of zero (0) and the sum of any number with zero (0) as at Key Stage 1. |
| **Various Representations and Translations** | Every specified representation provides some meanings on its essential nature of representation which can be produced by specified symbols and operations. Different representations have the different nature, use different symbols and operations. Every representation of specific mathematical idea has the limitation of interpretation on its nature, thus, thinking by using only one specified representation provides the limitation of reasoning and understanding. If one type of representation is translated to another type of representation, then the representation of idea can be interpreted in other ways. If the idea of specified representation with certain embedded nature is translated into various representations, a rich and comprehensive meaning and use will be produced. However, for making the translation meaningful, it is necessary to know the way of translations which consists with the correspondences between symbols and operations on different representations.<br><br>For example, proportional number lines only functions for teachers and students who know well how to represent the proportionality on the tape diagrams and number lines, and so on. If they know what it is, they can use it for explanation to produce the expression.<br><br>Comprehensive learning of mathematics by using various representations and translations are necessary however students have to learn how to represent it in other representations and translate, at first, as well as between expressions and situations. |

**Mathematical Ways of Thinking**

Mathematical ways of thinking support the student's thinking process by and for themselves.

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| **Generalisation and Specialisation** | Generalisation is to consider the general under the given conditions of situations through extending set or reducing conditions. Any task in mathematics textbooks is usually explained by using some examples which are known as special cases, however, it is usually preferred to discuss the general ideas. Given conditions are usually unclear in textbooks, however if students are asked to consider other cases such as by saying 'for example', the conditions become clear because we may need to consider the condition and set for producing other example. In the upper grades, variable, domain, range, parameter, discreet, continuous, zero, finite and infinite become the terms of set to consider the conditions for general.<br><br>Specialisation is to consider the thinkable or more simple example on situations and necessary to find the hidden conditions. Considering general with thinkable example is called generalisable-special example. In mathematics, general theory is usually stronger than local theory. It is one of the objectives for generalisation and specialisation.<br><br>In mathematical inquiry, the process is usually on going from special cases to a general case to establish a stronger theory. Thus, so many cases, task sequence of every unit in textbooks is progressed from special cases to general for generating |

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| | simple procedures, exceptionally the tasks for exercise and training the procedures which usually progress from general to special because students already learned the general.<br><br>For students engaging in generalisation and specialisation by and for themselves, students have to produce examples. Thus, developing students who say 'for example' by and for themselves is a minimum requirement for teaching mathematics. |
| **Extension and Integration** | Extension means extend the structure beyond the known set on the process of the enlargement of set. On this meaning, extension can be seen as a generalization.<br><br>The product of multiplication is usually increased if both the factors are natural numbers. However, if we extend it into fractions and decimals, there are cases where the products decease.<br><br>On the extension of structure beyond set, learned knowledge produce the misconception which is explained by the over generalisation of learned knowledge. In mathematics curriculum, we cannot initiate numbers from fractions and decimals instead of natural numbers. Thus, producing misconception is an inevitable nature of mathematics curriculum and its learning. Even through it is a source of difficulty for students to learn mathematics, it is the most necessary opportunity to think mathematically and to justify the permanent ideas to be extended. After experiencing the extension with misconceptions, if students learn what ideas can be extended, it is a moment of integration.<br><br>Extension and Integration is a mathematical process for mathematisation to reorganise mathematics. It implicates that the school mathematics curriculum is a kind of nets which connect local theory of mathematics as knot even though their strings/paths include various inconsistencies with contradictions. It is a long-term principle for task sequence to establish relearning on the spiral curriculum<br><br>In order to develop the way of thinking for extension and integration, teachers need efforts to clarify the repetition of both, the same patterns for extensions and the reflections after every extension in classroom. For example, there are repetitions for the extensions of numbers from Key Stage 1 to 3. On every extension of numbers, there are discussions of the existence of number with quantity, the comparison with equality, greater and lesser, and constructing operations of numbers with the permanence of form. Through the reflection of every repetition, students are able to learn what should be done on the extension of numbers. |
| **Inductive, Analogical and Deductive Reasoning** | These three reasoning are general ways of reasoning as for the components of logical reasoning in any subject in our life, however mathematics is the best subjects in schools to teach them.<br><br>**Inductive reasoning** is the reasoning to generalise the limited number of cases to the whole set or situations. Three considerable cases or more will be the minimum to be considered inductive reasoning. To promote inductive reasoning, teachers usually provide the table for finding the patterns, however it is not the way to develop the inductive reasoning. To promote inductive reasoning, students need to consider various possible parameters in a situation and choose two or further parameters by fixing other parameters. Then, consider the relationship in the situation for knowing the cause and effect, and subsequently, set the ways to check the cases by well ordering of the cause and get the effect, follow by recording the data in a table. To promote inductive reasoning, teachers usually provide the table first for finding the patterns, however it is not the way to develop the inductive reasoning. It is the concern of students because to develop inductive reasoning, students have to |

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| | learn the ordering of natural numbers is vital because it is the cause of effect and beautifulness of patters that is originated from the order of natural number. Table is a tool for finding pattern but students cannot produce their inductive reasoning by and for themselves as long as it is given by teachers. Students have to learn the way of ordering for finding the pattern, inductively because it is the manner of experiment in Science and Engineering.<br><br>**Analogical reasoning** is the reasoning to apply known ideas to the unknown set or situations when we recognise the similarity with the known set or situations. Depending on the context it is called abduction. Most of the reasoning to find ways of solution for unknown-problem solving by using what we already knew is analogical reasoning. In analogical reasoning, it is to recognising similarity between the unknown-problem and the known problems.<br><br>Even though the rule of translations between different representations are not well established, analogical reasoning may function as a metaphor for understanding. Many teachers explain operations by using diagrams. It appears meaningful to provide the hint for solving but most of students cannot use the hint by and for themselves because students do not recognise the similarity as in their own analogy. To develop analogical reasoning, the most necessary way is to develop the habit to use what students learned before by and for themselves. Providing assisting tasks before posing the unknown problem is also used as a strategy to find similarity.<br><br>**Deductive reasoning** is the reasoning with components of already approved notions and given by using 'if… then' and logics for propositions such as transitivity rule. 'If not' also functions for proof by contradiction as well as counter examples. In cases where the rules of translations are well established, the translations of various representations still function under their limitations too. Various methods for proving such as a complete induction is also done by deductive reasoning.<br><br>For finding the ways of explaining and proving, inductive and analogical reasoning are necessary, and analytical reasoning, thinking backward from conclusion to the given, is also used but these reasoning do not allow to write at the formal proof by deductive reasoning. Arithmetic and algebraic operation can be seen as automatised deductive reasoning. Most of students do it just by recognising the structure of expression intuitively without explaining why. For clarifying the reason, teachers are necessary to ask why?<br><br>For developing the three reasoning, knowing objectives of reasoning are necessary. Inductive reasoning is applied to find general hypothesises. Analogical reasoning is applied to challenge unknown problem solving. Deductive reasoning is applied to explain or proof in general on local system. |
| **Abstracting, Concretising and Embodiment** | Abstracting and concretising are changing perspectives relatively by changing representations such as expressions. Abstracting is usually done for making clear a structure. Concretising is usually done for making ideas meaningful by concrete objects. For numerical expressions, manipulative and diagrams function as concrete. For algebraic expression, numerical expressions function as concrete objects. For both examples, abstract representations and concrete representations do not correspond one-to-one on translation because concrete representation usually have some limitations but concrete representations function as metaphors of abstract ideas.<br><br>Embodiment functions in both abstracting and concretising. When abstract ideas can be concretised, it implicates those abstract ideas are embedded with some specified concrete ideas. When concrete ideas can be abstracted, it implicates concrete ideas are embedded into abstract ideas. Both the embodiments function |

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| | for understanding ideas using metaphors but their translations are usually limited only for corresponding contents. In the case abstract and concrete objects are well translated on both directions, both of them function as mathematical representation. Otherwise, embodiment is a useful word for explaining the metaphor on the given context in process to establish mathematical representations in general. |
| **Objectifying operations for symbolising and establishing new operations on mathematisation** | A mathematical representation can be characterised by its symbols and operations with specified purpose and context. In the process of mathematisation, lower-level operational matters are usually objectified for new symbolising and its operations.<br><br>Until Key Stage 2, numbers do not mean the positive and the negative number. The number in red on financial matter is large if the number is 'large'. The number in black on financial matter is large if the number is 'large'. Here, the meaning of 'large' are defined at the opposite number rays, thus cannot be compared the numbers in red and black easily. At Key Stage 3, as for integration, we have to alternate new symbols and operations. We represent the red number by the negative symbol '-' and the black number by the positive symbol '+' and integrate the one direction for comparison into one dimensional number line.<br><br>Here, on Key Stage 3, 'larger' for comparison (operational matter on number rays) on lower level become the object of higher level to produce the comparison (operational matter on number line) for new number symbols with positive and negative.<br><br>It is the process of mathematisation by objectifying the operational matter to establish new symbols and operations. The process of abstracting from concrete can be seen as the process of mathematisation. |
| **Relational and Functional Thinking** | Relational and functional thinking are ways of thinking that can be represented by relation and function if we need describing them by mathematical notation. It has been used as major terminologies to explain mathematical thinking at the Klein Movement, 100 years ago. Relation in pure mathematics is a basic axiom. In relation to functional thinking, it is known as the ordered pairs between two sets. Mapping is a relationship and ordering is also a relationship and it supports keeping the structure or not. Equivalence class is a relationship in which axiom is defined by reflective, symmetric and transitive properties.<br><br>Function is a binary relation between two sets that associates every element of the first set to exactly one element to the second set. The second set is usually numbers in mathematics and both sets are numbers in education. Relational thinking is the thinking which can be represented by various representations such as graphs. It is the activity for students who do not know such representations, and teachers have to teach such representations on the necessity of students to engage in their activity: It is a welcome for students to create their necessary informal representation such as graph and diagram by themselves.<br><br>Historically, characteristics of functional thinking was described by Hamley (1934). He defined functional thinking by four elements: Class, Order, Variable and Correspondence, Class is a set which has possibility to include equivalence in operations such as 5 can be seen as the value of 1+4, 2+3, 3+2, 4+1. Order is discussed within a set and between sets. Variables are domain and range which are not the same as the origin set and the destination set. Correspondence is discussed between domain and range. |

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| | Functional thinking is useful to predict and control a situation. In elementary level, proportionality and operations are used for functional thinking on this objective. For example, proportional number line is a representation of functional thinking. For knowing the properties of each function, tables and graphs are useful. Teachers usually provide the tables and the graph papers from the beginning of experiment for finding patterns and properties of function. However, this is not the way to develop functional thinking because teachers take over the opportunities from students to consider and fix the sets, orders, variables, correspondences and so on by themselves.<br><br>Rate of change is used to check if a function has linearity. In the case if it is constant, it is a liner function. If not, otherwise. The limit of the rate of change for finding the tangent is the definition of differentiation. To show pattern beyond order, semi-log graph and logarithm graph are in Physics and Engineering.<br><br>Correlation in statistics is a relation but not necessary functions as a causal relation. |
| **Thinking Forward and Backward** | Thinking forward and backward are the terminologies of Polya. In Pappus of Alexandria ($4^{th}$ Century A. D.), thinking forward corresponds to synthesis and thinking backward corresponds to analysis. Synthesis is the deductive reasoning (proving) from the given and known whereas analysis is the reasoning from the conclusion for finding the possible ways of reasoning from the given and known.<br><br>From Ancient Greece, analysis is a method of heuristics which was the ways to find adjoin lines on construction problem, valance for area and volume problem. Descartes used unknown *x* for algebraic problem. Leibniz used unknown limit *x* of the function for calculus. These are hypothetic-heuristic reasoning beginning from the conclusion, such as if the construction is achieved, if the valance is kept, if the unknown *x* is given, and if the limit of *x* existed. Since analysis began from hypothesis, without proving from the given, people used to believe that analysis produced tautology which is not allowed to write in the system. On the other hands, modern mathematics system itself begins from the axiom as presupposition. Thus, if the ways of analytic reasoning become a part of presupposition, it is allowed in a written form. On this reason, the unknown *x* is able to be written in Algebra and the limit of *x* is able to be written in Calculus. However, in school mathematics, before such reformations of mathematics, analysis, a method of heuristics, do not allow to be written in a part of proof. It is a reason why some mathematics textbooks look very difficult to understand because they have to be written in the form of deductive reasoning for the construction of the system from axiom which does not include heuristics and ways of findings. On this consequence, there are old fashioned textbooks which just oriented to exercise the procedures as rules without explaining why.<br><br>Current school textbooks, which orients to write the problem solving process with various solutions as well as misconception by using what already learned, include the heuristics such as thinking backward and so on. In the standards of Key Stage 1 and 2, addition and subtraction are inverse operation, and multiplication and division are inverse operation as for verification of answers on operations. Such ideas are reformulated at the algebra in Key Stage 3.<br><br>In the case where 'If your saying (conclusion) is true, it produces contradiction which we already knew' is known as dialectic in communication and it was formalised as the proof by contradiction in mathematics. It is also the way of analysis for thinking backward but accepted as the way of proving by counter example and Reductio ad absurdum. In mathematical communication, thinking backward is a part. Without |

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| | the preparation of lesson plan which includes thinking backward, teachers cannot realise the classroom communication including misconception because the counter example is the component for the proof by contradiction. Though the communication of both objective and way of reasoning such as thinking backwards by using what students already learned, we are able to develop students' mathematical thinking. |

**Mathematical Activities**

Mathematical Activities usually explain the teaching and learning process but also embedded the mathematical ideas, thinking, values and so on. As for style of teaching approach, they are usually enhanced however the style of teaching itself does not make clear them.

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| **Problem Solving** | Pure mathematicians inquire problems that are never solved yet and they develop new theorems for solving their problems. It produces a part of system. Such authentic activity is the model of problem solving in education because it usually embedded rich ideas, ways of thinking and values in mathematics. As mathematicians usually pose problems for themselves, the activity includes problem posing and reflection which is necessary for establishing new theories.<br><br>In education, there are two major approaches for embedding them in learning.<br><br>The first one is setting the time, unit or project for the problem solving. Here, solving problem itself is an objective for students. It focused on heuristics: it is usually observed unexpectedly and to plan it is inevitably not easy. On this difficulty, the problem solving tasks are usually provided in two types. The first type focuses on mathematical modelling from the real world. The second type focuses on open ended tasks for students because it provides the opportunity for various solutions.<br><br>The second one is called the problem solving approach, tries to teach content through problem solving in classes. This case is only possible if teachers prepare the task sequence that enabling students to challenge the unknown-task by using what students already learned (ZPD). In problem solving approach, the tasks given by teachers are planned for students where they are able to learn the content, mathematical ideas and ways of thinking. For teachers, solving the tasks itself is not the objective of their classes but students reveal the objectives of teaching by teachers through recognising problematic as an unknown and finding the way of solutions. For students to be able to learn by and for themselves, it is necessary to plan the class with the preparation of future learning as well as by using learned knowledge. Textbooks such as the Japanese textbooks equipped task sequence for this purpose. In such textbooks, heuristics is not an accidental matter but a purposeful matter because every task in the textbook will be solved by utilising the already learned representation and so on. It is called a 'guided discovery' under ZPD because it expects well-learned students on the learning trajectory and never expects the genius students on producing unknown ideas.<br><br>For observers, the way of teaching by the problem solving approach in a class cannot be distinguished with the open-ended approach. The open approach is not necessary to prepare the task sequence because it is characterised by an independent open-ended task. If teachers set the open-ended task independently, it is the first approach. If teachers set the task sequence of open ended tasks for learning mathematics itself, it is the problem solving approach, the second one. Both of the approaches were known as Japanese innovation in textbooks since 1934 for elementary level and since 1943 for secondary level which was done by Prof. Shimada Shigeru and others. Prof. Nobuhiko Nohoda was known as he theorized the open approach with open-ended task and task sequence. Currently, |

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| | Japanese textbooks until the middle school level equipped the task sequence for problem solving approach.<br><br>Even though problem solving in education resemble activities of authentic mathematicians, it is actually not the same because problems of mathematicians are usually unsolvable beyond years and decades, while tasks in classroom can be explained by teachers who posed the problems. When teachers refer problem solving in education, it includes various objectives such as developing mathematical ideas, thinking, values and attitudes. These terminologies are used in education to develop students who learn mathematics by and for themselves while mathematicians only use some of them. On teacher education, if teachers only learned the content of mathematics, they may lack the opportunity to learn the necessary terminology. If teachers do not know it, the higher order thinking becomes a black box which cannot be explained. |
| **Exploration and Inquiry** | Exploration has been enhanced in finding hypothesis by using technology such as Dynamic Geometry Software and Graphing Software. These software provide the environment for students to explore invariant in diversity, easily. Exploration on the environment support to produce hypothesis.<br><br>Inquiry which includes exploration orients to the justification and proving through reflections.<br><br>Both exploration and inquiry enhance questioning by students. Thus, the process and finding will be depending on students' questioning sequence, not likely on problem solving by the task which teachers are able to design before the class. |
| **Mathematical Modelling, Programming and Mathematisation** | Mathematical modelling is a necessary way to solve real world problems in the world. Mathematical model is hypothetically set by using mathematics to represent the situation of the problem. Mathematical answers based on the model are confirmed in the situation through interpretation. Modelling enhanced various possibility to apply various representations in mathematics.<br><br>In education, modelling has been enhanced for the problem solving after the new math movement which recognised school mathematics with set and structure. In this era of Artificial Intelligence (AI) and Big Data, modelling is done through programming with algorithms and it is also a part of mathematics.<br><br>Before mathematical modelling, mathematics had functioned as the metaphor, theory and language for the nature. In Ancient Greece, music and astronomy were exactly theorised under geometric representations. Today, mathematics itself has various theories with algebraic representations, thus, modelling means hypothetical approach for the real world by using universal mathematical language.<br><br>Mathematisation has two usages: The first usage is for science and engineering in the establishment of mathematical model and produce new mathematics theories based on the model. In physics, mathematical problem solving of nature has produced the various theorisation in mathematics for solving problems in general. The second usage is used in education as the principle of mathematics curriculum sequence which enhances reorganisation of mathematical experiences. It includes the process of extension and integration by using the prior learned knowledge. From the perspective of mathematical problem solving, second usage aimed to develop or reinvent mathematics by students through the solving mathematics problems with support of teachers, textbook and curriculum. For mathematisation, setting appropriate task sequence has the key role to develop or revise known mathematical ideas, concepts and procedure. |

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| **Conjecturing, Justifying and Proving** | Conjecturing, justifying and proving have been on the context of proof and refutation. In education, students conjecture hypothesis with reasoning on exemplar. Conjecture is conceived through generalisation and justifying with appropriate conditions. Proving includes not only the formal proof in a local system but also the various ways of explanations. Counter example is a way of refutation. Counter example is meaningful because it sets off the reasoning from which if the conclusion is true. Proving is the activity to reach proof. On this meaning, probing is not the proof itself. Collecting and explaining with examples can be also included. |
| **Conceptualisation and Proceduralisation** | Conceptual knowledge is the knowledge to explain the meaning and why, and used for the conscious reasoning, and procedural knowledge is the skilful knowledge used for unconscious-automatised reasoning. A unit of mathematics textbook usually begins from the initiations of new conceptual knowledge by using learned procedural-conceptual knowledge which is called conceptualisation. After the initiations, new conceptual knowledge formulates the new procedural knowledge for convenience and the exercises produce proficiency of procedure which is called proceduralisation.<br><br>In mathematics, the proposition format 'if…, then…' is the basic format to represent knowledge, however in school mathematics, it is impossible to make clear the proposition from the beginning because 'if' part can be clarified later. For example, 'number' changes the meaning several times in school curriculum. In multiplication, products become large if it is […] number. Until numbers are extended to decimal, […] part cannot be learned. On this problematic, it is normal that students meet the difficulty in their learning and challenge to produce appropriate knowledge if they do have a chance to over-generalise knowledge for knowing […] part Misconception (overgeneralized conception) is usually appear in the process of learning. It is a nature of mathematics curriculum.<br><br>On this reason for producing exact knowledge in mathematics, conceptualisation and proceduralisation is a journey that continues recursively in mathematics learning to change the view of mathematics. It is the opportunity to learn necessary mathematical ideas and ways of thinking, and to develop value and attitude. The process of mathematisation also can be seen from the perspective of conceptualisation and proceduralisation on the context of number and algebra. |
| **Representations and Sharing** | A mathematical representation consists of symbols, operations, and objective (context). Solving equations in algebra can be seen as specific ordered elements of every equation which has the same answer. Order of equations shows the context that it is the process of solving. Operation of equations between an equation to another equation can be explained by the property of equality. Each equation is a symbolic sentence.<br><br>If operations of representation are missing, it is not a mathematical representation even through it has some artistic images such as diagrams. When students draw diagrams, it is sharable if the rule of the drawing (operation) are shared. In a classroom, students produce their own images in diagrams. It is helpful to encourage for their explanation by themselves but every explanation is independent until knowing the hidden ideas as in the comparison. Other students cannot re-present it until the ways of drawing (operation) is shared. Thus, comparison of various representations is a part of the process to recognise ideas for producing symbol and operations, and translations as in mathematical representations.<br><br>In mathematics, different representations use different symbols and operations. If there is a rule for correspondence between symbols and operations on different |

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| | representations, it can be translated and produce rich meanings. It is the way to produce a mathematics system.<br><br>In mathematics, representation system can be defined universally, which is the product of convention by mathematicians. For teachers, it looks far for students' activity in the classroom, however it is the opportunity for students to reinvent the representation and its system through considering the why and how. For example, to produce metre as the measurement quantity includes such activities: There are historical episodes on why and how 'm' was defined by Condorcet and others in the middle of the French Revolution. On the area of Engineering, Informatic and Science, applied mathematicians usually try to produce new measurements based on their necessity of research and development to conceptualise the idea mathematically and operationally. Setting the measurable quantity is a part of mathematical modelling for real-world problem solving because if it is measurable, we can apply known mathematics. |

# APPENDIX C

**Strand: Mathematical Process – Humanity for Key Stage 1**

Enjoyable mathematical activities are designed to bridge the standards in different strands. Exploration of various number sequences, skip counting, addition and subtraction operations help to develop number sense that is essential to support explanations of contextual scenarios and mathematical ideas. Mathematical ways of posing questions in daily life are also necessary to learn at this stage. Ability to select simple, general and reasonable ideas enables effective future learning. Application of number sense provides facility for preparing sustainable life. The use of ICT tools and other technological tools provide convenience in daily life. At the initial stage, concrete model manipulation is enjoyable, however, drawing diagram is most necessary for explaining complicated situations by using simple representation.

**Standards:**

Enjoying problem solving through various questioning for four operations in situations
Enjoying measuring through setting and using the units in various situations
Using blocks as models and its diagram for performing operations in base ten
Enjoying tiling with various shapes and colours
Explaining ideas using various appropriate representations
Selecting simple, general and reasonable ideas which can apply for future learning
Preparing sustainable life with number sense
Utilising ICT tools such as calculators as well as other tools such as notebooks and other instruments such as clocks
Promoting creative and global citizenship for sustainable development of neighbourhood using mathematics

Enjoying problem solving through various questioning for four operations in situations[1]

i. In addition situations, pose questions for 'altogether' and 'increase'
ii. In subtraction situations, pose questions for 'remainder' and 'difference'
iii. In multiplication situations, pose questions for 'number of groups'
iv. In division situations, pose questions for 'partitive' and 'quotative'
v. Enjoy questioning in various situations by using combination of operations
vi. In operations, pose questions to find the easier ways of calculation
vii. Use posing questions for four operations on measurements in daily life

Enjoying measuring through setting and using the units on various situations[2]

i. Compare directly and indirectly
ii. Set different tentative units for measuring
iii. Give appropriate names (denominations) for counting units
iv. Use measurement for communication in daily life
v. Use tables and diagrams for showing the data of measures

Using blocks as models and its diagram for performing operations in base ten[3]

i. Show increasing and decreasing patterns using blocks
ii. Show base ten system using blocks, a unit cube is 1, a bar stick is 10 and a flat block represents 100
iii. Explain addition and subtraction algorithm in vertical form using base ten block model
iv. Explain multiplication table with number of grouped blocks

[1] *It is related to Numbers and Operations and Quantity and Measurement both in Key Stage 1.*
[2] *It is related to Quantity and Measurement and Pattern and Data Representations both in Key Stage 1.*
[3] *It is related to Patterns and Data Representations and Numbers and Operations both in Key Stage 1.*

v. Explain division using equal distribution of blocks and repeated subtraction of blocks
vi. Use the number of blocks for measurement in daily life

Enjoying tiling with various shapes and colours[4]

i. Appreciate to produce beautiful designs with various shapes and find the pattern to explain it
ii. Reflect, rotate and translate to produce patterns
iii. Cut and paste various shapes and colours to form a box and ball such as develop the globe from map

Explaining ideas using various appropriate representations[5]

i. Explain four operations using pictures, diagrams, blocks and expressions for developing ideas
ii. Explain measurement using measuring tools, tape diagrams, container and paper folding for sharing ideas
iii. Make decision on how to explain figures and solids by using manipulative objects or diagrams or only verbal explanation
iv. Explain patterns using diagrams, numbers, tables and expressions with blank box
v. Ask questions using terms such as why, how, what, if and if not, and reply using examples and 'for example' in discussion
vi. Change the representation and translate it appropriately in daily life

Selecting simple, general and reasonable ideas which can apply for future learning[6]

i. Discuss the argument for easier ways for addition and subtraction algorithm in vertical form
ii. Extend the algorithm to large numbers for convenience and fluency
iii. Use the pattern of increase in multiplication table for convenience
iv. Use multiplication tables for finding the answers of division

Applying number sense[7] acquired in Key Stage 1 for preparing sustainable life[8]

i. Use mathematics in situations with minimum and sequential use of resources
ii. Estimate for efficient use of resources in situations
iii. Maximise the use of resources through appropriate arrangement in space
iv. Understand equally likely of resources in situations

Utilising ICT tools such as calculators as well as other tools such as notebooks and other instruments such as clocks[9]

i. Use calculators for multi addition in situations
ii. Use mental calculations for estimations
iii. Use a balance scale to produce equality and inequality
iv. Use cups, tapes, stop watches, and weighing scales for measuring distances and mass

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*[4] It is related to Shapes, Figures and Solids and Pattern and Data Representations both in Key Stage 1.*
*[5] It is related to all strands in Key Stage 1.*
*[6] It is related to Numbers and Operations and Pattern and Data Representations both in Key Stage 1.*
*[7] It is related to Numbers and Operations, Pattern and Data Representations and Quantity and Measurement all included in Key Stage 1.*
*[8] Sustainable development goals were crafted at the 70th Session of the United Nations General Assembly and indicated them as universal value in education.*
*[9] STEM education is enhanced. Mathematics is the major and base subject for STEM Education in Key Stage 1 hence, technological contents are included in Mathematics.*

v. Use calculators to explain the process of calculation by solving backward and understand the relationship of addition and subtraction, and multiplication and division.
vi. Enjoy using notebooks to exchange learning from each other such as mathematics journal writing
vii. Enjoy presentations with board writing
viii. Use various tools for conjecturing and justifying

Promoting creative and global citizenship for sustainable development of neighbourhood using mathematics

i. Utilise notebooks and journal books to record and find good ideas and share with others
ii. Prepare and present ideas using posters to promote good practices in neighbourhood
iii. Listen to other's ideas and asking questions for better creativity
iv. Utilise information, properties and models as basis for reasoning
v. Utilise practical arts and outdoor studies to investigate local issues for improving welfare of life

# APPENDIX D

The four strands on content learning standards and the strand on Mathematical Process-Humanity of Key Stage Two

## KEY STAGE 2

Key Stage 2 (KS2) can be learned based on the Key Stage 1. This stage provides the extension of numbers and operations, measurement and relations, plane figures and space figures as well as data handling and graphs. This stage enables the extension of the four operations to daily use of numbers such as decimal and fraction and allows the use of mathematical terminologies, performing investigations and establish the ground for analysing, evaluating and creating in learners' life. Appreciating the beauty of the structure of mathematics will enable them to enjoy and sustain their learning which provides basis for Key Stage 3.

### Strand: Extension of Numbers and Operations

Numbers are extended to multi-digits, fractions and decimals. Multiplication and division algorithms are completed with fluency. Fraction becomes numbers through the redefinition as a quotient instead of part-whole relationship. Multiplication and division of decimals and fractions are also explored to produce procedures for calculation. Various representations are used to elaborate and produce meaning for the calculation. Number sense such as approximating numbers, relative size of numbers and values are enhanced for practical reasoning in the appropriate context of life.

Topics:

Extending Numbers with Base-ten Up to Billion and also to Thousandths with Three Digit Numeral System Gradually
Making Decision of Operations on Situations with Several Steps and Integrate them in One Expression and Think About the Order of Calculations and Produce the Rule (PEMDAS)
Producing the Standard Algorithm for Vertical Form Division with Whole Numbers
Extending the Vertical Form Addition and Subtraction with Decimals to Hundredths
Extending the Vertical Form Multiplication and Division with Decimals and Find the Appropriate Place Value such as Product, Quotient and Remainder
Using Multiples and Divisors for Convenience
Introducing Improper and Mixed Fractions and Extending to Addition and Subtraction of Fractions to Dissimilar Fractions
Extending Fractions as Numbers and Integrate
Extending Multiplication and Division to Fractions

**Extending Numbers with Base-ten Up to Billion and also to Thousandths with Three Digit Numeral System Gradually**

Extending numbers using base-ten system up to billion[1] with three-digit numeral system[2]

i. Adopt the three-digit numeral system, extend numbers up to billion with the idea of relative size of numbers
ii. Compare numbers such as larger, smaller with base[3]-ten system of place values through visualisation of relative size of numbers using cube, plane(flat), bar(long) and unit

[1] *Billion is too large for counting and it is introduced in the three-digit system under relative size of number.*
[2] *In British system, it is referred as short scale.*
[3] *Metric system names of units are discussed under Measurement and Relations.*

Extending decimal numbers to hundredths, and to thousandths[4]

i. Use the idea of quantity and fractions, extend decimal numbers from tenths to hundredths
ii. Compare decimal numbers such as larger, smaller with base-ten system of place value
iii. Adopt the ways of extension up to thousandths and so on, and compare the relative sizes

**Making Decision of Operations on Situations with Several Steps and Integrate them in One Expression and Think about the Order of Calculations and Produce the Rule (PEMDAS)**

Finding the easier ways of calculations using the idea of various rules of calculations[5] such as the associative, commutative and distributive rule

i. Find the easier ways of addition and subtraction and use it, if necessary, such as answer is the same if add same number to the subtrahend and minuend
ii. Find the easier ways of multiplication and division and use them in convenient ways such as 10 times of multiplicand produce the product 10 times
iii. Use associative, commutative and distributive rules of addition and multiplication for easier ways of calculation, however, commutative property does not work in subtraction and division
iv. Appreciate the use of simplifying rules of calculations

Thinking about the order of calculations in situations and produce rules and order of operations

i. Integrate several steps of calculation into one mathematical sentence
ii. Produce the rule of PEMDAS and apply it to the several steps situation
iii. Think about the easier order of calculation and acquiring fluency of PEMDAS and rules with appreciation

**Producing the Standard Algorithm Using Vertical Form Division with Whole Numbers**

Knowing the properties of division and use it for easier way of calculation

i. Find the easier ways of division and use it, if necessary, such as answer is the same if multiplying the same number to the dividend and divisor
ii. For confirmation of answer of division, use the relationship among divisor, quotient and remainder and appreciate the relationship

Knowing the algorithm of division in vertical form and acquiring fluency

i. Know the division algorithm with tentative quotient and confirm the algorithm by the relationship among divisor, quotient and remainder
ii. Interpret meaning of quotient and remainder in situations
iii. Acquire fluency for division algorithm up to 3-digit whole number divided by 2-digit
iv. Think about situations with or without remainder in relation to situations for quotative and partitive division

[4] *Under the three-digit system, if we teach until thousandths, we can extend by three-digit.*

[5] *In Measurement and Relations, use of constant sum, difference, product and quotient are described. (e.g.25 -21 = 4, 26–22 = 4, 27– 23 = 4)*

**Extending the Vertical Form Addition and Subtraction with Decimals to Hundredths**

Extending the vertical form addition and subtraction in decimals to hundredths

i. Extend the vertical form addition and subtraction to hundredths[6] place and explain it with models
ii. Appreciate the use of addition and subtraction of decimals in life

**Extending the Vertical Form Multiplication and Division with Decimals and Find the Appropriate Place Value Such as Product, Quotient and Remainder**

Extending the multiplication from the whole number to decimal numbers

i. Extend the meaning of multiplication with the idea of measurement by the number of unit length for multiplication of decimal numbers and use diagrams such as number lines to explain them with appreciation in situations
ii. Extend the vertical form multiplication of decimals up to 3 digits by 2 digits with consideration of the decimal places step by step
iii. Obtain fluency using multiplication of decimals with sensible use of calculators in life
iv. Develop number sense in multiplication of decimals[7] such as comparing sizes of products before multiplying

Extending the division from the whole number to decimal numbers

i. Understand how to represent division situations using diagrams such as number lines, and extend the diagram of decimal numbers for explaining division by decimal numbers
ii. Extend the division algorithm in vertical form of decimal numbers and interpret the meaning of decimal places of quotient and remainder with situations
iii. Acquire fluency in division algorithm of decimals up to 3 digits by 2 digits with consideration of decimal places step by step
iv. Obtain fluency using division of decimals with sensible use of calculators in life
v. Develop number sense of division in decimals such as comparing sizes of quotient before multiplying
vi. Distinguish situations with decimal numbers of multiplication and division

**Using Multiples and Divisors for Convenience**

Using multiples and divisors for convenience with appreciation to enrich number sense

i. Understand set of numbers by using multiples and divisors
ii. Find common multiples and appreciate the use in situations, and enrich number sense with figural representations such as arrangement of rectangles to produce a square
iii. Find common divisor and appreciate its use in situations, and enrich number sense with figural representations such as dividing a rectangle into pieces of square
iv. Understand numbers as composite of multiplication of numbers as factors[8]
v. Appreciate ideas of prime, even and odd numbers in situations using multiples and divisors
vi. Acquire number sense to see the convenient use of multiples and divisors

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[6] *Discussion of decimals to hundredths is related to the use of money. It is a minimum requirement. If teaching to hundredths, further extension of place value can be understood.*

[7] *Applying the idea of multiplication into ratio, percent and proportion is discussed in Measurement and Relations.*

[8] *This idea is related to the strand on Measurement and Relations in Key Stage 2 at the area of rectangle.*

**Introducing Improper and Mixed Fractions and Extending to Addition and Subtraction of Fractions to Dissimilar Fractions**

Extending fractions to improper, mixed and equivalent fractions

i. Extend fractions to improper and mixed fractions using number line of more than one by measuring with unit fraction[9]
ii. Find ways to determine equivalent fractions with number lines and with the idea of multiple of numerator and denominator
iii. Compare fractions using number line and the idea of multiple

Extending addition and subtraction of similar fractions to improper and mixed fractions, and dissimilar fractions

i. Extend addition and subtraction of similar fractions to proper and mixed fractions with explanations using models and diagrams
ii. Extend addition and subtraction into dissimilar fractions with explanations using diagrams and common divisors
iii. Acquire fluency of addition and subtraction of fractions with appreciation of idea to produce the same denominators

**Extending Fractions as Numbers and Integrate[10]**

Seeing fractions[11] as decimals and seeing decimals as fractions

i. See fractions as decimals using division and define quotient from the case of indivisible
ii. See decimals as fractions such as hundredths is per hundred
iii. Compare decimals and fractions and ordering them on a number line

**Extending Multiplication and Division to Fractions**

Extending multiplication to fraction

i. Extend multiplication to fraction with situations using diagrams such as number lines, and find the simple algorithm for the multiplication of fractions, step by step
ii. Acquire fluency with multiplication of fractions
iii. Develop number sense[12] of multiplication of fractions such as comparing sizes of products before multiplying

Extending division to fraction

i. Extend division to fraction with situations using diagrams such as number lines, step by step
ii. Acquire fluency with division of fractions
iii. Develop number sense of division of fractions such as comparing sizes of quotients before dividing

**Strand: Measurement and Relations**

Additive quantity such as angles, areas and volume and relational quantities such as population density, and speed are introduced. Additive quantity can be introduced by establishment of the standard unit which is the same way as the Quantity and Measurement of Key Stage 1. Relations of quantities in situations are discussed

*[9] Extension of fraction more than one is done by using the fraction with quantity for situation such as $\frac{4}{3}m$*

*[10] Selecting the appropriate denomination of quantities and units for fraction in the context ($\frac{2}{3}m$ is two of $\frac{1}{3}m$ and the whole is 1m, however $\frac{3}{3}m$ is $3x(\frac{1}{3})m$; the structure is the same as tens is ten of units and discussed under Measurement and Relations.*

*[11] Fraction as ratio is introduced in Measurement and Relations.*

*[12] Applying the idea of the multiplication of fraction into ratio, proportion, percentage and base is discussed in Measurement and Relations.*

with patterns such as sum is constant, difference is constant, product is constant and quotient is constant using tables and represented by mathematical sentences and letters. Proportion and ratio are introduced with representations of diagrams, graphs and tables for multiplication, and connected with decimals and fractions. Percent is introduced with diagrams in relation to ratio and proportion. Relational quantity is produced by different quantities with understanding of ratio. Area of a circle is discussed through a proportional relationship between radius and the circumference. Ideas of ratio and proportion are fluently applied for real world problem solving.

Topics:

Introducing Angle and Measuring it
Exploring and Utilising Constant Relation
Extending Measurement of Area in Relation to Perimeter
Extending Measurement of Volume in Relation to Surface
Approximating with Quantities
Extending Proportional Reasoning to Ratio and Proportion
Producing New Quantities Per Unit
Investigating the Area of a Circle
Exchanging Local Currencies in ASEAN Community
Extending the Relation of Time and Use of Calendar
Converting Quantities in Various System of Units
Showing Relationship Using Venn Diagram

**Introducing Angle and Measuring It[13]**

Introducing angle by rotation, enabling measure and acquire fluency using the protractor

i. Compare the extent of rotation and introduce degree as a unit for measuring angle
ii. Recognise right angle is 90 degrees, and adjacent angle of two right angles is 180 degrees, and 4 right angles is 360 degrees
iii. Acquire fluency in measuring angles using the protractor
iv. Draw equivalent angles with addition and subtraction using multiples of 90 degrees
v. Appreciate measurement of angles in geometrical shapes and situations in life[14]

**Exploring and Utilising Constant Relation**

Exploring equal constant relation with utilisation of letters to represent placeholders[15]

i. Explore two possible unknown numbers such that their sum (or difference /product/quotient) is constant,[16] for example □+Δ=12 (□ and Δ are placeholders).
ii. Use letters instead of placeholder[17] (empty box) to derive equivalent relation
iii. Understand the laws for operations (e.g. associative, commutative and distributive etc.) to explain the simpler way of calculation
iv. Appreciate the use of diagrams such as number lines and area to represent relation when finding solutions

[13] *Right angle is learned at Key Stage 1 in Shapes, Figures and Solids for explaining the properties of figures.*
[14] *Conservation of angles will be re-learnt in triangle under Key Stage 2 Plane Figures and Space Figures.*
[15] *The idea for the use of Numbers and Operations (Key Stage 2) in finding the easier ways of calculations with the idea of rules of calculations.*
[16] *Constants of multiplication and division, corresponds proportionality in multiplication table at Key Stage 1 under number and operations. Constant of addition and subtraction are treated in Key Stage 1 under Pattern and Data Representations.*
[17] *Place holder is introduced in Key Stage 1.*

**Extending Measurement of Area in Relation to Perimeter**

Introducing area and produce formula for the area of a rectangle

i. Compare extent of area and introduce its unit, and distinguish it from perimeter
ii. Introduce one square centimetre as a unit for area and its operation using addition and subtraction
iii. Investigate area of rectangles and squares and produce the formula of area[18]
iv. Extend square centimetre to square metre and to square kilometre for measure of large areas
v. Convert units and use appropriate units of area with fluency
vi. Draw the equivalent size of rectangular area based on a given area with the composite numbers[19]
vii. Appreciate the use of areas in daily life such as comparing of land sizes.

Extending area of a rectangle to other figures to derive formulae

i. Explore and derived formula for the area of parallelogram by changing its shape to rectangle without changing its area
ii. Explore and derived formula for the area of a triangle by bisecting a rectangle into two triangles without changing its area
iii. Appreciate the idea of changing or dividing shapes of rectangle, parallelogram, or/and triangle for deriving the area of other figures
iv. Use formulae to calculate areas in daily life

**Extending Measurement of Volume in Relation to Surface**

Introducing volume from area and derive formula for cuboid

i. Compare the extent of volume and introduce its unit, and distinguish it from surface
ii. Introduce one cubic centimetre as unit for volume and its addition and subtraction
iii. Investigate volume of cuboid and cube and produce the formulae
iv. Extend cubic centimetre to cubic metre to measure large volume
v. Convert units and use appropriate units of volume with fluency
vi. Appreciate the use of volume in life such as comparison of capacity of containers

Extending volume of cuboid to other space figures to derive formula

i. Extend the formula for the volume of a cuboid as base area x height for exploring space figures such as prism and cylinder
ii. Extend the formula for the volume of a prism and a cylinder to explore and derive the volume formula of pyramid and cone
iii. Use the formulae to calculate volume in daily life

**Approximating with Quantities**

Approximating numbers with quantities depending on the necessity of contexts

i. Understand the ways of rounding such as round off, round up and round down
ii. Use rounding as approximation for making decision on the quantity with related context
iii. Critique over approximation beyond the context with a sense of quantity such as based on relative size of a unit

[18] *Multiplications were studied in Key Stage 1 Number and Operations.*
[19] *The idea of composite numbers such as 2 times 10 equals 5 times 4 is related to factors in extending the numbers and operations at the same Key Stage 2.*

**Extending Proportional Reasoning to Ratio[20] and Proportion**

Extending proportional reasoning to ratio and percent

i. Understand ratio as a relationship between two same quantities or between two different quantities (the later idea is rate)[21]
ii. Express the value of ratio by quotient such as the rate of two different quantities[22]
iii. Understand percent as the value of ratio with the same quantities[23]
iv. Understand proportion with ratio
v. Apply the rule of three[24] in using ratio

Extending proportional reasoning to proportion

i. Extend proportional reasoning to multiplication tables as an equal ratio and understand proportions
ii. Understand proportion using multiple and constant quotient, not changing the value of ratio[25]
iii. Demonstrate simple inverse proportion by constant product[26]
iv. Express proportion in mathematical sentence using letters and graph[27]
v. Use properties of proportionality to predict and explain phenomenon in daily life

**Producing New Quantities by Per Unit**

Producing new quantities by per unit with the idea of average such as population density and speed

i. Introduce average as a unit for distribution and comparison of different sets of values
ii. Introduce population density with the idea of average and appreciate it for comparison
iii. Introduce speed with the idea of average and appreciate it for comparison
iv. Appreciate using diagrams such as number lines and tables to decide the operations situations of measurement per unit quantity
v. Comparing on the context of different quantities with the idea of average as rate[28]
vi. Apply the idea of measurement per unit quantity in different context[29]

**Investigating the Area of Circle**

Areas of a circle are discussed through the relationship between the radius and circumference

i. Investigate relationship between the diameter of a circle and its circumference using the idea of proportion
ii. Investigate area of a circle by transforming into a triangle or parallelogram and find the formula of the circle

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*[20] Band graph and pie chart for representing ratio are discussed under Key Stage 2 under the strand Data Handling and Graphs.*
*[21] Ratio of different quantities is a rate. Ratio of the same quantities has a narrow meaning of ratio.*
*[22] The value of fraction as a ratio is not necessary a part of a whole in situations. Fraction as a ratio is usually used in the context of multiplication situation, where denominator is the base or a unit for comparison.*
*[23] Percent is used in Data Handling and Graphs.*
*[24] Rule of three is the method on the table to find one unknown term from the three known terms using proportional reasoning such as*

$$\begin{matrix} a & c \\ ? & d \end{matrix}$$

*[25] Enlargement is discussed in Key Stage 2 under Plane Figures and Space Figures. The graph is treated at Key Stage 2 under the strand Data Handling and Graphs.*
*[26] Proportion and Inverse proportion are necessary in Key Stage 3 in science.*
*[27] This will be discussed in detail in same Key Stage 2 under Data Handling and Graphs.*
*[28] On number and operations Key Stage 2, rate is the value of division as quotient.*
*[29] Using measurement per unit quantity with fluency to make logical judgment in daily life, refer to Key Stage 2 Data Handling and Graphs.*

iii. Estimate the area of a inscribed and circumscribed shapes based on known formula of areas[30]
iv. Enjoy to estimate the area of irregular shapes with fluency in life

**Exchanging Local Currency with Currency in ASEAN Community**

Exchanging local currency in ASEAN community with the idea of rate

i. Extend the use of ratio for currency exchange (rate of exchange)
ii. Apply the four operations for money in appropriate notation in life
iii. Appreciate the value of money

**Extending the Relation of Time and Use of Calendar in Life**

Extending the relation of time and use of calendar in life

i. Convert time in 12-hour system with abbreviation a.m. and p.m. to 24-hour system and vice versa
ii. Investigate the numbers in calendar to relate days, weeks, months and year using the idea of number patterns
iii. Appreciate the significance of various calendars in life

**Converting Quantities in Various System of Units**

Converting measurement quantities based on international and non-international system with the idea of base-10

i. Convert measurement system of metre and kilogram with prefixes deci-, centi-, and milli-, and with deca, hecto-, and kilo-
ii. Convert measurement system of litre with cubic centimetre
iii. Convert measurement system of area using are (a) and hectare (ha) with square meter
iv. Convert measurement of local quantities with standard quantities
v. Understand the unit system with power, such as metre, square metre and cubic metre

**Showing Relationship Using Venn Diagram**

Using Venn diagram to show relationships of numbers and figures

i. Show relationship of squares, rectangles, rhombus, parallelogram, trapezoid and quadrilateral by using a Venn diagram
ii. Show relationship of numbers

**Strand: Plane Figures and Space Figures**

Through tessellation, figures can be extended through plane figures. Parallelogram and perpendicular lines are tools to explain properties of triangles and quadrilaterals as plane figures. They are also needed for identifying and recognising symmetry and congruence parallelogram and perpendicular lines. Plane figures are used to produce solids in space and vice versa. Opening faces of solids would produce plane figures which are referred as nets. Activities related to building solids from plane figures are emphasised and encouraged to facilitate finding the area of a circle through numerous sectors of the circle to construct a rectangle. Circles are used for explaining the nets of cylinders.

[30] *Relationship on polygons and circles are discussed in Key Stage 2 under Plane Figures and Space Figures.*

Topics:

Exploring Figures with their Components in the Plane
Exploring Solids with their Components in Relation to the Plane
Exploring Figures with Congruence, Symmetry and Enlargement

**Exploring Figures with their Components in the Plane**

Exploring figures with their components in the plane and use their properties

i. Examine parallel lines and perpendicular lines by drawing with instruments
ii. Examine quadrilaterals using parallel and perpendicular lines, and identify parallelogram, rhombus, trapezium through discussion
iii. Find properties of figures through tessellations such as a triangle where the sum of the angles is 180 degrees, a straight angle
iv. Extend figures to polygons and expand it to circles with knowing and using its properties

**Exploring Solids with their Components in Relation to the Plane**

Exploring rectangular prisms and cubes with their components

i. Identify relationship between faces, edges and vertices for drawing of sketch
ii. Explore nets of rectangular prism and find the corresponding position between the components
iii. Explore the perpendicularity and parallelism between faces of a rectangular prism
iv. Explain positions in rectangular prisms with the idea of 3-dimensions

Extending rectangular prism to other solids such as prisms and cylinders

i. Extend the number of relationships between faces, edges and vertices for drawing of sketch
ii. Explore nets of prisms and cylinders, and find the corresponding position between components
iii. Distinguish prism and cylinder by the relationship of their faces

**Exploring Figures with Congruence, Symmetry and Enlargement**

Exploring the properties of congruence

i. Explore properties of figures which fit by overlapping and identify conditions of congruency with corresponding points and sides
ii. Draw congruent figures using minimum conditions and confirm by measuring angles and sides
iii. Appreciate the power of congruent figures by tessellation

Exploring the properties of symmetry

i. Explore the properties of figures which reflect and identify conditions of symmetry with line and its correspondence
ii. Draw symmetrical figures using conditions in appropriate location
iii. Appreciate the power of symmetry in designs

Exploring the properties of enlargement[31]

i. Explore properties of figures in finding the centre of enlargement in simple case such as a rectangle
ii. Draw enlargement of rectangle using ratio (multiplication of the value of ratio)[32]
iii. Appreciate the power of enlargement in interpretation of map

[31] *General case will be discussed in Key Stage 3, Space and Geometry.*
[32] *Ratio and rate are discussed in Key Stage 2 under Measurement and Relations.*

**Strand: Data Handling and Graphs**

The process of simple data handling is introduced through data representation such as using table, bar graph, line graph, bar chart and pie chart. Graphs are utilised depending on the qualitative and quantitative data used such as bar graph is for distinguishing and counting in every category. The discussion of producing the line graph includes taking data at specific intervals, suitable scale used and slope. Histogram is necessary for interpreting the data representation of social study and science, and is also used as a special type of bar graph. Average is introduced based on the idea of ratio for making the dispersion of bar chart even, and used for summarising and comparing data on a table. *Problem-Plan–Data- Analysis–Conclusion* (PPDAC) cycles are experienced through the process of data handling by using those data representation skills. Those skills are necessary for learning of sustainable development.

Topics

Arranging Tables for Data Representations
Drawing and Reading Graphs for Analysing Data
Using Graphs in PPDAC Cycle (*Employing the Problem - Plan - Data - Analysis - Conclusion*) Appropriately
Applying Data Handling for Sustainable Development

**Arranging Tables for Data Representations**

Exploring and arranging tables for data representations

i. Explore how to collect multi category data based on a situation
ii. Explore how to arrange and read multi category data on appropriate tables.
iii. Appreciate the use of multi category tables in situations

**Drawing and Reading Graphs for Analysing Data**

Drawing and reading line graphs for knowing the visualised pattern as basis for tendency of change

i. Introduce line graphs based on appropriate situations such as rainfall, temperature and others
ii. Distinguish line graph from bar graph for observation such as increase, decrease, and no-change
iii. Introduce the graph of proportion using the idea of line graph and read the gradient by constant ratio[33]
iv. Appreciate the line graph in various situations

Drawing and reading band graph and pie chart for representing ratio in a whole[34]

i. Explore how to scale the band or circle for representing ratio or percent
ii. Use the band graph and pie chart for comparison of different groups
iii. Appreciate the band graph and pie chart in a situation

Reading histogram[35] for analysing frequency distribution

i. Draw a simple histogram[36] from frequency table on situations
ii. Read various histograms for analysing data distribution
iii. Use mean[37] to compare different groups in the same situation with histograms

[33] *Proportions are learnt in the Key Stage 2 under Measurement and Relations.*
[34] *Ratio is learnt under Key Stage 2 Measurement and Relations.*
[35] *How to draw histogram is discussed in Key Stage 3 under Statistics and Probability. Reading histograms is necessary in social studies and science.*
[36] *Using ICT for drawing graph will be mentioned in mathematical activities.*
[37] *Mean is introduced as average in Key Stage 2 under Measurement and Relations.*

**Using Graphs in PPDAC[38] Cycle Appropriately**

Identifying appropriate graphs for data handling in PPDAC cycle

i. Analyse a situation and discuss the expected outcomes before collecting data to clarify the purpose of a survey
ii. Plan the survey for the intended purpose
iii. Collect the data based on the purpose of the survey
iv. Use appropriate graphical representation which is most suitable for the purpose
v. Appreciate the use of graphs before making the conclusion

**Applying Data Handling for Sustainable Living**

Applying data handling for sustainable development[39] and appreciate the power of data handling for predicting the future.

i. Read data related to sustainable development such as emergency preparedness and resiliency, food and energy security, world weather warming, inclusion and human connectivity in society, and lifelong learning in the changing society such as TVET (Technical and Vocational Education and Training) and adopting positive views for changing the society
ii. Understand the idea of probability as ratio and percentage in reading the data for situations related to sustainable development such as weather report and risk analysis
iii. Experience implementing a project of reasonable size in data handling for the purposes of achieving sustainable development and appreciate the power of data handling

**Strand: Mathematical Processes – Humanity**

As a follow up of Key Stage 1, activities are designed to enable an appreciation of knowledge and skills learned and the ways of learning such as applying knowledge of number sense to solve daily problems. Mathematical processes such as communication and reasoning are used to provide explanation for mathematical problems and modelling. The ability to connect and reason mathematical ideas would trigger an excitement among learners. Discussions of misconceptions are usually enjoyable and challenging. Mathematics learning usually begins from situations at Key Stage 1. In Key Stage 2, the development of mathematics is possible through discussions for the extension of the forms. Appreciation of ideas and representations learned become part of the enjoyable activities. Through the consistent use of representations such as diagrams, application of learning becomes meaningful.

Standards:

Enjoying problem solving through various questioning and conjecturing for extension of operations into decimals and fractions with proportionality and new quantities such as area and volume
Enjoying measuring through settings and using the area and volume in situations
Using ratio and rate in situations
Using number lines, tables, and area diagrams for representing operations and relationships in situations
Establishing the idea of proportion to integrate various relations with consistency of representations
Enjoying tiling with various figures and blocks
Producing valuable explanation based on established knowledge, shareable representations and examples
Performing activities of grouping and enjoy representing with Venn diagram
Experiencing PPDAC (Problem-Plan-Data-Analysis-Conclusion) cycle and modelling cycle in simple projects in life
Preparing sustainable life with number sense and mathematical representations
Utilising ICT tools as well as notebooks and other technological tools
Promoting creative and global citizenship for sustainable development of community using mathematics

[38] *PPDAC itself will be described in the mathematical activities later.*
[39] *This standard which is related to SDG as inter subject content between social studies and science.*

Enjoying problem solving through various questioning and conjecturing for extension of operations into decimals and fractions with proportionality and new quantities such as area and volume[40]

i. Pose questions to develop division algorithm in vertical form using multiplication and subtraction
ii. Pose questions to develop multiplication and division of decimal numbers using the idea of proportionality with tables and number lines
iii. Pose questions to develop multiplication and division of fractions using the idea of proportionality with tables, area diagrams and number lines
iv. Pose questions to extend multiplication and division algorithm in vertical form to decimal numbers and discuss about decimal points
v. Pose questions to use decimals and fractions in situations
vi. Pose questions to use area and volume in life
vii. Pose questions to use ratio and rate in life
viii. Pose conjectures based on ideas learned such as when multiplying, the answer becomes larger

Enjoying measuring through settings and using the area and volume in situations

i. Compare directly and indirectly areas and volumes
ii. Set tentative units from difference for measuring area and volume[41]
iii. Give the formula for the area and volume for counting units
iv. Use measurement for communication in daily life

Using ratio and rate in situations[42]

i. Understand division as partitive (between different quantities) and quotative (between same quantity) in situations
ii. Develop the idea of ratio and rate utilising the idea of average and per unit with tables and number lines
iii. Communicate using the idea of population density and velocity in life

Using number lines, tables, and area diagrams for representing operations and relations in situations[43]

i. Represent proportionality on number lines with the idea of multiplication tables
ii. Use number lines, tables, and area diagrams for explaining operations and relations of proportionality in situations

Establishing the idea of proportion to integrate various relations with consistency of representations[44]

i. Use the idea of proportion as the relation of various quantities in life
ii. Identify through the idea of proportion using tables, letters, and graphs
iii. Adopt the idea of proportion to angles, arcs and area of circles
iv. Adopt the idea of proportion to area and volume
v. Adopt the idea of proportion to enlargement
vi. Use ratio for data handling such as percent and understand the difficulties to extend it to proportion

Enjoying tiling with various figures and blocks[45]

i. Appreciate to produce parallel lines with tessellation of figures
ii. Explain the properties of figures in tessellations by reflections, rotations and translations
iii. Develop nets from solids and explain the properties of solids by each of the component figures

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[40] *This is connected to the three strands, Extension of Numbers and Operations, Measurement and Relations, and Plane Figures and Space Figures.*
[41] *Euclidean algorithm is a method of finding the largest common divisor of two numbers.*
[42] *Ratio and proportion bridge multiplication and division in situation of two quantities with reference to Extension of Numbers and Operations and Measurement and Relations.*
[43] *This is a bridge to the Extension of Numbers and Operations and Measurement and Relations.*
[44] *Bridge to the three strands, Measurement and Relations, Plane Figures and Space Figures and Data Handling and Graphs.*
[45] *Connected to the two strands, Measurement and Relations, and Plane Figures and Space Figures.*

iv. Use the idea of tiling for calculating the area and volume

Producing valuable explanations based on established knowledge, shareable representations and examples

i. Establish the habit of explanation by referring to prior learning and ask questions using terms such as why, how, what, if and if not, and reply using examples and 'for example' in discussion
ii. Assessing the appropriateness of explanations using representations such as generality, simplicity and clarity
iii. Use other's ideas to produce better understanding
iv. Use inductive reasoning for extending formulae

Performing activities of grouping and enjoy representing with Venn diagram

i. Use the idea of Venn diagram for social study
ii. Understand classifications based on characteristics and represent by using Venn diagrams

Experiencing PPDAC (Problem-Plan-Data-Analysis-Conclusion) cycle and modelling cycle in simple projects in life

i. Understand the problem of context
ii. Plan appropriate strategies to solve the problem
iii. Gather data and analyse using appropriate methods and tools
iv. Draw conclusion with justification based on data analysis

Preparing sustainable life with number sense and mathematical representations[46]

i. Use minimum and sequential use of resources in situations
ii. Use data with number sense such as order of quantity and percentage for the discussion of matters related to sustainable development
iii. Estimate the efficient use of resources in situations
iv. Maximise the use of resources through appropriate arrangement in a space such as a room
v. Understand "equally likely" of resources in situations

Utilising ICT tools as well as notebooks and other technological tools

i. Use internet data for the discussion of matters related to sustainable development
ii. Distinguish appropriate or inappropriate qualitative and quantitative data for using ICT
iii. Use calculators for organising data such as average
iv. Use calculators for operations in necessary context
v. Use projectors for sharing ideas as well as board writing
vi. Enjoy using of notebooks to exchange learning experiences between each other such as in mathematics journal writing
vii. Use protractors, triangular compasses, straight edges, clinometers for drawing and measuring
viii. Use the idea of proportionality to use mechanism such as rotate once and move twice (wheels, gears)
ix. Use various tools for conjecturing and justifying

Promoting creative and global citizenship for sustainable development of community using mathematics

i. Utilise notebooks, journal books and appropriate ICT tools to record and find good ideas and share with others
ii. Prepare and present ideas using posters and projectors to promote good practices in the community
iii. Listen to other's ideas and ask questions for better designs
iv. Utilise information, properties, models and visible representations as the basis for reasoning
v. Utilise practical arts, home economics and outdoor studies to investigate local issues for improving welfare of life

[46] *It is related to Numbers and Operations, Pattern and Data Representations and Quantity and Measurement all under Key Stage 1.*

# APPENDIX E

The four strands of content learning standards and the strand on Mathematical Process-Humanity of Key Stage Three

**KEY STAGE 3**

Key Stage 3 (KS3) can be developed based on Key Stage 2. This stage focus on numbers and algebra, relations and functions, space and geometry, and statistics and probability. Symbolic representations allow the dealing of abstract ideas and concepts that enhance critical and creative thinking through the application of knowledge. Understanding and using mathematical concepts and principles in this stage through discussions, dialogue, and arguments enable learners to participate in contemporary societal, economic, technological, political, environmental and mathematical issues. This stage is the basis for the creation of a better future with predictions. It bridges further mathematics learning in various job demands.

**Strand: Numbers and Algebra**

Numbers are extended to positive and negative numbers and square roots. Algebraic expressions are already introduced by the mathematical sentences and symbols at Key Stage 2. At Key Stage 3, algebra is operated by expressions and equations until the second degree. On the extension from numbers to symbolic algebra, various possible ways of calculations are explored until their appropriateness is established. Like and unlike terms are introduced in an algebraic sentence and in simplifying expressions. Properties of equations are introduced for finding simple equivalent and solving equations with fluency. Substitution, addition and subtraction of equations enable further operations of simultaneous equations. Expansion and factorisation enable further operations of the polynomials. Finally, quadratic equations can be solved using various operations.

Topics:

Extending Numbers to Positive and Negative Numbers
Utilising Letters for Algebraic Expressions and Equations
Algebraic Expressions, Monomials and Polynomials and Simultaneous Equations
Expansion and Factorisation of Polynomials
Extending Numbers with Square Roots
Solving Quadratic Equations

**Extending Numbers to Positive and Negative Numbers**

Extending numbers to positive and negative numbers and integrate four operations into addition and multiplication

i. Understand the necessity and significance of extending numbers to positive and negative numbers in relation to directed numbers with quantity
ii. Compare numbers which is greater or less than on the extended number line and use absolute value for distance from zero
iii. Extend operations to positive and negative numbers and explain the reason
iv. Get efficiency on calculation in relation to algebraic sum

**Utilising Letters for Algebraic Expressions and Equations**

Extending the utilisation of letters for general representation of situations and find ways to simplify algebraic expressions

i. Appreciate the utilisation of letter for general representation of situations to see the expression as process and value

ii. Find ways to simplify expressions using distributive law and figural explanations, establish the calculation with like and unlike letter
iii. Acquire fluency of simplifying an expression and appreciate it for representing the pattern of situation

Thinking about set of numbers in algebraic expression with letters as variables and represent them with equality and inequality

i. Recognise numbers as positive and negative numbers, and explain integers as a part of numbers
ii. Represent a set of numbers using variables with equality and inequality
iii. Translate given sets of numbers on the number lines using interval and inequality notations
iv. Appreciate redefining of even and odd numbers using letters to represent different sets of variables

Thinking about how to solve simple linear equation

i. Review the answers of equations from the set of numbers and thinking backwards
ii. Know the properties of equations which keep the set of answers of equation
iii. Appreciate the efficient use of the properties of equations to solve linear equation
iv. Use equations based on life situations to develop fluency, to solve equation, and interpret the solution

**Algebraic Expressions, Monomials and Polynomials and Simultaneous Equations**

Thinking about the calculations of monomial and polynomials, simple case[1]

i. Introduce terms, monomials and polynomials
ii. Introduce a number raised to the power of two as square, and a number raised to the third power as cube
iii. Get fluency for the calculation of polynomials such as combining like terms and the use of the four operations in simple cases

Thinking about how to solve simultaneous equations in the case of linear equations

i. Understand the meaning of solution of linear equations and simultaneous linear equations as a pair of numbers
ii. Know the substitution and elimination methods of solving simultaneous linear equations
iii. Get fluency for selecting the methods from the form of the simultaneous linear equations
iv. Appreciate simultaneous linear equations in situations

**Expansion and Factorisation of Polynomials**

Acquisition to see the polynomials in the second degree with expansion and factorisation and use it

i. Use the distributive law to explain the formulae for expansion and explain them on diagrams
ii. Acquire proficiency for selecting and using the appropriate formulae
iii. Use the expansion formulae to factorise the second degree expression and recognise both formulae with inverse operation
iv. Solve simple second-degree equation using the factorisation and apply it in life situations

**Extending Numbers with Square Roots**

Extending numbers with square roots and calculate the square roots algebraically

i. Define square root and discuss ways to estimate the nearest value of a square root by Sandwich Theorem
ii. Understand that some square roots cannot be represented as fractions

[1] *Simple case may vary from and depending on the countries based on the mapping of curriculum.*

iii. Compare square roots using number line and understand that the order does not change but the differences between two consecutive square roots varied
iv. Think about multiplication and addition of square root and understand the algebraic way of calculation which is similar to polynomial
v. Appreciate square roots in applying to situations in life[2]

**Solving Quadratic Equations**

Solving simple second-degree equation using the factorisation and apply on the situation

i. Find the answers of simple second-degree quadratic equations by substitution and explore by completing the square, quadratic formula, and factorisation[3]
ii. Get fluency to select the appropriate ways for solving quadratic equations
iii. Apply quadratic equations in life situations

**Strand: Relations and Functions**

Relationships are represented by equations and system of equations. Functional relations are treated amongst situation, table, and equation of function are introduced based on patterns and relations with algebraic representation on Key Stage 2 and Key Stage 3. Solution of simple equation is done by equivalence deduction based on algebra learnt earlier. Two variables in simultaneous equations as simple system of equations are solved by substitution and additive-subtractive methods. Three representations, table, equation and graph, are used as methods to analyse the properties of every function. Proportion and inverse proportions are redefined with those representations mentioned. Proportional function is extended to line functions. The comparisons of inverse proportion and line functions are made clear by the property of linearity with 'constant ratio of change'. The concept of proportion is extended to function of $y = ax^2$. Ways of translations between table and equation, equation and graph, graph and table are specific skills for every function with fluency.

Topics:

Extending Proportion and Inverse Proportion to Functions with Variables
Exploring Linear Function in Relation to Proportions
Exploring Simple Quadratic Function
Generalising Functions

**Extending Proportion and Inverse Proportion to Functions with Variables**

Extending proportion and inverse proportion to functions with variables on positive and negative numbers

i. Extend proportions to positive and negative numbers, using tables and equations on situations
ii. Plot set of points as graph for proportions defined in ordered pairs $(x, y)$ in the coordinate plane using appropriate scales precisely[4]
iii. Introduce inverse proportion using tables, equations and graphs
iv. Introduce function as correspondences of two variables in situations
v. Explore the property of proportional function with comparison of inverse proportional function
vi. Appreciate proportion and inverse proportion functions in life

[2] *Pythagorean Theorem is discussed in Space and Geometry strand.*
[3] *The graph of quadratic equation will be treated in Relations and Functions strand.*
[4] *Utilising ICT is recommended in mathematical activities.*

**Exploring Linear Function in Relation to Proportions**

Exploring linear function in relation to proportion and inverse proportions

i. Identify linear functions based on situations represented by tables and compare it with proportional functions
ii. Explore properties of linear function represented by tables, equations and graphs and compare it with direct and inverse proportional functions
iii. Acquire fluency to translate the rate of change of a linear function represented in table, as coefficient in an expression and gradient in a graph[5]
iv. Acquire fluency to translate *y* values of $x = 0$ in a table, constant in an expression, and $y$ intercept in graph
v. Apply the graphs of linear functions to solve simultaneous equations
vi. Apply the linear function for data representation on situations to determine best fit line

**Exploring Simple Quadratic Function**

Exploring quadratic function $y = ax^2$ in relation to linear function

i. Identify the quadratic function on situations using tables and comparing it with linear function
ii. Explore properties of quadratic function using tables, equations as well as graphs and comparing it with linear function
iii. Apply the quadratic function on situations in daily life and appreciate it

**Generalising Functions**

Generalising functions with various representations[6] on situations

i. Distinguish domain, range and intervals and is appropriately used for explaining function
ii. Use various situation for generalising ideas of functions such as moving point A and moving point B with time
iii. Compose a graph as a function of two or more graphs with different domains in a situation
iv. Introduce situations of step-functions[7] with graph for generalisation the idea of function which cannot be represented by equation

**Strand: Space Geometry**

Space and Geometry provide the ways of reasoning for exploring properties in geometry and produce the ways of argument to explain justifications of visual reasoning. The calculations of angles are not just simple calculation but also the ways of using the geometric propositions to justify answers through explaining why it is correct based on basic properties. By explaining the relationship of figures using transformation, the properties of congruency and describing similarity are identified and described. Finding the value of angles and building arguments for proving are means for developing the habit of reasoning in the properties of plane figures. The conditions of congruence and similarities, properties of circles, are also used to explain and prove appropriateness of geometric conjectures in relation to triangles, quadrilaterals, and circles. Dynamic geometric software as well as simple compass and ruler are used for conjecturing. It shows general ideas from consistency in variations. Counter example is also found as special case from variations.

[5] *Family of linear functions are recommended to use ICT tool under mathematical activities strand.*
[6] *Utilising functions as model in daily life which is necessary for STEM education.*
[7] *Intervals are taught in the Key Stage 3 on Numbers and Algebra.*

Topics

Exploring Angles, Construction and Designs in Geometry
Exploring Space with its Components
Exploring Ways of Argument for Proving and Its Application in Geometry

**Exploring Angles, Construction and Designs in Geometry**

Exploring angles to explain simple properties on the plane geometry and do the simple geometrical Construction[8]

i. Explain how to determine the value of angles using the geometrical properties of parallel lines, intersecting lines, and properties of figures
ii. Use ruler and compass to construct a simple figure such as perpendicular lines and bisectors
iii. Appreciate the process of reasoning that utilises the properties of angles and their congruency in simple geometrical constructions

Exploring the relationship of figures using congruency and enlargement for designs

i. Explore the congruence of figures through reflection, rotation and translation and explain the congruency using line of symmetry, point of symmetry and parallel lines
ii. Explore similarity of figures with enlargement using points, ratio, and correspondences
iii. Enjoy using transformations in creating designs

**Exploring Space with its Components**

Exploring space by using the properties of planes, lines and their combinations to form solids

i. Explore the properties produced by planes, lines and their combinations, such as parallel lines produced by intersection of parallel planes with another plane
ii. Produce solids by combining planes such as nets and motion such as rotation, reflection and translation
iii. Recognise the space of an object based on its properties and projection

**Exploring Ways of Argument for Proving and its Application in Geometry[9]**

Exploring properties of congruency and similarity on plane geometry

i. Explore ways of arguments using the congruence of two triangles and appreciate the logic of argument in simple proving
ii. Explore ways of arguments using the similarity of two triangles based on ratio and angles and appreciate the logic of arguments in simple proving
iii. Explore the proof of the properties of circles such as inscribed angles, and intercepted arcs
iv. Appreciate proving through making the order of proven propositions to find new propositions

Exploring Pythagorean theorem in solving problems in plane geometry and spaces

i. Explore the proving of Pythagorean theorem using diagram and use it in solving problems involving plane figures
ii. Apply Pythagorean theorem on prism by viewing the figures through faces.
iii. Explore the situations for simple trigonometry using special angles in relation to the Pythagorean theorem
iv. Appreciate the use of Pythagorean theorem in life[10]

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*[8] Simple geometric construction is discussed by the ruler and compass with reasoning. Dynamic Geometric software usually draws entire circles. For knowing invariant dynamic geometric software is useful.*

*[9] Dynamic Geometry software is useful to find the invariant properties which is discussed in Mathematical Processes and Humanity strand in Key Stage 3.*

*[10] Pythagorean theorem is used for re-understanding the topic on square root under Key Stage 3 Numbers and Algebra.*

## Strand: Statistics and Probability

Data handling are extended to explore the dispersion of histogram with mean, median, mode and range. Exploratory data analysis (EDA) attempting to represent and visualise the structure from the given data using Information Communication and Technology (ICT) is enhanced. Histogram shows different dispersion if we change the class. Probability is introduced as ratio with the law of large numbers. Sample space with assumption of equal probability becomes the point of discussion. Logical analysis to understand whole possible cases such as a tree diagram is introduced for knowing the ways to represent logical reasoning. Histogram can be seen relatively and produce frequency distribution polygon. Difference between sample and population is discussed. Boxplots with quartile is an extension of median and range is used for comparisons of distributions. Using skills of statistics and probability make problem solving in situations possible. Analysing and identifying the trends in situations for making decisions are necessary such as issues for sustainable living.

Topics:

Exploring Distribution with the Understanding of Variability
Exploring Probability with Law of Large Numbers and Sample Space
Exploring Statistics with Sampling

### Exploring Distribution with the Understanding of Variability

Exploring distribution with histograms, central tendency and variability

i. Use histogram with different class intervals to show different distribution of the same set of data
ii. Identify alternative ways to show distribution such as dot plots, box-plot and frequency polygon
iii. Investigate central tendencies such as mean, median, mode[11] and their relationships in a distribution
iv. Investigate dispersion such as range and inter-quartile range in a distribution
v. Appreciate the analysis of variability through the finding of the hidden structure of distribution on situations using the measure of central tendency and dispersion

### Exploring Probability with Law of Large Numbers and Sample Space

Exploring probability with descriptive statistics, law of large numbers and sample space

i. Experiment with tossing coins and dice to explore the distribution of the relative frequency and understand the law of large numbers
ii. Use the idea of equally likely outcomes to infer the value of probability[12]
iii. Analyse sample space of situations represented by a table to determine the probability and use it for predicting occurrence
iv. Use various representations such as table, tree diagram, histogram and frequency polygon for finding probability
v. Analyse data related to issues on sustainable development and use probability to infer and predict future events

*[11] Mean, median and quartile are fixed depending on the data. However, mode changes depending on the class.*
*[12] The probability here is called equiprobability when all possible cases are equally likely. In the upper grade level, probability will be redefined based on distributions.*

**Exploring Statistics with Sampling**

Exploring sampling with the understanding of randomness

i. Discuss the hidden hypothesis behind sample and population
ii. Use randomness to explain sampling
iii. Analyse the data exploratory such as dividing the original into two for knowing better data representations and discuss appropriateness such as regrouping
iv. Appreciate data sampling in a situation with sustainable development

**Strand: Mathematical Processes – Humanity**

Critical argument in mathematics is enhanced through communication with others beyond Key Stage 2. This proposed challenging activities will promote metacognitive thinking at different level of arguments to make sense of mathematics. Translating real life activities into mathematical models and solving problems using appropriate strategies are emphasised in functional situations. The process of doing mathematical activities involves patience that develops perseverance in learners and takes responsibility of one's own learning. At this stage, the habitual practice of self-learning will eventually develop confidence, thus, opportunity for challenges to extend mathematics and the ability to plan sequence of future learning are also enhanced.

Standards:

Enjoying problem solving through various questioning and conjecturing for extension of operations into algebra, space and geometry, relationship and functions, and statistics and probability
Enjoying measuring space using calculations with various formulae
Producing proof in geometry and algebra
Utilising tables, graphs and expressions in situations
Using diagrams for exploring possible and various cases logically
Exploring graphs of functions by rotation, by symmetry and by translation of proportional function
Understanding ways for extension of numbers
Designing sustainable life with mathematics
Utilising ICT tools as well as other technological tools
Promoting creative and global citizenship for sustainable development of society in mathematics

Enjoying problem solving through various questioning and conjecturing for extension of operations into algebra, space and geometry, relationship and functions, and statistics and probability[13]

i. Pose questions to extend numbers and operations into positive and negative numbers, algebraic operations, and further extension into polynomial operations, and numbers with square roots
ii. Pose questions to solve linear equations, simultaneous equations and simple quadratic equations
iii. Pose assumptions in geometry as objects of argument and proof
iv. Pose questions to transform three dimensional objects into two dimensional shapes and vice versa
v. Pose questions in relations and functions for knowing properties of different types of function
vi. Pose questions to exploratory of data handling for knowing structure of distribution
vii. Pose questions that apply PPDAC in relation to statistical problem solving
viii. Pose questions in relation to "equally likely" events
ix. Pose assumptions to discuss hypothesis based on sample and population
x. Pose conjecturing such as if $x$ increases and $y$ decreases then it is inverse proportion

Enjoying measuring space using calculations with various formulae[14]

i. Extend the number line to positive and negative numbers and compare the size of numbers with the idea of absolute value
ii. Derive the square root using unit squared paper through the idea of area
iii. Explain the expansion of polynomials using area diagrams

[13] *Connected to the three strands, namely Numbers and Algebra, Relations and Functions, and Space and Geometry.*
[14] *Connected to the three strands, namely Numbers and Algebra, Relations and Functions, and Space and Geometry.*

iv. Use the projection of space figures to plane figures using the Pythagorean theorem
v. Apply similarity and simple trigonometry for measurement
vi. Use common factors to explain factorisation of the area of a rectangle based on the area of a square

Producing proof in geometry and algebra

i. Have an assumption through exploration and produce propositions
ii. Justify the proposition using examples and counter examples for understanding
iii. Rewrite propositions from sentences to mathematical expressions by using letters and diagrams
iv. Search the ways of proving by thinking backward from the conclusion and thinking forward from the given
v. Show the proof and critique for the shareable and reasonableness
vi. Deduce other propositions in the process of proving and after proving using what if and what if not
vii. Adapt ways of proving to other similar propositions of proof
viii. Explain the written proof in geometry and algebra by the known
ix. Revise others' explanation meaningfully

Utilising tables, graphs and expressions in situations[15]

i. Explore the properties of functions by using tables, graphs and expressions and establish the fluency of connections among them for interpreting functions in context
ii. Analyse distribution of raw data by using tables, graphs and expressions in daily life

Using diagrams for exploring possible and various cases logically[16]

i. Use number line with inequality to identify range and set
ii. Use circle to identify relationship between the circumference and the central angle (acute, obtuse and right)
iii. Use rectangle and rotating a point on the side rectangle to draw the graph of the area
iv. Use tree diagram for thinking about all possible cases sequentially

Exploring graph of functions by rotation, by symmetry and by translation of proportional function[17]

i. Use the slope of a graph for the proportional function to rotate the graph or to determine the point of intersection
ii. Explore to know the nature of two simultaneous equations by using translation
iii. Use the $y$-axis, $x$-axis and $y=x$ as the line of symmetry to explore proportional function
iv. Explain the graph of linear function by translation of proportional function.

Understanding ways for extension of numbers[18]

i. Extend the numbers based on the necessity of solving equations such as $x + 5 = 3$ and , and show examples for demonstrating the existence such as on the number lines, and understand it as set
ii. Compare the size of number and identify how to explain the order of numbers and its equivalence
iii. Extend operations for keeping the form[19] beyond the limitations of meaning[20]

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[15] *Connected to the strand on Relations and Functions.*
[16] *Connected to the two strands on Relations and Functions and Space and Geometry.*
[17] *Connected to the two strands on Numbers and Algebra and Relations and Functions.*
[18] *Connected to the strand on Numbers and Algebra.*
[19] *There are three meanings of form: (1) Permanence of form means "keep the pattern of operation" such as (-3)x(+2)=-6, (-3)x(+1)=-3, (-3)x0=0, and (-3)x(-1)=+3, and (-3)x(-2)=+6. Here, the product of the pattern increases by 3; (2) The form means "Principle of the permanence of equivalence form" which means to keep the law of commutativity, associativity and distributivity; and (3) The form means the axiom of field in Algebra. Normally, in education, we only treat (1) and( 2).*
[20] *For the extension of numbers to positive and negative numbers, beyond the limitations of meaning such as subtract smaller number from larger number. For the extension of numbers to irrational number, beyond the limitation of meaning such as rational number is quotient (value of division).*

Designing models for sustainability using mathematics[21]

i. Discuss and utilise probabilities in life such as weather forecasting for planning
ii. Design cost saving lifestyle models through comparison of data such as cost of electricity, water consumption, and survey
iii. Plan emergency evacuation such as heavy raining and landslide where the calculations on the amount of water in barrel per minute exceeds the maximum standards
iv. Forecast the future with mathematics

Utilising ICT tools as well as other technological tools

i. Use dynamic geometry software for assumption, specialisation and generalisation
ii. Use graphing tool for comparison of the graph and knowing properties of function
iii. Use data to analyse statistics with software
iv. Use internet data for the discussion of sustainable development
v. Use calculators for operations at necessary context
vi. Use projector for sharing ideas such as project survey, reporting and presentation
vii. Use the idea of function to control mechanism
viii. Use ICT tools for conjecturing and justifying to produce the object of proving.

Promoting creative and global citizenship for sustainable development of society using mathematics

i. Utilise notebooks, journal books and appropriate ICT tools to wisely record and produce good ideas for sharing with others
ii. Prepare and present ideas using posters, projectors, pamphlets and social media to promote good practices in society
iii. Promote the beautifulness, reasonableness and simplicity of mathematics through contextual situations in the society
iv. Listen to other's ideas and ask questions for better designs, craftsmanship and innovations
v. Utilise information, properties, models and visible representations as the basis for making intelligent decisions
vi. Utilise practical arts, home economics, financial mathematics and outdoor studies to investigate local issues for improving welfare of life

[21] *Connected to the three strands, Numbers and Algebra, Relations and Functions and Space and Geometry.*