Elementary School Teaching Guide for the Japanese Course of Study : Mathematics (Grade 1-6)

with the English translation on the opposite page

小学校学習指導要領解説 算数編 日英対訳

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Preface

For a number of years, many Japanese educators have been engaging in international cooperation with their educational experiences. English translation of '*The Teaching Guide of the Course of Study for Elementary School Mathematics*[4]*'* as a guidebook on Japanese curriculum standards[5] is contained in this material along with original Japanese sentences. This was made for those who participate in international education cooperation abroad with Japanese educational background. The translation work was conducted as a part of the project "Preparing the Educational Information of Japan for Japan Overseas Cooperation Volunteers (JOCV)" (Representative: Mariko Sato / CRICED, University of Tsukuba) of International Cooperation Initiative by Ministry of Education, Culture, Sports, Science and Technology (MEXT). The following is the background and how to use of this book.

1) Revision of the Curriculum Standard in Japan[6]

In Japan, curriculum standards are determined by the Course of Study and School Education Law (Act). School Education Law defines the number of classes and subjects.

Japanese curriculum standards are designed within the frameworks designated by superior laws and councils and course of study is set by the committee for each subject established under Curriculum Council. The course of study is developed based on the information given from the academic achievement survey conducted by MEXT, lesson studies at laboratory schools, international comparative studies of students' achievement, and researches on the results of educational studies by academic societies.

Fundamental Law of Education was amended in December 22, 2006, under the following objective of the law.

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4 In Japan, Elementary School Mathematics has been translated as 'Arithmetic' in English. In Japanese, since 1941, the name of the elementary school mathematics has been called 'San Su', which means Elementary School Mathematics. Before 1941, it was called 'Sanjutsu', Arithmetic. After World War II in the Occupation, there was no one word in English translation for Elementary School mathematics, thus Ministry of Education still used Arithmetic for Elementary School Mathematics. In this book, we preferred mathematics instead of Elementary School Mathematics or Arithmetic as for the translation of 'San Su'

5 'Course of Study' is an English translation for 'Gakushu Shidou Youryo' adopted in the Occupation after World War II. Today it could be regarded as Curriculum Standards.

6 The revision process of Japanese curriculum standards and school curriculum is well descried following articles: Isoda, M., Stephen, M. Ohyara, Y., Miyakawa, T. (2007). *Japanese Lesson Study in Mathematics*. NJ: World Scientific.

“We, the citizens of Japan, desire to further develop the democratic and cultural state we have built through our untiring efforts, and contribute to the peace of the world and the improvement of the welfare of humanity. To realize these ideals, we shall esteem individual dignity, and endeavor to bring up people who long for truth and justice, honor the public spirit, and are rich in humanity and creativity, while promoting an education which transmits tradition and aims at the creation of a new culture. We hereby enact this Act, in accordance with the spirit of the Constitution of Japan, in order to establish the foundations of education and promote an education that opens the way to our country’s future.”

Under the amended Fundamental Law of Education, School Education Law (Act) was revised on June 27, 2007, and the goal of the school education was changed as follows.

“Particular attention must be paid to achieve basic knowledge and skills, to cultivate thinking, decision making and expressing ability to solve problems by using those knowledge and skills, and to nurture the attitude to willingly pursue leaning in order to lay foundation for lifelong learning.”

In accordance with this revision of School Education Law, the new course of study for compulsory education was announced on March 20, 2008. Shift to the revised version started in April, 2009 and it will be implemented completely from April, 2011.

While the Course of Study is regarded as a legal document, MEXT has published guides for the course of study, though they are not legally binding, to explain the gist of it. ‘*The Teaching Guide of the Course of Study for Elementary School Mathematics’* is one of those guidebooks.

2) How to use this book

This book shows the composition of Guidebook for the curriculum standards in Japan and the elementary mathematics curriculum in Japan. There are roughly two ways of using this book.

The first is to use the book as a reference book to learn verbal expressions in English-speaking countries by listing parallel translation in Japanese and in English. Half of in-service teachers sent as Japan Overseas Cooperation Volunteers (JOCV) are elementary school teachers, and half of them directly work on improvement of mathematics education at elementary school. They are expected for the first time in their life to work and develop activities in English. This book is useful to help those teachers to find expression when they talk about education in English. Even if they are posted in non-English speaking area, knowing English expression could be a clue to find right expression in the native tongue of the area.

Another usage is as an evidence document. The Course of Study as an evidence document is not for everyday reading even in Japan. There are a number of teachers who believe they can teach without reading curriculum standards all around the world. In Japan, on the other hand, textbooks are developed based on the curriculum standards of education. Therefore, when it comes to discussing why a theme is taught in a classroom, typical documents for explaining the reason as evidence are guidebooks for the Course of Study like this book.

At present, thousands of educators are trained within and outside of Japan by teacher trainers with background in Japanese education. On the training, it is required to explain situation in Japan precisely. When holding teacher training outside of Japan, when training trainees and exchange students in Japan, and when conducting cross-national research on education, teacher trainers often use a phrase "in Japanese cases". In some cases, personal experiences which would not gain support in Japan were transmitted to abroad as "Japanese case".

For example, JICA recognizes following characteristics in Japanese Education: "child-centered approach", "well-sequenced curriculum", and "Improvement through lesson studies". To discuss questions like "How 'children-centered approach' was described in Japanese education curriculum documents?" and "What type of education curriculum is described as 'well-sequenced curriculum'?", this translation of *The Teaching Guide of the Course of Study for Elementary School Mathematics* will become the foundation.

I would like to express my sincere appreciation to the team of translators without whom this project would not have been completed in such a timely fashion: Akihiko Takahashi, Ph.D., DePaul University. Tad Watanabe, Ph.D., Kennesaw State University. Makoto Yoshida, Ph.D., William Patterson University, Natsumi Takezawa, Wolfram Research, Inc., Yoshi Takezawa, Wolfram Research, Inc.,Thomas McDougal., DePaul University.

At the same time, I would like to deeply acknowledge Shigeo Yoshikawa, Inspector, Ministry of Education, Japan who gave the editor necessary information, guidance and clear suggestion for translation based on the edition of the ministry of education[8], Japan, and Mari Usami and Masaru Sanuki, Ph.D. , CRICED, University of Tsukuba who support this translation through the back translation to Japanese from English translation and compared it with the original Japanese document.

Without their contributions, suggestions and supports, I could not complete this work.

24,December,2009.

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Editor of the English translation of
The Guide of the Course of Study for Elementary School MATHEMATICS
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8 http://www.mext.go.jp/component/a_menu/education/micro_detail/__icsFiles/afieldfile/2009/04/21/1261037_4.pdf

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Chapter 1. General Overview

1. History of the Revision

The 21st century is the so-called "era of the knowledge-based society" where new knowledge, information, and technology have become dramatically important in various aspects of life, such as politics, the economy, and culture. This process of moving to a knowledge-based society and globalization will intensify international competition for ideas, knowledge, and human resources, and at the same time emphasize the importance of coexistence and global cooperation among different cultures and civilizations. In this atmosphere, it will become more and more important to nurture a "zest for living" that emphasizes scholastic ability, a rich heart and mind, and the harmony of a healthy body.

On the other hand, various studies such as the Program for International Student Assessment (PISA) study done by the Organization for Economic Co-operation and Development (OECD) indicate the following about students in Japan:

(1) difficulty in reading comprehension and with writing problems that require thinking, decision-making, and expressing;
(2) problems in the widening distribution of scores in reading comprehension, that are reflections of problems at home where not enough time is spent studying, and there is a lack of motivation to learn, as well as a poor study and living environment;
(3) lack of confidence in their ability; feelings of insecurity about the future; and lowered physical strength.

Consequently, to enrich the education for students who will live in the 21st century, in February 2005 the Ministry of Education, Culture, Sports, Science and Technology-Japan (MEXT) requested the Central Education Council to reform the standards for the curriculum, and also to improve the quality and skill of teachers and the infrastructure of education. In April of the same year, discussion on these subjects commenced.

During this period, the Fundamental Law of Education and the School of Education Law were amended, and the balance between knowledge, morality, and health was reaffirmed (the Fundamental Law of Education, Article 2,1), as were the importance of fundamental and basic knowledge and skills, of ability to think, to make decisions, and to express, and of motivation for learning (School Education Law, Article 30,2). For school education, it is now stated in the law that it is necessary to promote these objectives in a harmonious way. Based on the amendment of the laws, these fundamental issues in education were discussed in the Central Education Council. The discussion lasted 2 years and 10 months, and in January 2008 the report, *The Reform of the Curriculum for Kindergarten, Elementary Schools, Lower Secondary Schools, Upper Secondary Schools and Special-Needs Schools*, was submitted.

Based on the problems our students are facing, this report gives the following points that were considered as the basis for reform as well as directions for each school level and subject:

(1) Reform of the curriculum based on the amendment of the Fundamental Law of Education
(2) Establishing a shared concept of "zest for life"
(3) Acquiring fundamental and basic knowledge and skills
(4) Fostering the ability to think, to make decisions, and to express
(5) Allocating necessary hours of classes to acquire solid academic ability
(6) Motivating students to learn and helping them develop sound study habits
(7) Enriching teaching for fostering rich minds and good physical health.

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Specifically regarding (1), in order to develop Japanese people who are courageous and rich in spirit and who can lead the world of the 21st century, the Fundamental Law of Education was amended for the first time in approximately 60 years, and new ideals for future education were established. In the School Education Law amended by the Fundamental Law of Education, the new objectives of compulsory education were ordained, and reform of the curriculum is required to be fully grounded in the revision of the objectives of each school level. In (3), fundamental and basic skills such as reading, writing, and calculation are to be completely acquired, so that they can form the foundation for future learning as appropriate to each developmental stage. For example, in grades 1 through 4, the emphasis is given on understanding obtained through experience and repeated practice. Building on this foundation, to foster the ability to think, to make decisions, and to express described in (4), learning activities, such as experiments, observations, and writing reports and essays, in which knowledge and skills are utilized are enhanced according to each developmental stage. In addition, in order to develop language ability to support these activities, it was pointed out that in each subject it is necessary to acquire basic Japanese language skills – reading out loud, reading silently, reading and writing kanji characters – followed by learning activities such as recording, summarizing, describing, and writing essays. In order to foster a rich mind and good physical health (7), in addition to enriching character education and physical education classes, by emphasizing language skills in Japanese language class and other classes and by nurturing learning through experience, it was suggested that it is necessary to let students gain confidence in life by interacting with others, society, nature, and the environment. Based on the report of 3/28/2008, the School Education Law Enforcement Regulation was amended, and educational guidelines for kindergarten and elementary and lower secondary schools were made public.

Educational guidelines for elementary schools will be implemented ahead of schedule for courses such as mathematics and science from 4/1/2009, and will be fully implemented by 4/1/2011.

2. Basic Policies for the Revision in Mathematics

The revision of the curriculum for mathematics was based on the main reform policies in mathematics stated in the report of the Central Education Council.

○ In mathematics, considering its challenges, mathematical activities must be further enriched so that students will acquire fundamental and basic knowledge and skills, develop the ability to think and express mathematically, and increase their motivation to learn throughout elementary, lower secondary, and upper secondary schools, according to their developmental stage.

The first policy states a very important objective in mathematics education throughout elementary, lower secondary, and upper secondary schools. The recent Japanese study of the curriculum and the international study of academic competitiveness indicate no decline in Japanese students' technical skills such as calculation, but the students find difficulty in interpreting the meaning of a problem and they do not sufficiently use their acquired skills and knowledge in learning and daily life. To correct these problems, the following concrete policy was adopted:

○ The fundamental and basic knowledge and skills of numbers, quantities, and geometrical figures are the foundation for daily living and learning. With the advancement of scientific technology, it is actively being discussed more than ever that mathematics and science education should meet international competitive levels. Consequently, to firmly establish the fundamental and basic knowledge and skills of numbers, quantities, and geometrical figures in students, while retaining the importance of a systematic nature of mathematics, the curriculum may adopt repeated learning (spiral) according to developmental stages and grade levels of students by overlapping some of the content across grades.

The second policy is about organizing the curriculum by incorporating a spiral structure in the curriculum according to the developmental stages and grade levels of students in order to firmly establish fundamental and basic knowledge and skills. It is a characteristic of mathematics that its contents are systematically organized and a continuous learning path is clearly defined. Considering this characteristic, this policy aims to develop a curriculum that provides better connections across grades within the same content domain, and gradually increases the levels of complexity.

○ Mathematical thinking and expression play an important role in rational and logical thinking as well as in intellectual communication. For this reason, instructional content and activities that foster mathematical thinking and expression should be clearly indicated. We will enrich the kind of teaching where students are taught to think systematically, in logical steps, by reasoning, and to understand the connections among words, numbers, algebraic expressions, figures, tables, and graphs. This kind of teaching will also allow students to learn appropriate usage, problem-solving, how to explain one's ideas clearly, and how to express and communicate one's ideas to others.

The third policy is about fostering mathematical thinking and exposition. In mathematics, as in spoken language, there are many ways to express ideas, using numbers, algebraic expressions, figures, tables, or graphs. It is important to enrich learning activities where these methods are used to teach how to think, explain, and express one's ideas.

◯ It is important that we motivate students to learn mathematics, and students should experience the meaning of learning and utility of what was learned. For this reason, the following objectives are emphasized:

- To help students understand numbers, quantities, and geometrical figures through learning activities which serve as a basis for understanding their meanings.
- To help students feel progress in learning, such as a depth and broadening of understanding through repeated learning (spiral) that is designed according to the developmental stage and grade level of each student.
- To help students apply what has been learned to activities in daily life, to the study of other subjects, and to learning more advanced mathematics.

The fourth policy is about increasing the motivation for learning. The objectives that are emphasized are as follows: to enable students to develop an understanding of number and geometrical figures through experience with foundational learning activities at appropriate levels, to help them feel the progress in their learning by broadening and deepening their understanding through the spiral structure of the curriculum, and to help them apply what they have learned and acquired to activities in their daily life, in the study of other subjects, or in learning more advanced mathematics.

◯ Mathematical activities play an important role in helping students acquire fundamental and basic knowledge and skills, in increasing students' ability to think and express mathematically, and in enabling students to feel a joy and purpose in learning mathematics. To enrich the teaching of mathematics through mathematical activities — with experiential activities and an emphasis on language — concrete examples of mathematical activities should be provided in the curriculum for elementary and lower secondary schools; and in upper secondary schools, project-based learning is introduced in the required subjects and in more popular elective courses.

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The fifth policy is about enriching mathematical activities. The phrase "mathematical activity" was first introduced in the Course of Study (COS), 1988. Along with defining the meaning of mathematical activities, the revised COS also describes instructional activities in each grade.

In addition to the five policies described above, the report indicates the following for reforming mathematics in elementary schools.

○ In elementary schools, the emphasis is placed on enriching mathematical activities and building fundamental and basic knowledge and skills while fostering a rich sense for numbers, quantities, and geometrical figures. It also raises the students ability to think and express mathematically by using what has already been learned in daily life. To achieve these goals, the following reforms will be made:

a. The structure of the domains are unchanged; they are "Numbers and Calculations", "Quantities and Measurements", "Geometrical Figures", and "Mathematical Relations". To emphasize the importance of thinking and expressing mathematically — using words, algebraic expressions, tables, and graphs — the domain "Mathematical Relations" is introduced in lower grades.

b. To enable students to acquire knowledge and skills regarding quantities and geometrical figures, and to foster their ability to think and express mathematically, a part of the contents is overlapped between grades, emphasizing the importance of a systematic path of mathematics. This means contents are gradually developed, giving an opportunity to review what has been learned. Teaching by this repeated learning (spiral) is applied according to developmental stages and grades.

c. In order to emphasize the importance of mathematical activities, the contents in each grade now has a section that describes these activities. In doing this, special consideration is given to the connection between elementary schools and lower secondary schools. Concrete examples of mathematical activities are now included; for example, activities that promote the understanding of quantities and geometrical figures using manipulative; activities that require application of knowledge and skills in real-world situations; and activities in which students explore and explain methods of problem solving.

The report states the following about the reforms in each domain of mathematics while clarifying those that need to be emphasized:

d. Under the domain "Numbers and Calculations", the report emphasizes understanding how to express integers[1], decimal numbers, and fractions, and developing a rich sense for numbers. It also emphasizes:

   - Understanding the meaning of calculation;
   - Thinking about the method of calculation;
   - Acquiring and using calculation skills.

   For example, in grades 1 and 2, in order to understand fractions, fundamental mathematical activities such as folding a paper in half are carried out. Through a curriculum that uses a repeated learning method (spiral), students are expected to completely acquire calculation skills of integers in grades 1 through 4, and in grades 3 through 6 they should develop calculation skills of decimal numbers and fractions. In grades 3 and 4, estimations of results are taught, and students are expected to estimate the results of calculations and make appropriate reasoning.

e. In the "Quantities and Measurement" domain, understanding various units of quantities and their measurements, enriching a sense for the size of quantities, and explaining how to find quantities such as area, are considered important.

   For example, in lower grades, direct comparison of quantities such as the length and area of actual objects is taught. In grades 3 through 6, students are taught to investigate and summarize conclusions about the relationship between units. In grades 5 and 6, using previously learned knowledge about area, calculations of the area of a rhombus and trapezoid are taught.

f. In the "geometrical figures" domain, understanding the meaning and properties of geometrical figures, enriching a sense for geometrical figures, and becoming able to use the skills to view geometrical figures in learning and daily life are considered to be important.

   For example, in all grade levels, students are asked to draw, construct, or tile with various geometrical figures, and they are asked to compare their shapes and sizes. The geometric plane and solid figures are taught in a well-balanced manner. In grades 5 and 6, congruence, enlargement, and reduction of geometrical figures are taught.

g. In the "Mathematical Relations" domain, expressing quantitative relationships with words, numbers, algebraic expressions, tables, and graphs, and raising the idea of a function as the change and correlation between two quantities are considered to be important.

   For example, in grades 1 and 2, using simple tables and graphs, students are taught to represent phenomena and situations they experience. In grades 3 through 6, students are taught to write algebraic expressions with □ and with letters. In grades 5 and 6, the curriculum for teaching the concepts of proportion and inverse proportion are enriched.

[1]The elementary school mathematics curriculum does not deal with negative integers. Thus, "integers" here refers to positive integers and 0, which are typically referred to as "whole numbers"

3. Main Points of the Revision in Mathematics

(1) Objectives

The following describes the objectives of mathematics instruction.

Through mathematical activities, to help pupils acquire basic and fundamental knowledge and skills regarding numbers, quantities and geometrical figures, to foster their ability to think and express with good perspectives and logically on matters of everyday life, to help pupils find pleasure in mathematical activities and appreciate the value of mathematical approaches, and to foster an attitude to willingly make use of mathematics in their daily lives as well as in their learning.

α **"Through mathematical activities"**

In this revision, the phrase "through mathematical activities" is used at the top of this section for the first time. The objectives to which the term "through mathematical activities" refers have the same structure as in the previous COS. "Mathematical activities" mean various activities where students willingly and purposefully work on mathematics.

Mathematical activities can include various activities. Hands-on activities, experimental activities, physical activities, and activities that use concrete objects are often considered to be typical examples of mathematical activities. But there are others.

Thinking about mathematical problems, building on mathematical knowledge and applying that knowledge, representing and explaining what students think – these do not deal with concrete objects but are included in mathematical activities.

β **"Foster their ability to think and express with good perspectives and logically"**

In this revision, the phrase "(ability) to express" was added to the statement, "foster their ability to think and express with good perspectives and logically". The ability to think and the ability to express are considered to be complementary. In the process of expressing their thoughts, students may realize their own good points or errors in their ideas. By expressing thoughts, they become better able to organize logical steps and produce better ideas. In class, they can express various ideas and they can learn from each other. For this reason, thinking and expressing are mentioned in parallel.

γ **"Foster an attitude to willingly make use of mathematics in their daily lives as well as in their learning"**

In this revision, solidifying fundamental and basic knowledge and skills and using what has been acquired are emphasized. Mathematical knowledge can be used in many daily life situations and in one's studies. It is not limited to studying other subjects but it can be applied to the future study of mathematics. In this revision, the term "studies" was added, and the term "apply" was changed to "make a use of" to make these points.

(2) The Objectives and Content in Each Grade

① Objectives and Structure of the Contents

In mathematics, the objectives and content are shown in each grade, and are described in the order below:

1. Objectives
2. Content
   - A. Numbers and Calculations
   - B. Quantities and Measurements
   - C. Geometrical figures
   - D. Mathematical Relations

     Mathematical Activities

     Terms and Symbols
3. Handling the Content

The content taught in each grade is categorized into four domains: "A. Numbers and Calculations", "B. Quantities and Measurements", "C. Geometrical figures", and "D. Mathematical Relations". This is to make it easy to see the complete content of the mathematics curricula, and to make its connections and range intelligible.

In this revision, we include domain D, "Mathematical Relations" in grades 1 and 2. Due to this change, the contents in all six grades now consist of the same four domains. Furthermore, in this revision, following the four domains, "mathematical activities" has been added for each grade.

For the configuration of curricular content for elementary schools, a summary is shown in Table 1, page 27, for each grade and each domain. The curricular content for lower secondary school is shown in Table 2, page 29.

② Mathematical activities

As previously noted, "mathematical activities" mean various activities in which students willingly and purposefully work on mathematics.

What is meant by "willingly engage in mathematics with purpose" is trying to find new properties or to create new ways of thinking or to solve concrete problems. It is necessary to teach students to have a clear purpose and to willingly engage in learning in order for them to understand the meaning of quantities and geometrical figures, to enhance their ability to think, to make decisions, and to express, and for them to feel the joy and meaning of learning mathematics. For this reason, activities where students just listen to teachers' explanations or complete practice problems are not included in mathematical activities.

Mathematical activities can include various activities. Hands-on activities, experimental activities, physical activities, and activities that use concrete objects are often considered to be typical examples of mathematical activities. But there are others. Thinking about mathematical problems, building on mathematical knowledge and applying that knowledge, representing and explaining what students think – these do not deal with concrete objects but are included in mathematical activities.

In this revision, in order to enrich mathematical activities and to clarify their role in mathematics lessons, actual examples are included in the content of each grade.

In the beginning of the "Mathematical Activities" section in each grade, the following statement will be found:

> (1) The content listed in "A. Numbers and Calculations", "B. Quantities and Measurements", "C. Geometrical Figures" and "D. Mathematical Relations" should be taught through, for example, the following mathematical activities.

This statement suggests that it is necessary to teach all the content in each curricular domain by conducting mathematical activities. However, that does not mean that we deny the appropriateness of teachers explaining something or assigning practice problems, as these are naturally necessary.

The mathematical activities listed in the curricular content are considered typical activities students can do. There are so many different activities depending on content and learning progress, it is impossible to list all of them. As implied by the word "examples," the mathematical activities stated there can be implemented as they are, or similar activities can be added. Furthermore, activities not mentioned can be created and introduced by schools and teachers.

The summary of mathematical activities from grade 1 to grade 6 is shown below.

Grade 1

a. Activities to count concrete objects
b. Activities to express the meaning and ways of calculation
c. Activities to compare quantities
d. Activities to find and to compose shapes
e. Activities to represent situations in algebraic expressions

Grade 2

a. Activities to find situations where integers are used
b. Activities to find rules in the multiplication (*ku-ku*) table
c. Activities to estimate size
d. Activities to draw geometrical figures; to construct them; tessellations
e. Activities to draw figures; to write algebraic expressions; explaining with them

Grade 3

a. Activities to think of ways to calculate and to explain them
b. Activities to compare decimal numbers and fractions
c. Activities to learn relations between units
d. Activities to draw geometrical figures such as equilateral triangles
e. Activities to sort items and to make tables

Grade 4

a. Activities to estimate results of calculations and to make judgments
b. Activities to think about how areas can be calculated and to explain one's ideas
c. Activities to measure areas
d. Activities to tessellate (tile) with figures such as parallelograms and learning properties of figures
e. Activities to investigate relationships among quantities in their surroundings

Grade 5

a. Activities to think of ways to calculate and to explain them
b. Activities to think about how areas can be calculated and to explain one's ideas
c. Activities to construct and to draw congruent figures
d. Activities to explain properties of geometrical figures inductively and deductively
e. Activities to choose and to use appropriate graphs and tables for different purposes

Grade 6

a. Activities to think of ways to calculate and to explain them
b. Activities to learn relations between units
c. Activities to find scaled drawings and symmetric figures
d. Activities to solve problems by using proportions

③ Improvement of the content of each domain

The following is a summary of the main changes of contents in this revision – enhancements, additions, or repositionings – organized by the domains.

A. Numbers and Calculations

The meaning, representation, and calculations of integers are mainly taught as before, in grades 1 through 4. In this revision, in order to promote learning of fundamental and basic contents, a major emphasis is placed on the spiral structure across grades. This allows curricula to devise more effective ways to connect contents within the same content strand across grades and to gradually raise the level of sophistication. For example, in grade 1, "a simple representation of 3-digit numbers," and "addition and subtraction with simple 2-digit numbers" are introduced; in grade 2, "addition and subtraction with simple 3-digit numbers," and "multiplication between a simple 2-digit number and 1-digit number" are introduced; in grade 3, "simple division whose quotient becomes a 2-digit number" is introduced; and in grade 4, "the acquisition of calculation skills of integers" is noted.

Instruction on the meaning and representation of decimal numbers and fractions is included mainly in grades 3 through 6. As with integers, the spiral structure across the grades is emphasized. For example: in grade 2, simple fractions are introduced; in grade 3, "addition and subtraction with simple decimal numbers" and "addition and subtraction with simple fractions" are introduced; in grade 4, "multiplication and division with simple decimal numbers" is introduced; in grade 5, "multiplication and division with simple fractions" is introduced; and in grade 6, "the acquisition of calculation skills of decimal numbers and fractions" is noted.

As for the range of calculation with integers, "addition and subtraction with 4-digit numbers" and "multiplication between 3-digit numbers and 2-digit numbers" are now newly noted. For decimal numbers and fractions, the hadome restriction[1] is now removed.

The instructional content of "estimating the results of calculation" is included in grade 4, and stresses the students' ability to appropriately reason about ways of calculating and coming up with results based on estimates.

B. Quantities and Measurements

In addition to "length," "comparing areas and volumes" is noted in grade 1, and it is hoped that this will become the foundation for understanding units and measurements in grade 2 and beyond. The instructional content moved between grades includes, for example, "Units of volume (liter, etc.)" in grade 2; "Units of volume (cubic centimeters, etc.)" and "per-unit quantities" in grade 5; and "finding the area of a circle" in grade 6. The new instructional content in grade 6, "finding volumes of prisms and cylinders" and "understanding units in the metric system and how they work," is noted.

C. Geometrical figures

Plane figures and solid figures can be taught with sufficient balance in each grade. In grade 1, shapes that appear in the everyday life of students are used, and both plane figures and solid figures are taught. The instructional contents that have been moved between grades are: "square, rectangle, and right triangle" and "box shape" in grade 2; "isosceles triangle and equilateral triangle," "angle," "circle and sphere" in grade 3; "parallelogram, rhombus, and trapezoid" and "cube and rectangular parallelepiped" in grade 4; and "prisms and cylinders" in grade 5. The instructional contents that have been moved from lower secondary schools is "congruence of figures" in grade 5 and "scaled drawing" and "symmetric figures" in grade 6. These are noted to address the continuity of instructional contents between elementary and lower secondary schools. As new contents, "how to represent the location of a thing" in grade 4, and "polygon and regular polygon" in grade 5 are noted.

D. Mathematical Relations

In this revision, the instructional content for algebraic expressions includes: "algebraic expressions for addition/subtraction" in grade 1; "algebraic expressions representing the relationship between addition and subtraction" and "algebraic expressions for multiplication" in grade 2; "algebraic expressions for division" and "algebraic expressions with □" in grade 3; "algebraic expressions that use □ and △," (partially moved from lower secondary school) in grade 4; and "algebraic expressions with letters" in grade 6.

In regard to functions, "simple proportional relationships" in grade 5 is noted, and connects to "proportional relationships" in grade 6 so that the level of complexity may be gradually increased. Also, in grade 6, "Inverse proportional relationships" (partially moved from lower secondary schools) is noted.

As for organizing data, "representing quantities using pictures and diagrams" is noted in grade 1; "simple tables and graphs" is in grade 2; and "frequency distribution" and "possible outcomes" (partially moved from lower secondary schools) are noted in grade 6.

[1] *Hadome* restriction means explicitly stating which items to teach and which not to teach.

Table 1: Structure of the content of mathematics in elementary school

[] indicates new content; {} indicates overlapping contents between grades due to the spiral structure of the curriculum. The underline indicates content moved between grades.

	A. Numbers and Calculations	B. Quantities and Measurements	C. Geometrical figures	D. Mathematical Relations	Mathematical activities
Grade 1	**Meaning and Representation of integers** ・2-digit numbers and simple 3-digit numbers **Addition and subtraction of integers** Addition and subtraction of simple 2-digit numbers	**Comparing sizes of quantities** ・comparing length, comparing [area and volume] **Reading clock times** (moved from grade 2)	**Geometrical figures** ・observing and composing the shapes of familiar objects (plane figures, solid figures)	**Representation with algebraic expressions** ・representing situations where addition and subtraction are used by using algebraic expressions (moved from "A. Numbers and Calculations") **Representing the number of objects using pictures or figures**	a. Activities to count concrete objects b. Activities to express the meaning and ways of calculation c. Activities to compare quantities d. Activities to find and compose various shapes e. Activities to express situations with algebraic expressions
Grade 2	**Representation of numbers such as integers** ・3-digit numbers, 4-digit numbers, 10000, simple fractions (1/2, 1/4 etc.), etc. Addition and subtraction of integers ・Addition and subtraction of 2-digit numbers and addition and subtraction of simple 3-digit numbers **Multiplication of integers** ・multiplication table (ku-ku) [up to 9×9] ・multiplication of simple 2-digit numbers	**Units and measurements of quantities** ・units of length (mm, cm, m) ・units of volume, (mℓ, dℓ, ℓ) (moved from grade 3) **Units of time: (days, hours, minutes)** (moved from grade 3)	**Geometrical figures** ・triangles, quadrilateral ・squares, rectangles, right triangles (moved from grade 3) ・shape of a box (moved from grade 3)	**Representation with algebraic expressions** ・relationship between addition and subtraction (moved from "A. Numbers and Calculations") ・representing situations where multiplication is used by using algebraic expressions (moved from "A. Numbers and Calculations") **Simple tables and graphs** (moved from "A. Numbers and Calculations")	a. Activities to find situations where integers are used b. Activities to find rules from multiplication tables. c. Activities to estimate the sizes of quantities d. Activities to draw and construct geometrical figures. e. Activities to express relationships by using figures, diagrams and algebraic expressions
Grade 3	**Representation of integers** ・the unit of ten-thousands (man in Japanese), 100 million (oku in Japanese) Addition and subtraction of integers ・addition and subtraction of 3-digit numbers and [4-digit numbers] Multiplication of integers ・multiplication of 2-digit numbers and 3-digit numbers, (for example, [3-digit number × 2-digit number]) Division of integers ・division in simple cases when the divisors are 1-digit numbers (the quotients are 1-digit or 2-digit numbers) Decimal numbers (moved from grade 4) ・The meaning and the representation of decimal numbers, addition and subtraction of decimal numbers (the tenths place) Fractions (moved from grade 4 and 5) ・The meaning and the representation of fractions, simple addition and subtraction of fractions Soroban (Japanese Abacus) ・The representations of numbers on soroban addition and subtraction	**Various units and measurements** ・Units of length (km) and weight (g, kg, [ton]) Measurement using instruments Units of time(seconds), calculations with time	**Geometrical figures** ・isosceles triangle, equilateral triangle (moved from grade 4) ・angle (moved from) ・circle, sphere (moved from grade 4)	**Representation with algebraic expressions**・representing situations where divisions are used, by using algebraic expressions (moved from "A. Numbers and Calculations") ・[making connections between algebraic expressions and diagrams; algebraic expressions that use □] Tables and bar graphs	a. Activities to explore and explain ways of calculation b. Activities to compare sizes of decimal numbers and fractions c. Activities to investigate the relationships among the units d. Activities to construct geometrical figures such as equilateral triangles e. Activities to organize data and to represent the data in tables

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	A. Numbers and Calculations	B. Quantities and Measurements	C. Geometrical figures	D. Mathematical Relations	Mathematical activities
Grade 4	**Representation of integers** The units such as hundred million (oku in Japanese) and trillion (cho in Japanese), Round numbers ・round numbers, rounding, estimate the results of four basic calculation (moved from grades 5 and 6) **Division of integers** ・division in the cases where the divisor is a 2-digit number. Acquisition and utilization of the four basic operations with integers **Calculation of decimal numbers** ・addition and subtraction of decimal numbers (the tenths and hundredths places, etc.) ・multiplication and division of decimal numbers (decimal number × integer, decimal number ÷ integer) (moved from grade 5) Calculation of fractions ・addition and subtraction of fractions with like denominators (proper fraction, [improper fraction]), etc. (moved from grade 5) Japanese Abacus ・addition and subtraction	**Area** ・Units of area (cm^2, m^2, km^2, [a, ha] and measurements ・finding areas of squares and rectangles Units of angles (degree (°))	**Geometrical figures** ・relationships of parallelism and perpendicularity of lines (moved from grade 5) ・parallelogram, rhombus, trapezoid (moved from grade 5) ・cube, rectangular parallelepiped (moved from grade 6) ・[representing the location of an object]	**relationships between two numbers/quantities as they vary simultaneously** ・represent how the numbers/quantities vary on a broken-line graph and to interpret the features of their variation Representation in algebraic expressions ・algebraic expressions that contain some of the four basic operations and, expressions with ( ), formulas ・[expressions with □, △] Properties of the four basic operations (moved from grade 5) Gathering and organizing data ・table with two viewpoints, broken-line graphs	a. Activities to estimate the result of calculations and to make proper decisions b. Activities to explore and explain ways to determine the area of geometrical figures c. Activities to actually measure the area d. Activities to investigate features of geometric figures, such as parallelograms, by tessellation. e. Activities to investigate how quantities in everyday life relate to each other
Grade 5	**Properties of integers** ・even and odd numbers, divisors and multiples (moved from the six grade), [prime numbers] Numeration system for integers and decimal numbers Calculation of decimal numbers ・multiplication and division of decimal numbers (the tenths and [hundredths] places, etc.) **Calculation of fractions** ・addition and subtraction of fractions with different denominators (proper fraction, [improper fraction]), (moved from grade 6) ・multiplication and division of fractions (fraction×integer, fraction÷integer)	**Area** ・finding areas of triangles and parallelograms ・[finding areas of rhombi and trapezoids] **Volume** (moved from grade 6) ・units of volume (cm3, m3) and measurement ・finding volumes of cubes and rectangular parallelepipeds **Mean of measurements** **Per-unit-quantity** (example: population density) (moved from grade 6)	**Geometrical figures** ・[polygons and regular polygons] ・[congruence of figures (partially moved from lower secondary schools)] ・properties of geometrical figures ・ratio of circumference of a circle to its diameter (pi) ・prisms, cylinders (moved from grade 6	Simple proportional relations Observation and examination of Mathematical Relations ・studying a relation of two quantities that are represented by a simple algebraic expressions Percentage **Pie charts, Bar graphs**	a. thinking about ways of calculating and explaining them b. thinking about ways to calculate areas and explaining them c. drawing and constructing congruent figures d. explaining properties of geometrical figures inductively and deductively e. selecting a graph or a table depending on each objective and applying them
Grade 6	**Calculation of fractions** ・multiplication and division of fractions ([calculations that involve both fractions and decimal numbers], etc.) Consolidation and utilization of the four basic operations with decimal numbers and fractions	**Approximate shape** ・approximate areas, etc. **Area** (moved from grade 5) ・finding area of a circle **Volume** ・[finding volume of prisms and cylinders (moved from lower secondary school)] **Speed** ・meaning and representation, finding speed [Metric system, how it works]	**Geometrical figures** ・[reduced figures and enlarged figures (moved from lower secondary schools)] ・[symmetric figures (moved from lower secondary schools)]	**Ratio** proportional relationships and [inversely proportional relationships (partially moved from lower secondary schools)] [algebraic expressions by using letters such as a, x (partially moved from lower secondary schools)] **Ways to examine data** ・the average of data ・[“frequency distribution”] [Possible outcomes (moved from lower secondary schools)]	a. Activities to explore and explain the meaning of calculations b. Activities to find how the units are related c. Activities to find reduced figures, enlarged figures and symmetric figures d. Activities to solve problems by using the proportional relationships.

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	A. Numbers and Algebraic expressions	B. Geometrical figures	C. Functions	D. Data Handling	Mathematical activities
Grade 7	**Positive numbers, Negative numbers** a. necessity and meaning of positive and negative numbers [“Set of numbers and four fundamental operations”] b. Meaning of four basic operations with positive and negative numbers c. Four basic operations with positive and negative numbers d. Using positive and negative numbers **Algebraic expressions using letters** a. Necessity and meaning of using letters b. how to express multiplication and division c. Addition and subtraction with linear expressions d. representing with algebraic expressions with letters [“representations with inequality”] **Linear equations with one unknown** a. Necessity and meaning of equations and the meaning of their solutions b. Properties of equations and how to solve them c. Solving and using linear equations [“Proportional expressions”]	**Plane figures** a. Fundamental methods for constructing figures and their application b.[moving basic figures] **Space figures** a. Positional relationship between straight lines and planes b. structure of space figures and their representation on a plane [“projection drawing”] c. measurements in basic figures [“surface area and volume of sphere”]	**Direct proportion, Inverse proportion** a. Meaning of functional relationships (moved from the eighth grade) b. Meaning of direct proportion and inverse proportion c. Meaning of coordinates d. tables, algebraic expressions, and graphs of direct proportion and inverse proportion e. applying direct proportion and inverse proportion	**Dispersion of data and representative values of data** (approximations and errors; using the notation $a \times 10^n$) a.[Necessity and meaning of histograms and representative values] b.[applying histograms and representative values]	In learning content within and across domains, opportunities to do mathematical activities such as shown below should be implemented. a. Activities for finding out the properties of numbers and geometrical figures based on previously learned mathematics. b. Activities for making use of mathematics in daily life c. Activities for explaining and communicating each other in one’s own way by using mathematical representations

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	A. Numbers and Algebraic expressions	B. Geometrical figures	C. Functions	D. Data Handling	Mathematical activities
Grade 8	**Calculation of the four basic operations with expressions using letters** a. Calculation of addition and subtraction with simple polynomials, as well as multiplication and division with monomials b. representing and interpreting algebraic expressions using letters. c. Transforming algebraic expressions according to the purpose **Simultaneous linear equations with two unknowns** a. Necessity and meaning of linear equations with two unknowns and the meaning their solutions b. Meaning of simultaneous linear equations with two unknowns and the meaning of their solutions c. solving simultaneous equations and applying them	**Basic plane figures and properties of parallel lines** a. properties of parallel lines and angles b. properties of angles of polygons **Congruence of plane figures** a. Congruence of plane figures, conditions for congruence of triangles b. Necessity, meaning and methods of proof c. basic properties of triangles and parallelograms	**Linear functions** a. phenomena and linear functions b. tables, algebraic expressions and graphs of linear functions c. linear equations with two unknowns and functions d. Using linear functions	**Probability** a Necessity and meaning of probability and finding the probability b. Using probabilities	In learning content within and across domains, opportunities to do mathematical activities such as shown below should be implemented. a. Activities for finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics. b. Activities for making use of mathematics in daily life and society c. Activities for explaining and communicating each other in an evidenced, coherent and logical manner by using mathematical representations

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	A. Numbers and Algebraic expressions	B. Geometrical figures	C. Functions	D. Data Handling	Mathematical activities
Grade 9	**Square root** a. Necessity and meaning of square roots [“rational numbers, irrational numbers”] b. calculations of expressions with square roots c. using square roots **Expanding and factoring algebraic expressions** a. multiplication and division of polynomials and monomials b. expanding and factoring of simple expressions c. representing and explaining through expressions using letters **Quadratic equations** a. Necessity and meaning of quadratic equations and the meaning of their solutions b. solving quadratic equations by factoring and completing squares c.[Solving quadratic equations by using the quadratic formula] d. Using quadratic equations	**Similarity of plane figures** a. Similarity of plane figures and conditions for similar triangles b. Basic properties of geometrical figures c. Parallel lines and ratios of line segments. d.[Relationships between the scale factor, the ratio of areas and the ratio of volumes of similar geometric figures] e. Using the properties of similar geometrical figures Inscribed angle and central angle a. Relationship between the inscribed angle and the central angles, and its proof (moved from eighth grade) [“converse of the Central Angle Theorem”] b Using the relationship between an inscribed angle and a central angle (moved from the seventh grade) **Pythagorean Theorem** a. Pythagorean Theorem and its proof b. Using the Pythagorean Theorem	**Function** $y = ax^2$ a. Phenomena and the function $y = ax^2$ b. Tables, algebraic expressions, and graphs of the function $y = ax^2$ c. Using the function $y = ax^2$ d. [Various concrete phenomena and functional relationships]	**Sample survey** [a. Necessity and meaning of a sample survey] [b. Carrying our sample surveys.]	

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Chapter 2. The Objectives and Content of Mathematics

"Section 3: Elementary School Mathematics" in the Course of Study is composed of the following:

1. OVERALL OBJECTIVES
2. OBJECTIVES AND CONTENT FOR EACH GRADE
3. SYLLABUS DESIGN AND HANDLING THE CONTENT

In this chapter, the objectives of mathematics and the objectives for each grade are explained first. Then, the objectives and main content in each of four domains, "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometrical Figures," and "D. Mathematical Relations" are explained.

Section 1: OVERALL OBJECTIVES

1. Objectives of the subject

In the objectives of the subject, skills, capabilities, and attitudes that we need to develop in students through the study of mathematics are outlined. This section clarifies the role mathematics plays in guiding human development in elementary education.

The objectives in teaching mathematics are described below:

> Through mathematical activities, to help pupils acquire basic and fundamental knowledge and skills regarding numbers, quantities and geometrical figures, to foster their ability to think and express with good perspectives and logically on matters of everyday life, to help pupils find pleasure in mathematical activities and appreciate the value of mathematical approaches, and to foster an attitude to willingly make use of mathematics in their daily lives as well as in their learning.

Even though the parts of objectives are closely related to each other, we will explain the objectives in five separate parts to make it easier to understand the points of the objectives clearly.

(1) Through mathematical activities

The objective begins with the phrase "Through mathematical activities", and it affects the overall objectives of mathematics. It states basic ideas about how to proceed with the course of study to achieve the objectives listed below.

Mathematical activities are various activities related to mathematics where students engage willingly and purposefully.

"Students engage willingly and purposefully," means students try to find new properties or new ways of thinking or try to solve a concrete problem. In order to help students experience the meaning of quantities and geometrical figures, raise their ability to think, make decisions and express ideas, and let them feel the joy and meaning in learning mathematics through mathematical activities, it is necessary to teach in such a way that students will engage in an activity willingly and purposefully. For this reason, simply listening to what teachers say and doing practice problems are not included in the definition of mathematical activities.

Mathematical activities can include various activities, and although physical hands-on, experimental activities are often mentioned, there are others. Thinking about mathematical problems, developing and applying one's knowledge, and presenting and explaining what they think are all activities that do not deal with concrete objects, but they are included in mathematical activities.

By incorporating mathematical activities, the following improvements can be expected in mathematics lessons:

- Mathematics lessons become more centered on students' activities and more proactive.
- Mathematics lessons become more fun to students.
- Mathematics lessons become easier to understand to students.
- Mathematics lessons become more compelling.
- Mathematics lessons become creative and exploratory.
- Mathematics becomes more connected to daily life and natural phenomena.
- It becomes easier to think about activities that relate mathematics and other subjects and Integrated Study.

(2) To help pupils acquire basic and fundamental knowledge and skills regarding numbers, quantities and geometrical figures

The basic and fundamental knowledge and skills students acquire in mathematics, as well as their abilities in language, provide the foundation for daily life and future studies. They form the basis for daily activities and learning in other subjects and Integrated Study. They also form the foundation of future studies in mathematics. In this regard, they are important.

Knowledge and skills can include: understanding the meaning, concepts, principles, and laws behind quantities and geometrical figures; methods for concisely expressing quantities and geometrical figures using simple algebraic expressions and symbols; and methods for using tools to measure quantities or to draw geometrical figures. It should also be noted that any knowledge and skills students acquire is often built upon knowledge and skills previously acquired.

What is meant by "acquire" is that students grasp the meaning of quantities and geometrical figures, are conscious of their understanding, and are able to use their knowledge and skills according to their goals in daily life and in their studies.

If students are forced to memorize steps of calculation mechanically, without understanding the meaning, or if teaching is done in a way that only emphasizes the formal processing of calculations, the values of knowledge and skills are significantly reduced. It is important to instruct students to understand the concept of calculation, and use it appropriately depending upon the objectives.

(3) To foster their ability to think and express with good perspectives and logically on matters of everyday life

This part of the objective describes the cultivation of the ability to think and represent. In this revision, the phrase, "the ability to think and express" was added to indicate the ability to express ideas. The ability to think and the ability to express are complementary to one another. In the process of expressing ideas, students may notice good points or find an error, and by expressing their ideas, they often learn to carry out their thinking in logical steps and create better ideas. In class, students present various ideas, and they learn from each other. With this background, the ability to think and the ability to represent are both explicitly stated.

"Phenomena in their daily lives" here means a wide range of phenomena to which mathematics can be applied in their real life situations or learning situations. In these situations, if students encounter some difficulty in the goals they want to achieve, the phenomena become their problems. In order to solve these problems, while finding new ways to get results, students need to have good perspectives and to think logically.

In attempting to have good perspectives about the methods and results of problem solving, observing each part of the problem or the entire problem in its entirety and conducting trials and experiments are quite often useful. The inductive method in which various examples are studied and the commonality is sought, and the analogical thinking where results are inferred from the similar cases, can also be used. Having good perspectives is important to carry out problem solving steps appropriately and efficiently.

To indicate if the steps for solving problems and the results are correct, students are required to organize their thinking in logical steps. That is, each step of the process is clearly justified before moving on to the next. The deductive method in which explanation is given based on some assumption is well known, but inductive and analogical reasoning are also considered to be logical thinking methods since students need to justify each step of their reasoning.

The important aims that the subject of mathematics focuses on are to enhance the ability to think in logical steps with foresight in the process of problem solving, reasoning, and making inferences. These are also considered to be the aims of other subjects. But in mathematics, students frequently encounter more situations where they need to think inductively or deductively. Furthermore, in consideration of the systematic and objective nature of the subject, mathematics can contribute most greatly.

In teaching in each grade, it is important to cultivate the ability to think in organized logical steps with good perspectives, according to the developmental stages of the students and appropriately within the content of each grade. It is important to implement learning activities where students can express their thoughts and explain to their friends by using things such as words, numbers, algebraic expressions, diagrams, tables, and graphs.

(4) To help pupils find pleasure in mathematical activities and appreciate the value of mathematical approaches

This part addresses the affective aspect of the objectives in mathematics. The comparative studies of the International Association for the Evaluation of Educational Achievement (IEA) reported that the percent of Japanese students who say they like mathematics is less than in other nations, and this trend has not yet reversed in recent years. In teaching mathematics, it is very important to create lessons where students can find pleasure, interest, and wonder of mathematics. The statement in the objectives, "recognize the joy of mathematical activities", addresses this point.

There are various mathematical activities that can help students find pleasure in activities, for example, activities that associate daily life experience with mathematics, hands-on activities that involve making things, experimental activities that can include checking the actual size of numbers and quantities, exploration activities that can include finding hidden patterns in the multiplication table, and extension activities in which new problems are created from the solved problems. Students are by nature energetic and enjoy activities. It is important to create joyful mathematics classes by incorporating mathematical activities that are rooted in the nature of students.

The latter part states "to appreciate the value of mathematical approaches.' To make students appreciate the value of mathematical approach, it is important to create learning activities where students can grasp and process daily phenomena mathematically. To grasp phenomena mathematically is to investigate and to explore, for example, by paying attention to elements such as numbers, quantities, geometrical figures in phenomena, focusing on mathematical reasoning such as ideas of functions like changes and correspondences and ideas of sets that can clarify the target.

To appreciate the value is to recognize the merits of mathematics and the meaning of learning mathematics; it can boost a student's motivation to learn and lead to deeper understanding, and it can also foster a more positive attitude towards mathematics. In this respect, it is especially important that the teacher's instruction should lead students to actively engage with the subject.

In regard to merits, there are those that are contained in the knowledge and skills of quantities and geometrical figures, and there are those contained in thinking, judgment, and representation. What are these merits? They can be usability, simplicity, generality, accuracy, efficiency, expandability, and beauty.

For example, in mathematics, using numbers, students investigate how many things there are, and compare their size. This idea can be applied to many situations in students' daily life. This is a merit of "numbers" that are related to the idea of usability. Integers[2] can be represented by the base-10 notation system. And this notation has the value of expressing a size by the location of each numeral. Through this property, a number can be expressed simply, and two numbers can be compared. This merit in the ways of representation is related to usability, simplicity, and generality.

As discussed here, it is important to research teaching materials by clarifying the merits in each method. It is also important to be creative in teaching so that students may notice these merits.

(5) To foster an attitude to willingly make use of mathematics in their daily lives as well as in their learning

This part describes the goal of fostering an attitude in students that encourages them to use mathematics. In this revision, two points are emphasized: acquiring fundamental and basic knowledge and skills, and using acquired knowledge and skills. Mathematics can be used in various daily life and learning situations. By applying what students have learned in mathematics to various daily life and learning situations, the study becomes meaningful, and students can really experience and appreciate the merits of mathematics. What was meant by "daily life and learning situations" can be interpreted broadly.

This could be their family time, school life, and life in their communities, and can also include their future life in society. About learning, of course, what has been learned in mathematics can be used in other subjects, but it is important to apply this knowledge to the future study of mathematics. In mathematics, it is possible to create new knowledge and methods from what was previously learned. Also, during the Integrated Study, students can engage in exploring various learning activities by applying the knowledge, skills, thinking ability, reasoning ability, and representation ability acquired in mathematics.

[2]The elementary school mathematics curriculum does not deal with negative integers. Thus, "integers" here refers to positive integers and 0, which are typically referred to as "whole numbers."

2 Objectives of each grade

In mathematics, the content and objectives are indicated in each grade. In the objectives of each grade, it primarily describes the main points that teaching should focus on. Since each grade has the four domains, "A. Numbers and Calculations," "B. Quantities and Measurements, " "C. Geometrical Figures," and "D. Mathematical Relations," objectives in each grade corresponds to the domains.

The Objectives of Grade 1

(1) Through activities using concrete objects and so on, to help pupils enrich their number sense. To help them understand the meaning and the representations of numbers, and to help them understand addition and subtraction, and explore ways of the calculations, and use the calculations.

(2) Through activities using concrete objects and so on, to help pupils enrich their experiences that will form the foundation for understanding quantities and measurements, and enrich their sense of quantities.

(3) Through activities using concrete objects and so on, to help pupils enrich their experiences that will form the foundation for understanding geometrical figures, and enrich their sense of geometrical figures.

(4) Through activities using concrete objects and so on, to help pupils represent numbers and quantities as well as their relations by using words, numbers, algebraic expressions, figures and diagrams and interpret such representations.

In "A. Numbers and Calculations" in grade 1, the meaning and representation of numbers up to 2-digit numbers are taught. As a way to spiral across grades, 3-digit numbers are taught in simple cases. In calculation, addition between two 1-digit numbers and subtraction that is the inverse of these facts are taught so that the students acquire the skills of adding and subtracting accurately and reliably. As a spiral across grades, simple cases of sums and differences involving 2-digit numbers are taught.

In "B. Quantities and Measurements," students are taught to directly compare length, area, and volume. The aim of these activities is to develop the foundation for learning units and measurements. Also, students will be taught to how to tell time in their daily lives.

In "C. Geometrical Figures," students are taught about the shapes of objects in daily life, and both planar and solid figures are dealt with. The aim of these activities is to develop the foundation for learning geometrical figures.

In "D. Mathematical Relations," students are taught to represent cases where addition and subtraction can be applied by using algebraic expressions, and to represent the number of objects in diagrams or in pictures.

The Objectives of Grade 2

(1) Through activities using concrete objects and so on, to help pupils enrich their number sense. To help them deepen their understanding of the meaning and the representations of numbers, as well as their understanding of addition and subtraction, and use the calculations. Furthermore, to help them understand the meaning of multiplication, explore ways of the calculation, and use the calculation.

(2) Through activities using concrete objects and so on, to help pupils understand the units and measurements of length and volume and so on, and enrich their sense of quantities

(3) Through activities using concrete objects and so on, to help pupils understand geometrical such as triangles and quadrilaterals, and enrich their sense of geometrical figures.

(4) Through activities using concrete objects and so on, to help pupils represent numbers and quantities as well as their relations by using words, numbers, algebraic expressions, figures, diagrams, tables, and graphs, and interpret such representations.

In "A. Numbers and Calculations," the meaning and representation of numbers up to 4 digits is taught, as well as 1 *man* [10000]. Simple fractions such as $\frac{1}{2}$ and $\frac{1}{4}$ are taught, and the aim is for learning activities here to become the foundation for learning fractions. In calculation, the main focus is on addition of two 2-digit numbers and on subtraction that is the inverse of those facts, and on basic multiplication (*ku-ku*) [1×1 through 9×9]; therefore these skills should be solidly acquired. As a way to spiral across grades, simple cases of addition and subtraction of 3-digit numbers and multiplication between 2-digit and 1-digit numbers are also taught.

In "B. Quantities and Measurements," units of length (e.g. meters) and their measurement, and units of volume (e.g. liters) and their measurement are taught. Telling time (on a clock) is also taught.

In "C. Geometrical Figures," students are taught about triangles, quadrilaterals, squares, rectangles, and right triangles as plane figures, and the box as a solid figure.

In "D. Mathematical Relations," students are taught to represent the relationship between addition and subtraction, and cases in which to apply multiplication by using algebraic expressions. Also, students are taught to represent data by using simple tables and graphs.

The Objectives of Grade 3

(1) To help pupils use addition and subtraction appropriately, to help them deepen their understanding of multiplication and use the calculation appropriately. To help them understand the meaning of division, explore ways of the calculation, and use the calculation. Furthermore, to help them understand the meaning and the representations of decimal numbers and fractions.

(2) To help pupils understand the units and measurements of length, weight and time.

(3) To help pupils understand geometrical figures such as isosceles triangles and equilateral triangles by paying attention to the elements that compose the geometrical figures.

(4) To help pupils represent numbers and quantities as well as their relations by using words, numbers, algebraic expressions, figures, diagrams, tables, and graphs, and interpret such representations.

In "A. Numbers and Calculations", students are taught about the unit of man [ten-thousand] and the relative size of numbers as a representation of integers. In calculation of integers, the following processes are taught: addition and subtraction of 3- and 4-digit numbers, multiplication of 2- and 3-digit numbers by 1- or 2-digit numbers and division in which both the divisors and the quotients are 1-digit. Teaching about the meaning and representation of decimal numbers and fractions begins formally from grade 3. In grade 2, they are exposed to some basic materials, so teaching should be done using these experiences. Furthermore, addition and subtraction of decimal numbers through the tenths place and simple cases of addition and subtraction of fractions are taught. Also, students are taught to represent numbers on the abacus and to add and subtract using the abacus.

In "B. Quantities and Measurements," units of length (kilometers) and their measurement, and units of weight (e.g. grams) and their measurement are taught. Also, students are taught about units of time (seconds) and how to determine clock time and elapsed time.

In "C. Geometrical Figures," students are taught about isosceles triangles, equilateral triangles, and circles as plane figures, and spheres as solid figures.

In "D. Mathematical Relations," students are taught how to represent cases where division may be applied by using algebraic expressions; to represent quantitative relationships in algebraic expressions by using a symbol such as □; and to interpret and draw bar graphs.

The Objectives of Grade 4

(1) To help pupils deepen their understanding of division and use the calculation appropriately. To help them deepen their understanding of the meaning and the representations of decimal numbers and fractions, understand the meaning of addition and subtraction of decimal numbers and fractions, explore ways of addition and subtraction, and use them. Moreover, to help them understand round numbers and use them according to their purposes.

(2) To help pupils understand the units and measurements of area, determine the area of geometrical figures, and understand the unit and measurements of angle.

(3) To help pupils understand plane figures, such as parallelograms and rhombuses, and solid figures, such as rectangular parallelepiped, by paying attention to the elements that compose the geometric figures and the relationships of those elements.

(4) To help pupils represent numbers and quantities as well as their relations by using words, numbers, algebraic expressions, figures, diagrams, tables, and graphs, and investigate such representations.

In "A. Numbers and Calculations," students are taught the units of *oku* [hundred million] and *cho* [trillion] as representations of integers. In the calculation of integers, division of 2- and 3-digit numbers by 1- and 2-digit numbers is taught. Grade 4 is the year in which the four arithmetic operations of integers are completed and summarized, therefore, teaching should be done in a way that solidifies students' calculation skills of integers and enhances their ability to apply their skills. In decimal numbers, calculation of the sum and the difference of decimal numbers are taught. Also, multiplying and dividing decimal numbers by integers is taught. Addition and subtraction of fractions with like denominators is taught. Furthermore, students are taught to understand round numbers and to estimate the results of the four arithmetic operations. Also, students are taught about addition and subtraction using the abacus.

In "B. Quantities and Measurements," units of area (e.g. m^2) and their measurements, as well as how to determine the area of squares and rectangles are taught. Also, students are taught about the unit of angle measure (degree (°)) and how to measure angles.

In "C. Geometrical Figures," students are taught about parallelograms, rhombuses, and trapezoids as plane figures, and cubes and rectangular parallelepipeds as solid figures. Also, students are taught about how to represent the position of an object in space.

In "D. Mathematical Relations," students are taught about changing quantities and broken-line graphs, algebraic expressions that represent the relationship among quantities, properties of the four arithmetic operations, and how to sort, organize, and represent data.

The Objectives of Grade 5

(1) To help pupils deepen their understanding of properties of integers. To help them deepen their understanding of the meaning of multiplication and division of decimal numbers as well as the meaning of addition and subtraction of fractions, explore ways of the calculation, and use the calculations.

(2) To help pupils determine the area of geometrical figures such as triangles and parallelograms and the volume of solid figures such as rectangular parallelepiped. To help them understand the average of measured value and the ratio of two quantities of different types.

(3) To help pupils deepen their understanding of plane figures, and to help them understand solid figures such as prisms.

(4) To help pupils investigate relationships between numbers/quantities. To help them investigate the features of data by using percentages and pie graphs.

In "A. Numbers and Calculations," even and odd numbers, and factors and multiples are taught as the properties of integers. Also, the notation system for integers and decimal numbers is taught. With decimal numbers, multiplication division are taught. With fractions, addition and subtraction with unlike denominators are taught. In addition, students are taught about multiplying and dividing fractions by whole integers.

In "B. Quantities and Measurements.́ ways to determine the area of triangles, parallelograms, rhombuses, and trapezoids are taught. Also, students are taught about units of volume (e.g. m^3) and their measurements, and ways to determine the volume of cubes and rectangular parallelepipeds. Furthermore, the mean of measurements and per-unit quantities are dealt with.

In "C. Geometrical Figures," polygons and regular polygons are taught as plane figures, and prisms and cylinders as solid figures. Also, the congruence of figures and recognition of the properties of geometrical figures are taught.

In "D. Mathematical Relations," simple cases of proportional relationships are taught. This will provide the basis for understanding proportion in grade 6. Also, algebraic expressions that express quantitative relationships, and percentage and pie charts, are taught.

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The Objectives in Grade 6

(1) To help pupils deepen their understanding of the meaning of multiplication and division of fractions, explore ways of the calculations, and use the calculations.

(2) To help pupils determine the area of circles and the volume of solid figures such as prisms. To help them understand speed and determine it.

(3) To help pupils understand reduced figures, enlarged figures and symmetric figures, and deepen their understanding of geometrical figures.

(4) To help pupils understand ratio and direct proportion, and use the idea of a function when exploring the relationships of numbers/quantities, and represent the relationships in algebraic expressions. To help them explore the distribution of data and investigate it statistically.

In grade 6 "A. Numbers and Calculations," multiplication and division of fractions are taught. Also, since the learning of the four arithmetic operations of decimal numbers and fractions is completed, teaching should be done in the way that solidifies students' calculation skills with these types of numbers and enhances their ability to apply these skills.

In "B. Quantities and Measurements," students are taught how to approximate shapes and areas, how to determine the area of a circle, and how to determine the volume of prisms and cylinders. Ways of determining speed and the system of the metric units are also taught in this grade.

In "C. Geometrical Figures," students are taught about reduced and enlarged figures and symmetric figures.

In "D. Mathematical Relations," students are taught about ratios, direct and inverse proportional relationships, and algebraic expressions with letters. In data handling, mean, frequency distribution, and analysis of all possible outcomes for actual events are taught in this grade.

Section 2: The Content of Mathematics

1 Structure of the Content

The contents of mathematics are organized in four domains: "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometrical Figures," and "D. Mathematical Relations." They are configured in this way to make it easier to see the overall structure and to clarify systematic and developmental paths. In this revision, "mathematical activities" is added following the four domains.

The three domains, A, B, and C, represent respectively, numbers, quantities, and geometrical figures, which are targets of learning in mathematics. The domain "A. Numbers and Calculations" consists of the contents that describe the meaning and representation of numbers such as integers, decimal numbers, and fractions, and methods of calculation. The domain "B. Quantities and Measurements" is mainly about units and measurements of quantities surrounding us in daily life. The domain "C. Geometrical Figures" is mainly about the meaning and properties of plane figures and solid figures, and the structures of geometrical figures. In each of these domains, it is important to conduct teaching by closely connecting understanding of the meaning of numbers, quantities, and geometrical figures with activities such as calculation, measurement, and composition.

The domain "D. Mathematical Relations" involves ways of thinking and methods that can be commonly used in handling quantities and geometrical figures. In this domain, the main contents include some ideas of functions such as change and correspondences, representation by algebraic expressions, and tables and graphs.

2 Overview of the contents of each domain

A. Numbers and Calculation

(1) Objectives of "A. Numbers and Calculations"

In this domain, the objectives are to understand the meaning and representation of integers, decimal numbers, and fractions, and to have a rich sense of numbers. Also, students will understand the meaning of calculations with integers, decimal numbers, and fractions and learn to think about ways to calculate with them, acquire calculation skills and the ability to apply them. Additionally, it is also an important object in this domain for students to find pleasure in mathematical activity and the values of mathematical manipulation by acquiring the ability to think mathematically.

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(2) Overview of the contents of "A. Numbers and Calculations"

The table below shows the main contents in each grade categorized by numbers and calculation separately.

Grade	Numbers	Calculations
Grade 1	· 2-digit numbers · Simple 3-digit numbers	· Addition of 1-digit numbers and its inverse, subtraction · Addition/subtraction of simple 2-digit numbers.
Grade 2	· 4-digit numbers (numbers up to 1 *man* [10000]) · Base-ten notation system · Simple fractions	· Addition of 2-digit numbers and its inverse, subtraction · Addition/subtraction of simple 3-digit numbers. · Multiplication table (*ku-ku* — 1×1 through 9×9) · Multiplying simple 2-digit number by 1-digit number
Grade 3	· Units of man (numbers up to 1 *oku* [100,000,000]) · Decimal numbers (the tenths place) · Fractions	·Addition/Subtraction of integers (3-digit, 4-digit numbers) · Multiplication of integers (2-digit numbers, 3-digit numbers) · Division of integers (divisors and quotients are 1-digit numbers) · Simple division (divisors are 1-digit numbers and quotients are 2-digit numbers) (mental calculation) · Calculation by using an abacus · Addition/subtraction of simple decimal numbers and fractions.
Grade 4	· Units of *oku* [100 million] and *cho* [1 trillion] · Round numbers · Decimal numbers · Fractions (proper fraction, improper fraction, mixed fraction)	· Division of integers (Divisors are 1- or 2-digit numbers, and dividends are 2-digit or 3-digit numbers) · Estimating results of calculations (simple mental calculation) · Mastery of calculation skills of integers · Calculation by using an abacus · Addition/subtraction of decimal numbers · Multiplication and division of decimal numbers in which multipliers and divisors are integers · Addition/subtraction of fractions with like denominators.
Grade 5	·Even numbers, odd numbers · Divisors, multiples (greatest common divisor, least common multiple) · Prime numbers	· Multiplication and division of decimal numbers in which multipliers and divisors are decimal numbers · Addition/subtraction of fractions with unlike denominators · Multiplication and division of fractions in which multipliers and divisors are integers
Grade6	· Reciprocal	· Multiplication and division of fractions in which multipliers and divisors are fractions · Mastery of calculation skills of decimal numbers and fractions

In grade 1, the meaning and representation of 1-digit and 2-digit numbers are taught. As a way to spiral across grades, simple cases of 3-digit numbers are also taught. In calculation, additions of 1-digit numbers and subtraction that is inverse of those facts are taught, and it is necessary that students acquire these calculation skills. As a way to spiral across grades, simple cases of sums and differences involving 2-digit numbers are also taught.

In grade 2, the meaning and representation of numbers up to 4-digits is taught. The unit, *man*, [10,000] is also taught. Simple fractions such as $\frac{1}{2}$ and $\frac{1}{4}$ are taught in this grade, and should become the foundation for learning fractions in the future. In calculations, the main contents are in the addition and subtraction of 2-digit numbers and the multiplication table (*ku-ku*). It is necessary that students acquire these calculation skills. As a way to spiral across grades, simple cases of addition and subtraction of 3-digit numbers and multiplication between 2-digit and 1-digit numbers are taught in this grade.

In grade 3, as representations of integers, the units of man, and the relative sizes of numbers are taught. In calculation of integers, students are taught addition and subtraction of 3-digit and 4-digit numbers, multiplying 2-digit and 3-digit numbers by 1-digit or 2-digit numbers, and division where both divisors and quotients are 1-digit numbers. Learning about the meaning and representation of decimal numbers and fractions begins in earnest in grade 3. In grade 2, students have done learning activities on the basics of fractions, and teaching should make use of those experiences. Furthermore, addition and subtraction of decimal numbers in the tenths place is taught in this grade. Also, the use of the abacus to represent numbers and to do addition and subtraction is taught.

In grade 4, as representations of integers, the units of *oku* [hundred thousand] and *cho* [trillion] are taught. In the calculation of integers, students are taught about division in which divisors are 1-digit or 2-digit numbers and dividends are 2-digit or 3-digit numbers. Grade 4 is where the study of the four arithmetic operations of integers is complete and to be summarized, so students should securely acquire "calculation skills of integers and enhance their abilities to apply these skills." With decimal numbers, addition and subtraction, and multiplication and division with integer multipliers and divisors are taught. Regarding fractions, addition and subtraction of fractions with like denominators are taught. Furthermore, students are taught to understand round numbers and to become able to estimate the results of their calculations. Also, addition and subtraction using an abacus is taught.

In grade 5, even numbers, odd numbers, multiples and divisors are taught as properties of integers. Also, students are taught about the notation system for whole and decimal numbers. Multiplication and division of decimal numbers are taught. In the case of fractions, addition and subtraction of fractions with unlike denominators are taught.

In grade 6, multiplication and division of fractions are taught. Grade 6 is where the study of the four arithmetic operations of decimal numbers and fractions is complete and to be summarized, so students should securely acquire "calculation skills with decimal numbers and fractions."

(3) Main contents

① Numbers

a Integers

Integers are used to represent a number of things or to represent an order in a series. Integers are represented by the base-ten system. In grade 1 they are simply called "numbers" in teaching. Beginning in grade 3 they are referred to as "integers" to distinguish them from decimal numbers and fractions.

In grade 1, students are taught to compare numbers of objects in groups by making one-to-one correspondence. They are taught to correctly count or represent the number and order of objects, to put numbers in sequence, and to consider a number as a sum or difference of other numbers. Students are expected to understand the meaning of integers.

In grade 2, students are taught to count objects by rearranging them into groups of the same size or by classifying them, to represent numbers in the base-ten notation system, to treat 10 and 100 as units, and to consider a number as a product of other numbers. Through these activities, students are expected to enhance their abilities to use integers.

In grade 3, students are taught about the unit of *man* [10,000] and the workings of the base-ten notation system. Students are expected to deepen their understanding in the representation of integers, and to enhance their ability to use numbers.

In grade 4, students are taught about the units of *oku* [hundred million] and *cho* [trillion] to deepen their understanding of the base-ten notation system. Also, they are introduced to the concept of rounding and learn about round numbers, and students are expected to use these skills appropriately.

In grade 5, even and odd numbers and multiples and divisors are taught to deepen their understanding of the properties of integers.

b. Decimal numbers and fractions

In grade 3, decimal numbers are used to represent the size of fractional parts. Ways to represent decimal numbers with the tenths place are taught.

In grades 4 and 5, additional place values such as the hundredths and the thousandths are taught and students will deepen their understanding about how decimal numbers are represented in the base-10 notation system.

Teaching about fractions begins in grade 2. In this grade, simple fractions such as $\frac{1}{2}$ and $\frac{1}{4}$ are taught and these learning activities become the basis for the future understanding of fractions.

In grade 3, students are taught that fractions are used to express an amount obtained as a result of equal partitioning and a fractional part of a number, and can be considered as a collection of unit fractions. For example, $\frac{4}{3}$ is four units of the unit fraction, $\frac{1}{3}$.

In grade 4, students are taught to be aware that some fractions are equal in size in simple cases. Also, they are taught that a fraction that is less than 1 is called a proper fraction, a fraction that is greater than or equal to 1 is called an improper fraction, and an improper fraction represented by the sum of an integer and a proper fraction is called a mixed number.

In grade 5, students are taught to express the results of dividing an integer by another integer as a fraction. For example, $2 \div 3 = \frac{2}{3}$. Also, expressing decimal numbers in fraction form and comparing the size of fractions with unlike denominators is taught with the expectation that students' understanding of fractions will be deepened.

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② Calculations

Understanding the meaning of calculations, having the ability to think about methods of calculation, acquiring calculation skills, and the ability to apply them are considered to be important objectives in teaching calculations

a. Addition and subtraction

[Addition and subtraction of integers]

In grade 1, students are taught to become aware of situations where addition and subtraction may be applied. Students are taught to think about ways to calculate addition 1-digit numbers and subtraction that are the inverse of those addition facts, and they are expected to master those calculations.

In grade 2, in addition to deepening their understanding about situations where addition and subtraction are applied, students are taught to investigate the properties of these operations, and by using those properties students are expected to think about the method of calculation and to verify the results. Also, students are expected to master addition of 2-digit numbers and subtraction that are their inverse.

In grade 3, students should master addition and subtraction of integers, and they are expected to enhance the ability to use addition and subtraction appropriately.

[Addition and subtraction of decimal numbers]

In grade 3, students are taught to understand the meaning of addition and subtraction of decimal numbers up to the tenths digits; they are expected to think about methods of calculation and to be able to calculate.

In grade 4, students deepen their understanding of addition and subtraction of decimal numbers and they are expected to master the calculation.

[Addition and subtraction of fractions]

In grade 3, students are taught to understand the meaning of addition and subtraction of fractions in simple cases and to think about the method of calculation.

In grade 4, students are taught to understand the meaning of addition and subtraction of fractions with like denominators, and to be able to apply the operations.

In grade 5, students are taught to understand the meaning of addition and subtraction of fractions with unlike denominators. For students to add and subtract fractions with unlike denominators, it is necessary to teach how to find a common denominator.

b. Multiplication and division

[Multiplication and division of integers]

In grade 2, students are taught to become aware of situations where multiplication may be applied, and to understand the meaning and the method of calculation. For example, students can understand multiplication as the operation to determine the total number when given the number in a group and the number of groups, or the amount that is so many times as many as the base amount. They can determine the product by repeated addition. The multiplication table (*ku-ku*) is taught, and it is important that students become proficient with multiplication of 1-digit numbers. Furthermore, to deepen their understanding of multiplication facts, students are taught simple cases of multiplication between 2-digit numbers and 1-digit numbers.

In grade 3, students are taught to become able to securely multiply by 2-digit numbers. Students are also expected to understand the meaning of division and the situations in which division applies. Division includes the case of *partitive division* and the case of *quotative division*. *Partitive division* finds one part of an equally divided number or quantity. *Quotative division* finds how many times one quantity is of another quantity. The methods of calculation for these two are identical and, therefore, are treated as one operation. Students are taught to become aware that division is the inverse of multiplication. By checking the results of division by multiplication, students are expected to deepen their understanding of the relationship between multiplication and division, and to become able to do these calculations securely.

In grade 4, it is important that students become proficient with divisions whose divisors are 2 digit numbers.

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[Multiplication and division of decimal numbers]

In grade 4, students are expected to become able to think about the ways of multiplying and dividing decimal numbers when the multipliers and divisors are integers, and to become able to do these calculations.

In grade 5, students are expected to become able to multiply and divide when the multipliers and divisors are decimal numbers. When the multipliers are decimal numbers, students can no longer treat multiplication as repeated additions, so it is necessary to extend the meaning of multiplication. Similarly, the meaning of division must be extended when the divisor becomes decimal numbers.

[Multiplication and division of fractions]

In grade 5, students are expected become able to think about multiplication and division of fractions when the multipliers and divisors are integers.

In grade 6, students are expected to do multiplication and division when the multipliers and divisors are fractions. In dealing with the cases in which multipliers and divisors are fractions, the extended meanings of multiplication and division that was developed in Grade 5 with decimal numbers can be directly applied, therefore it is necessary to consider how these ideas are related. Also, students are taught to consider division by a fraction as multiplication by its reciprocal, and to become aware that multiplications of decimal numbers and fractions can be converted to calculations of fractions.

c. Round Numbers and Estimation

In grade 4, students are taught about the meaning of round numbers and the process of rounding. Also, they are taught to estimate results of the four arithmetic operations appropriate to particular objectives.

Approximate numbers and estimates relate to various contents taught in grade 4 and beyond. It is important that students become able to estimate the results of calculations, and use it to judge the accuracy and the appropriateness of calculations.

B. Quantities and Measurements

(1) Objectives of "B. Quantities and Measurements"

The main objectives of the domain are to gain understanding about units and measurements of various quantities which have close ties with students' daily life, to develop the skills to measure, and to foster a rich sense of quantities.

Quantities taught in mathematics include length, area, volume, time, weight, angle, and speed. Units and measurements appropriate for each quantity are taught. Students are taught to become aware of usefulness of expressing size by using units, and they should become able to choose a unit purposefully.

(2) Overview of the contents of "B. Quantities and Measurements"

The table below shows the main contents in each grade categorized in two by units of quantities, and comparisons/measurements/etc. of quantities. The units indicated in ( ) are those that are listed in the section, "Handling the Content"

	Units of quantities	Comparisons/Measurements/etc.
Grade 1		· Direct comparisons in length, area, and volume · Clock reading
Grade 2	· Units of length (mm, cm, m) · Units of volume ($m\ell$, $d\ell$, ℓ) · Units of time (day, hour, minute)	· Measurements of length and volume
Grade 3	· Units of length (km) · Units of weight (g, kg) [t] · Units of time (second)	· Measurements of length and weight · Measuring by choosing appropriate units · Calculation of clock time and elapsed time
Grade 4	· Units of area (cm^2, m^2, km^2) [a, ha] · Units of angle (°)	· Determining area (squares, rectangles) · Measurements of angles
Grade 5	· Units of volume (cm^3, m^3)	· Determining area (triangles, parallelograms, trapezoids, rhombuses) · Determining volume (cubes, rectangular parallelepipeds) · Mean of measurements · Determining per-unit quantities
Grade 6		· Approximating shapes and approximate area · Determining area (circles) · Determining volume (prisms, cylinders) · Determining speed · The system of the Metric units

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Many contents in "B. Quantities and Measurements" are closely related to contents in other domains. For example, to express quantities, integers, decimal numbers, and fractions are needed. Area and volume are measurements of plane and solid figures. Furthermore, algebraic expressions needed to determine area and volume are related to the concepts of proportion and functions. It is important to give special considerations to relations with contents in other domains in teaching these contents.

In grade 1, the main objective is to provide students with rich experiences that will become the basis for learning quantities and measurements. Students are taught to compare length, area, and volume, and to use objects around them as units of measure and to compare sizes by considering how many such units there are. Also, students are taught to read clocks in the context of their daily life.

In grade 2, regarding length and volume, students are taught about the meaning of standard units that are shared by everyone, and measurement using the standard units. As units of length, *millimeter (mm)*, *centimeter (cm)*, and *meter (m)* are taught; as units of volume, *milliliter (mℓ)*, *deciliter (dℓ)*, and *liter (ℓ)* are taught. As units of time, *day, hour, minute*, and the relationship among them are taught.

In grade 3, as a unit of length, *kilometer (km)* is taught. As units of weight, *gram (g)* and *kilogram (kg)* are taught. A unit of time, *second* and calculation of clock time and elapsed time are taught.

In grade 4, regarding area, students are taught to understand the meaning of measuring area and the units used to express area. As units of area, *square centimeter (cm^2)*, *square meter (m^2)*, and *square kilometer (km^2)* are taught. Students are taught about ways to determine the area of planar figures (squares, rectangles).Students are taught to consider the size of an angle as an amount of rotation. A unit of angle measure (degree (°)) is taught.

In grade 5, students are taught to think about the ways to determine area of triangles, parallelograms, rhombuses, and trapezoids. Students are taught to understand the meaning and units of volume. Units of volume, *cubic centimeter (cm^3)*, *cubic meter (m^3)* are taught, and students are taught to think about the ways to determine volume of solid figures (cubes, rectangular parallelepipeds). Also, the mean of measurements and per-unit quantities (ratio of two different quantities) — for example, population density — are taught in this grade.

In grade 6, students are taught to estimate the area of shapes in their surroundings by approximating them with familiar figures. Also, students are taught to think about the ways to determine the area of a circle. Regarding volume, students are taught to think about the ways to determine the volume of prisms and cylinders. Students already learned about per-unit quantities in grade 5, but speed is taught in grade 6. Also in grade 6, students review units of quantities they have studied, and are expected to understand how the metric system works.

(3) Remarks on the main contents

a. Quantities taught in mathematics

Length is an easy quantity to grasp visually. In grade 1, students are taught ideas such as direct comparison of lengths. In grade 2, students are expected to understand the meaning of standard units that are used by many people. The units of length *millimeter (mm)*, *centimeter (cm)*, and *meter (m)* are taught and measurement with these units is taught. In grade 3, the unit of length *kilometer (km)* is taught. Also, students are taught to become able to choose appropriate units and tools to measure quantities.

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Area is the size of a surface that extends. In grade 1, students are taught to directly compare area by activities such as overlapping actual objects. In grade 4, units of area *square centimeter (cm^2)*, *square meter (m^2)*, and *square kilometer (km^2)* and how to measure area are taught. Also, students are taught to think about the ways to determine area of planar figures (squares, rectangles). In grade 5, based on what students have learned in grade 4, they are expected to think about the ways to determine area of triangles, parallelogram, rhombus, and trapezoid. In grade 6, students are taught to estimate the area of shapes in their surroundings by approximating them with familiar figures. Also, students are taught to think about the ways to determine area of a circle.

Volume is considered as a quantity of containers such as boxes that students can find in their daily lives. In grade 1, students are taught to directly compare volume, for example, by overlapping objects. In grade 2, the units of volume *milliliter (mℓ)*, *deciliter (dℓ)*, *liter (ℓ)* and how to measure volume are taught. In grade 5, the units of volume *cubic centimeter (cm^3)*, *cubic meter (m^3)* and how to determine volume are taught. Also, students are taught to think about the ways to determine the volume of solid figures (cubes, rectangular parallelepipeds). In grade 6, by using what students learned to grade 5, students are taught to think about the ways to determine volume of prisms and cylinders.

Time is taught by relating it to events and experience in daily life. In grade 1, students are taught to become able to read the clock. In grade 2, the units of time *days, hours*, and *minutes* are taught and students are expected to understand relations among them. In grade 3, the unit of time *seconds* is taught as well as calculating clock time and elapsed time.

Weight is a quantity that is impossible to grasp visually, and it is important to have activities in which students can actually experience it—for example, by having an object in hand or wearing it. In grade 3, units of weight *gram (g)* and *kilogram (kg)* and how to measure weight are taught.

The size of an angle is considered as an amount of rotation. In grade 4, the unit of angle measure *degree* and how to measure angles are taught. Angles as geometrical figures are treated in the contents of "C. Geometrical Figures" in grade 3.

Quantities that can be considered as ratio between two quantities, such as population density and speed, are taught in grades 5 and 6. *Population density* is a quantity that is expressed as a ratio between two quantities, a number of people and area, and it can be considered as the number of people per unit area. *Speed* is a quantity that is a ratio between distance and time, and it can be considered as the distance traveled per unit time.

b. Meaning and properties of quantities

A quantity expresses size of an object.

A number of objects can be expressed in integers, for example by counting them. On the other hand, in measuring length of strings or weight of water, the quantity can be divided infinitely and cannot always be expressed in integers.

There are various types of quantities, such as length, area, volume, weight, angle, and speed. In introducing each quantity for the first time, it is important to have activities in which the quantities of actual objects are compared. By actually comparing them, it becomes easier for students to grasp what quantity is being compared, and the meaning of the quantity will become gradually clear to students. When comparing size, it will help students understand the meaning of quantities if terms describing the size of quantity are used. For length, for example, "long/short"; for area, "wide/narrow"[3]; for volume, "large/small" (more/less); for weight, "heavy/light"; for angle, "large/small"; for speed, "fast/slow."

[3]The size of area can be expressed by using terms like wide/narrow in Japanese. The meanings of these Japanese terms are similar to large/small.

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Quantities such as length, area, volume, weight and angle have the following properties:

We will use volume as an example. Imagine that there is a container with 2 liters of juice in it. Even if we move this juice to some other container, or divide it into several containers, the overall amount of the juice does not change and remains 2 liters. Reshaping something or moving it to other places, or dividing it into several groups does not change the overall amount of it. We call this property "preservation of quantity."

Based on the preservation of quantity, addition is verified. If we add 300 g of clay to 500 g of clay, it becomes 800 g of clay. This illustrates addition in weight.

Also, if we cut a parallelogram into two pieces and rearranges them to form a rectangle, we can explain that the area of the original parallelogram is equal to the area of the rectangle. In the explanation, "preservation of quantity" and "addition" is applied.

c. Comparing size of quantities

With regard to quantities that have fundamental properties, there are stages in teaching: direct comparison, indirect comparison, comparison using arbitrary units, and comparisons using standard units.

In direct comparisons, two sizes are directly compared. For example, when we compare the length of two pencils A and B, we align them at one end and judge relative length from the position of the other ends.

In indirect comparisons, the sizes of A and B are indirectly compared, using some other thing which is the same size as A (or B). For example, if we want to compare the adjacent sides of a rectangular table, we cut paper strips in the same lengths as the two sides of the table, and then compare the lengths of the tapes.

By doing direct comparisons or indirect comparisons it should become clear to students which quantity among many attributes of a concrete object they are trying to compare.

In measurement using arbitrary units, the size of A and B are quantified by using an object possessing the appropriate attribute to find out how many times of that object A and B are. For example, to represent the length and the width of a table may be quantified by using the length of a pencil; if the length of a table is four times as long as a pencil, and the width is six times as long as the pencil, we know that the width is longer than the length by twice a pencil's length. So, as illustrated in this example, by quantifying the sizes of objects, comparisons can be made more clearly.

If a quantity just needs to be quantified, arbitrary units are sufficient. But in a society, the size of units should be clear to everyone, so students should be taught to become aware that common units are necessary. So, in comparison using standard units, A and B are measured by using standard units that are commonly used throughout the country (world).

d. Units of quantities

A unit is the basic size that is used to express size of a quantity.

Size of an object can be expressed in terms of how many of the unit (how many times as much) the object is. For example, with length, $3m$ is three of $1m$; with volume, $\frac{2}{3}$ liters is $\frac{2}{3}$ times as much as 1 liter.

A unit of volume, a cubic meter (m^3) is based on a unit of length, meter (m). One cubic meter is the volume of a cube whose edge is 1 meter. Units that are created by multiplying or dividing basic units are sometime called *derived units*. Speed and population density are expressed as ratios between two different quantities and are examples of derived units.

Units can be determined freely, but it is more convenient if we use units that many people commonly use. Such units are called standard units or fundamental units. The meter for length and the liter for volume are examples of standard units. In Japan, they are set by the law called "measurement law"; internationally, they are determined by the International System of Units.

e. Measurements of quantities

Measurement means to investigate and to determine the size of a quantity.

Sometimes measurement tools are used for measuring. To measure length, a ruler or a measuring tape can be used, and to measure the volume of a liquid, a measuring cup can be used. It is important that students be able to choose an appropriate unit and tool according to the size of the quantity they are measuring.

The area of a plane figure is expressed as the number of a basic unit (such as 1 square centimeter) or expressed as how many times as many (times as much) as the basic unit. To determine area, instead of using a tool it is a calculated using quantity such as the length of a side. Determining volume of solid figures is done in the same way.

f. Sense of the sizes of quantities

One of the objectives of teaching quantities and measurements is to enrich students' sense of the sizes of quantities. It is important for students to have good sense of the sizes of various quantities and apply this rich sense appropriately. For example, if we take the example of length, we can illustrate this idea in the following:

- When looking at a pencil, estimate its length by saying something like, "It is about 20cm."
- Choose an appropriate unit and measurement tool for the object to be measured, such as: "To measure this, I can use a 30cm ruler."
- Approximate the size of a basic unit: "1 meter is about this much."
- Understand the approximate size of concrete objects based on things students find in daily life: "The diameter of 1 yen coin is about 2cm."

In teaching, it is important to implement hands-on and experiential activities where students can investigate and verify the size of concrete objects, and develop a rich sense of quantities. Also, it is effective to conduct activities involving comparisons and measurements in a variety of situations.

C. Geometrical Figures

(1) Objectives of "C. Geometrical Figures"

In this domain, the main objectives are to understand the meaning and properties of plane figures and solid figures, to enrich one's sense of geometrical figures, and to acquire the ability to think and express ideas mathematically through the process of identifying and explaining properties of geometrical figures. The sense of geometrical figures means an ability to recognize the shapes of objects and a sense of the characteristics of a shape and of its properties. It is important to enrich the sense of geometrical figures through activities that include observation and construction.

(2) Overview of the contents of "C. Geometrical Figures"

The table below shows the main contents in each grade.

Grade	Understanding geometrical figures	Components of geometrical figures	Viewpoint and methods for investigating geometrical figures
Grade 1	· Shapes of objects from students' everyday lives		· Activities involving observation and composition · Words such as "front and back", "right and left", and "above and below"
Grade 2	· Triangles, Quadrilaterals · Squares, Rectangles, Right triangles · Box-shaped objects	· Straight line, right angle, vertex, side[3], face	· Activities involving observation and composition · Focusing on components of shapes · Investigating length of sides · Focusing on right angles
Grade 3	· Isosceles triangle, equilateral triangle · Circle, sphere	· Angle, center, radius, diameter	· Activities involving observation and composition · Focusing on components of shapes · Compare length of sides · Focusing on shape of angles
Grade 4	· Parallelogram, rhombus, trapezoid · Cube, parallelepiped	· Diagonal, plane	· Activities involving observation and composition · Relationships of straight lines: parallel and perpendicular lines · Drawing sketches and nets · Expressing a position of an object
Grade 5	· Polygon and regular polygon · Prism and cylinder	· Base, side face[4]	· Activities involving observation and composition · Congruence of geometrical figures · Identifying properties of geometrical figures · Relationships between circumference of a circle and its diameter (π) · Drawing sketches and nets
Grade 6			· Activities involving observation and composition · Reduced and enlarged figures · Symmetric figures (line symmetry and point symmetry)

[3] "edge" for solid figures. In Japanese, the word for sides (plane figures) and edges (solid figures) are the same.
[4] "side face" in this document is a direct translation of the Japanese mathematical term that means lateral surface of prism, cylinder, pyramid, and cone.

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In grade 1, the main objective is to provide students with enriched experiences that will form the basis for understanding geometrical figures. Students are taught to recognize shapes and to grasp the characteristics of shapes.

In grade 2, students need to focus on components of geometrical figures and are expected to understand geometrical figures such as triangles and quadrilaterals.

In grade 3, students need to focus on components of geometrical figures and are expected to understand geometrical figures such as isosceles and equilateral triangles.

In grade 4, students need to focus on components of geometrical figures and positional relationships. They are expected to understand plane figures such as parallelograms and rhombuses, and solid figures such as rectangular parallelepipeds.

In grade 5, students are taught how to identify properties of geometrical figures. By using those properties, they are expected to deepen their understanding of plane figures and solid figures such as prisms.

In grade 6, students deepen their understanding of geometrical figures through understanding enlarged and reduced figures and symmetric figures.

(3) Main contents

① Understanding geometrical figures

a. Plane figures

In grade 1, students carry out activities such as observing and composing shapes that are familiar in their everyday life, and they are taught how to recognize and grasp the characteristics of shapes. Students are expected to be able to call shapes by names such as "triangle," "quadrilateral," and "circle,"[5] and to be able to study their characteristics.

In grade 2, by focusing on components of geometrical figures, students are taught to understand geometrical figures such as triangles and quadrilaterals. For example, students should recognize a triangle as a geometrical figure surrounded by three straight lines, and a quadrilateral as a geometrical figure surrounded by four straight lines. Also, they should recognize a square as a quadrilateral whose four sides are equal in length, and whose four angles are right angles. Other plane figures taught are rectangles and right triangles.

In grade 3, by focusing on components of geometrical figures, students are taught to understand geometrical figures such as isosceles triangles, equilateral triangles, and circles. An isosceles triangle is a triangle that has two sides of the same length. If students fold an isosceles triangle, they find the base angles overlap exactly. By doing this type of activity, students are expected to be able to focus on properties of geometrical figures.

In grade 4, students are taught to understand the positional relationships of straight lines such as parallelism and perpendicularity. By understanding these relationships, students can deepen their understanding of previously learned geometrical figures such as squares and rectangles. Students are taught about parallelograms, rhombuses, and trapezoids, and they are expected to investigate the properties of these geometrical figures. Students are also taught the new term, "diagonal."

In grade 5, students are taught about polygons and regular polygons. Students should recognize that equilateral triangles and squares that they learned about previously can be recognized as instances of regular polygons. They are also expected to understand congruence, where two geometrical figures have the same size and shape. Students are taught to find properties of geometrical figures; for example, the sum of the measures of the three interior angles of a triangle is 180°, and the sum of the measures of the four interior angles of a quadrilateral is 360°. Also, students should be able to explain the reason for these properties.

[5] In Japanese, there are informal (children's) words for triangles, quadrilaterals, and circles. Those are the words used here, not the formal mathematical terms. The formal terms are introduced in Grade 2.

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In grade 6, students are taught to understand enlarged and reduced figures and symmetric figures. For example, isosceles triangles and squares, studied previously, have line symmetry, and parallelograms have point symmetry. By being able to look at geometrical figures in this way, students are expected to deepen their understanding of plane

b. Solid figures

In grade 1, students do activities such as observation and composition of shapes they find in daily life. They are taught how to recognize and grasp the characteristics of shapes. For example, students are expected to be able to call shapes by "box shape" and "ball shape" and be able to investigate their characteristics.

In grade 2, students are taught about the shape of a box. Also, the following new terms are taught: "vertex," "side,"[6] and "face."

In grade 3, a sphere is taught as a solid figure. Students are taught to understand the sphere and its center, radius, and diameter, in comparison to the circle.

In grade 4, cubes and rectangular parallelepipeds are taught. With respect to rectangular parallelepipeds, students are expected to understand the relationships of parallelism and perpendicularity of straight lines and planes. Students are also taught how to draw sketches and nets of rectangular parallelepipeds and cubes.

In grade 5, prisms and cylinders are taught. Students are also taught how to draw sketches and nets of prisms and cylinders. The new terms, "base" and "side face" are taught. In grade 6, there is no content about solid figures in the domain "C. Geometrical Figures." However, determining volume of prisms and cylinders is taught in the domain "B. Quantities and Measurements."

② Components of geometrical figures

By focusing on components of plane and solid figures, students can deepen their understanding of geometrical figures.

In grade 2, students are taught about straight lines, right angles, vertices, sides, and faces. These are components of plane and solid figures. Using these components, students can construct geometrical figures and explain their meaning. For example, by using concrete objects such as three sticks, students can make a triangle. By doing this type of activity, it becomes easier to explain that "a geometrical figure that is surrounded by three straight lines is a triangle," and it helps students understand the meaning of geometrical figures.

In grade 3, by investigating the length of sides that are components of geometrical figures, students can perceive geometrical figures in such a way that a triangle whose two sides are equal in length is called an isosceles triangle and a triangle whose three sides are equal in length is called an equilateral triangle. Also, by folding a paper isosceles triangle, students can grasp the property that the sizes of two angles are equal in an isosceles triangle. Here, the focus is also on the component, which is an angle, of a geometrical figure. Similarly, in grade 3, the focus is on the following components of a circle: center, diameter, and radius. If students are asked, "Draw a circle whose radius is 5cm," every student is expected to be able to draw a figure with the same size and shape.

In grade 4, the term "diagonal" is taught, and by focusing on diagonals students are expected to be able to investigate the properties of a parallelogram, rhombus, and trapezoid. For example, with a rhombus, students can find that the two diagonals intersect perpendicularly. Similarly, in grade 4, in conjunction with cubes and rectangular parallelepipeds, positional relationships of lines and planes are taught.

In grade 5, the terms, "base" and "side face" are taught in relation to prisms and cylinders. By focusing on these components — the base and side face — students are able to draw nets of prisms and cylinders.

[6]See footnote #2.

③ Viewpoint for analyzing geometrical figures

a. Observation and Composition

In the content of geometrical figures for each grade, activities such as observation and composition are mentioned. Through these activities, students are able to understand the meaning of geometrical figures and to find and verify their properties.

In grade 1, students can observe various concrete objects around them, and learn to recognize shapes, grasp their characteristics, and give theses shapes names on their own. Also, students are taught to create concrete objects around them using solids such as blocks or boxes.

In grade 2, students can find objects among the things around them that have the shapes of triangles, quadrilaterals, squares, rectangles or right triangles. Students are taught to do activities where they make geometrical figures, for example, by connecting dots in grids, using sticks, folding and cutting paper, and placing colored geometric tiles.

In grade 3, students are taught how to draw isosceles triangles, equilateral triangles, and circles by using a ruler and compass. Also, students are taught about isosceles triangles, equilateral triangles, and circles by observing tessellations of isosceles and equilateral triangles and designs that are created with circles.

In grade 4, through observations of quadrilaterals such as parallelograms, rhombuses, and trapezoids students will learn to categorize geometrical figures with common properties, and investigate properties of each individual class of shape. Also, based on definitions and properties of each geometrical figure, students are taught to draw figures using a ruler and compass.

In grade 5, students are taught to deepen their understanding of plane figures through construction of congruent geometrical figures while focusing on lengths of sides and sizes of angles.

In grade 6, students are expected to enrich their sense of geometrical figures by observing and reviewing previously-learned geometrical figures from the viewpoints of scaled figures and symmetry.

Actual activities for composing geometrical figures can include: folding or cutting paper; moving, decomposing, or changing geometrical figures; and drawing geometrical figures using a ruler and compass. These activities are examples of hands-on and experimental activities for learning about geometrical figures. The significance of these activities is as follows:

- These activities help students understand the meaning and terms of geometrical figures. For example, by drawing an isosceles triangle, students can understand that two sides are equal in length.
- In problem solving involving geometrical figures, students can grasp a problem or have good perspectives to solve a problem. For example, students can find a solution by drawing a circle with a certain radius in a problem where one finds an area where obstacles should not be placed when opening a door.
- Students can discover properties of geometrical figures and verify and explain those properties. For example, to find the sum of the interior angles of a quadrilateral, students can think about the problem by breaking the quadrilateral into triangles.
- Students can apply properties of geometrical figures in daily life and learning. For example, when measuring the horizontal length of a window frame, using properties of a rectangle, students need to measure only the bottom side.
- Students can acquire knowledge and master and retain skills. For example, when drawing designs with circles using a compass, students become aware that, in a circle, one can cut the circumference into 6 equal-lengths by the radius of the circle, and at the same time, students can acquire skill with the compass.

- Students can acquire ways to observe and analyze geometrical figures. For example, by drawing diagonals in rhombuses, students can view the rhombus as a geometrical figure that has line symmetry with the diagonals as axes of symmetry.

Composing geometrical figures can be a fun activity for students, but it is important that students understand the objectives of the activities, act according to those objectives, and summarize and verify what has been made clear to them.

b. Finding properties of geometrical figures and explaining them

In grade 2, students are taught "there are many sizes but all squares have the same shape," and "in a rectangle, opposite sides have equal length."

In grade 3, students are taught that "the sizes of two angles of an isosceles triangle are equal" and "all three angles of an equilateral triangle are equal."

In grade 4, students are taught that "in a parallelogram, lengths of opposite sides and sizes of opposite angles are equal," and in a rhombus, "opposite sides are parallel to each other, and sizes of opposite angles are equal, and the two diagonals intersect perpendicularly and bisect each other."

In grade 5, it is important to foster the ability to think logically by finding and explaining properties of triangles and quadrilaterals.

For logical thinking may be inductive thinking where generalizations are developed from the commonality among various examples, analogical thinking where new ideas are inferred from their similarities to previously learned cases, or deductive thinking where the validity of an idea is explained based on previously validated ideas.

When investigating the sum of the measures of the interior angles of a quadrilateral, some students can recall the method they used in finding the sum of the measures of the interior angles of a triangle — drawing many triangles — and wonder if they can use a similar method, while others may think that since the sum of the measures of four angles of a square or a rectangle is 360, that the of angles in a general quadrilateral might also be 360°. These are examples of analogical thinking. The inductive thinking is used where a student conclude that the sum of interior angles of a quadrilateral is 360° by actually drawing various quadrilaterals and measuring their interior angles. Based on the fact that the sum of three angles of a triangle is 180°, since a quadrilateral can be divided into two triangles, students can think that the sum of the angles of a quadrilateral is twice as big. This is an example of the deductive thinking.

The ability to think logically must be fostered in all grade levels and in every domain.

In grade 6, students are expected to review previously learned geometrical figures from the viewpoints of line symmetry and point symmetry.

c. Drawing sketches and nets

In grade 2, by cutting open a box or constructing a box from an expanded figure, students are taught to understand that a solid figure is composed of plane figures, and to recognize how faces are connected to each other.

In grade 4, when learning about cubes and rectangular parallelepipeds through observation of these solid figures, students are taught to focus on the positional relationships of vertices, edges, and faces based on a sketch, and to understand the correspondence of vertices, edges, and faces of a solid figure and its net. When teaching this, students should draw sketches and nets, so that they become aware of the merits of representing solid figures in the plane.

In grade 5, while learning about prisms and cylinders using sketches and nets, students deepen their understanding about the properties of solid figures.

d. How to grasp shapes and sizes of geometrical figures

In grade 5, congruence is taught through investigating geometrical figures from the viewpoints of shape and size.

In grade 6, students are taught to grasp reduced figures and enlarged figures as geometrical figures that are same shapes but different sizes, as well as relationships among geometrical figures by focusing on the sizes of corresponding angles and the ratios of lengths of corresponding sides.

Also, line symmetry and point symmetry are taught. A figure that has line symmetry can overlap itself completely when it is folded along a straight line. Isosceles triangles, equilateral triangles, squares, rectangles and rhombuses are figures with line symmetry. A figure that has point symmetry can overlap itself completely when it is rotated by 180° around a certain point. Squares, rectangles, rhombuses, and parallelograms have point symmetry.

D. Mathematical Relations

(1) Objectives of "D. Mathematical Relations"

The objective of this domain is for students to acquire mathematical thinking and methods that are used while understanding and applying the contents in the domains, "A. Numbers and Calculations," "B. Quantities and Measurements," and "C. Geometrical Figures." This domain also includes the objective of acquiring mathematical thinking and methods for investigating and representing quantities and geometrical figures. In this revision, the ability to think, to make decisions, and to express ideas by using words, numbers, algebraic expressions, diagrams, tables and graphs are emphasized. Therefore, the domain "D. Mathematical Relations" is added from grades 1 and 2 to enhance these abilities.

In this domain, the main contents are "ideas of functions," "algebraic expressions and their interpretation," and "organizing and interpreting data." In each area, it is important to enable students to represent and to investigate quantities and their relations by using numbers, algebraic expressions, diagrams, tables, and graphs, and to express and investigate them with words, to make decisions about them, and to explain own ideas. Especially in grades 1 and 2, the contents, "algebraic expressions and interpreting them," and "sorting data and interpreting them" (which were previously located in "A. Numbers and Calculations") are moved to "D. Mathematical Relations" in order to improve the organization and enrichment of the content.

The idea of functions describes a way of thinking whereby a problem is solved by focusing on patterns of change or correspondence involving quantities or geometrical figures. It is especially important to investigate relationships between two quantities that change simultaneously, and interpret and express characteristics and tendencies of the relationship.

Algebraic expressions are sometimes called the language of mathematics, and they play important roles in expressing objects and their relationships accurately and concisely so that they can be understood. It is important to be able to interpret algebraic expressions and to use them in relationship to words and diagrams.

With respect to organizing and interpreting data, it is important for students to be able to collect and sort data according to one's objectives, and to express the data clearly by using tools such as tables and graphs, and to investigate their characteristics and interpret them. Also, it is important to be able to choose appropriate graphs and tables for the purpose, to use them together by relating them, and to understand how to read and apply them.

(2) Overview of the contents of "D. Mathematical Relations"

The table below shows the main contents in each grade, categorized by "Ideas of functions," "Algebraic expressions and their interpretation," and "Organizing and interpreting data." (The items under "Ideas of functions" in Grades 1 through 3 are the related ideas from the domain, "A. Numbers and Calculations.")

Grade	Ideas of functions	Algebraic expressions and their	Organizing and interpreting data
Grade 1	· Correspondence between two objects · Sizes of numbers (large/small), order of numbers · Seeing a number as sum or difference of numbers	· Algebraic expressions of addition and subtraction and how to interpret them	· Representing and interpreting numbers of objects using pictures and diagrams
Grade 2	· Sizes of numbers (large/small), order of numbers · Seeing a number as product of other · How a product changes when the multiplier increases by 1.	· Relationship between addition and subtraction · Algebraic expressions of multiplication and how to interpret them · Algebraic expressions with ( ) and symbols like □	· Organize quantities in daily life and represent them in simple tables or graphs, and interpret them
Grade 3	· Including the case in which the multiplicand or multiplier is zero, how a product changes when the multiplier increases or decreases 1	· Algebraic expressions of division and to interpret them · Representing quantitative relations in algebraic expressions, relating algebraic expressions to diagrams · Algebraic expressions with symbols like	· Organize and represent data clearly in tables or graphs, and interpret data represented in tables or graphs · Drawing and interpreting bar graphs
Grade 4	· Relations between two quantities and broken-line graphs	· Algebraic expressions that have a combination of the four arithmetic operations and ( ). · Idea of formulas and applying them · Algebraic expressions with symbols like □ and △ · Summary of properties of four arithmetic operations	Organize data from two viewpoints and investigate them · Drawing and interpreting broken-line graphs
Grade 5	· Simple cases of proportional relationship	· Algebraic expressions to express relationships between quantities	· Percentage · gathering and organizing data · Pie chart and percentage bar graph
Grade 6	· Ratios · Investigating proportional relations by using algebraic expressions, graphs, and tables · Solving problems using proportional relationships · Inverse proportion	·Algebraic expressions with letters such as a, x	· The mean of data · Tables and graphs that represent frequency distribution · Investigate possible outcomes

In grade 1, algebraic expressions of addition and subtraction and their interpretation are taught. Also, students are taught to represent numbers of objects by using pictures and diagrams and to be able to interpret such representations.

In grade 2, students are taught about representing and explaining the mutual relationship between addition and subtraction, and algebraic expressions of multiplication and their interpretation. Students are also taught to organize numbers and quantities from their everyday lives, represent them by using simple tables and graphs, and interpret such representations.

In grade 3, algebraic expressions of division and how to interpret them are taught, as well as how to find relationships between algebraic expressions that represent relationships of quantities and diagrams, and how to represent relationships with algebraic expressions that use symbols such as □. Also, expressing phenomena using tables and bar graphs, and how to interpret tables and bar graphs, are taught.

In grade 4, students are taught to represent the relationship between two quantities that change simultaneously on broken-line graphs and to interpret their characteristics. Also, students are taught to understand and use algebraic expressions that represent quantitative relationships, and algebraic expressions that use □ and △. Students are taught to gather, sort, and organize data according to their purposes, to clearly represent data in tables and broken-line graphs, and to investigate features of data.

In grade 5, students are taught to investigate simple cases of proportional relationships by using tables. Also, students are taught about the relationship between two quantities expressed in a simple algebraic expression. Students are also expected to understand percentage, and to gather, sort, and organize data according to their purposes, to represent data in pie charts and percentage bar graphs, and to investigate the features of this data.

In grade 6, students are taught to investigate the features of ratios and proportional relationships by using algebraic expressions, tables, and graphs; to solve a problem by using proportional relationships; and to become aware of inversely proportional relationships. Also, they are taught about algebraic expressions that use letters such as "a" and "x." They are also taught about the mean of data, the distribution of data, and systematic analysis of all possible outcomes.

(3) Explanation of Main contents

① Ideas of functions

Ideas of functions are ways of thinking that gives considerations to patterns of change and correspondence involving quantities and geometrical figures as we solve problems. By using ideas of functions, it is important that students will deepen their understanding of quantities and geometrical figures, use these ideas effectively, investigate two simultaneously changing quantities, and interpret their features and tendencies.

To make the best use of the ideas of functions, it is important to consider the following:

First, one must focus on how an aspect about quantities and geometrical figures in a certain situations relates to other aspects. For example, when one quantity changes, does the other quantity change? Or if one quantity is determined, does it mean the other quantity is also determined? If some components of a geometrical figure are determined, does it mean the other components or matters are determined? By focusing on these relationships, students are able to investigate any dependency relationship between two aspects. This is the first step in forming ideas of functions. In considering this, it is also important to clarify the range of the aspect one is investigating.

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Second, it involves the investigation of the characteristic of change and correspondence between two aspects. It is often possible to find relationships such as regularity in the way the quantities changes simultaneously. By expressing quantities and their relationships in words, numbers, algebraic expressions, diagrams, tables, and graphs, students can interpret situations of regularity in change and correspondence more specifically.

Third, the recognition of regularity in change and correspondence obtained in the process above needs to be used for solving problems, and for enabling students to express and explain the steps in their thinking, and their results.

In grades 1 through 3, students are taught to make a correspondence between two objects, to grasp a number as sum or difference of two numbers, to grasp a number as product of two numbers, and to think about how a product changes when a multiplier increases by 1. These contents relate to the contents in "A. Numbers and Calculations," and exhibit the ideas of functions used here.

In grade 4, students will discover relationships between two simultaneously changing quantities from the events in daily life, and they will represent these relationships in tables and broken-line graphs, while interpreting their features.

In grade 5, simple cases of proportional relationships are investigated by using tables to describe relationships between two simultaneously changing quantities.

In grade 6, students are taught to understand ratios, and they are also taught to investigate proportional relationships as one type of relationship between two simultaneously changing quantities using algebraic expressions, tables and graphs. They are taught to solve problems using proportional relationships; inversely proportional relationships are also taught in this grade.

② Algebraic expressions and their interpretation

There are words, numbers, algebraic expressions, diagrams, tables and graphs that work as methods for representing quantities and their relationships in daily life. Among them, algebraic expressions are excellent for representing phenomena and relationships concisely, clearly, accurately, and in general terms. In teaching algebraic expressions, students must develop their ability to represent matters and relationships in algebraic expressions by associating them with concrete situations. Furthermore, it is important that students become able to interpret situations through algebraic expressions, and to represent them by using words or diagrams, and to process and promote thinking by using algebraic expressions. It is also important that students become able to explain their ideas by relating algebraic expressions to words, diagrams, tables, and graphs, and to clearly communicate these ideas to one another.

Some algebraic expressions are like $2+3$, $\square \times 3$, $x-5$ while others contain an equal sign, such as $2+3=5$, $\square \times 3 = 12$, $a \times b = b \times a$. There are also algebraic expressions that contain words such as (unit cost) $\times$ (number of items) = (total cost). With algebraic expressions like those in the first set of examples, it is important for students to be able to see that the expression like $2+3$ represents a quantity in a certain situation. Also, students should see that algebraic expressions with equal signs such as $\square + 3 = \triangle$, $x+3=8$, represent relationships between quantities in certain situations.

Additionally, some algebraic expressions may contain ( ), and others represent more than one phenomena in a single expression (compound expression).

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The following are some uses of algebraic expressions:

(a) to express things and relations concisely, clearly, accurately, and generally;

(b) to manipulate formally apart from the concrete meaning they are expressing;

(c) to interpret a concrete situation or relationship, and to examine them more accurately;

(d) to express one's own way of thinking, and to communicate it with other people.

Interpreting algebraic expressions involves the following:

(a) interpreting a concrete situation corresponding to the expression;

(b) generalizing the phenomena or the relations that the expression represents;

(c) expanding the range of numbers in an expression (e.g., expanding form integers to decimal numbers), that is, to interpret it broadly;

(d) understanding the thinking process behind problem solving from the given algebraic expression;

(e) interpreting algebraic expressions in correspondence with models like a number line.

In grade 1, students are taught to represent situations where addition and subtraction are used in algebraic expressions and to interpret these expressions. For example, if the situation is described as "3 children were playing and 4 more children came to join them," students are taught to write 3+4 to represent the situation. But, since concrete numbers are used in grades 1 and 2 and calculation immediately leads to one number, students rarely become aware of the fact that $3 + 4$ represents a concrete phenomenon. Therefore, it is important to teach students to focus on the meaning of algebraic expressions instead of paying attention solely to getting results.

Also, in grade 2, students are taught to explain the mutual relationship between addition and subtraction, and to be able to represent multiplicative situations with algebraic expressions. In this revision, it was decided to teach algebraic expressions with letters and symbols such as □. In grade 2, this is mentioned in "Handling the Content"-(2).

In grade 3, students are taught to represent division situations with algebraic expressions, and to interpret them. They are also taught to associate algebraic expressions that represent quantitative relations with diagrams, and to represent it in a mathematical expression with a symbol such as □, which acts just like letters . For example, an answer to $12 \div 3$ can be written as □ in $3 \times \square = 12$.

In grade 4, students are taught to deepen their understanding of algebraic expressions that represent quantitative relations. They are also taught to construct algebraic expressions using □ and △, which acts just like letters.

In grade 5, students are taught about relationships of quantities that are represented by simple algebraic expressions.

In grade 6, students are taught to represent algebraic expressions with letters such as "a" and "x".

③ Organizing and interpreting data

The main objectives here are to collect, sort and organize data according to one's purpose, to represent them clearly in tables and graphs, and to investigate and interpret the characteristics of the data. Through these activities, it is also important to foster an attitude for making good judgments and rational predictions. This is especially important in modern society, which is flooded with much information. If students can see how mathematics can be applied in this way, they can appreciate its value.

To enhance students' ability to collect, sort, organize, represent, and interpret data, it is important that students acquire knowledge and skills, and learn ways of thinking and representation, and learn ways of interpreting data through activities such as the following:

(a) Define purposes and gather the necessary data appropriately.

(b) Sort and organize data by making tables and graphs, and find the mean and percentage in order to understand the characteristics and tendencies of the data.

(c) By focusing on the characteristics and tendencies of data, make predictions and decisions about the situation, apply it to problem solving, and express and explain the thinking process and results.

In order for students to learn to sort, organize, and interpret data, students in grade 1 learn about the number of objects and compare numbers by using concrete objects. Also, they are taught to represent situations by using pictures and diagrams and to interpret them.

In grade 2, they are taught to sort and organize quantities surrounding them. They are taught to represent them by using simple tables and graphs, and they are taught to interpret tables and graphs.

In grade 3, they are taught to sort and organize quantities and represent them clearly by using tables and bar graphs as well as interpret tables and graphs.

In grade 4, they are taught to collect, sort, and organize data according to a purpose. They are taught to represent the data clearly using tables and broken-line graphs, and they are taught to investigate the characteristics of the data.

In grade 5, focusing on percentage as the ration of a part to the whole, students are taught to represent the data using pie charts and percentage bar graphs and to investigate their characteristics. It is important for the students to choose appropriate representation — table, bar graph, broken-line graph, pie chart, or percentage bar graph — according to their purpose; representing the data by in accordance with what they see in the tables and graphs; interpreting characteristics of the data; and making decisions.

In grade 6, students are taught about the mean and distribution of data. They are also taught how to represent data by using frequency tables and graphs, and about statistical investigation. Finally, regarding concrete phenomena, students are taught how to investigate possible outcomes by organizing them.

Chapter 3. Content by Grades

1. The Content of Grade 1

[A. Numbers and Calculations]

A (1) The meaning of numbers and their representation

1. Through activities such as counting the numbers of concrete objects, to help pupils understand the meaning of numbers and use numbers.

   a. To compare numbers of objects by making one-to-one correspondence between objects.
   b. To correctly count or represent the number and order of objects.
   c. To make a sequence of numbers and to put numbers on a number line by judging the size and the order of the numbers.
   d. To consider a number in relation to other numbers by regarding it as a sum or difference of other numbers
   e. To understand the representations of two-digit numbers.
   f. To get to know the representations of three-digit numbers in simple cases.
   g. To consider numbers using ten as a unit.

Mathematical Activities (1)

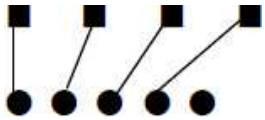
a. Activities to count concrete objects by making groups, to divide them equally, and to organize and represent them

[Terms/Symbols] ones place, tens place

Students are first taught to compare numbers of concrete objects through one-to-one correspondence. Then they are taught to correctly count those objects and to represent them by using numerals. Through these activities, students learn the size and order of numbers, and gradually learn the meaning and the system of numbers through 100. Also, students learn how to represent 3-digit numbers in simple cases.

a. Comparing numbers

When comparing the number of objects, one does not have to count them — one can judge the size of a number by making one-to-one correspondence. For example, in the diagram below, when you find a one-to-one correspondence between blocks ■ and marbles ●, you can understand that the number of marbles is greater than that of blocks.



When comparing objects that are difficult to compare directly, you can replace them with concrete objects. For example, when you count the number of sounds, the number of passing cars, or trees in the school yard — something you cannot manipulate by hand — you can replace the objects with something easy to count, e.g. making these objects correspond to marbles and then counting the number of the marbles.

In order to help students learn to count correctly by one-to-one correspondence between objects and number words, or to compare using numbers, it is important to use activities where correspondence between objects is made in this way.

b. Counting the number and order of objects

When counting the number of objects, it is important to understand what is being counted as a group. Then, make a one-to-one correspondence between what is being counted and the number words, "one, two, three, four, $\cdots$," reciting the number words in order, then represent the number of the objects by the last number word.

Also, in expressing order of objects, it is important to foster an understanding of numbers. When examining objects in order, if you make correspondences between the objects and the numbers, the numbers can be used to represent their positions. In this case, you will know that the number used to represent the last position is the same as the total number of the objects.

In the case of 0, it is necessary to help students gradually understand the following ideas.

① Through experiences such as not having any score in a "point game," or in which the number of concrete objects decreases one by one until nothing is left, students can understand that 0 stands for nothing. It is important that students perceive 0 as a number just like any other number.

② 0 represents a vacant place in the base-10 numeration system, such as the ones place of 70 or 80.

③ 0 represents the origin on a number line

c. Size and order of numbers and the number line

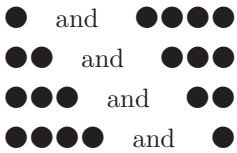
A *number line* represents numbers by the distances of points on the line from a reference point, to which 0 has been corresponded, using a selected length as the unit. By using this number line, the size, order, and sequence of numbers can be represented clearly.

It is good to introduce a number line in relation to the order of objects in a line. Also, students should gradually learn the number line with intervals other than 1, e.g., 5 or 10, or a number line starting at a number other than 0, e.g., 50.

The term "number line" is introduced in grade 3.

d. Considering a number as a sum or difference of other numbers

The starting point for understanding integers is in the process of counting objects. Soon, through activities, students start to grasp the structure of numbers by composing and decomposing them. The understanding of composition and decomposition of numbers is indispensable for establishing the meaning of numbers. For example, as shown in the drawing below, students start to perceive 5 marbles as a combination of parts.



Students also learn to see a number in relation to other numbers. For example, if you see 8 in relation to 10, since 8 is smaller than 10 by 2, you can represent it by the expression $10 - 2$.

This way of viewing numbers is also important because it forms the foundational knowledge for understanding regrouping with addition and subtraction. For example, in the case of $8 + 6$, you may think: "first, 8 is 2 less than 10. 6 is the sum of 2 and 4, and 8 and 2 make 10. This 10 and 4 make 14."

So it is important to be able to see numbers from various perspectives, thus enriching one's sense of numbers.

e. How to represent 2-digit numbers

In grade 1, the objective is to help students have a basic understanding of the principle of the base-10 numeration system.

It is important to introduce 2-digit numbers through activities such as counting concrete objects or representing numbers using concrete objects. Through these activities, students can understand that two-digit numbers are based on the idea of groups of ten and a remainder. This understanding enriches a sense of how numbers are composed.

When representing numbers using the base-10 numeration system, because different places are used to represent the size of units such as one, ten, and hundred, only a small set of symbols are necessary. In this system, judging the size of numbers or carrying out the four arithmetic operations with a computational algorithm (introduced from grade 2) becomes easy. It is important to help students understand the merit of this system as they study specific contents of numbers and calculations.

To foster an understanding of the base-10 numeration system, the terms "ones place" and "tens place" are introduced. For example, for number 43, its ones place is 3 and its tens place is 4. This means there are 3 ones and 4 tens.

f. How to represent 3-digit numbers in simple cases

Students learn how to represent 3-digit numbers in simple cases. Simple cases here mean numbers up to about 120.

Through activities with concrete objects such as making groups of 100, 10 and a remainder, students may solidify their understanding of the meaning and representation of 2-digit numbers. Moreover, these activities will lead smoothly to the study of 3-digit numbers taught in grade 2.

g. Considering numbers using ten as a unit

In grade 1, students learn how to represent numbers using the base-10 numeration system through such activities as making groups of ten. In grade 1, students learn to see the size of numbers using ten as a unit.

Here, thinking about numbers using ten as a unit means to see the group of tens in numbers such as "40 is four tens", or to see numbers as several tens such as "Six tens makes sixty."

By guiding students to see numbers in units of ten, you deepen their understanding of the structure of numbers, and make connections to a method of adding and subtracting numbers that can be seen as made up of units of ten.

Mathematical Activities (1) a

Activities to count concrete objects by making groups, to divide them equally, and to organize and represent them.

The objectives of these activities are to help students acquire the fundamental ability to count, to understand the basic idea of the base-10 numeration system, and to enrich their sense of numbers.

When counting concrete objects by grouping, activities such as counting by groups of 2, 5, and 10, or sorting and representing the concrete objects that students counted on a chart are used. In particular, counting objects by making groups of 10 becomes the foundation for understanding how the base-10 numeration system works. When teaching this, it is important to have students make a suitable group of numbers, count the objects using that group, and sort and represent what they counted, instead of counting pre-grouped objects. Also, to develop the process of counting, it is important to let students count objects in their everyday life, not only objects on the desk.

In cases where you want to equally divide concrete objects, use activities such as dividing the entire group into several groups of equal numbers or dividing the entire group into different numbers of equal groups. For example, explain how to divide 8 pencils by groups of twos or groups of fours. Explain how many people can get pencils if one divides the pencils equally by manipulating them and using charts, or sorting and representing the results of grouping using algebraic expressions. Through these activities, students see the number 8 from various perspectives and enrich their sense of numbers.

$2+2+2+2$

$4+4$

A (2) Addition and subtraction

(2) To help pupils understand the meaning of addition and subtraction and use the calculations.

a. To get to know situations where addition and subtraction are used.

b. To explore ways of addition of two one-digit numbers, and subtraction as the inverse operation, and to do these calculations accurately.

c. To explore ways of addition and subtraction of two-digit numbers and so on in simple cases.

Mathematical Activities (1)

b. Activities to express the meaning and ways of calculation by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

[Terms/Symbols] +, − , =

Students are expected to understand when to apply addition and subtraction to concrete phenomena as well as the meaning of addition and subtraction. Also, students are expected to think about how to carry out addition of a one-digit number with another one-digit number, and subtraction that are the inverse calculation, to understand the meaning of these calculations, to think about and explain how to carry out these operations for themselves, to actually carry out these calculations, and to utilize these calculations in their daily life. Furthermore, students are expected to think about how to carry out addition and subtraction of 2-digit numbers in simple cases and to be able to explain them. It is important that the calculation activities like “understanding the meaning of calculation,” “thinking about how to carry out calculations,” and “mastering calculations and appropriately applying them” are done in every grade.

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a. Cases where addition and subtraction are used, and their meanings

The following list provides examples of cases where addition and subtraction are used.

① Cases where addition is used.

(A) To obtain the result of adding something to an original number or amount, or to increment a certain amount from the original number or amount. (Increase)

(B) To find the total of two co-existing numbers or quantities. (Combine)

(C) To find the number or order that is a certain number of places following another number or order. (Addition involving ordinal numbers)

② Cases where subtraction is used.

(A) To find the result of removing or decrementing certain numbers or quantities from the original numbers or quantities. (Take away)

(B) To find the difference of two numbers or quantities. (Comparison)

(C) To determine the number or order that is a certain number of places preceding another number or order, or to find the difference of the two. (Subtraction involving ordinal numbers)

To teach these ideas, it is important to use concrete situations and to help students understand that in all of these cases the same addition or subtraction can be used. Therefore, gradually generalizing cases where addition and subtraction are used helps students understand the meaning concretely. And it is important to help students understand through such activities as concrete manipulation that addition is the calculation for finding the number of elements of two sets combined and that subtraction is the calculation for finding the number of elements of one group when the original group is separated into two.

b. Addition of 1-digit numbers and subtraction that are their inverse

For addition of a one-digit number with another one-digit number, and subtraction that is the inverse of this addition, there are two cases: one is an addition whose sum is less than or equal to 10, and the subtraction that is inverse of this addition. The other is an addition whose sum is greater than 10, and the subtraction that is the of this addition.

You can help students understand the former type of calculation through activities with concrete objects. For latter, help students think about these as “10 and more” through activities with concrete objects so that they will able to explain how to do this calculation logically. In either case, this will become the basis for addition and subtraction taught later, so it is important to teach it using such means as concrete objects so that students will understand the meaning of calculations, think about how to carry out these calculations, and become able to carry out these calculations without problems.

Especially in the case of subtraction, there are many possible ways to perform calculations. The principle methods are minuend decomposition and subtrahend decomposition. For example, in the case of $12-7$, minuend decomposition subtracts 7 from 10 and adds 2, as in $(10-7)+2$. Subtrahend decomposition subtracts numbers in steps, as in $(12-2)-5$. If you introduce activities with blocks, etc., the difference would be between taking away from a group of ten, or taking away from the ungrouped objects first. The choice of these two methods should be flexible, depending on the size of the numbers, but it is important to instruct students according to their level.

c. Simple cases of addition and subtraction involving 2-digit numbers

Based on additions and subtractions of a one-digit number and another one digit number and what has been learned so far about numbers, students are supposed to think about how to perform the addition and subtraction of two-digit numbers in simple cases.

Simple cases are as follows:

① Addition and subtraction of numbers that can be grasped using ten as a unit
Addition and subtraction of numbers that can be grasped using ten as a unit means, for example, calculations like $20 + 40$ or $70 - 30$. These calculations can be thought of as $2 + 4$ and $7 - 3$, when related to the idea of seeing numbers using ten as a unit.

② Addition and subtraction of 1 and 2-digit numbers without regrouping
Addition and subtraction of 2- and 1-digit numbers without regrouping here means, for example, addition like $13 + 4$ or $20 + 5$ and subtraction like $15 - 2$ or $38 - 8$.

By studying these simple cases of calculations of 1- and 2-digit numbers we aim at securing students' understanding of calculation with 1-digit numbers as well as their understanding of 2-digit numbers. Understanding how to carry out these calculations will be of help in grade 2 when they formally learn addition and subtraction of 2-digit numbers and subtraction that are the inverse of these additions.

[Mathematical Activities] (1) b

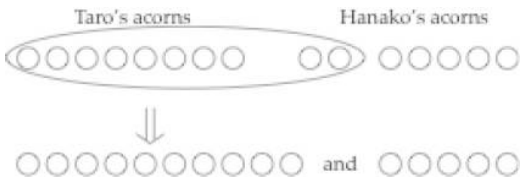
Activities to express the meaning and ways of calculation by using concrete objects, words, numbers, algebraic expressions, figures and diagrams

The objective of these activities is to help students understand the meaning of calculations based on concrete situations, to come up with and to represent new ways of calculation on their own using methods they have already learned.

For example, through a problem like, "Taro picked 8 acorns. Hanako picked 7 acorns. How many acorns do they have in total?" ask students to think about the meaning of the calculation and ways to calculate addition with regrouping.

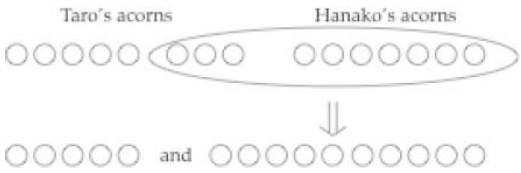
From this problem students can see that this is a case of finding the total of two co-existing numbers or quantities, which can be represented using the algebraic expression 8+7. Also, based on the meaning of the calculation in this case, we can think about the ways of calculation, such as in the following examples.

1) Decompose the 7 acorns Hanako picked.



Decompose 7 into 2 and 5. Adding 2 to 8 makes 10. Adding 10 and 5 makes 15.

2) Decompose the 8 acorns Taro picked.



Decompose 8 into 5 and 3. Adding 3 and 7 makes 10. Adding 10 and 5 makes 15.

Thus, we can think of this problem as $(8+2)+5$ by decomposing the addend, 7, or think of it as $5+(3+7)$ by decomposing the other addend, 8. By having students pay attention to the fact that both methods are making use of grouping by 10, we can help students develop the method of addition with regrouping by thinking about "ten and some more".

[B. Quantities and Measurements]

B (1) The foundation for understanding quantities and measurements

(1) Through activities such as comparing sizes of concrete objects, to help pupils enrich their experiences that will form the foundation for understanding quantities and measurements.

a. To directly compare length, area and volume.
b. To compare quantities by using familiar objects as a unit in terms of multiples of it.

Mathematical Activities (1)

c. Activities to compare directly the length, area and volume of familiar objects, and to compare them by using other objects.

The objective of grade 1 is to provide enriching experiences that will form the foundation for understanding the meaning of quantities such as length, area, and volume, and the meaning of measurement before actually measuring using units that are commonly used in their everyday life.

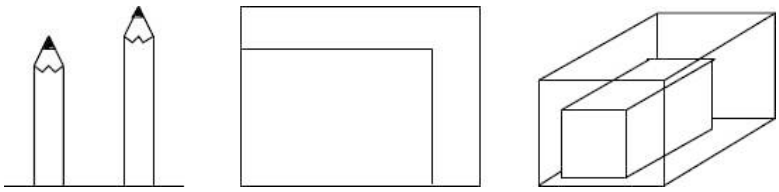
In grades 1 and 2, during size comparison activities, area may be called wideness[1] and volume may be called "capacity."

The important thing about teaching the meaning of a unit and measurement is to deepen the understanding of measurable attributes through activities in which the size of an object is compared directly or indirectly. Furthermore, it is necessary to have students understand that they can represent the size of an object by selecting a unit and counting the number of units. Through these activities, have students gradually understand that a certain amount can be divided into two or more parts without changing the amount as a whole, or that the original size can be reconstituted by adding up these parts.

[1]Wideness is a way for Japanese children to express the size of the space in everyday conversation.

a. Direct comparison of quantities

When comparing the size of objects, sometimes one can move objects, juxtapose one object with another to compare their sizes directly. For example, when you want to compare the length of two pencils, you can compare the length by moving the pencils and placing them side-by-side or superimposing one onto the other. In this case, since it is important to align the reference points, be sure to align one end of each pencil, and judge which one is longer by comparing the other ends of the pencils. As for comparing the area, when you can superimpose one object onto the other, if one object is completely covered by the other, you can see which is larger. Volume is more difficult to compare than length or area. But, for example, if you can put a small box inside a larger one, you can directly compare the volume of two boxes.



b. Comparing quantities using arbitrary units

When comparing the size of objects, sometimes it is difficult to move the objects in order to compare them directly. In this case, you can compare the size by using another object as a reference. Imagine comparing the length and width of a desk. In such cases, you may want to use a string or a stick to copy those lengths since the length and width of a desk cannot be moved around. By copying the length and width of a desk, you can compare these two lengths based on what was discussed earlier. When an appropriate object cannot be found to copy the length, one can use the length of an object such as a new pencil or a new eraser as a unit. We can then represent the length of each side by how many units will be required to make the same length, and compare the resulting numbers to compare the lengths of the sides.

As for area, even when you cannot superimpose one object onto the other, you can still compare area by covering the objects with color tiles and counting the number of those color tiles. It is important to make students aware of the meaning of area through such activities as covering with color tiles or shading individual squares on grid paper. Comparison of the volume of two objects can be done by filling one container with water and moving the water into another container, by filling two containers with water and moving water into the third container, or by counting how many glasses of water are in the container.

In this way, we can quantify an attribute of objects using everyday items as units and represent clearly the difference in the sizes of those objects.

[Mathematical Activities] (1) c

Activities to compare directly the length, area and volume of familiar objects, and to compare them by using other objects.

The objective of these activities is for students to be able to utilize the units and measurement of area and volume in their daily lives.

In this activity, students actually compare the length, area, or volume of objects at school and in their homes. For example, when comparing the area of two blankets, students may place one over the other (direct comparison). Or, when comparing the volume of water in bottles, students may fill each bottle with water and pour the water into tanks (indirect comparison). Or, when comparing the width and the length of a desk, students may represent each length using erasers and see how many erasers long the width and length of the desk are (measurement with arbitrary units).

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Through comparisons and measurements in various situations, students can understand what it means that the length is about how long an object extends, the area is about how spacious an object is, and the volume is about how much an object can contain. Thus, they will understand these attributes and how to measure them; moreover, these comparisons will help them acquire a sense of the size of quantitie

B (2) Clock reading

(2) To help pupils read clock times in their daily lives.

In the everyday life of a student, it becomes necessary to be able to tell time from relatively early on. In grade 1, students are taught how to tell time (hour, minute) from a clock with long and short hands. Also, it is important to help students become interested in time by relating time to activities in their daily life.

[C. Geometrical figures]

C (1) Foundational understanding of geometrical figures

(1) Through activities such as observing and composing the shapes of familiar objects, to help pupils enrich their experiences that will form the foundation for understanding geometrical figures.

a. To recognize the shapes of objects, and to grasp their features.
b. To express the position of an object by correctly using the words concerning direction and position such as “front and rear”, “right and left”, and “above and below”.

Mathematical Activities (1)

d. Activities to find various shapes of objects in familiar situations, and to compose or decompose the shapes by using concrete objects.

The objective of grade 1 is to provide rich experiences that will become the foundation for understanding plane figures and solid figures.

Students have experienced such everyday activities as piling up and tiling blocks and boxes, or folding, superimposing, and comparing pieces of paper while they were playing or in their daily life before entering school. Drawing on their experience, it is important to teach students to use shapes around them as objects of observation or composition, to find those shapes around them, actually pick them up, and create different shapes with them. Through these activities, students are gradually able to recognize those shapes regardless of their color, size, place or material, and understand their characteristics.

Activities where students observe and compose shapes are emphasized. At the same time, students will be taught to describe in words how they compose and decompose those shapes.

a. Shapes and ways to recognize their features

“To recognize the shapes of objects” means to be able to identify triangular, rectangular and circular shapes in such everyday concrete objects as floor tiles or tiled cobblestones by disregarding properties other than the shape. It also means the ability to find triangular, rectangular or circular shapes as the constituent parts of everyday solids such as boxes, tubes, and balls.

"To grasp the features [of shapes]" means to be able to recognize the characteristics of objects. For example: "triangles" and "rectangles" have corners but "circles" do not; "triangles" have three corners; the numbers of corners of "a triangle" and "a rectangle" are different when comparing these two. Furthermore, students may recognize characteristics of solid figures such as boxes have flat faces but the balls do not. Students are also taught about the functional aspects of solid figures such as a tube can roll over easily or be piled up depending on how they are placed; balls have a shape that rolls easily; or boxes have shapes that can be piled up.

To teach students to recognize the shapes of objects or to grasp the characteristics of shapes, it is important to have them do such activities as composing shapes in their daily life using solid figures such as blocks or boxes, or imagining the original shape from the shapes they constructed. Activities like presenting a shape and having students collect concrete objects that are similarly shaped, or asking students where they can find those shapes or what characteristics those shapes have, are also important. Activities like observing a box and copying its face on paper, and having students find those shapes in their surroundings, are also important.

Introducing these activities and gradually emphasizing the process of discovering the characteristics and properties of shapes will enrich students' experiences and will provide the foundation for understanding plane and solid figures.

b. Direction and Position

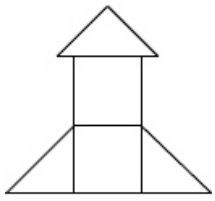
Observe how students express the direction and position of objects in their daily life, and teach them how to express the position of objects by using such words as "front" and "back," "left" and "right," and "top" and "bottom" correctly.

For example, have students be able to express two spatial relationships in the classroom, such as: "the clock on the wall is above the class schedule" or "the TV is to the left of the blackboard from my point of view." In this study, make sure students really understand directions and positions through such activities as lining in single file or aligning concrete objects.

[Mathematical Activities](1) d

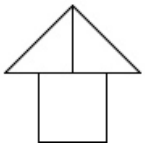
Activities to find various shapes of objects in familiar situations, and to compose or decompose the shapes by using concrete objects

The objective of this activity is to raise students' awareness of and interest in the shapes of objects, and to have them accumulate rich experiences that will form the foundation for the study of shapes.



Have students observe concrete objects such as boxes, tea canisters, or blocks and find the "rectangle" in boxes, the "circle" in tea canisters, and the "triangle" in blocks. They can also copy those shapes on paper and cut them out to make those shapes.

Students can do activities such as making the rocket shapes or house shapes shown at right by arranging shapes or color tiles they have made.



Students can make squares or triangles using sticks, etc.

Students can cut an origami paper into halves to make "triangles." Also, they can make a "rectangles" by combining two "triangles."

These activities not only provide students with rich experiences that will provide the foundation for understanding shapes, but also enrich their sense of shapes by familiarizing students with shapes by relating them to their daily lives. When arranging color tiles and making shapes, it is possible to introduce such activities as sliding, turning, and flipping color tiles.

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[D. Mathematical Relations]

D (1) The algebraic expressions of addition and subtraction

(1) To help pupils represent situations where addition and subtraction are used, by using algebraic expressions, and interpret these expressions.

Mathematical Activities (1)

e. Activities to represent by using algebraic expressions real situations associated with numbers and quantities, and connect the algebraic expressions to real situations.

[Terms/Symbols] +, − , =

It is important to emphasize representing the concrete situations in which addition or subtraction is applied using algebraic expressions with + or − symbols, interpreting those expressions in a concrete context, or interpreting algebraic expressions and representing them with diagrams or concrete objects.

Algebraic expressions are thought to represent situations or processes for finding answers, and are important as expressions unique to mathematics.

To interpret an algebraic expression means that, looking at an algebraic expression, students relate it to a corresponding concrete situation, and understand the relationship between quantities in that situation. Students should become able to express mathematical situations and quantities using words, diagrams, and concrete objects.

You can create activities for interpreting algebraic expressions in relation to concrete situations like the following example: Based on the algebraic expression $5 + 3 = 8$, students might create a story like, "There were 5 students playing in a sandbox. Three more students came. Now, there are 8 students playing." Or, from the expression $5 + 3$, students might create problems such as, "There were 5 students playing in a sandbox. Three more students came. How many students are playing now?" In this way, students can invent concrete situations using words, diagrams, and concrete objects for given algebraic expressions.

[Mathematical Activities] (1) e

Activities to represent by using algebraic expressions real situations associated with numbers and quantities, and connect the algebraic expressions to real situations

The objective of this activity is to enable students to interpret algebraic expressions with addition and subtraction, or to utilize those expressions by representing concrete situations or relating algebraic expressions to concrete situations.

To promote activities relating algebraic expressions of addition and subtraction with concrete objects in the classroom or in school or real situations in daily life, students are taught to find situations that can be expressed by those expressions and express them using words, pictures, and diagrams, and to represent the quantities found in real life using algebraic expressions, or to make mathematical problems.

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For example, a student can find the total number of morning glory seeds collected yesterday and today using addition. Or, from the equation 8—3 = 5, a student can tell a story like, "There were 8 students playing in a sandbox. Three students left. Now there are 5 students playing." Or, from the expression $6 - 3 + 7$, a student can make a mathematical problem like, "There were 6 squirrels. Three of them left. But another 7 came. How many squirrels are there now?" and represent the situation using pictures.

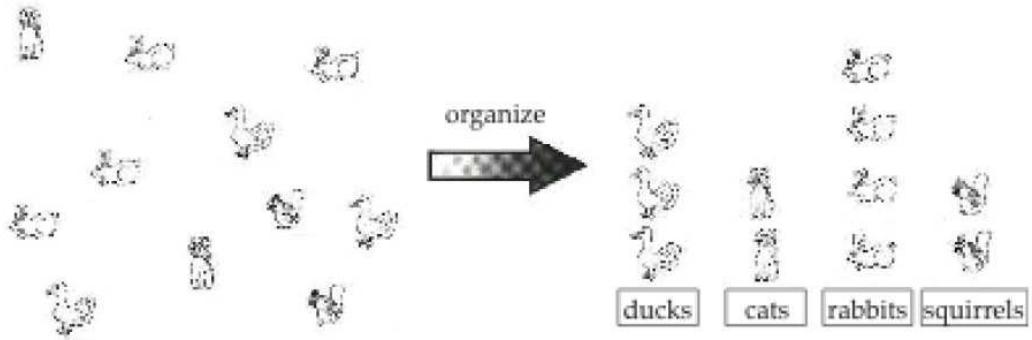
To foster a deeper understanding of algebraic expressions and to relate them with concrete situations through this kind of teaching is important in the study of the four arithmetic operations in the following grades.

D (2) Representation of quantities using pictures and diagrams

(2) To help pupils represent the number of objects using pictures or figures, and interpret them.

When counting a collection of objects that consist of different kinds of object — for example, ducks, cats, rabbits, and squirrels — students are expected to represent the objects using pictures or diagrams like the one below:

Also, students are expected to interpret the characteristics of the collection, such as where there are the most objects or fewest objects, from the pictures or diagrams.



[Mathematical Activities]

(1) The content listed in "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometrical Figures" and "D. Mathematical Relations" should be taught through, for example, the following mathematical activities:

a. Activities to count concrete objects by making groups, to divide them equally, and to organize and represent them.

b. Activities to express the meaning and ways of calculation by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

c. Activities to compare directly the length, area and volume of familiar objects, and to compare them by using other objects.

d. Activities to find various shapes of objects in familiar situations, and to compose or decompose the shapes by using concrete objects.

e. Activities to represent by using algebraic expressions real situations associated with numbers and quantities, and connect the algebraic expressions to real situations.

[Terms/symbols] ones place, tens place, +, —, =

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2. The Content of Grade 2

[A. Numbers and Calculations]

A (1) The meaning of numbers and their representation

(1) To help pupils understand the meaning and the representations of numbers, and extend their ability to use numbers.

a. To count objects by arranging them into groups of the same size, or by classifying them.

b. Up to four-digit numbers, to understand the representations of numbers, understand size and order of numbers by the decimal positional numeration system.

c. To understand relative size of numbers by regarding 10 or 100 as a unit. d. To consider a number in relation to other numbers, such as regarding it as a product of other numbers.

e. To get to know simple fractions such as $\frac{1}{2}$ and $\frac{1}{4}$.

Mathematical Activities (1)

a. Activities to find situations in everyday life where integers are used.

[Terms/Symbols] >, <

(Handling the Content)

(1) As for the content (1) in "A. Numbers and Calculations," the number 10,000[2] should be dealt with

The objectives of grade 2 are to broaden the range of numbers, understanding how to read and represent numbers up to 4 digits, and their size and order. The objectives of this grade are also to deepen an understanding that numbers can represent numbers of items and order, and to enrich the sense of numbers such as viewing numbers in relationship to other numbers, and seeing numbers as products of other numbers.

Furthermore, students are expected to understand fractions in simple cases through such activities as folding paper.

a. Counting by grouping and sorting

Since grade 1, students have been counting numbers of things by groups of two, five, or ten, or by paying attention to characteristics such as color, shape, position or type and sorting according to these characteristics, and by counting after sorting.

The activity of making groups of a suitable size, such as groups of 2, 5, or 10, leads to an understanding of the idea of multiplication. When counting numbers of many objects, the idea of making groups of ten, then making groups of ten of ten, and so on, leads to the idea of the base-10 numeration system.

[2] 10,000 called one man in Japanese numeration system instead of ten thousand.

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b. Base-10 numeration system

In grade 1, students were taught numbers through around 120. In grade 2, the objectives are to broaden the range of numbers up to 4-digits and to let students know how to represent them in the base-10 numeration system. Also, students are taught to understand the size and order of numbers to deepen their understanding of numbers. In order to do this, it is necessary to gradually expand the range of numbers, first considering 3-digit numbers and then 4-digit numbers.

The base-10 numeration system is the system where, when the number of a certain place becomes ten, it is traded with one of the next place, and the remaining part stays and becomes the digit of the original place. In this system, the same symbols are used in all places, and the size of the place is decided by its position. These are the characteristics of the system.

The characteristics of this numeration system can be understood when comparing it with the numbers are read. For example, when reading 8235, all the place values are stated. When reading 8000, we only say the necessary place value, but need to write 0 for each place even if it is not read.

Here, in relation with comparing the size of numbers, inequality signs “>” and “<” are introduced so that the relative magnitude of numbers can be represented easily. The term “inequality sign” is introduced in grade 3.

Note (1) of “Handling Content” states: “man [ten thousand] should be included.” This means to teach 10,000 as the number after 9999. It is important to take this idea into consideration so the study of numbers beyond 10,000 in grade 3 will be a natural continuation and extension.

c. Relative size of numbers

“To understand the relative size of numbers” means to grasp a numbers’ size in terms of units such as tens and hundreds. For example, it means to see 6000 as “600 groups of a group of 10” or “60 groups of a group of 100.” The objective here is to deepen pupils’ understanding of the structure of numbers and to enrich their number sense by grasping the relative size of numbers. It is important to help students understand the relative size of numbers through activities with concrete objects, and mere formal teaching should be avoided.

d. To see one number as a product of other numbers

One of the objectives is to help students understand the multiplicative structure of numbers through activities such as making equal groups when counting objects. For example, students are taught to grasp the entire size of a certain object by using a partial group of that object as a unit and then counting how many units (groups) there are.

For example, if you do an activity such as: “Arrange marbles so that it is easy to see that there are 12 marbles,” various ways of arranging them can be thought of. If marbles are arranged as shown in the diagram below, they can be represented by such algebraic expressions as 2×6, 6×2, 3×4 or 4×3. This helps students see one number as a product of other numbers, deepening their understanding of numbers, and enriching their feel for numbers.

2×6 or 6×2

3×4 or 4×3

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e. Simple Fractions

You can represent the number of objects using numbers like 1, 2, 3, 4, ..., but you cannot represent the size of an object cut into halves. Using fractions enables you to represent the size of an object cut into halves. When concrete objects such as a piece of origami or a rope are cut into half, you end up with half the size of the original. $\frac{1}{2}$ is read as "one half," and it means one of the two equally divided pieces. If you cut a concrete object in half, then cut the resulting two objects into halves, you end up with something $\frac{1}{4}$ the size of the original object. When you divide an object equally into four parts, one of them is the size of "one fourth." If you continue further, you get $\frac{1}{8}$ the original size.

In this way, teach students to make sizes of $\frac{1}{2}$ or $\frac{1}{4}$ using concrete objects, and teach them that numbers such as $\frac{1}{2}$ or $\frac{1}{4}$ are called fractions. The meaning of fractions and their representation will be taught formally starting in grade 3. The objective in grade 2 is for students to do activities that will become the foundation of the study of fractions, and to understand the meaning of fractions empirically.

[Mathematical Activities] (1) a

Activities to find situations in everyday life where integers are used The objective of this activity is for students to deepen their understanding of integers by finding them in everyday life and to discover for themselves the value of studying integers.

Students can find many examples of integers in daily life. For example, integers are used to represent numbers, and amounts and quantities in things like calendars, prices of commodities, or time. Also, numbers are used to represent the result of grouping and sorting objects, such as car license numbers and room numbers.

In this way, it is important for students to discover and observe situations where integers are used in everyday life, to introduce these situations to other students, and understand that integers are widely used in daily life.

A (2) Addition and Subtraction

(2) To help pupils deepen their understanding of addition and subtraction, and extend their ability to use the calculations.

a. To explore ways of addition of two-digit numbers, and subtraction as the inverse operation, to understand that these calculations are based on the basic calculations of one-digit numbers, and to do these calculations accurately. To understand the way of calculation using algorithms in column forms.

b. To explore ways of addition and subtraction of three-digit numbers and so on in simple cases.

c. To explore properties of addition and subtraction and to make use of the properties in order to explore ways to calculate or check the results.

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(Handling The Content)

(2) As for the content (2) in "A. Numbers and Calculations", and (1) in "D. Mathematical Relations", symbols such as ( ) and □ can be used when necessary.

(3) As for the content (2)-c in "A. Numbers and Calculations", commutative law and associative law should be dealt with.

Building on the instructional content of grade 1, the objectives of grade 2 are to deepen students' understanding of situations where addition and subtraction of 2-digit numbers are used; for students to find calculation methods for addition and subtraction by themselves; and for students to learn and be able to use computational algorithms.

Other objectives are for students to consider how to calculate addition and subtraction of 3-digit numbers in simple cases and to be able to explain the process.

a. Addition of 2-digit numbers and subtraction as the inverse calculation *

In grade 2, the first thing taught is addition of 2-digit numbers and subtraction that is the inverse calculation. One should help students discover how to perform calculations for themselves, based on basic addition of single-digit numbers and its inverse operation and simple cases of addition and subtraction of 2-digit numbers taught in grade 1. For example, with a calculation such as $28 + 57$, students should first understand the meaning of the calculation based on a concrete situation. Next, utilizing the calculation methods learned so far, students may obtain the sum of the ones digits, $8 + 7 = 15$, and the sum of the tens digits $20 + 50 = 70$ by thinking in terms of 10 as a unit. They can then reach the final answer of 85 by adding those sums. If this way of thinking is directly reflected in a paper-and-pencil algorithm —- the algorithm shown on the left, an expanded version of the commonly-used algorithm — then when students learn what it means to re-group, they will be able to understand the ordinary algorithm.

$$\begin{array}{rll} & 28 & \\ + & 57 & \\ \hline & 15 & \cdots\ 8 + 7 \\ + & 70 & \cdots\ 20 + 50 \\ \hline & 85 & \end{array}$$

$$\begin{array}{rl} & 28 \\ + & 57 \\ \hline & 85 \end{array}$$

As shown, the algorithm allows us to carry out the calculation mechanically by aligning the place values. This way of calculation is based on the base-10 numeration system and will become the basis for multiplication and division introduced later.

The same is true with teaching the subtraction algorithm. Ask students to first think about the meaning of the calculation as well as how to carry it out based on concrete situations.

When thinking about how to calculate, or checking the result of a calculation, it is important to estimate the approximate size and the number of digits in the result. This is also important in teaching calculation in later grades.

[3]*[Translators' note: i.e. subtraction, including cases of (3-digit) — (2-digit) = (2-digit).]

b. Simple cases of addition and subtraction of 3-digit numbers

Students are to think about how to calculate addition and subtraction of 3-digit numbers in simple cases based on the addition of 2-digit numbers and subtraction as its inverse, and on the study of numbers so far,

Examples of simple cases are as follows:

① Addition and subtraction of numbers that can be considered using hundred as a unit
Here, addition and subtraction of numbers that can be considered using hundred as a unit means, for example, calculations such as $800 + 700$ or 500—100. These calculations can be thought of as $8 + 7$ or $5 - 1$, by looking at those numbers using 100 as a unit.

② Addition and subtraction with one 3-digit number and one 2-digit number, etc.
Here, addition should not involve regrouping into the hundreds place: for example, $628 + 7$ and $234 + 57$. Subtraction should not require regrouping from the hundreds place; for example, $753 - 6$, $683 - 51$, or $546 - 27$.

By teaching simple 3-digit addition and subtraction, not only students' understanding of calculations of 2-digit numbers can be solidified, but also their understanding of numbers through 3 digits may be secured. Also, students' understanding of these calculations should be connected to the study of how to calculate addition and subtraction to 3- and 4-digit numbers in grade 3.

c. Properties of addition and subtraction

We often group numbers or change the order of calculation. For example, when calculating $25 + 19 + 1$, we think $25 + (19 + 1)$ to make this calculation more efficient. Also, we can check the results of calculations by comparing the results of $16 + 8$ and $8 + 16$. Help students pay attention to the properties of calculation through these activities.

Note (3) of "Handling the Content" says: "As for the content (2)-c in" "A. Numbers and Calculations," commutative law and associative law should be dealt with". Therefore, teach students the commutative and associative properties of addition. You can also have students think about the mutual relationship between addition and subtraction, and teach them to use subtraction for checking results of addition, or addition for checking results of subtraction. Checking of calculations should include both the results and the processes. This emphasizes the fact that properties of calculations should be utilized, not just memorized. A general study of properties of calculation is done in grade 4.

Note (2) of "Handling the Content" says, "( ) and □ can be used when necessary." This is to encourage the ingenuity of students as they are taught to deepen their understanding of addition and subtraction.

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A (3) Multiplication

(3) To help pupils understand the meaning of multiplication and use the calculation.

a. To get to know situations where multiplication is used.

b. To explore simple properties which hold for multiplication, and to make use of them for making the multiplication table up to 9 times9 and for checking the results of calculations.

c. To learn the multiplication table up to 9 times 9 and to multiply one-digit numbers accurately.

d. To explore ways of multiplication of a two-digit number and a one-digit number in simple cases.

Mathematical Activities (1)

b. Activities to find properties and rules of the multiplication by constructing and observing multiplication tables.

[Terms/Symbols] ×

(Handling the Content)

(4) As for the content (3)-b in "A. Numbers and Calculations," the way in which the product increases when the multiplier increases by 1 and commutative law should be dealt with.

One of the objectives of grade 2 is to understand the meaning of multiplication by examining concrete situations where multiplication is used. Also, other objectives of this grade are to compose multiplication tables based on this understanding, paying attention to properties of the multiplication table in the process; to master the multiplication table, i.e. to be able to do multiplication of 1-digit numbers with certainty; and to utilize these skills in everyday life and study.

In addition, other goals of this grade include being able to think about how to do simple cases of multiplication of a 2-digit and a 1-digit number and to explain these calculations based on simple properties that hold for multiplication.

a. Cases where multiplication is used, and its meaning

Multiplication is used to find how many objects there are in so many units when the number of object for one unit is known. That is, multiplication is used to simplify the expression of adding the same number over and over; in other words, a concise expression of repeated addition. Furthermore, the meaning of multiplication as repeated addition may be considered as a way to find the amount that is so many times as many as the base amount.

The multiplication table is not only simple in its representation, it is also easy to get results once students memorize the traditional chant that has been used in our country since ancient times.

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b. Properties of multiplication

Note (4) of "Handling the Content" says, "As for the content (3)-b in "A. Numbers and Calculations," the way in which the product increases when the multiplier increases by 1 and commutative law should be dealt with." Have students study, for themselves, the commutative property of multiplication, and the property that the product increases by the size of the multiplicand when the multiplier increments one at a time.

When composing the multiplication table, it is important for students to see that the product increases by the size of the multiplicand when the multiplier increases by 1, to see the commutative property of multiplication, and to use these facts effectively to compose multiplication tables or to check results of calculations.

c. Multiplication table

When teaching the multiplication table, it is important to have students comprehend it by composing the table for themselves as they discover patterns in the table.

The multiplication table is the basis for the multiplication and division taught in the later grades. Therefore, it is important to make a connection between the multiplication table and experiential activities and daily experiences when having students compose and understand the table. It is also important to help students master every row of the table so they can calculate correctly and use them in their everyday life and study. Students become proficient in using the multiplication table by utilizing it in their daily life and study.

d. Simple cases of multiplication of a 2-digit and 1-digit number

As simple cases of multiplication of a 2-digit number and a 1-digit number, teach multiplication of 2-digit numbers up to about 12. You can explain how to do such calculations based on the multiplication table and by utilizing the property of multiplication where the product increases by the size of the multiplicand when the multiplier increases by 1.

For example, take a case of multiplication where 4 is multiplied by a 2-digit number. First, from $4 \times 9 = 36$, which is in the four's row of the multiplication table, students see that $4 \times 10 = 40$ (4 greater than 36). Further, $4 \times 11 = 44$ (4 greater than 40) and $4 \times 12 = 48$ (4 greater than 44). Students can see how to obtain products this way. Also, one can see 10×4 means four 10's, hence 40. Also one can see $11 \times 4 = 44$ and $12 \times 4 = 48$, based on the properties which hold for multiplication or by drawing diagrams for explanation. Thinking about how to carry out simple cases of multiplication of a 2-digit and a 1-digit number provides the foundation for the study of multiplication of 2- and 3-digit numbers taught in grade 3.

Mathematical Activities (1) b

Activities to find properties and rules of the multiplication by constructing and observing multiplication tables

The objective of these activities is to understand the properties and patterns of computation through composing and examining the multiplication table.

For example, students may find, "when the multiplier increases by one, the product increases by three" when learning the three's facts. Help students investigate if something like this happens in other cases by developing the multiplication table or examining the completed multiplication table. Students can conclude inductively that "when the multiplier increases by one, the product increases by the size of the multiplicand." Also, while composing the multiplication table, students may find that the answers of "3×4" and "4×3" are both 12. From this and similar cases, students may inductively discover this property of calculation: "Even if the multiplier and the multiplicand are switched, the answer remains the same".

Students may find various patterns in the multiplication table, such as the sum of the 3's row and the 4's row is the 7's row, or multiplication of the same numbers such as 1×1, 2×2, 3×3, ... are aligned on the diagonal line. Teaching students to find various patterns in the multiplication table through composing and examining it will also teach them the joy of discovery.

[B. Quantities and Measurement]

B (1) Units of length and their measurement

(1) To help pupils understand the meaning of units and measurements of length and measure the length.

a. To get to know the units of length (millimeter [mm], centimeter [cm] and meter [m]).

Mathematical Activities (1)

c. Activities to estimate the length and volume of objects in everyday life and measure them by using units

[Terms/Symbols] unit

A *unit* is a size that will be the basis of measurement. *Measurement* means to quantify the size of an object based on a fixed quantity.

In grade 1, students learned the basic idea of measurement: to compare the size of objects in concrete situations, to decide the appropriate length for the unit, and to represent the size of the object by counting the number of units.

In grade 2, based on what was taught in grade 1, help students become aware that it is necessary to use standard units for length, not any arbitrary length around them (arbitrary unit). Also, help them understand the meaning of units and gain the ability to measure using those units.

A ruler is used to measure length. Students are expected to understand how tick marks are organized on the ruler.

a. Units of length (*mm, cm, m)*

For units of length, millimeter (mm), centimeter (cm), and meter (m) will be taught here. Help students realize they can choose an appropriate unit based on the size of the object they are measuring. Also, help students pay attention to the necessity of the unit with such prefixes as milli- (m) or centi- (c) to deal with shorter lengths, and help them understand clearly the meaning and the role of units.

[Mathematical Activities] (1) c

Activities to estimate the length and volume of objects in everyday life and measure them by using units

The objective of these activities is for students to experience and understand the meaning of length and the idea of measurement through the actual measuring of objects.

For example, when measuring objects in their surroundings, students can find objects about 1 m long, and actually measure the length of those objects using a meter stick.

By providing experiences with these types of measuring activities, help students understand the meaning quantities, how to measure them, and how to comprehend the standard units of length.

B (2) Units of volume and their measurement

(2) To help pupils understand the meaning of units and measurements of volume and measure the volume.

a. To get to know the units of volume (milliliter [$m\ell$], deciliter [$d\ell$] and liter [ℓ]).

[Mathematical Activities] (1)

c. Activities to estimate the length and volume of objects in everyday life and measure them by using units.

In grade 2, through the activities of measuring the volume of objects, help students realize the necessity of using standard units, not arbitrary quantities in their surroundings (arbitrary units); and help them understand the meaning of units and how to use them correctly while measuring.

In grade 1, students measured volume using an object of appropriate size in their surroundings. In grade 2, they will be taught how to use things like a 1 liter measuring cup.

a. Units of volume ($m\ell$, $d\ell$, ℓ)

Milliliter ($m\ell$), deciliter ($d\ell$), and liter (ℓ) are introduced as units of volume. Teach 1 deciliter (dl) as a unit that is made by equally dividing 1 liter (ℓ) into ten portions and taking one of them. Teach milliliter ($m\ell$) as a unit for representing volume that cannot be measured using deciliter ($d\ell$), and teach that one ℓ is equal to $1000m\ell$.

[Mathematical Activities] (1) c

Activities to estimate the length and volume of objects in everyday life and measure them by using units The objective of these activities is for students to experience and understand the meaning of volume and the idea of measurement through actually measuring objects. For example, when measuring the amount of water that a glass bottle and a plastic bottle can hold, first let students predict how much water each can hold, then actually measure the amount of water in each bottle. By providing opportunities to actually experience these kinds of measuring activities, help students understand the meaning of volume and how to measure it, and the sizes of standard units of volume.

B (3) Units of Time

(3) To help pupils understand time and use it.

a. To get to know days, hours and minutes and to understand the relationships between them.

Elapsed time is a quantity that represents the interval from one point in time to another. Though sometimes the difference between clock time and elapsed time is not clear in everyday life, these two are distinguished when teaching.

a. Units of elapsed time (days, hours, and minutes)

In grade 2, day, hour, and minute are introduced as units of time. Teach students to use these units appropriately in concrete situations. Also help students understand the relationships that a day consists of 24 hours and an hour consists of 60 minutes, and how to use them.

[C. Geometrical figures]

C (1) Figures such as triangles and quadrilaterals

(1) Through activities such as observing and composing the shapes of objects, to help pupils pay attention to the elements that compose geometrical figures, and understand geometrical figures.

a. To get to know triangles and quadrilaterals.

b. To get to know squares, rectangles, and right triangles.

c. To get to know objects that have the shape of a box.

Mathematical Activities (1)

d. Activities to draw and make squares, rectangles, and right triangles and tessellate them on a plane.

[Terms/Symbols] straight line, right angle, vertex, side, face

In grade 2, students are expected know figures such as triangles and quadrilaterals, with a focus on the components of these figures. Triangles, quadrilaterals, squares, rectangles, and right triangles are taught as plane figures. Also, objects in students' surroundings that have a box shape are used to do activities that will be the foundation for understanding the solid shapes.

a. Triangles and Quadrilaterals

In grade 1, students understand plane figures intuitively and label them with informal names.

In grade 2, students are taught that shapes surrounded by three straight lines are called triangles and shapes surrounded by four straight lines are called quadrilaterals. These rules classify various figures into triangles and quadrilaterals based on the number of their constituent components, i.e., sides. Students are also encouraged to pay attention to vertices of triangles and quadrilaterals.

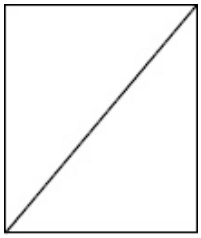
It is important to engage students in activities involving composing triangles and quadrilaterals of various shapes and sizes using concrete objects.

b. Squares, Rectangles, Right triangles

In grade 2, teach the meaning[4] and properties of squares and rectangles. Also, help students understand the meaning of a right angle through studying characteristics of a square and a triangle, finding shapes with right angles from their surroundings, or composing a right angle by folding paper.

Quadrilaterals that have four equal-length sides and four right angles are called squares. There are squares in many sizes, but their shapes are the same. Quadrilaterals that have four right angles are called rectangles. There are various shapes of rectangles with different combinations of length and width.

A right triangle is a triangle that can be made, for example, by dividing a square or a rectangle into two, as shown in the diagram at right.



c. Box shapes

In grade 2, teach about objects in students' surroundings that are box shaped. Also create activities that will provide the foundation for understanding solid shapes in grade 4, such as a cube and a rectangular parallelepiped.

Help students pay attention to the constituent components such as vertices, edges, and faces through observing concrete objects. If one observes a box-shaped object, one may notice there are faces with the shape of a square and faces with the shape of a rectangle. There are edges where two faces meet, and a vertex is a point where those edges meet.

Also, help students investigate the number of vertices, edges, and faces. To do this, it is recommended to introduce activities of observing box-shaped objects, and composing and decomposing them. For example, compose a box by gluing six rectangles or squares together, or composing a box shape using 12 sticks. Also, gathering paper boxes, cutting them open, and re-composing boxes again from the cut outs would be a good activity. This will lead students to realize that solid figures are composed of plane figures.

[Mathematical Activities] (1) d

Activities to draw and make squares, rectangles, and right triangles and tessellate them on a plane

The objective of these activities is for students to pay attention to the constituent components of figures so that they can understand the characteristics of squares, rectangles, and right triangles experientially through drawing them on dot paper like the grid at right, or by folding paper. For example, the following activities can be suggested:

[4] "Teaching the meaning of squares and rectangles" means to teach the pupil the definition of squares and rectangles in the ways that pupil can understand.

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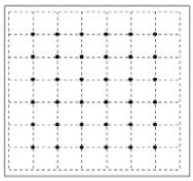
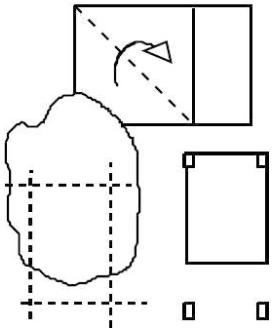
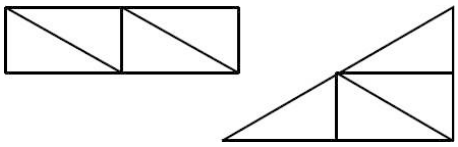


- Help students understand the elements composing figures by drawing squares, rectangles, and right triangles by connecting the points arranged on the grid like the one shown at right.



- By having students explain how to make a square by folding a rectangular piece of paper like the one at right, foster students' ability to develop an explanation that is based on the constituent components.
- As they fold the four corners of a piece of paper like the one at right to make a rectangle, help students to pay attention to the elements which make up this shape.
- Compose a right triangle by dividing a square or a rectangle along the diagonal, or compose a rectangle large right triangle from right triangles of the same size.

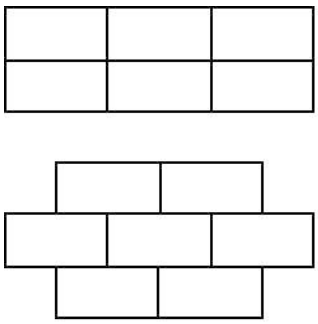


It is also important for students to identify triangles or quadrilaterals in concrete objects in their surroundings.

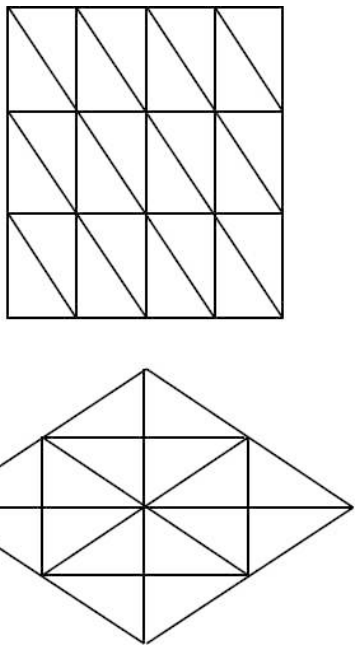
Furthermore, it is important to feel the broadening of a plane and the beauty of patterns generated by tessellations of the plane with squares, rectangles and right triangles.

For example, rectangles or right angle triangles can be used to tessellate the plane as shown in the diagrams below:

Tessellation with rectangles



Tessellation with right triangles



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[D. Mathematical Relations]

D (1) Mutual Relationship between Addition and Subtraction

(1) To help pupils understand the mutual relationships between addition and subtraction and explain them by using algebraic expressions.

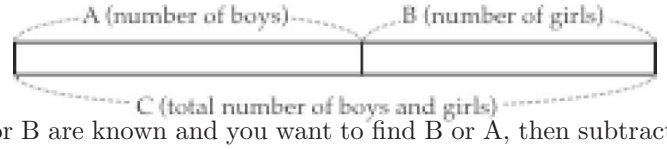
Mathematical Activities (1)

e. Activities to express and explain the mutual relationships between addition and subtraction by using figures, diagrams and algebraic expressions.

[Handling the Content]

(2) As for the content (2) in "A. Numbers and Calculations," and (1) in "D. Mathematical Relations," symbols such as ( ) and □ can be used when necessary.

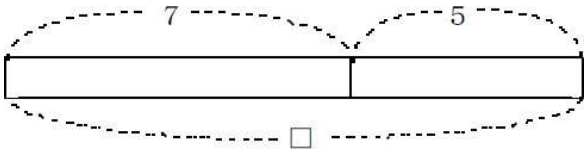
Suppose there are three quantities A, B, and C. If their relationship is something like the one in the diagram below, for example, then addition is the operation used to find C when A and B are known, i.e. $A + B = C$ or $B + A = C$.



Also, if C and either A or B are known and you want to find B or A, then subtraction is appropriate and can be represented $C - A = B$ or $C - B = A$. Addition and subtraction are mutually related with respect to which of the three quantities is to be found. This relationship between addition and subtraction is called the mutual relationship between addition and subtraction.

Teach students the mutual relationship between addition and subtraction using situations such as (1), (2), and (3) below. each of these situations, have students come up in front of the classroom and act out the situation, arrange concrete objects, represent the situation using diagrams and algebraic expressions, make a connection between the representation with concrete objects and the representation with diagrams or algebraic expressions, or explain those representations, so that students will deepen their understanding.

(1) Subtraction situations where addition is needed to find the missing quantity.
For example: "We had several apples. We ate 5 of them. Now we have 7 apples. How many apples did we have in the beginning?" This situation can be represented in a diagram as below, and □ can be found by $5 + 7$.



(2) Addition situations where subtraction is needed to find the missing quantity.
For example, take a situation like the following: "We had several apples. We were given 5 more apples. Now we have 12 apples. How many apples did we have in the beginning?"

(3) Missing subtrahend situation where subtraction is needed to find the missing quantity.
For example, take an example like the following: "We had 12 apples. We ate some of them. Now we have 7 apples. How many apples did we eat?"

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To solve these types of problems, it is important that students pay attention to the relationship between addition and subtraction, make use of it to understand the problem, to decide what kind of calculation to use, and to check the answers. It is also important that students can explain their solutions using algebraic expressions.

When explaining solutions using algebraic expressions, it is important that students can connect the numbers and the operations with the contexts — for example, with the problem in (1) above: "There were several apples and somebody ate 5 of them. Now there are 7 apples left" — returning (adding) 5 apples to the remaining 7 apples can be expressed by the algebraic expression $7 + 5$. Also, it is important to link the situation in (1) with the tape diagram mentioned above as part of the explanation. Through making these connections, students understand the mutual relationship between addition and subtraction, and grow in their ability to use algebraic expressions.

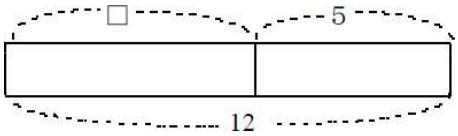
Moreover, it is necessary for students to deepen their understanding that algebraic expressions represent relationships among things and quantities concisely through this type of study.

Mathematical Activities (1) e

Activities to express and explain the mutual relationships between addition and subtraction by using figures, diagrams and algebraic expressions

The objective of these activities is to deepen the understanding of the mutual relationship between addition and subtraction. To do this, situations such as (1), (2), and (3) described above will be used — especially (1) and (2), which involves so-called "reverse thinking." Students will be challenged to think about how to solve these problems, represent these situations using diagrams and algebraic expressions, and explain them.

The following problem uses reverse thinking and is related to the mutual relationship between addition and subtraction. For example, the problem in (2) above, "We had several apples. Someone gave us 5 more. Now we have 12 apples. How many apples did we have in the beginning?" it is important to use a diagram like the one at right or to relate various representations. Furthermore, in situation (2), for example, one would think it is necessary to return (subtract) 5 apples from the current total of 12 apples; therefore, the algebraic expression $12 - 5$ is appropriate. In this manner, it is important to connect each part of the algebraic expression "12", "− ", and "5" to the situation, or to relate these parts of the algebraic expression to different parts of the diagram and be able to explain by using those relationships. It is important to foster students' ability to think and to explain how to solve problems through these kinds of activities.



When using diagrams, it is important for students to abstract the quantities in the situation by moving from representations with concrete objects to a diagram, fostering the understanding of the diagram experientially. Moreover, it is important that students use diagrams as "tools of thinking" and "tools of explanation" in mathematical activities. Also, help students use diagrams in relationship to algebraic expressions or expressions with words, so that they are able to think, interpret and explain them.

This type of teaching should also be done when teaching segment diagrams or number lines in later grades. It is important to deepen students' understanding of diagrams and algebraic expressions and their ability to represent, interpret, and explain them.

D (2) Multiplication expressions

(2) To help pupils represent situations where multiplication is used, by using algebraic expressions, and interpret these expressions.

[Terms/Symbols] ×

It is necessary to emphasize activities in which students represent concrete situations with algebraic expressions using the multiplication symbol, "×", interpret the expressions in view of these concrete situations, or interpret the expression and represent it using diagrams or concrete objects. Activities such as developing problems from multiplication expressions are important for deepening students' understanding of multiplication and their ability to use those expressions.

When teaching algebraic expressions, help students deepen their understanding of the meaning and appreciation for the simplicity and clarity of symbolic expressions by relating those expressions with phrases such as "each bag contains 5 oranges and there are 4 of them," diagrams with ○ or tapes, or representations with concrete objects.

When teaching how to interpret algebraic expressions, one can have students construct problems from a algebraic expression like 3×4, such as: "A pack contains three containers of pudding. There are four packs. How many containers of pudding are there in total?" To further foster the ability to interpret algebraic expressions, it is important to make the connection between algebraic expressions and concrete situations like this, and to represent the interpretation using ○'s, diagrams, or concrete objects.

D (3) Simple Tables and Graphs

(3) To help pupils organize and classify numbers and quantities in everyday life and represent them by using simple tables and graphs, and interpret these representations.

Students will be able to sort and organize numbers and quantities from their everyday lives, and express them by using simple tables and graphs. Here, simple tables mean those with one viewpoint, like the one below.

Number of Floowers Blooming

gound	punpkin	cucumber	eggplant
4	3	5	2

Number of Floowers Blooming

○		○	
○	○	○	
○	○	○	○
○	○	○	○
gound	punpkin	cucumber	eggplant

Simple graphs mean those with something like ○ to express quantity.

Students will be able to interpret characteristics from tables or graphs, such as which one has the most number of items. In doing so, it is more important to be able to interpret the characteristics than to be able to draw tables and graphs in a mechanical way. By expressing what one has sorted and counted using tables and graphs, the comparison of quantities and interpretation of differences become easier.

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[**Mathematical Activities**]

(1) The content listed in "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometrical Figures" and "D. Mathematical Relations" should be taught through, for example, the following mathematical activities:

a. Activities to find situations in everyday life where integers are used.

b. Activities to find properties and rules of the multiplication by constructing and observing multiplication tables.

c. Activities to estimate the length and volume of objects in everyday life and measure them by using units.

d. Activities to draw and make squares, rectangles, and right triangles and tessellate them on a plane.

e. Activities to express and explain the mutual relationships between addition and subtraction by using figures, diagrams and algebraic expressions.

[Terms/Symbols] unit, straight line, right angle, vertex, side, face, $\times$, $>$, $<$

3. The Content of Grade 3

[A. Numbers and Calculations]

A (1) How to represent numbers

(1) To help pupils deepen their understanding of the representations of integers, and extend their ability to use the numbers.

a. To get to know the unit of ten-thousands (man in Japanese).

b. To get to know numbers that are 10 or 100 times as many or $\frac{1}{10}$ of another number, and the representations of the numbers.

c. To deepen their understanding of relative size of numbers.

[Terms/Symbols] sign of inequality, number line

[Handling the Content]

(1) As for the content (1) in "A. Numbers and Calculations," the number100 million[4] should be dealt with.

In grade 3, teach *man* [ten thousand] as a unit, and help students deepen their understanding of how to represent numbers. When teaching large numbers such as ten *man* and a hundred *man*, consideration should be given to preparing concrete situations or devising teaching materials.

a. Unit of man

In grade 2, students learned 4-digit numbers in the base-10 numeration system and dealt with one *man* [ten thousand]. In grade 3, the representation of a whole number is extended and taught through the unit of *man*.

When teaching the unit of *man*, it is important to help students experience and understand its size. Help students grasp that the size of one man is ten groups of 1000, and one bigger than 9999. Also, convey the size of one "man" as 5000 plus 5000, or 100 times 100. To see one *man* through these various viewpoints helps students understand the size of this number. In teaching the size of one *man*, it is important to enrich the sense of the size of numbers by examining large numbers in students' everyday lives.

Also, regarding the representation of numbers, even with numbers bigger than one *man*, like ten *man*, one hundred man, and one thousand man, help students express these numbers as one, ten, hundred, and thousand using *man* as a unit.

As for numbers bigger than one *man*, since they have little experience with counting numbers bigger than one *man*, it is important to help students understand these numbers using the basic principles of the base-10 numeration system, and using a number-line that has been marked off by units of *man* to express those numbers on it,. At that time, teach the term number line.

Number (1) in Handling the Content indicates that "the number 100 million[4] should be dealt with." This is to serve as a way to sprial across grades, connecting it to the formal introduction to the unit of oku in grade 4.

[4]oku in Japanese

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b. 10 times, 100 times, $\frac{1}{10}$ times a number

Teach how to express a number that is 10 times, 100 times, or $\frac{1}{10}$ times a number, and help students deepen their understanding of the numeration system.

When teaching 10 times bigger or 100 times bigger than a number, it is necessary to help students pay attention to fact that the order of the digits in the number does not change and that the value of each digit is 10 times or 100 times greater than the original corresponding digit. For example, when 234 is made 10 times bigger, the 2 in the hundreds place goes to the thousands place, the 3 in the tens place goes to the hundreds place, and the 4 in the ones place goes to the tens place. In the same way, when making a number that is $\frac{1}{10}$ as big as another number, it is necessary to help students pay attention to the fact that the order of the digits in the number does not change and that the value each digit is $\frac{1}{10}$ of the original corresponding digits.

c. Relative size of numbers

In grade 3, broaden the range of numbers to include the unit of *man*. It is important to help students deepen their understanding of the relative size of numbers, and also enrich their sense of numbers, by using ten, hundred, thousand, and ten-thousand as units.

The calculation of $500 + 700$ can be seen as $5 + 7$, if one hundred is used as a unit. Also, since 800 can be seen as 8 if the unit of one hundred is used, then 800×5 can be seen as 8×5. Utilizing this way of viewing the relative size of numbers, students will be able to understand numbers, compare the size of numbers, and do calculations. Here, inequality signs will be introduced.

A (2) Addition and subtraction

(2) To help pupils add and subtract accurately, and extend their ability to use the calculations appropriately

a. To explore ways of addition and subtraction of 3- and 4-digit numbers and to understand that those calculations are based on basic calculations of 2-digit numbers and so on. To understand the way of calculations using algorithms in column forms.

b. To add and subtract accurately, and to use the calculations appropriately.

c. To explore properties of addition and subtraction and to make use of the properties in order to explore ways to calculate or check the results.

Mathematical Activities (1)

a. Activities to explore and explain the meaning and ways of calculating integers, decimal numbers

[Terms/Symbols] sign of equality

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Handling the Content

(2) As for the content (2) and (3) in "A. Numbers and Calculations," consideration should be given to enable pupils to do simple mental calculations.

(3) As for the content (2)-c in "A. Numbers and Calculations," commutative law and associative law should be dealt with.

One of the objectives in grade 3 is to think about how to carry out calculations of 3- and 4-digit numbers based on fundamental calculations students have learned previously, and for students to be able to do them reliably.

a. How to calculate addition and subtraction

Based on the addition and subtraction of 2-digit numbers, and simple cases of 3-digit numbers taught in grade 2, help students think about how to calculate addition and subtraction of 3- and 4-digit numbers.

For example, when thinking about how to calculate $154 + 172$, help students find the method of calculation for themselves by making use of their prior learning experience of $54 + 72 = 126$.

In doing so, it is necessary to teach students how to add and subtract 3 and 4-digit numbers by aligning the place values, based on the algorithms for the addition and subtraction of 2-digit numbers taught in grade 2.

Also, students will be taught to think about how to calculate addition and subtraction of 4-digit numbers based on the method they used for calculations of addition and subtraction of 3-digit numbers.

b. Mastering calculation of addition and subtraction

Students will acquire the skill of adding and subtracting 3- and 4-digit numbers reliably, and be able to apply those calculation skills as necessary in problem situations.

When students think about how to add and subtract 3- and 4-digit numbers or check the results of their calculations, it is necessary for the teacher to help students utilize estimation of the results. For example, when calculating $389 + 4897$, the answer 8787 might be obtained if the digits are not correctly aligned. Here, if one can see that 389 is approximately 400 and 4897 is approximately 5000, the answer will be about 5400, so one will notice that 8787 is a wrong answer. In situations like this, it is important for students to be able to make these kinds of estimations on their own.

Simple mental calculations indicated in Handling the Content A-(2) include addition of 2-digit numbers or subtraction that is their inverse. This type of calculation is often used in daily life and is even a necessary part of the process of multiplication and division. Mental calculations are often used to make estimates in daily life. It is important to consider these kinds of applications when teaching.

c. Properties of addition and subtraction

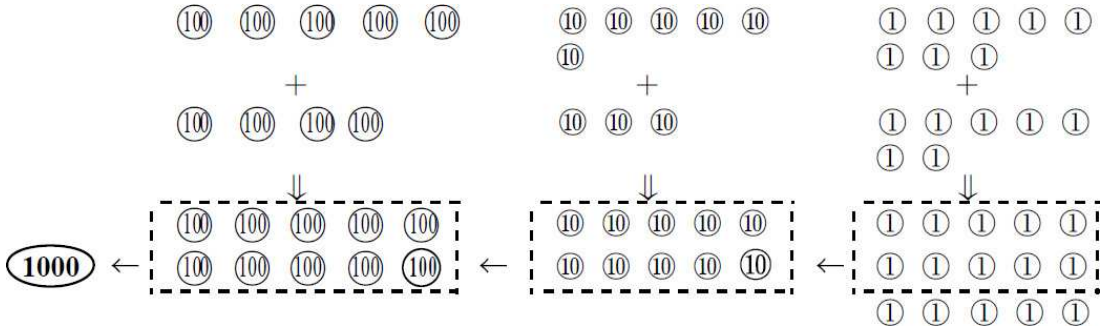
In grade 2, when devising how to carry out a calculation or how to check the result of a calculation, students were taught to examine the properties of addition and subtraction. In grade 3, when students think about how to calculate addition and subtraction of 3- and 4-digit numbers, or how to check their calculations, the students investigate the properties of calculations and become better at utilizing those properties. As indicated by note (3) of "Handling the Content," commutative and associative laws of addition are taught and students are encouraged to devise the idea such as $387 + (74 + 26)$ when calculating $387 + 74 + 26$. However, representing these ideas in algebraic expressions explicitly and describing and utilizing the properties of operations in general are in the content of grade 4.

Mathematical Activities (1) a

Activities to explore and explain the meaning and ways of calculating integers, decimal numbers and fractions by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

The objective of these activities is to help students think about the meaning of calculations, how to carry out calculations, and how to express their thoughts and ideas. Here, the meaning and methods for adding and subtracting 3- and 4-digit numbers will be explained.

For example, when calculating $568 + 437$, based on the addition of 2-digit numbers such as $68 + 37$ taught in grade 2, split the numbers into the hundreds place, the tens place, and the ones place, and calculate each place individually. When thinking about how to treat the re-grouped 1, the tens place will be "6 and 3 and1 re-grouped from the ones place, all together make 10," the hundreds place will be "5 and 4 and 1 re-grouped from the tens place, all together make 10," and in this way the answer is obtained. Students may represent this using the diagram as shown below, and express their explanation with words as well.



A (3) Multiplication

(3) To help pupils deepen their understanding of multiplication, multiply accurately, and extend their ability to use the calculation appropriately

a. To explore ways of multiplication of 2- or 3-digit numbers and 1- or2-digit numbers and understand that those calculations are based on basic multiplication of 1-digit numbers. To understand the way of calculation using algorithms in column forms.

b. To multiply accurately, and to use the calculation appropriately.

c. To explore properties of multiplication and to make use of the properties in order to explore ways to calculate or check the results.

Mathematical Activities (1)

a. Activities to explore and explain the meaning and ways of calculating integers, decimal numbers and fractions by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

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[Handling the Content]

(2) As for the content (2) and (3) in "A. Numbers and Calculations," consideration should be given to enable pupils to do simple mental calculations.

(4) As for the content (3) in "A. Numbers and Calculations," the calculations in cases where either the multiplier or the multiplicand is 0 should be dealt with.

(5) As for the content (3)-c in "A. Numbers and Calculations," commutative law, associative law and distributive law should be dealt with.

In grade 3, students are taught multiplication of 2- and 3-digit numbers by 1- and 2-digit numbers.

a. How to calculate multiplication

When teaching multiplication where a multiplier is 1-digit number, it is important to teach students to think about how to calculate on their own. For example, when thinking about how to calculate 23×4, we can see 23 as $20+3$ and split the calculation into 20×4 and 3×4. This idea is connected to the multiplication algorithm. It is important for students to use the base-10 numeration system and the multiplication table that they already learned as a basis for thinking about the multiplication algorithm. The same is true when teaching calculations in which a multiplican is a 3-digit number and a multiplier is a 1-digit number.

When the multiplier is a 2-digit number, it is possible for students to think how to calculate based on what they have previously learned: multiplication by a multiple of 10, and multiplication by a 1-digit number. For example, when calculating 23×45, one can view the multiplier 45 as $40 + 5$ and split the calculation into 23×40 and 23×5. When students think about how to multiply by a 2-digit number based on multiplication by multiples of ten and multiplication by a one-digit number, it is important for the students to have foresight about the result and the calculation process.

It is important for the students to master the calculation skill of multiplication learned here and become able to use this calculation in their daily life.

b. Utilizing multiplication

There are simple cases of multiplication where the multiplier or multiplicand is the number of people or of some object. Multiplication can also be used in such cases as, "the price of the ribbon is 85 yen per meter. When you buy 25 meters of this ribbon, how much will it cost?" and in such cases as, "A string is divided into 4 equal lengths. Each of these lengths is 9 centimeters. How long was the string to begin with?" multiplication can be used as the inverse calculation of division. It is important to teach students how to judge situations where multiplication is used and become able to use it appropriately.

Note (4) of "Handling the Content" says, "the calculations in cases where either the multiplier or the multiplicand is 0 should be dealt with." For example, when playing darts, if one hits the zero point area 3 times, it can be represented as 0×3. If one cannot hit the 3-point area, it can be represented as 3×0. Students can find the answer 0×3 by thinking about concrete situations or by returning to the meaning of multiplication and think of it as $0 + 0 + 0 = 0$. The answer 3×0 can be found from concrete situations or by remembering multiplication principles: by writing down $3 \times 3 = 9$, $3 \times 2 = 6$, $3 \times 1 = 3$ and finding the products decrease by 3, students will be able to derive their answer, $3 \times 0 = 0$. This type of multiplication with 0 can also be used in calculations such as 30×86 or 54×60.

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Note (2) of "Handling the Content" states, "consideration should be given to enable pupils to do simple mental calculations." This is because mental calculation may be needed in the process of multiplication and division. This is also because mental calculation is used in estimating or finding results even in daily life. "Simple calculations" here means calculations up to multiplication of a 2-digit multiplicand by a 1-digit multiplier. When teaching this, consideration should be taken so that it will not be too much of a burden for students. Also, when teaching, it is important to help students make the best use of mental calculations when applying algorithms or making estimates.

c. Properties of multiplication

In grade 2, students learned the commutative law, and how the product changes each time the multiplier increases by one. In grade 3, students learn commutative and associative laws of multiplication as described in "Handling the Content"-(5). Also, based on the way the product changes as the multiplier increases by one, students explore that the product increases or decreases by the multiplicand when the multiplier increases or decreases by one as represented in $a \times (b \pm 1) = a \times b \pm a$. It is impotant to enable students to use this property. Also, by investigating and working out the distributive law such as $a \times (b \pm c) = a \times b \pm a \times c$, students can understand that this has been used as the basis for the computational algorithm. For example, when considering the calculation of 24×3 thinking of 24×3 as combination of 20×3 and 4×3 is an example of the use of the distributive law.

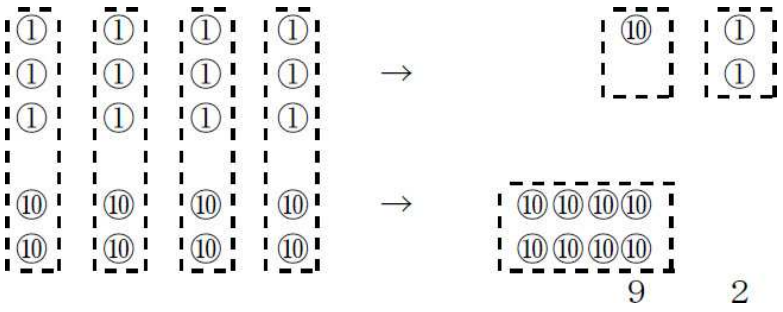
It is important that students are taught to appreciate the merits of using these properties, and make the best use of them when thinking about how to calculate, when explaining their calculations, and when checking their results. To summarize these properties and to understand that these hold for the calculation of decimal numbers as well is one of the contents of the fifth grade.

Mathematical Activities (1) a

Activities to explore and explain the meaning and ways of calculating integers, decimal numbers and fractions by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

The objective of these activities is for students to think about the meaning of the calculation and how to carry it out, and to represent their thinking processes. Here, doing calculations using algorithms is explained.

In the calculation of 23×4, for example, the multiplicand 23 can be split into "20 and 3" and further to "2 tens and 3 ones." When each number is multiplied by 4, you get "8 tens" and "12 ones," thus "$80 + 12$, with answer 92." Students are expected to represent this process using a diagram such as the one below, and become able to explain it with words as well.



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A (4) Division

(4) To help pupils understand the meaning of division, and use the calculation.

a. To get to know situations where division is used. Also, to get to know remainders.

b. To understand the relationships between division and multiplication and between division and subtraction.

c. To divide accurately when both the divisors and the quotients are1-digit numbers.

d. To explore ways of division in simple cases when the divisors are1-digit numbers and the quotients are 2-digit numbers.

Mathematical Activities (1)

a. Activities to explore and explain the meaning and ways of calculating integers, decimal numbers and fractions by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

[Terms/Symbols] ÷

Division is introduced in grade 3.

a. Cases where division is used, and its meaning

Generally, there are two concrete cases where division is used.

One is a case of *quotative division* where one has to find how many times as much one number or quantity is as the other number or quantity. The other case, called *partitive division*, is to find one part of an equally divided number or quantity. Quotative division can be thought of as division based on the idea of repeated subtraction. For example, the meanings of $12 \div 3$ are: dividing twelve candies equally by three people (partitive division), and dividing twelve candies into groups of three (quotative division).

When comparing quotative division and partitive division, quotative division may be easier to represent with manipulation of concrete materials, and corresponds better with the word, nozoku [removal]. Partitive division is better represented as *warizan* [dividing].[5] When introducing division, it is important to know these characteristics well. It is also important to introduce activities that involve manipulation of concrete objects such as counters, or activities that use every day objects.

In addition, in division, there are some cases that cannot be divided evenly. In such cases, teach student that remainders can be generated.

b. The relationships between division and multiplication and between division and subtraction

Division can be thought of as the inverse of multiplication. Therefore, as it relates to multiplication, it is important to clarify which of the two values is being sought, the one corresponding to the multiplier or the one corresponding to the multiplicand. Partitive division is where $\square$ in $3 \times \square = 12$ is sought, and quotative division is where $\square$ in $3 \times \square = 12$ is sought. It is important for students to realize that when we divide in the real world, we can divide things in a partitive way or a quotative way; students should thereby understand that both types of division can be expressed by the same algebraic expression.

[5] Although not a literal translation, the idea expressed here may correspond to the English phrases "go into" and "share/divide." When we say "how many times does 4 go into 12?" it matches more with the quotitive division, while the expression "share 12 items among 4 people" matches with the partitive division.

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When a remainder is involved, as, for example, in a situation such as "There are 13 cards, and these cards are to be divided equally among four people," $13 \div 4$ can be thought of as seeking $\square$, which is a whole number in $4 \times \square$ or $\square \times 4$, is less than and nearest to 13. And solving this problem means finding the whole number $\square$ that satisfies these conditions and also its remainder. We can think of $13 \div 4$ as finding the largest number of times the cards are dealt, with the remainder being smaller than the divisor. Students are expected to understand these ideas.

c. Calculating division

Teach division where the divisors and quotients are both 1-digit numbers. These are calculations that can be done by using the multiplication table once to find a quotient, such as $48 \div 6$ and $13 \div 4$. These types of calculations will be necessary for teaching division later on where quotients are 2-digit numbers and division with decimals, therefore, it will also be necessary for students to acquire these skills securely.

d. Simple cases of division where the divisor is a 1-digit number and the quotient is a 2-digit number

Utilizing division where divisors and quotients are 1-digit numbers, teach simple cases of division where the divisors are 1-digit numbers and quotients are 2-digit numbersHere, teach simple cases calculations such as the following.

One case is division where the dividend is a multiple of ten (10, 20, 30, $\cdots$) and the tens place of the dividend is divisible by a divisor as in $80 \div 4$ or $90 \div 3$. As for $80 \div 4$, it is important for students to understand for themselves, based on the idea of units, that 80 is "8 tens" and when divided by 4, the answer is "2 tens."

The other case is division where each digit in the 2-digit dividend is divisible by the sivisor, such as $69 \div 3$. In the case of $69 \div 3$, based on the idea of units, students can see for themselves this calculation in the same way as the multiplication of 2-digit numbers, and, split 69 into 60 and 9, since $60 \div 3 = 20$ and $9 \div 3 = 3$, the answer is 23.

It is important to teach children to think about ways of calculating because it ensures computation skills and the understanding of the meaning of division where the divisor and quotient are both 1-digit numbers, and fosters the ability to utilize knowledge and skills already learned.

Mathematical Activities (1) a, "Activities to explore and explain the meaning and ways of calculating integers, decimal numbers and fractions by using concrete objects, words, numbers, algebraic expressions, figures and diagrams ," is described in other contents.

A (5) The meaning of decimal numbers and their representation

(5) To help pupils understand the meaning and the representations of decimal numbers.

a. To use decimal numbers in expressing size of fractional part. To get to know the representations of decimal numbers to the tenths place.

b. To understand the meaning of addition and subtraction of decimal numbers through the tenths place. To explore ways of the calculations and do the calculations through the tenths place.

Mathematical Activities (1)

a. Activities to explore and explain the meaning and ways of calculating integers, decimal numbers and fractions by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

b. Activities to express decimal numbers and fractions by using concrete objects, figures, diagrams and number lines and to compare sizes.

[Terms/Symbols] decimal pointt, tenth place, number line

[Handling the Content]

(6) As for the content (5) and (6) in "A. Numbers and Calculations," thedecimal numbers 0.1 and the fraction $\frac{1}{10}$ and so on should be dealt with by using number lines with each other

In grade 2, the fact that length or volume measurements are represented in units such as "9 cm and 2 mm" or "3ℓ and 6$d\ell$" was taught. In grade 3, based on these experiences, students will understand the use of decimal numbers to represent quantities that are fractional parts of units, and become able to use them appropriately.

Utilizing decimal numbers enables us to express quantities less than 1 liter — as, for example, 0.8 liters, or 2 liters and 5 deciliters together as 2.5 liters. When teaching this idea, help students understand that the unit structure of quantity measure has a base-10 structure.

Another objectives here is to have students understand that decimal numbers can be used in addition and subtraction just like whole numbers and think about the process of calculations, and be able to do the calculations.

a. The meaning of decimal numbers and their representation

Since decimal numbers often become necessary in relation with measurement, it is possible to introduce decimal numbers to represent quantities that are fractional parts.

One of the ideas of decimal numbers is to extend the base-10 numeration system to numbers smaller than 1. For integers, when the quantity of a particular unit reaches ten, it is expressed as the following unit. However, in the case of decimals, a certain unit (1) is equally divided into ten parts to make a new unit (0.1). The size of a quantity is represented by how many of these new units. Here, the meaning of, and the term "tenths place" is introduced. Instead of the "tenths place," it is sometimes called the "first decimal place."

Expressing decimal numbers on a number line on which integers are also placed is important for deepening students' understanding of decimal numbers. Teach decimal numbers in relation to the number line of integers. For example, teach that 3.6 is between integers 3 and 4, and furthermore, that the interval between 3 and 4 is divided evenly into ten parts, and that this point (3.6) is at the 6th tick mark.

"Handling the Content"-(6) is discussed in the comment of "A-(6) The meaning and representation of fractions."

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b. Addition and subtraction of decimal numbers

Teach students to understand the meaning of addition and subtraction of decimal numbers, to think about how to calculate them, and to be able to carry out those calculations.

The following methods can be utilized when students think about how to calculate addition and subtraction of decimal numbers.

① Think about calculation for addition and subtraction of decimal numbers by relating them to the number line.

② Using relative size of numbers, represent decimal numbers with whole numbers, and calculate with those whole numbers.

③ By aligning decimal points, the unit of each digit is aligned as well. This enables one to calculate the whole number parts and the decimal parts separately

Since 1 is 10 of the $\frac{1}{10}$ unit, calculations involving regrouping can be done in the same way as when calculating with whole numbers.

Ultimately, addition and subtraction of decimal numbers should be done based on the understanding of the mechanism of decimal numbers, by aligning the decimal points, and by calculating the whole number parts and decimal parts separately, as mentioned in (3) above. It is important for students to understand that calculations with decimal numbers can be done based on the same principle and in the same way as calculations with whole numbers.

Mathematical Activities (1) a, "Activities to explore and explain the meaning and ways of calculating integers, decimal numbers and fractions by using concrete objects, words, numbers, algebraic expressions, figures and diagrams," and b, "Activities to express decimal numbers and fractions by using concrete objects, figures, diagrams and number lines and to compare sizes," are discussed in the explanation of other "Handling the Content."

(6) The meaning and representation of fractions

(6) To help pupils understand the meaning and the representations of fractions.

a. To use fractions in expressing size of fractional parts or quantities obtained as a result of equal partitioning. To get to know the representations of fractions.

b. To get know that a fraction can be represented as a collection of unit fractions.

c. To understand the meaning of addition and subtraction of fractions in simple cases, and to explore ways of the calculations.

Mathematical Activities (1)

a. Activities to explore and explain the meaning and ways of calculating integers, decimal numbers and fractions by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

b. Activities to express decimal numbers and fractions by using concrete objects, figures, diagrams and number lines and to compare sizes.

[Terms/Symbols] number line, denominator, numerator

[Handling the Content]

(6) As for the content (5) and (6) in "A. Numbers and Calculations," the decimal numbers 0.1 and the fraction $\frac{1}{10}$ and so on should be dealt with by using number lines with each other.

In grade 2, students learned simple fractions such as $\frac{1}{2}$ and $\frac{1}{4}$. Based on this teaching, students have gained fundamental experiences to understand fractions. Building further on those learning experiences, students are expected to securely acquire the understanding of the meaning and the representation of fractions in grade 3. Students will also learn that, just like whole numbers, addition and subtraction of fractions is possible. They are expected to think about how to carry out these calculations and be able to do the calculations.

a. The meaning and the representation of fractions

Fractions are used to represent sizes of quantities that are formed by equally dividing a whole or sizes of fractional parts of a unit. There are various ways to interpret the meaning of a fraction, depending on the point of view. Taking $\frac{2}{3}$ as an example, the interpretations are as follows:

① expressing the quantity comprised of two of the three equal parts of a concrete object

② expressing a measured quantity, such as $\frac{2}{3}$ liter or $\frac{2}{3}$ meter

③ expressing twice the unit that is obtained when 1 is partitioned into three equal parts (i.e. the unit fraction $\frac{1}{3}$)

④ expressing the relative size of A when B is considered as 1, in usage such as "A is $\frac{2}{3}$ of B"

⑤ expressing the result (quotient) of whole number division, "$2 \div 3$."

These distinctions are a matter of convenience. Often several of these ideas are used together when teaching. The terms denominator and numerator are introduced here.

In grade 3, ways of thinking, such as in ①, ②, and ③ are used. ④ and ⑤ are handled in grade 5.

b. How many of a unit fraction

Contrary to decimal numbers, which are represented using prespecified units such as $\frac{1}{10}$, we can choose any fraction suitable for the unit such as $\frac{1}{3}$, $\frac{1}{4}$ or $\frac{1}{5}$. This is one of the merits of fractions. Fractions whose numerators are 1 — such as $\frac{1}{3}$, $\frac{1}{4}$ or $\frac{1}{5}$ — are called unit fractions. Fractions can be thought of as multiples of a unit fraction. For example, $\frac{2}{3}$ is twice the size of $\frac{1}{3}$. $\frac{2}{3}$ is a fraction smaller than 1. $\frac{4}{3}$ is a fraction four times bigger than $\frac{1}{3}$, and greater than 1.

It is necessary to use some ingenuity in teaching, such as putting fractions on the number line so that students can grasp the size of fractions.

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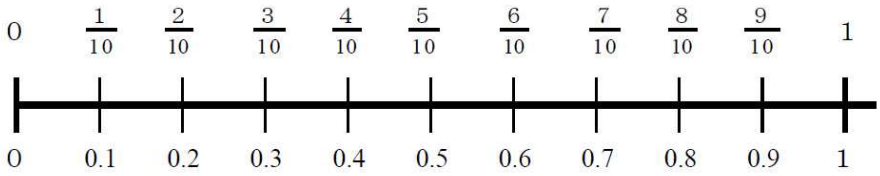
Note (6) of "Handling the Content" says, "the decimal numbers 0.1 and the fraction $\frac{1}{10}$ and so on should be dealt with by using number lines with each other." Consideration should be given so that students can visually experience and understand that 0.1 and $\frac{1}{10}$ are the same size by placing them on the top and below of the same number line after studying decimal numbers and fractions.



c. Addition and subtraction of fractions in simple cases

Students are expected to understand meaning of addition and subtraction of fractions with like denominator, and to think about how to calculate them. For simple cases, addition and subtraction of proper fractions are taught. Also, addition with sums up to 1 and subtraction as the inverse of this addition are taught.

Regarding the method of calculation, if one thinks about how to carry out addition and subtraction of fractions as addition and subtraction of numbers of unit fractions, one can see this can be done in the same way as whole numbers.

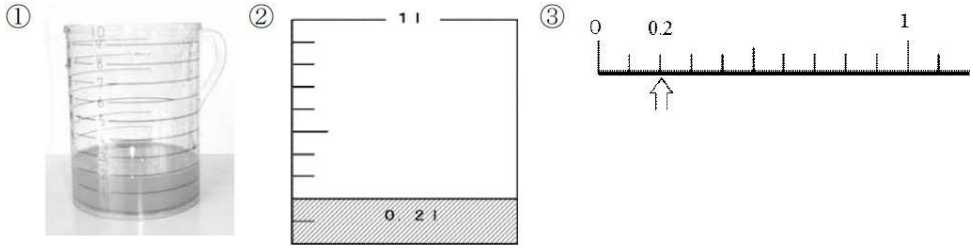
For example, in a problem involving length that requires one to find the sum of $\frac{1}{5}$ meter and $\frac{2}{5}$ meter, students are expected to be able to think about and explain how the result becomes three times $\frac{1}{5}$ meter (three times the unit fraction). "Mathematical Activities (1) a" "Activities to explore and explain the meaning and ways of calculating integers, decimal numbers and fractions by using concrete objects, words, numbers, algebraic expressions, figures and diagrams " is described in other contents.

[Mathematical Activities] (1) b

Activities to express decimal numbers and fractions by using concrete objects, figures, diagrams and number lines and to compare sizes

The objective of these activities is to have students experience and understand the meaning and the size of decimal numbers and fractions intuitively. When introducing decimal numbers and fractions, the instruction should help students understand their meaning and size in relation to concrete objects, such as shown below.

For example, when teaching decimal numbers, help students realize that the amount of liquid in a 1 liter measuring cup like (1) is two of ten equally divided portions, and express this using a diagram like (2), and using that diagram as a clue, express it on the number line like (3). Through sequences of activities like these, help students understand that 0.1 is a unit that is the result of dividing 1 into ten equal parts, and help them to represent how many of this unit (0.1) is used.



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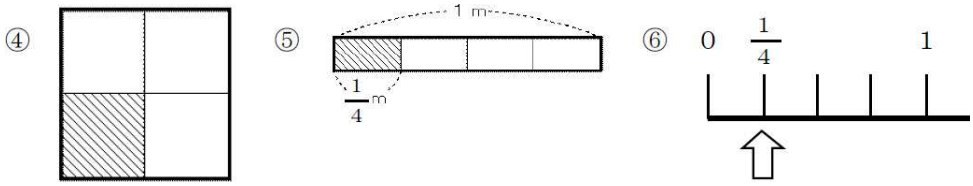
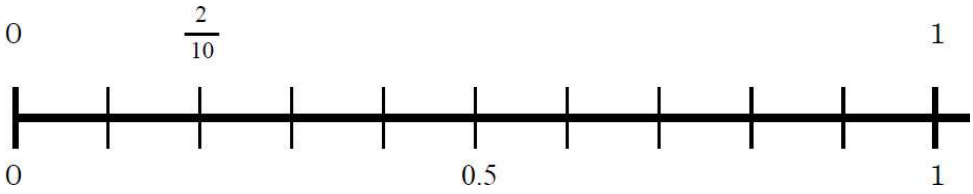
It is also important to include activities where students locate another decimal number on the number line and compare the size of the two decimal numbers by their positions. This is important to help students understand the meaning and size of decimal numbers, and their addition and subtraction.

For fractions, as for decimals, help students see that they represent numbers as compositions of units. Such an understanding may be developed through a series of activities like the following: Represent their previous experience that half of a half of an origami paper is one of four equal parts of the paper, with a diagram such as (4). Students relate this diagram to a tape diagram such as (5) and eventually to the number line such as (6). From these experiences, students will see that other fractions may represent numbers by how many unit fractions they comprise — this example, the unit is $\frac{1}{4}$ which is obtained when 1 is divided into four equal parts.



After studying decimal numbers and fractions, help students realize that using number line makes it easier to compare the sizes of a decimal number and a fraction. For example, if you put a decimal number (0.5) and a fraction $\frac{2}{10}$ on a number line, as shown below, it is easy to compare them.



A (7) The abacus

(7) To help pupils get to know the representations of numbers on soroban(Japanese abacus) and do simple addition and subtraction by using soroban.

a. To get to know the representations of numbers on soroban.

b. To get to know ways of addition and subtraction.

Mathematical Activities (1)

a. Activities to explore and explain the meaning and ways of calculating integers, decimal numbers and fractions by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

The abacus is a tool for calculation that has been used for a long time in Japan. It is a very useful tool for representing numbers or doing calculations. One can represent whole numbers and decimal numbers on an abacus by assigning place values, and manipulating the beads on it. Once one understands the mechanism of an abacus, one can perform addition, subtraction, multiplication, and division using it. The abacus is taught in grades 3 and 4.

a. How to represent numbers on an abacus

In grade 3, teach how to represent whole numbers and decimal numbers on abacus. Students will become able to represent whole numbers and decimal numbers according to the base-10 numeration system. Whole numbers up to the *man* [ten thousands] place and decimal numbers to the thenths place are taught.

b. How to do calculations using the abacus

As for calculations, teach simple cases of addition and subtraction. It is important for students to understand how to move beads on abacus to calculate. For whole numbers, teach addition and subtraction of 1- and 2-digit numbers. Students start with such calculations as "$2 + 1$" or "$3 - 1$" which can be done just by manipulating the specified number of beads, and proceed to calculations such as "$4 + 3$" or "$6 - 4$" which involve composing and decomposing 5, and "$8 + 9$" or "$15 - 7$" which involve regrouping. Calculations involving the man place such as "3 *man* [thirty thousand] + 5 man [fifty thousand]" are taught as well. As for decimal numbers, simple addition and subtraction up to the tenths place such as "$2.6 + 0.3$" are taught.

Mathematical Activities (1) a "Activities to explore and explain the meaning and ways of calculating integers, decimal numbers and fractions by using concrete objects, words, numbers, algebraic expressions, figures and diagrams " is described in other contents.

B. Quantities and Measurements

B (1) Units and Measurements of length and weight

(1) To help pupils deepen their understanding of length, and to help them understand the meaning of units and measurements of weight, and measure the weight.

a. To get to know the unit of length (kilometer [km]).

b. To get to know the units of weight (gram [g] and kilogram [kg]).

Mathematical Activities (1)

c. Activities to investigate the relationships among the units within each quantity such as length, volume, and weight.

[Handling the Content]

(7) As for the content (1)-b in "B. Quantities and Measurements," the unit of "ton"[t] should be touched upon.

The objectives of grade 3 are to deepen the understanding of length, to understand weight based on the study of length and volume studied from grade 1, to understand the standard units of weight, and to become able to measure weight using measuring instruments.

a. Units of length (km)

In grade 3, kilometer (km) is introduced as a unit of length. Since it is hard to see the actual length of 1 kilometer, it is necessary to use its relationship to amounts such as ten times 100 m or 100 times 10 m so that students can understand how long 1 km is. Also, it is important to help students develop a willing attitude to investigate such questions as what they can find at places that are 1 kilometer from school, or where the unit of kilometer is used.

b. Units of weight (g, kg) and measurements

Students have experienced whether something is heavier or lighter than something else when holding or putting it on. Also, they have seen or heard about measuring weight using scales on occasions such as grocery shopping and measuring body weight at school.

In teaching about weight, based on these experiences that students have had, help them understand, in the same way as with other quantities, weight can be measured in terms of how many of a unit of weight. Also, it is important to enrich the sense of fundamental quantities of weight through activities such as holding objects that weigh 1 kilogram.

When weighing objects, if those objects cannot be weighed directly, containers may be used. In this case, the relationship "(net weight) = (total weight) − (weight of the container)" is used.

Mathematical Activities (1) c

Activities to investigate the relationships among the units within each quantity such as length, volume, and weight

The objective of these activities are to help students become aware of the relationships common to each set of units based on the study of the units of length, volume and weight they experienced up to grade 3, and to deepen their understanding of units.

In these activities, students will examine the relationships common to each set of units, for example, by using a table that summarizes the units of length (millimeter, centimeter, meter, and kilometer), volume (milliliter, deciliter, and liter), and weight (gram and kilogram) that they have learned up to grade 3. Through these investigations, following points can be made:

- Both units of length and units of weight have units that involve kilo-
- Both units of length and units of volume have units that involve milli-
- 1 kilometer is 1000 meters; that is, 1000 times 1 meter
- 1 kilogram is 1000 grams, that is, 1000 times 1 gram
- 1 liter is 1000 milliliters; that is, 1000 times 1 liter
- 1 meter is 1000 millimeters; that is, 1000 times 1 millimeter

In grade 6, units taught through grade 6 will be summarized in the study of "Units and structure of the metric system."

Note (7) of "Handling the Content" says that, in addition to gram and kilogram, the unit of weight *ton (t)* should be touched upon. Here, teach that the unit of ton is often used in daily life to express heavy objects and that 1 ton is 1000 kilograms.

B (2) Selecting appropriate units or measuring instruments

(2) To help pupils estimate length and weight and measure them by selecting appropriate units and instruments according to their purposes.

The objective here is to enable students to make efficient measurements and represent them appropriately in everyday life.

When one measures some quantity, one should estimate approximately how big the object is, and select the appropriate unit and measuring instrument. For example, teach that when measuring the length of a curve, such as the distance around a tree trunk, one would use a measuring tape, and when weighing an object, one would estimate the approximate weight and select the appropriate measuring instrument. When teaching, care should be taken so that students understand that it is desirable for the measure of a quantity to be expressed with numbers that are easy to use by selecting an appropriate unit.

B (3) Clock time and elapsed time

(3) To help pupils understand time.

a. To get to know seconds.

b. To determine clock time and elapsed time which are necessary in their daily lives.

Elapsed time is a quantity that, unlike length, area, or volume, is difficult to capture by observing. Also, it is difficult to actually use a selected size as a unit and directly measure somthing with it. So, it is important to help students gradually understand time by recognizing elapsed time in concrete situations, and making connections with activities and experiences in everyday life.

a. Units of time: second

The second is introduced as a unit of time. The relationship between minute and second, that one minute is 60 seconds, and the ability to use these units is also taught here. It is important to teach where this unit, second, is used in real life.

b. Determining clock time and elapsed time

Students will become able to determine clock time and elapsed time in concrete situations in their daily lives where necessary. Their understanding will deepen through activities with concrete objects, such as actually moving clock hands. When determining the time by calculation, care should be taken so that concrete situations in daily life are provided in which telling time is needed, and unnecessarily complicated conversions of units of time should be avoided.

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C. Geometrical figures

C (1) Geometrical figures such as isosceles and equilateral triangles

(1) Through activities such as observing and composing geometrical figures, to help pupils pay attention to the elements that compose geometrical figures and understand geometrical figures.

a. To get to know isosceles triangles and equilateral triangles.

b. To get to know angles.

c. To get to know circles and spheres. Also, to get to know the center, radius and diameter of such figures.

Mathematical Activities (1)

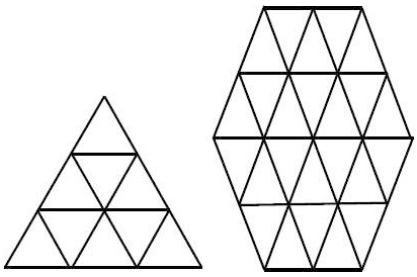
d. Activities to construct isosceles triangles and equilateral triangles using a ruler and a pair of compasses.

In grade 3, isosceles triangles and equilateral triangles are introduced. Students will also understand angles in relation to these triangles.

a. Isosceles triangles and equilateral triangles

Help students grasp the features of triangles by paying special attention to the lengths of the sides of triangles. A triangle that has two sides of equal length is called an isosceles triangle, and a triangle that has three sides of equal length is called an equilateral triangle. Help students understand isosceles and equilateral triangles through activities involving construction using compasses and rulers. Also help students to verify that an isosceles triangle has two equal-sized angles and an equilateral triangle has three equal-sized angles by engaging them in activities such as examining these triangles or cutting triangles drawn on paper and folding them.

Furthermore, help students enrich their sense of geometrical figures by engaging them in activities such as: tiling a plane surface using congruent isosceles triangles or equilateral triangles in order to understand that these geometrical figures can tessellate; observing a pattern of tiled triangles and recognizing other geometrical figures in it; or realizing that a plane figure can extend, and that there is beauty in geometrical figures.



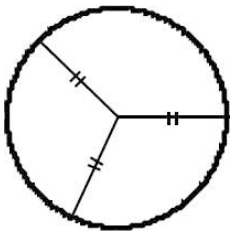
b. Angles

Right angles have been taught in grade 2. In grade 3, teach that the shape formed by two sides coming from the same vertex is called an angle. Students will be able to compare the size of angles by overlapping two angles. Students will be taught that, if they cut out isosceles triangles or equilateral triangles and fold them by overlapping the sides of equal length, they can see that the overlapping two angles are the same size.

The measurement of angles and the unit used to express the size of angles are taught in grade 4.

c. Circles and spheres

In grade 1, students were taught that circles and spheres are things that have a round shape or a ball-like shape. In grade 3, students will understand circles through activities such as observation, sorting, composing, and drawing, and spheres will be understood through observation.



As for circles, as shown in the figure above, help students understand that all points on the circle are equidistant from the center.

Radius is defined to be the line from the center to the circle. *Diameter* is a straight line that passes through the center, from one point on the circle to another point on the circle.

Furthermore, help students recognize that there are infinitely many radii or diameters through activities such as drawing. Also, help students get used to manipulating compasses through activities such as making designs with circles, and help them see the beauty of circles. Also, help students get interested in circles through activities such as drawing big circles in the playground, or finding circle-shaped objects in their daily lives.

The compass is used in conjunction with this content. The compass is not only used to draw circles, but also as a tool for measuring and copying an equal length of a line segment. Thus, it can be used to compare length.

Activities such as finding the center of a circle by folding a circle made of paper, or making a top, would be effective for students to recognize the properties of circle.

As for spheres, help students understand through manipulating or observing models that the section of a sphere cut by a plane is always a circle, and that the cut section is largest when a sphere is cut exactly in half.

The length of the diameter of a sphere such as a ball can be investigated with activities like placing the ball between two solid figures such as rectangular parallelpipeds.

Mathematical Activities (1) d

Activities to construct isosceles triangles and equilateral triangles using a ruler and a pair of compasses

The objective of these activities is for students to develop the ability to utilize the methods for constructing geometrical figures using ruler and compass according to the purpose and situation, through activities like drawing isosceles triangles and equilateral triangles using a compass and ruler.

These activities are also important for helping students become good at using a ruler and compass when focusing on the constituent elements of geometrical figures or understanding their properties.

The following methods can be used to draw isosceles triangles with a compass and ruler:

① Using a given condition (such as the base)
When segment BC is given, draw with a compass an arc whose center is B, and draw another arc with the same radius centered at C. Then, connect the intersection point, A, with points B and C.

② Using circles
Draw a circle with a compass. Using the property of a circle that the radii are the same length no matter where they are drawn, connect two points on the circle and the center.

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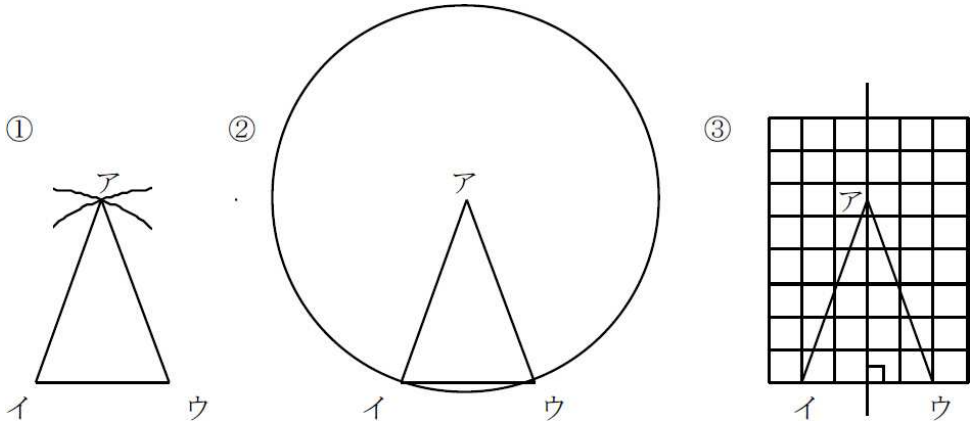
There is yet another way of drawing isosceles triangles using grid paper.

③ Using grid paper
Choose a segment BC on a grid, and then connect points B and C with point A on the perpendicular bisector of segment BC.



Equilateral triangles can be constructed using origami paper as shown in the diagram below:

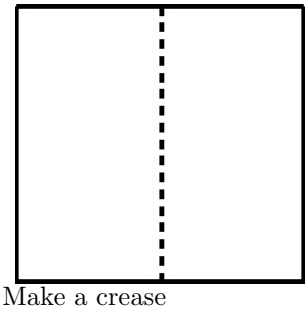
① Fold origami paper into halves (see the picture below) and put a crease in the paper.

② Overlap the vertex on the lower right corner on the crease.

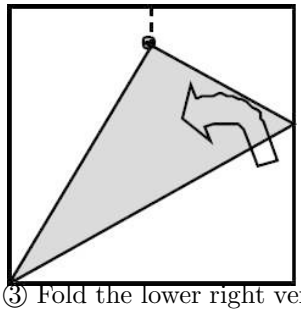
③ Mark the point where the vertex and the crease overlap.

④ Connect the lower vertices of the origami paper and the point marked above.

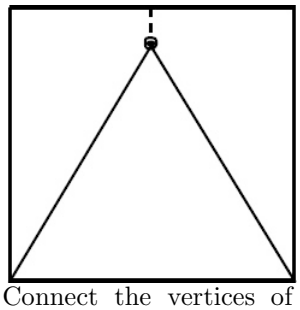
[Composing an equilateral triangle from origami paper]



① Make a crease



②, ③ Fold the lower right vertex to the crease and mark the point where they meet



④ Connect the vertices of the origami paper and the point of the mark.

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[D. Mathematical Relations]

D (1) Algebraic expressions of division

(1) To help pupils represent situations where divisions are used, by using algebraic expressions, and interpret these expressions.

Terms/Symbols ÷

In grade 3, students will become able to understand algebraic expressions with the symbol "÷" for division. When teaching, emphasis should be put on representing quantitative relations using algebraic expressions and interpreting algebraic expressions in the same way as was done when teaching addition, subtraction, and multiplication.

When teaching representation using algebraic expressions, while deepening students' understanding of the meaning of algebraic expressions by connecting them to phrases such as "Divide 12 candies equally among three people," to diagrams that use ◯ or a tape, or to the manipulation of concrete objects, help students to appreciate the conciseness and clarity of expressions using the ÷ symbol.

In grade 3, to interpret algebraic expressions means to grasp concrete quantitative relations from them. For example, from the algebraic expression $15 \div 3$, one can imagine a problem such as "There are 15 oranges. If you divide them so that each person gets 3 oranges, how many people can get oranges?" It is important to connect algebraic expressions with concrete situations such as this.

Also, it is important to help students understand the meaning of expressions of division in relation to those of multiplication such as "the multiplier and multiplicand in multiplication correspond to the divisor in division," while linking them to words or diagrams.

D (2) Algebraic expressions that represent quantitative relations

(2) To help pupils understand algebraic expressions that represent the relationships between numbers/quantities and use these expressions.

a. To represent the relationships between numbers/quantities in algebraic expressions, and to make connections between algebraic expressions and diagrams.

b. To represent numbers and quantities by using □, to represent the relationships between numbers/quantities in algebraic expressions, and to explore the expressions by substituting numbers for the □.

As for representing with and interpreting algebraic expressions, from addition in grade 1 through subtraction, multiplication, and division, students have been taught to understand the meaning of the algebraic expressions and to be aware that the algebraic expression is an excellent way to represent quantitative relationships with conciseness, clarity, precision, and generality. The objective of grade 3 is to raise awareness of algebraic expressions and to help students master the representation and interpretation of algebraic expressions.

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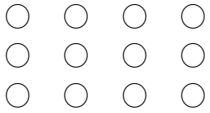
a. Relating algebraic expressions to diagrams

In grade 3, when teaching algebraic expressions, students are expected to be able to represent quantities and quantitative relationships using algebraic expressions in relation to concrete situations. At the same time, they are expected to master skills for utilizing algebraic expressions: to be able to interpret the meaning of the situation that the algebraic expression is representing; to be able to explain their own thinking through the use of algebraic expressions; and to be able to work and think using algebraic expressions.

When teaching diagrams, help students become able to represent quantities and quantitative relationships using diagrams, to interpret the quantitative relationships represented by diagrams, and to explain one's own ideas using diagrams.

Moreover, students are expected to understand that an algebraic expression and a diagram may represent the same thing.

$3 \times 4 \quad \leftrightarrow$ 

When teaching, it is important to help students relate algebraic expressions and diagrams through activities where students interpret quantitative relationships represented in diagrams and express them in algebraic expressions, and where students interpret quantitative relationships in algebraic expressions and express them in diagrams.

Also, it is important for students to deepen their understanding of the mutual relationships between addition and subtraction, and between multiplication and division, and to become able to explain those relationships by relating algebraic expressions and diagrams.

b. Algebraic expressions using □

In grade 2, in relation to deepening the understanding of addition and subtraction, the use of ( ) and □ is taken up in "Handling the Content".

In grade 3, students are expected to be able to express mathematical relationships according to the context by using symbols such as □ to represent unknown quantities, and to be able to investigate numbers that correspond to □. There are two cases where symbols such as □ are used: to represent an unknown quantity and to represent a variable. In grade 3, the focus should be on representing unknown quantities. Students are expected to be able to precisely understand quantitative relationships through writing algebraic expressions using symbols such as □, and through making the connection between writing such expressions and representing quantitative relationships in diagrams. When teaching, consideration should be given to use □ at first as the placeholder where students fill in numerals, and then gradually use □ as a symbol that represents an unknown numerical quantity so that they will understand the role of □, which is similar to using letters. When examining which number should be placed in □, for example, in the algebraic expression $\square + 8 = 17$, we can think of a method to fill in □ with 1, 2, 3, etc., in order. Another method is to estimate a suitable number for □ and fill in □ with 8, and then with 9. The most efficient method would be to use the inverse calculation based on the mutual relationships among the four arithmetic operations. It is important to help students understand, through a sufficient number of these mathematical activities, not only that the symbol □ represents 9, but also that the algebraic expression $\square + 8$ itself represents the quantity 17.

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D (3) Tables and bar graphs

(3) To help pupils organize and classify data, and represent them clearly by using tables and graphs, and interpret these representations.

a. To get to know how to interpret and draw bar graphs.

Mathematical Activities (1)

e. Activities to organize data from perspectives, such as dates, locations, and to represent the data in tables.

In grade 2, students were taught to represent what they sorted and organized in tables or simple graphs with pictures and diagrams. The main objectives in grade 3 are, based on what students have learned so far, for students to be able to decide on a viewpoint according to a purpose, then, sort and organize data, represent the data in tables and graphs, and interpret those data.

Also, simple 2-dimensional tables will be introduced, and students will be taught to see tables from two viewpoints such as time and place. As for tables, since they classify, sort, and represent data in many different ways, it is important for students to understand the characteristics of each way and become able to use them appropriately depending on the purpose.

Also, while making connections to tables, students are expected to be able to recognize the characteristics of a bar graph. Bar graphs enable students to capture the differences in the quantity of data at a glance, and help students find this fact out for themselves through representing data.

a. How to interpret and draw bar graphs

The objectives of the study of bar graphs in grade 3 are to interpret the different sizes and differences between the sizes of quantities, to determine maximum and minimum values, and to interpret the relations between categories or the overall characteristics of data, building on what students learned in grade 2.

When teaching, it is important to teach correctly the way of choosing appropriate items and ordering them, choosing a title for the graph, etc. while taking advantage of students' ingenuity to represent what they are thinking in an easy-to-understand manner.

As for choosing and interpreting the scale of the axis, it is important to use skills in measuring with a ruler or labeling a number line. The primary focus will be on the cases where the smallest interval is 1, 10, or 100. However, it is also necessary to use 2 or 5 to be the size of the smallest interval, depending on purposes, and to interpret them.

In doing so, it is important to enable students to use appropriate scales depending on the purpose or the size of the grid paper through such activities as comparing several graphs that use different scales on the same grid paper, or selecting the most suitable grid paper from several different kinds.

Mathematical Activities (1) e

Activities to organize data from perspectives, such as dates, locations, and to represent the data in tables

The objective of these activities is for students to acquire the basic and fundamental knowledge and skills for creating and interpreting tables. These activities will help students understand the meaning of tables and how to represent data using tables by choosing appropriate categories for sorting the date such as, day of the week, time and location, etc., and organize and sort data in a skillfully and purposefully.

When engaging in activities, it is important to take care that students foster an attitude and ability to know exactly what the task is, willingly gather data accordingly, and sort and organize the gathered data according to their own perspective. To do this, it is important to emphasize activities such as checking data to see if there are any missing or overlapping items, or verify if there are any errors in counting to minimize errors. It is necessary to emphasize the importance of the sum total column and help students utilize their knowledge and skills of tables, for example, checking to see if the numbers in the sum total column and the numbers of the data are in agreement.

When teaching, the following points should be taken into consideration:

① To make the purpose clear, to consider the conditions for the collection of data, and to select the perspective according to the purpose.

② To decide which categories to use and how to classify data so that there is no missing or overlapping data.

In regard to tables, there are many ways of sorting and various methods of presentation, so it is important for students to be able to use tables according to their purpose.

[**Mathematical Activities**]

(1) The content listed in "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometrical Figures" and "D. Mathematical Relations" should be taught through, for example, the following mathematical activities:

a. Activities to explore and explain the meaning and ways of calculating integers, decimal numbers and fractions by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

b. Activities to express decimal numbers and fractions by using concrete objects, figures, diagrams and number lines and to compare sizes.

c. Activities to investigate the relationships among the units within each quantity such as length, volume, and weight.

d. Activities to construct isosceles triangles and equilateral triangles using a ruler and a pair of compasses.

e. Activities to organize data from perspectives, such as dates, locations, and to represent the data in tables.

[Terms/symbols] sign of equality, sign of inequality, decimal point, tenth place, number line, denominator, numerator, ÷

4. The Content of Grade 4

[A. Numbers and Calculations]

A (1) Representing Integers

(1) To help pupils deepen their understanding that integers are represented by the decimal positional numeration system.

a. To understand units such as hundred million (oku in Japanese) and trillion (cho in Japanese), and to summarize the decimal positional numeration system.

[Handling the Content]

As for the content (1) in "A. Numbers and Calculations," cases when large numbers are expressed by spliting them into 3-digit groups should be touched upon.

In grade 3, the base-10 numeration system including the unit of *man* [ten thousand] was taught.

In grade 4, teach students the units of *oku* [hundred million] and *cho* [trillion] and deepen their understanding of the base-10 numeration system.

a. Units of hundred million (oku in Japanese) and trillion (*cho* in Japanese)

When the range of numbers reaches *oku* (hundred million) or *cho* (trillion), it is hard to imagine the size of these numbers. Therefore, when teaching these numbers it is necessary to give some concrete examples, such as the population or budget of some country. When representing numbers bigger than 1000 *man*, the new unit *oku* is used to represent 10 times 1000 *man*. In addition to that, we introduce another new unit, *cho*, to represent 10 times 1000 oku. In Japan's numeration system, one, ten, hundred, and thousand are used and then repeated, with the new units of *man*, *oku*, and *cho* introduced every 4th place. Thanks to this system, we can represent numbers with *man* digits using a small number of units.

tho- hu- ten one usand ndred	tho- hu- ten one usand ndred	tho- hu- ten one usand ndred	tho- hu- ten one usand ndred
cho [trillion]	oku [hundred million]	man [ten thousand]	

Integers are represented using the base-10 numeration system. This means that numbers are written based on the fundamental principles shown below:

① When the quantity of each unit becomes ten, it is replaced by a new unit (the idea of base-10 notation).

② Instead of using different symbols for different units, use places to indicate the differences (the idea of positional notation).

The structure of this notation system enables us to represent any number in any size using 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Note A-(1)-c of "Handling the Content" A-(1)-c talks about cases where large numbers are represented with delimiters every three digits. When expressing large numbers such as the population or budget of a country, sometimes a comma is inserted every three digits. It is necessary to teach students to read numbers represented with these commas.

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A (2) Round numbers and rounding

(2) To help pupils understand round numbers and use them according to their purposes.

a. To get to know cases where the use of round numbers is appropriate.

b. To get to know the rounding to the nearest integer.

c. To estimate the results of four rules of calculation according to one's purpose.

Mathematical Activities (1)

a. Activities to estimate the result of calculations according to one's purpose, and to make proper decisions about the way of the calculations and the results

Terms/Symbols sum, difference, product, quotient, more than or equal, less than or equal, less than

[Handling the Content]

(2) As for the content (2)-c, (3) and (4) in "A. Numbers and Calculations," consideration should be given to enable pupils to do simple mental calculations. Also, consideration should be given to enable pupils to make use of mental calculation when doing the calculation using algorithms in column forms and doing estimation.

The objectives of grade 4 are to understand the meaning of approximate (rounded) numbers, to manipulate numbers skillfully, and to be able to use approximate numbers for appropriate purposes. It is also important to teach so that students appreciate the significance of approximate numbers: approximate numbers help us see the size of numbers at a glance, they help us make judgment and manipulation of numbers easy, and they help us work with some foresight.

Teach phrases such as "greater than or equal to", "less than or equal to", and "less than," when teaching about approximate numbers and rounding. For example, "greater than or equal to 5" means "5 or greater", and "less than or equal to 4" means "4 or less". Also, "less than 10" means "9 or less".

Students are expected to be able to use terms sum, difference, product, and quotient when referring to the results of the four arithmetic operations.

a. Cases where approximate numbers are used

Approximate numbers are used in cases such as the following:

① In cases where approximate numbers are used on purpose even when the exact number is known. For example, when reporting numbers of people in a baseball stadium, an approximate number is used.

② In cases such as comparing populations of cities using bar graphs. The length of each bar represents the approximate population.

③ In cases where it is difficult to find the exact value. For example, if you want to represent the current population of our country at this moment, an approximate number will be used.

It is important for students to understand how to use approximate numbers by clarifying their purpose. When using approximate numbers in everyday life, it is important to judge how many digits to use, according to the purpose. For example, when drawing a graph, the smallest interval is determined by the size of the paper used, and this, in turn, will determine how many digits to use.

b. Rounding

Rounding is most widely used to find approximate numbers. The procedure can be described as reading the fractional part to the nearest tick mark on a scale while measuring. Technically, rounding can be performed by just looking at the next digit to the right of the place to which the number is being rounded: if it is less than or equal to four, then round down; if it is greater than or equal to five, round up. For example, when rounding 42948 to the nearest thousands place, look at the digit "9" which is in the hundreds place, judge that it will be rounded up, and represent the number as 43000.

c. Estimating the results of the four arithmetic operations

Here, teach the estimation of the sum, difference, product, and quotient using approximate numbers. When estimating the results of computations, as pointed out in "Handling the Content"-(2), mental calculation should be utilized.

It is important to estimate a sum, difference, product, and quotient to develop a good perspective or to avoid making a major error. In our daily life, there are many occasions where it is convenient to estimate a sum, difference, product, or quotient using approximate numbers. For example, when we go shopping, if we know approximately how much the cost will be, we can think of how to pay appropriately. Here, students are expected to become able to estimate a sum, difference, product, and quotient appropriately using approximate numbers in concrete situations.

"According to one's purpose" means to clarify for why approximate number is to be used, to use precision appropriate to the purpose, or to judge the approximate size of the answer.

Estimating the product and quotient plays an important role when estimating the quotient of a division whose divisor is a 2-digit number.

When teaching estimation, it is important to have students judge and calculate a approximate number to an appropriate place, according to a concrete problematic situation. Consideration should be taken so that students will not repeat only formal manipulation.

Mathematical Activities (1) a

Activities to estimate the result of calculations according to one's purpose, and to make proper decisions about the way of the calculations and the results

The objective of thiese activities is to utilize estimation in our daily life and learning environment.

For example, let's think about the problem: "A field trip fee is 198 yen per person. There are 97 people. How much will the total fee be?" If you see 198 as about 200, and 97 as about 100, you can use 200×100, and so the estimated total fee would be 20000 yen.

The problem "Decide if you can buy three items costing 225 yen, 280 yen, and 340 yen with 1000 yen" would be another example. When answering this problem, if you over-estimate these prices and use 300 yen, 300 yen, and 400 yen respectively, the estimated total is $300 + 300 + 400 = 1000$. Since you over-estimated, the actual total price should be less than 1000, hence you can buy these three things with 1000 yen.

A (3) Division of whole numbers

(3) To help pupils deepen their understanding of division of integers, divide accurately, and extend their ability to use the calculation appropriately

a. To explore ways of division in the cases where the divisor is a1-digit or 2-digit number and the dividend is a 2-digit or 3-digit number, and to understand that these calculations are based on the basic calculations. Also, to understand the way of calculation using algorithms in column forms.

b. To divide accurately, and to use the calculation appropriately.

c. To investigate the relationships between dividend, divisor, quotient and remainder and to put them in the following formula:

$$(\text{dividend}) = (\text{divisor}) \times (\text{quotient}) + (\text{remainder})$$

d. To explore properties of division and to make use of the properties in order to explore ways to calculate or check the results.

Terms/Symbols quotient

[Handling the Content]

(2) Consideration should be given to enable pupils to do simple mental calculations. Also, consideration should be given to enable pupils to make use of mental calculation when doing the calculation using algorithms in column forms and doing estimation.

(3) As for the content (4)-d in "A. Numbers and Calculations," it should be dealt with that the quotient remains unchanged when the dividend and the divisor are multiplied or divided by the same number.

In grade 4, teach students the computational algorithm of division of whole numbers.

a. How to divide

For division, in grade 3, students learned how to find the quotient by using the multiplication table once, in calculations such as $12 \div 3$ or $13 \div 3$. They also learned how to handle division where the divisor is a 1-digit number and the dividend is a 2-digit number in simple cases like $80 \div 4$ and $69 \div 3$.

In grade 4, calculations where the divisor is a 1- or 2-digit number and the dividend is a 2- or 3-digit number will be introduced. When the divisor is a 1-digit number, cases such as $96 \div 8$ and $962 \div 4$ where a 1-digit number divides a 2- or 3-digit number will be taught, and the computational algorithm will be introduced as well so that students will understand the meaning of division and how to calculate it. When teaching, encourage students to think for themselves about ways to calculate and notice that calculation is based on the division they learned in grade 3. Also, help them see that when dividing, subtraction and multiplication are also used. For example, in the calculation of $96 \div 8$, we can split 96 into 90 and 6, and see $90 \div 8$ as $9 \div 8$, re-constitute the remainder of 1 as 10, add 6 to this 10, and divide the resulting 16 by 8. This viewpoint is related to the computational algorithm.

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Teach the division of a 2- or 3-digit number by a 2-digit divisor such as $99 \div 12$ or $567 \div 24$. Here, it would be difficult for students to understand if you taught the procedure of the calculation mechanically. When teaching, consideration should be taken so that students fully understand each step of the calculation. It is necessary for students to be able to use knowledge of the relative size of numbers in the calculation of division. Also, estimation will be necessary to find the quotient at each step. Abilities to estimate calculations and to do simple mental computations will be utilized. There are times when an estimated quotient is too large or too small and needs to be modified. It is especially important to carefully teach what to do when modifying the estimate of quotients.

b. Using division

The skill of performing division where the divisor is a 1- or 2-digit number and the dividend is a 2- or 3-digit number should be mastered, and students should be able to use this skill whenever necessary.

For example, division is used in the following problem: "A ribbon is 96 meters. Another ribbon is 24 meters. How many times as long is the former as the latter?" Here "the base quantity" and "the quantity to be compared" are known and "how many times" is asked. Division is also used in problems such as, "The yellow ribbon is 72 meters long and four times longer than the white ribbon. How many meters long is the white ribbon?" Here the "quantity to be compared" and "how many times" are known and "the unit quantity" is asked.

Note (2) of "Handling the Content" talks about simple mental calculations. Simple calculations are those such as $48 \div 2$ that are the inverse of multiplication of 2-digit numbers and 1-digit numbers. Consideration should be taken so that this will not be too burdensome for students.

c. The relationship among dividend, divisor, quotient, and remainder

Division with a remainder was taught in grade 3. Therefore, for example, students were taught to express such calculations in the form "$30 \div 4 = 7$ remainder 2." In grade 4, examine the relationships among the dividend, divisor, quotient, and remainder, and summarize the relationship in the following form:

$$\text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}$$

Students need to be aware that the remainder is less than the quotient. Also, students are expected to use the relationship among dividend, divisor, quotient, and remainder when checking the answers of calculations.

d. Properties of division

Note (3) of "Handling the Content" indicates that the property "the quotient remains unchanged when the dividend and the divisor are multiplied or divided by the same number" should be taught. This can be represented by the following algebraic expressions:

if	a	$\div$	b	$=$	c
then	$(a \times m)$	$\div$	$(b \times m)$	$=$	c
and	$(a \div m)$	$\div$	$(b \div m)$	$=$	c

It is important to help students so that they can examine this property of division for themselves. For that purpose, it is necessary to teach activities where students create algebraic expressions of division with the same quotients. By doing such activities, students will become aware that the division of $350 \div 50$ can be thought of as $35 \div 5$, for example, and be able to willingly examine other similar cases. This property can be utilized in various cases such as the division of decimal numbers or fractions as it relates to numbers and calculations.

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Also, it can be said that the property of division is also used to estimate a quotient when using approximate numbers. This property is further utilized in grade 5 when considering the division of decimal numbers and in grade 6 when considering division of fractions.

When teaching the property of division, it is desirable to teach it in situations where division is used, or when checking how to do calculations.

A (4) Solidifying the ability to use whole number computations

(4) To help pupils consolidate the ability to calculate integers and extend their ability to use the calculations.

[Handling the Content]

(2) As for the content (2)-c, (3) and (4) in "A. Numbers and Calculations," consideration should be given to enable pupils to do simple mental calculations. Also, consideration should be given to enable pupils to make use of mental calculation when doing the calculation using algorithms in column forms and doing estimation.

The four arithmetic operations of whole numbers are taught from grade 1 through grade 4. In grade 4, students are expected to solidify their abilities in computing with whole numbers and extend their abilities in using them.

The ability to compute with whole numbers includes understanding the meaning of computation and thinking about how to carry out computations. In order to judge what kind of computation is necessary in situations involving numerical quantities, it is necessary to understand the meanings of the computations. It is also important to know how to compute and to exercise flexibility in finding the appropriate computation for the situation.

The ability to utilize computation includes mastering the basic and fundamental computational skills and utilizing them in everyday living and studying. Computational skills students have acquired so far form the basis of computations that will be necessary in daily life and study. They also form the basis for continuing on to more complicated computations.

Help students master the basic and fundamental computational skills and utilize the meaning and methods of computations they have acquired up to this point when thinking about how to perform calculations of numbers with many digits.

Also, it is an objective in this grade to connect calculations to concrete situations and utilize them in daily life and study.

A (5) The structure of decimal numbers and their calculation

(5) To help pupils deepen their understanding of decimal numbers as well as their understanding of addition and subtraction of decimal numbers, and to help them understand the meaning of multiplication and division in decimal numbers, and use the calculations.

a. To get to know the fact that decimal numbers are represented in the same manner as integers, and to deepen their understanding of the relative size of numbers.

b. To explore ways of addition and subtraction of decimal numbers and to do the calculations.

c. To explore ways of multiplication and division of decimal numbers in cases where multipliers and divisors are integers, and to do the calculations.

[Handling the Content]

(4) As for the content (5)-c in "A. Numbers and Calculations," the case where a quotient of two integers is expressed as a decimal numbers should be included.

In grade 4, the objectives are to teach that decimal numbers are represented in the same manner as whole numbers and to teach addition and subtraction of decimal numbers. Also, in multiplication and division, by teaching cases where the multiplier or divisor is a whole number, students will deepen their understanding of decimal numbers.

a. The structure and relative size of decimal numbers

In grade 3, decimal numbers up to the tenths place were taught. In grade 4, by teaching the use of $\frac{1}{100}$. and $\frac{1}{1000}$ as units that numbers smaller than $\frac{1}{10}$ can be represented. Also, since decimal numbers are based on the base-10 numeration system just like the whole numbers, a place to the right of another place has $\frac{1}{10}$ the value of the place to its left. A place to the left of another place has ten times the value of the place to its right. It is important to be aware of this relationship, and that comparing the size of decimals or calculating them can be done in the same way as whole numbers. The hundredths place is also called the second decimal place.

Even in cases of decimal numbers, it is also important to deepen an understanding of the relative size of numbers. The relative size of numbers means to pick a certain unit and see how many of that units a certain number is made up of. In the case of decimal numbers, for example, help students get used to seeing 1.68 as 168 of 0.01. To foster such a viewpoint not only deepens the understanding of decimal numbers, but helps in thinking about how to multiply and divide decimal numbers effectively.

b. Addition and subtraction of decimal numbers

In grade 3, addition and subtraction of decimal numbers through the first decimal place was taught. In grade 4, teach addition and subtraction of a broader range of decimal numbers.

When adding and subtracting decimal numbers, it is important to understand the mechanics of decimal numbers, such as aligning the decimal points and calculating place by place. It is important to teach so that students understand that the calculation of decimal numbers is based on the same principle and can be done in the same way as whole numbers, following the same procedures.

In the calculation of decimal numbers, 0.1 is ten times the unit of $\frac{1}{100}$, so we can do the calculations that involve regrouping the same way as we do calculations of numbers to the first decimal place.

For example, when doing the algorithm of $3.7 + 2.48$, if 0.01 is taken as the basic unit, one can use the same principle as calculating whole numbers. But, aligning the end of the numbers instead of place values is a common mistake. So, it is important to align place values by using the decimal point as the reference mark in the way shown below. Then, you can calculate place by place thinking of the empty place as 0. In this way, calculating decimal numbers can be done in the same way as whole numbers.

$$\begin{array}{rrrrr} & 3 & . & 7 & \\ + & 2 & . & 4 & 8 \\ \hline & 6 & . & 1 & 8 \end{array}$$

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c. Multiplication and division of decimal numbers by a whole number multiplier or divisor

Students are expected to understand the meaning of multiplication and division of decimal numbers where the multiplier or divisor is a whole number. Multiplication is used when the amount corresponding to a unit is known and the total amount for a certain number of units has to be found. That is, multiplication can be thought of as adding the same number several times. For example, 0.1×3 has the meaning $0.1 + 0.1 + 0.1$. Multiplication can be used as a simple representation of repeated addition. Multiplication can also be thought of as a calculation for finding the quantity corresponding to so many times as much as the base quantity. The meaning of division is the inverse of multiplication, and can be explained as either finding how many times as much or finding the base quantity. Students are expected to learn where to place the decimal point in the product and the quotient by comparing them with cases of whole numbers. For example, since 1.2 is 12 of 0.1, 1.2×4 means 48 of 0.1. Also, 31.6 is 316 of 0.1, so $31.6 \div 4$ means 79 of 0.1.

Note (4) of "Handling the Content" says, "the case where a quotient of two integers is expressed as a decimal numbers should be included." When a whole number is divided by a whole number, the result may or may not be a whole number. Teach students that when a whole number cannot be divided by a whole number, one can still go on dividing.

A (6) Addition and subtraction of fractions with like denominators

(6) To help pupils deepen their understanding of fractions, and to help them understand the meaning of addition and subtraction of fractions with the same denominators, and use the calculations.

a. To pay attention to the fact that there are fractions that are the same in size in simple cases.

b. To explore ways of addition and subtraction of fractions with the same denominators, and to do the calculations.

Terms/Symbols proper fraction, improper fraction, mixed fraction

The objectives of grade 4 are to help students deepen their understanding of the meaning and representation of fractions, understand the meaning of addition and subtraction of fractions with like denominators, and be able to calculate them.

Teach the terms and meanings of *proper fraction*, *improper fraction*, and *mixed fraction*. Also, teach addition and subtraction of proper fractions, improper fractions and mixed fractions.

A proper fraction is a fraction whose numerator is less than its denominator, such as $\frac{1}{2}$ or $\frac{3}{5}$. An improper fraction is a fraction whose numerator is greater than or equal to its denominator, such as $\frac{2}{2}$ or $\frac{7}{5}$. A mixed fraction is a combination of a whole number and a proper fraction, such as $1\frac{2}{5}$, which is the sum of 1 and $\frac{2}{5}$. It is easier to see the size of an improper fraction if it is converted to a mixed fraction. For example, if $\frac{25}{3}$ is converted to $8\frac{1}{3}$, one can easily see that it is a number greater than 8. It is important to enrich students' sense of the size of fractions in this way.

Sometimes it is easier to compute with fractions if they are represented as proper fractions or as improper fractions. For example, multiplication and division of fractions taught in grade 6 will be easier to compute if they are represented as improper fractions rather than mixed fractions.

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a. Fractions representing the same number

There are fractions such as $\frac{1}{2}$ and $\frac{2}{4}$ whose representations are different but are the same size. In grade 4, students are expected to notice simple cases of fractions of equal size. For example, students are expected to notice the fractions on a number line that have different representations but equal sizes.

Comparing fractions by finding a common denominator is taught in grade 5.

b. Addition and subtraction of fractions

Simple cases of addition and subtraction of fractions were taught in grade 3.

In grade 4, students are expected to think about how to add and subtract fractions with like denominators and to be able to actually perform the calculation.

For example, with $\frac{3}{5}+\frac{4}{5}$, which is an addition of proper fractions, since there are $\frac{7}{5}$, the result can be represented as $\frac{7}{5}$.

Addition of improper fractions can be thought of in the same way. For example, there are 13 $\frac{1}{5}$s in $\frac{7}{5}+\frac{6}{5}$, so the result can be represented as $\frac{13}{5}$ or $2\frac{3}{5}$.

Students are also expected to think about how to add and subtract mixed fractions such as $1\frac{1}{5}+2\frac{3}{5}$, and actually be able to do the calculation. One of the ways to do this calculation is to separate mixed fractions into whole number parts and fractional parts, calculate the whole number parts and fractional parts separately, and then combine the results, such as $(1+2)+(\frac{1}{5}+\frac{3}{5})$. Another way is to convert mixed fractions into improper fractions such as $\frac{6}{5}+\frac{13}{5}$ and then do the calculation.

A (7) The abacus

(7) To help pupils add and subtract using soroban (Japanese abacus).

In grade 3, ways of representing numbers on the soroban [abacus] and addition and subtraction of whole and decimal numbers are taught. In grade 4, students are expected to deepen their understanding of the workings of the abacus. Numbers are represented based on the base-10 numeration system on the abacus. Students are expected to be able to represent the whole numbers up to the unit *oku* [hundred million] or *cho* [trillion] and through the hundredths place for decimal numbers on the abacus.

As for calculations, students are expected to understand how to calculate by manipulating beads on an abacus. For whole numbers, students are expected to carry out addition and subtraction of 2-digit numbers. Also, they are expected to perform simple calculations involving units of *oku* or *cho* such as "2 *oku* +6 *oku*" or "10 *cho* +20 *cho*." As for decimal numbers, students are expected to perform simple addition and subtraction of decimal numbers up to the hundredths place such as $0.02+0.85$.

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[B. Quantities and Measurements]

B (1) Units and measurement of area

(1) To help pupils understand the meaning of units and measurements of area, and determine the area by calculation.

a. To get to know the units of area (square centimeter [cm^2], square meter [m^2], square kilometer [km^2]).

b . To explore ways to determine the area of squares and rectangles.

Mathematical Activities (1)

b. Activities to explore and explain ways to determine the area of geometrical figures composed of rectangles by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

c. Activities to actually measure the area of objects found in everyday life.

[Handling the Content]

(5) As for the content (1)-a in "B. Quantities and Measurements," units"are"[a] and "hectare"[ha] should be touched upon.

The objectives of grade 4 are to understand the meaning of units and measures of area, and to understand how to determine the area of squares and rectangles based on the study of area in grade 1, and to be able to actually use these units and measures. Note that everyday words such as "broad" and "narrow" may be used in phrases such as "a broad/narrow road" and do not necessarily mean the size of the area. The quantities studied so far were measured using measuring instruments, but area is found by calculations using the length of the sides, etc. and is not measured by a measuring tool. Students need to be aware of this fact.

a. Units of area (cm^2, m^2, km^2) and measurement

Students are expected to appreciate the merits of representing the size of an area using numbers in the same way as in the study of measuring length and volume. Teach students to quantify the size of an area by deciding on a basic unit and counting how many units are in the area. For example, when measuring area, it is convenient to use a square whose side is 1 centimeter as a unit.

Square centimeter (cm^2), *square meter* (m^2) and *square kilometer* (km^2) are introduced as units of area. When teaching these units, consideration should be taken so that students understand that it is convenient to use the units of square meter and square kilometer when measuring large areas. For example, if you use the square centimeter as a unit for measuring the area of a classroom or a gym, the numerical value would be too large to handle with ease. Students are expected to understand that they should use units such as square meter in such cases.

Also, in relation to this content, students will examine the areas of things around them as a mathematical activity. Furthermore, students will enrich their sense of area by engaging in activities such as examining the area of their home town using km^2.

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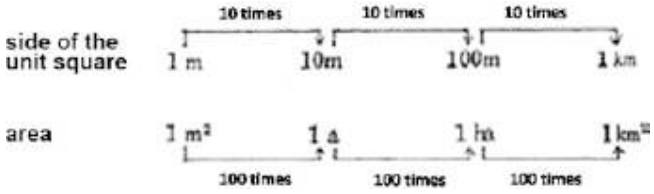
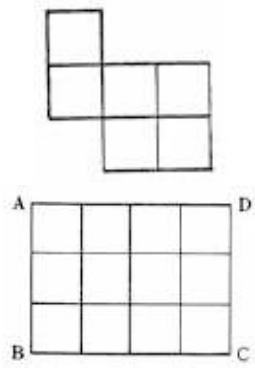
In relation with this content, note (5) of "Handling the Content" says "units"are"[a] and "hectare"[ha] should be touched upon." The units *are* and *hectare* relate to the study in other subject areas such as social studies. Here, consideration should be taken so that students understand that *are* and *hectare* are used when representing the area of a field because if you use the square meter as a unit, the numerical value may become too large. When teaching these facts, the relationship between square meter, square kilometer, *are* and *hectare* should also be taught. That is, the side of the unit square of 1 a is ten times as long as that of $1m^2$, the side of the unit square of 1 ha is ten times as long as that of 1 a, and the side of the unit square of $1km^2$ is ten times as long as that of 1 ha, and the area becomes 100 times each time.



b. How to find the area of squares and rectangles

The area of the shape at right can be found by counting the number of unit squares. Here, if you move the unit square on the top left to the bottom left, the entire shape becomes rectangular. One of the objectives here is to help students understand that the area of a rectangle can be calculated if the lengths of its sides are known. For example, when you want to know the area of the rectangular shape at right, if you think about the meaning of the area, you can see that you can tile the entire area by unit squares and count the number of the squares. Since the unit squares are tiled regularly, you can use multiplication and easily find the area. If you measure the length and width of this rectangle using 1 centimeter as a unit, the area is represented by (width)×(length) (or (length)×(width)) with the unit, cm^2. Hence, you get this formula:



$$\text{Area of rectangle} = \text{length} \times \text{width (or width} \times \text{length)}$$

Also, by using this formula together with concrete diagrams, you can have students think about how the area changes when the side of a rectangle becomes twice or three times longer. It is important for students to deepen their understanding from the viewpoint of algebraic expressions, and to avoid mixing up the area and the perimeter.

The above-mentioned ideas will be the basis in grade 5 for finding the areas of a figure such as triangles and parallelograms, or volumes of cubes and rectangular parallelpipeds.

[Mathematical Activities](1) b

Activities to explore and explain ways to determine the area of geometrical figures composed of rectangles by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

The objective of these activities is to help students experience and understand that if they use their understanding of geometrical figures, the area of complex figures may be calculated more easily using the formulas for areas of squares and rectangles. Another objective of these activities is to foster students' disposition to make logical explanations based on what they have learned so far. "Figures composed of rectangles" means shapes such as L-shape figures or concave figures.

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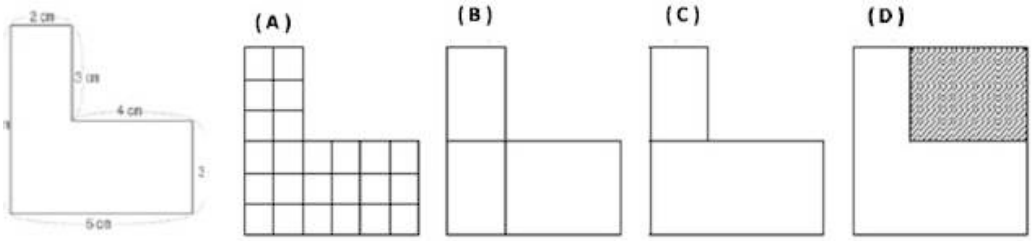
Explaining ways to determine the areas of L-shaped figures such as the one on the lower left, for example, by using words, numbers, algebraic expressions, or diagrams means the following:



When finding the area of this shape, one explanation would be to draw grid lines such as (a) and say, "I counted the number of $1cm^2$ squares and found the area of the shape is $24cm^2$."This explanation is based on the definition of area. Another explanation would be to divide the shape into three rectangles as in (b) or two rectangles as in (c), and decide the area of each shape by computation. Or, from the area of a large rectangle, you can subtract that of a small rectangle as in (d).

[Mathematical Activities] (1) c

Activities to actually measure the area of objects found in everyday life. The objectives of these activities are to develop the sense for the size of units, and to understand how to determine area more thoroughly by actually determining area and using units in various everyday situations. It is also the goal for students to feel and experience the value of studying area through these kinds of activities.

There are students who make an error and think $1m^2 = 100cm^2$ by analogy and from their experience with the conversion of the units of length, even though they know that $1m^2 = 10000cm^2$ through computation. This kind of error can be prevented if one has a sense of how big $100cm^2$ actually is and really understands the meaning of $1m^2$. So measuring the areas of squares and rectangles found in the students' environment will help. The objects can be origami paper, the surface of their desks, the floor of the classroom, flowerbeds, or the gym. When doing these activities, it is necessary to have students have a sense of purpose according to their actual level of experience and help them appreciate that the study of area is useful in their daily life, for example, through the comparison of the areas of their classroom and the library.

B (2) Size of Angles

(2) To help pupils understand the meaning of unit and measurements of angle, and measure angles.

a. To regard the size of an angle as the amount of turn.

b. To get to know the unit of angle measurement (degree [°]).

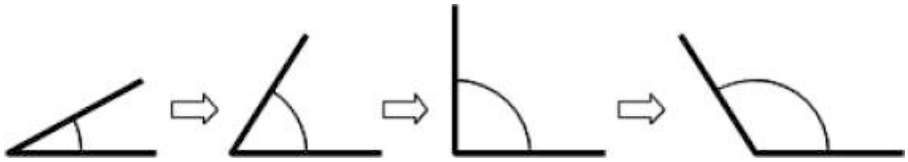
In the domain of "C. Geometrical figures," right angles were taught in grade 2. In grade 3, students focused on angles that were the same size in relation to figures such as isosceles triangles.

In grade 4, teach students to see angles as rotations, and the use of the degree (°) as the unit of their measurements.

a. The size of a rotation

The shape formed by two sides from one vertex is called an angle. The amount of rotation of one side, with a vertex as the center, is the size of the angle. The size of an angle is considered to be how much the two sides are apart. Teach about the size of an angle as described above.

The size of two angles can be compared directly by overlapping those angles. It is necessary to pay attention to the fact that the size of an angle has nothing to do with the length of the sides.



b. The unit of angle measure

Degree (°) is introduced as the unit of angle measure. Teach that the right angle is 90° and that one full turn is 360°.

Students will learn how to measure an angle or how to draw angles of a given size using a protractor.

In addition, students will enrich their sense of angles such as the ability to judge if the size of an angle is bigger than 90° using a right angle as a benchmark.

[C. Geometrical figures]

C[1] parallelograms, rhombuses and trapezoids

(1) Through activities such as observing and composing geometrical figures, to help pupils pay attention to the elements that compose geometrical figures as well as their positional relationships, and deepen their understanding of geometrical figures.

a. To understand the relationships such as parallelism and perpendicularity of straight lines.

b. To get to know parallelograms, rhombuses and trapezoids.

Mathematical Activities (1)

d. Activities to investigate features of geometric figures, such as parallelograms, rhombuses and trapezoids, by tesselating them on a plane.

Terms/Symbols Parallel, perpendicular, diagonal line

In grade 4, teach the constituent elements of geometrical figures such as parallel and perpendicular lines. Also teach about parallelograms, rhombuses and trapezoids.

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a. Parallel and perpendicular lines

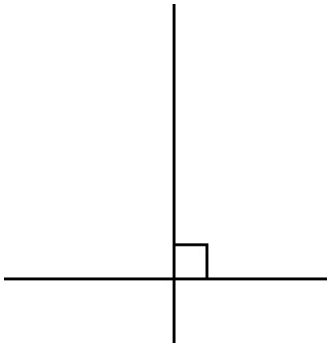
In grade 2, squares and rectangles were taught. Since students carried out activities where they observed and constructed squares and rectangles, they have gained experience that will form the basis for understanding parallel and perpendicular lines.

In regard to two parallel lines, one can say, "When two lines do not cross no matter how far they are extended, these two are parallel." But the expression "no matter how far they are extended" and its meaning may be difficult for students to understand. So, it may be better to define the perpendicular relation first and then define the parallel relation as follows.

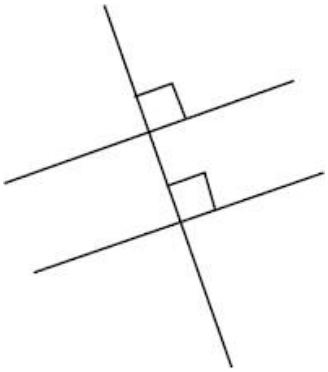
① If two lines cross at right angles, it is said these two lines are *perpendicular*.

② When there are two straight lines that are perpendicular to another line, it is said those two lines are *parallel*.

＜two lines are perpendicular＞



＜two lines are parallel＞



It is a good idea to include activities such as finding two parallel lines or two perpendicular lines by observing concrete objects around students. Also, activities such as making two parallel or perpendicular lines using concrete objects such as sticks or drawing two parallel or perpendicular lines using two set squares would be helpful. In addition, when considering figures such as parallelograms, help students deepen their understanding of figures by examining the relationship of opposite sides or how diagonals intersect.

If we define parallel lines as we did in ② above, the fact that "two lines do not cross no matter how far they are extended" becomes one of the properties of parallel lines. Another property of parallel lines is "the distance between parallel lines is always the same."

As for perpendicularity, it is necessary to pay attention to the fact that perpendicularity represents a positional relationship between two lines and thus differs from the right angle, which is a shape.

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b. Parallelograms, rhombuses, and trapezoids

By paying attention to the positional relationships and lengths of the sides, one can see the properties of quadrilaterals in the following examples, and be able to classify quadrilaterals.

A quadrilateral with two pairs of opposite sides that are parallel is called a *parallelogram*.

A quadrilateral whose four sides are equal in length is called a *rhombus*.

A quadrilateral with one pair of opposite sides that are parallel is called a *trapezoid*.

Students are expected to know these names of quadrilaterals and be able to use them. It is important to do activities such as the following to help students understand these figures: sort and organize quadrilaterals by common properties by observing various quadrilaterals students construct themselves; examine the property of each figure; and draw shapes and differentiate them based on their definitions or properties. Also, teach students to find concrete objects from their surroundings that are shaped like a parallelogram, rhombus, or trapezoid.

Teach the term and meaning of *diagonal*. A *diagonal* of a quadrilateral is a line segment that connects two opposite vertices.

Parallelograms have the property that opposite sides are the same length. They also have the property that opposite angles are the same size. Another property of parallelograms is that the two diagonal lines bisect each other. These properties can be verified by activities such as cutting a parallelogram along its diagonals into several triangles and overlapping them.

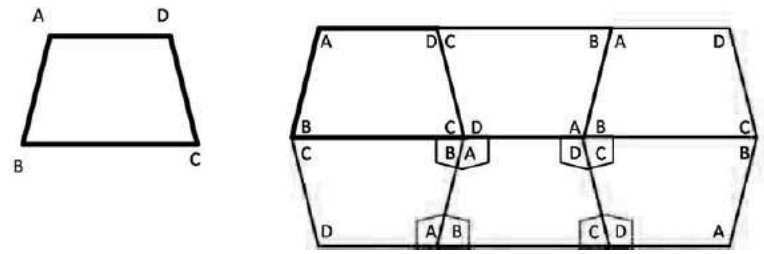
As for rhombuses, one of their properties is that opposite angles are the same sizes. Drawing a rhombus and cutting and folding it can verify this property. Also, students are expected to notice properties of rhombuses such as the fact that the two diagonals are perpendicular to each other and bisect each other. Teach students to pay attention to the fact that a rhombus is a quadrilateral that has all the properties of a parallelogram.

Mathematical Activities (1) d

Activities to investigate features of geometric figures, such as parallelograms, rhombuses and trapezoids, by tessellating them on a plane

The objective of these activities is for students to deepen their understanding of shapes experientially by focusing on the constituent elements of shapes and their positional relationships.

The objective of these activities is to enrich the sense of and ways to observe geometrical figures. Students can verify that a plane surface can be tiled with parallelograms, rhombuses, and trapezoids, and identify other geometrical figures or notice the properties of parallel lines in the resulting tiling patterns.



For example, if one tiles a plane with trapezoid ABCD, the result will be the image above. This verifies that the plane can be tiled by trapezoids. One can find parallelograms in the resulting pattern. Also, one may notice that if angles A and B or angles C and D are combined, the results are 180° in both cases.

Moreover, coloring tiled shapes and adding various designs to them will teach the students the beauty of geometrical figures.

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C (2) Solid figures such as cubes and rectangular parallelpipeds

(2) Through activities such as observing and composing geometrical figures, to help pupils understand solid geometrical figures.

a. To get to know cubes and rectangular parallelepiped.

b. To understand such relationships as parallelism and perpendicularity of straight lines and planes in connection with rectangular parallelepiped.

Terms/Symbols plane

[Handling the Content]

(6) As for the content (2)-a in "C. Geometrical Figures," drawing sketches and nets should be dealt with.

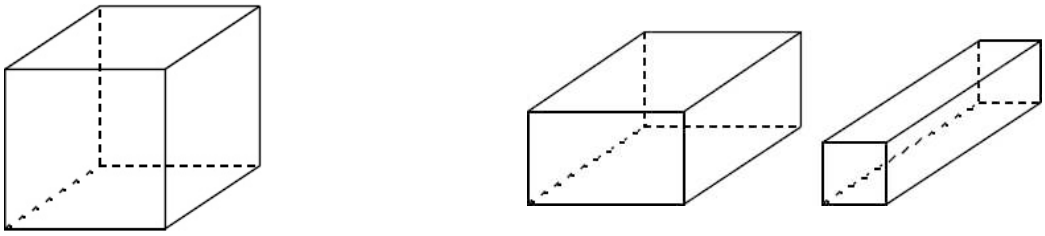
In grade 2, students were taught to focus on the constituent elements of solid shapes by observing or composing box shapes. The objective of grade 4 is to know about cubes and rectangular parallelpipeds, and to understand solid shapes. Also, students are expected to understand the parallel and perpendicular relationships between lines and planes as they examine rectangular parallelpipeds, and to enrich their ways of considering geometrical figures through observing, composing, and decomposing shapes.

The term *plane* is shown in "Terms/Symbols." It is important for students to be able to express, explain and represent using this term.

a. Cubes and rectangular parallelpipeds

A *cube* is a solid shape that is enclosed by six squares. A *rectangular parallelpiped* is a shape that is enclosed by six rectangles.

[cube] [rectangular parallelpiped]



As for cubes and rectangular parallelpipeds, teach students to focus on the characteristics of these shapes by considering their components such as the number of edges, faces, and vertices, the shapes of their faces, and parallel and perpendicular relationships of edges and faces. Students are expected to develop their understanding through composing and decomposing geometrical figures.

Also, note (6) of "Handling the Content" says that "drawing sketches and nets should be dealt with" in relation to cubes and rectangular parallelpipeds. This is because one of the objectives is to help students understand how edges or faces connect, and how their spatial relationships work, through representing cubes and rectangular parallelpipeds with sketches and nets.

Sketches and nets are one of the ways to represent solid figures on a plane. Teaching students the value of this is important. Consideration should be taken in relation to plane figures when teaching this. It is important to teach that not only one, but several nets can be drawn from one solid figure. It is also important to help students imagine the solid figure that results from a given net.

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b. Parallel and perpendicular relationships of lines and planes

Students are expected to understand the parallel and perpendicular relationships among lines and planes in relation to rectangular parallelpipeds. As for the edges and faces of a rectangular parallelpiped, it is important for students to understand that of 12 edges, there are 3 groups of 4 parallel edges each, that each edge is perpendicular to two faces, and that three edges which meet at one vertex are perpendicular to one another. Students can understand these relationships through activities such as observing, composing, and decomposing.

C (3) How to represent the position of objects

(3) To help pupils understand how to represent the position of an object.

As for the position of an object, representing positions using words such as “front” and “rear,” “right” and “left,” and “above” and “below” was taught in grade 1.

In grade 4, students are expected to understand how to represent the position of objects on a plane or in space.

To express a position in a plane, two elements (vertical and horizontal distance) are necessary. For example, to describe a position to place a flag on the floor of a gym, you can select one of the four corners of the gym as the referent point then represent the point that can be reached by marching 3 meters forward and 4 meters sideways as (forward 3 meters, sideways 4 meters).

Also, to represent an object in space, 3 elements (vertical distance, horizontal distance, and height) are necessary. If one thinks of the classroom as a rectangular parallelpiped and wants to represent an object dangling from the ceiling of the classroom, and the object is located 3 meters and 4 meters from two edges connected to a common corner of the classroom floor and its height is 2 meters from the floor, one could represent the place as (3 meters, 4 meters, 2 meters), for example.

[D. Mathematical Relations]

D (1) Two quantities that vary simultaneously

(1) To help pupils represent and explore the relationships between two numbers/quantities as they vary simultaneously

a. To represent how the numbers/quantities vary on a broken-line graph and to interpret the features of their variation.

[Mathematical Activities](1)

e. Activities to find two quantities in everyday life that vary in proportion to each other, and to represent and investigate the relationships of numbers/quantities in tables and graphs.

The objective of grade 4 is to foster the ability to pay attention to quantities that vary simultaneously in concrete settings and to make their relationship clear by representing them using tables and graphs.

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a. How quantities vary, and broken-line graphs

In grade 4, students are expected to make tables, express functional relationships using broken-line graphs, and interpret characteristics in the change of two quantities that are in a functional relationship represented in broken-line graphs. Elaborate on the idea of function through these activities.

To express the changes using a broken-line graph means to place one of the two quantities that are in a functional relationship on the horizontal axis and the other on the vertical axis, to express those two quantities using points on the coordinate plane, and to show how each part changes or how the graph as a whole changes visually by connecting those points.

Also, to interpret characteristics of change on a broken-line graph means to visually perceive the change of one of the two quantities which vary simultaneously while the other increases, and to make the relationship between two varying quantities clear. To do this, it is necessary to see if quantities increase or decrease from the gradient of line.

Mathematical Activities (1) e

Activities to find two quantities in everyday life that vary in proportion to each other, and to represent and investigate the relationships of numbers/quantities in tables and graphs.

The aim of these activities are to be able to utilize tables and graphs, for example, to find two quantities that vary simultaneously from the surroundings and to represent these quantities using tables or broken-line graphs, and to investigate the relationships between these quantities.

In solving problems involving quantities or geometrical figures, it is important to relate what one wants to find to other quantities or figures. For example, it is important to know the relationship between the unknown and other quantities or to know what quantity (or quantities) will determine another quantity.

Also, it is necessary to make relationships clear between two simultaneously changing quantities. To do this, it is necessary to do activities such as finding many pairs of those two quantities and sorting and organizing those pairs in a table, or representing them using graphs and investigating their relationships.

It is important to enhance students' functional thinking and their disposition to consider phenomena statistically, as well as to help them experience the usefulness of such perspectives. It is also important to foster the disposition to willingly use these perspectives in their daily life and study.

D (2) Algebraic expressions representing quantitative relations

(2) To help pupils understand the algebraic expressions that represent the relationships between numbers/quantities, and use these expressions.

a. To understand algebraic expressions that contain some of the four basic operations and parentheses ( ), and to calculate them accurately.

b. To understand the idea of formulas and to use them.

c. To represent numbers and quantities by using □ and △, to represent the relationships between numbers/quantities in algebraic expressions, and to explore the expressions by substituting numbers for the □ and △.

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Students have been taught to represent addition, subtraction, multiplication, and division using algebraic expressions and to interpret those expressions through grade 3. The main contents of this grade are to understand algebraic expressions composed of a mixture of those four operations as well as expressions including parentheses, to generalize quantitative relationships using formulas, and to represent quantitative relations in algebraic expressions with symbols such as □ and △.

The objectives of grade 4 are to represent quantitative relationships in algebraic expressions, to improve in their ability to interpret algebraic expressions, to understand the order of operations, and to utilize algebraic expressions appropriately. Moreover, students should acquire ways of thinking about formulas based on already-learned algebraic expressions and on expressions representing concrete situations.

a. Expressions including some of the four operations and parentheses.

The objectives of grade 4 are not only to calculate algebraic expressions but also to be able to represent relationships between quantities using expressions including a mixture of operations and parentheses; to interpret such algebraic expressions; to appreciate the merits of algebraic expressions; and to use them appropriately.

Expressions that contain some of the four operations or parentheses were dealt with previously. The main objective is to help students understand that parentheses are sometimes used to represent one quantity, and expressions that include multiplication or division represent one quantity. Students are to solidify their understanding of these things through various problems, by representing these relationships in expressions, and by interpreting problems and general relationships from expressions.

When teaching, help students understand the rules of calculation — that multiplication and division are calculated before addition and subtraction, and expressions in parentheses should be calculated before anything else — so that they master these rules. Moreover, it is important to understand that by representing relationships of quantities using expressions that contain more than one of the four operations and parentheses, the relationships can be represented concisely. It is also important to understand the merit of this, and to be able to represent relationships between quantities in one expression using the four operations and parentheses.

b. Formulas

In grade 4, in expressing the relationships between concrete quantities by using formulas, students are expected to represent quantitative relations using algebraic expressions; to be able to interpret those expressions; and to appreciate the merit of formulas since they can be used in other situations.

Formulas taught in grade 4 are not limited to those that are generally considered as formulas. They also include expressions with words used naturally when writing equations to solve concrete problems. As for general formulas, it is important to understand that they represent quantitative relationships using words, and these words can be replaced by various numbers.

As for formulas, it is necessary to emphasize the importance of the process of their derivation: making many combinations of numbers, finding the common relationships and rules among those numbers, and representing the relations and the rules using expressions with words.

In order to help students understand that formulas represent general relationships among quantities, it is necessary to utilize activities of substituting various numbers in formulas in concrete situations. Moreover, it is important to represent quantitative relations using formulas and to interpret formulas in concrete situations. Consideration should be taken so that students understand the merit of formulas that generalize quantitative relationships through these activities.

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In grade 4, the formula for finding the area of rectangle is introduced. For example, the idea of generalizing and making a formula — such as (area of rectangle)=(height)×(width) — is often used in mathematics and various other fields. It is important to help students understand that formulas represent general relationships that hold for all numbers. It is also important for students to understand that the formula may be used to find a height, given area and width, as well as finding an area, given height and width.

c. Expressions with □ or △

In grade 3, students learned to represent unknown quantities using □, or to write a algebraic expression using □. The objective of grade 4, based on past understanding, is to become able to appropriately use algebraic expressions including symbols such as □ and △ to represent variable quantities. In grade 4, there are cases where expressions with □ or △ are used; for example, representing the relationship between the length of a side and the perimeter of the square using □ × 4=△. Representing variables using □ or △ in an expression has the merit of simplifying how relations between quantities may be expressed.

When teaching, consideration should be taken so that students understand that various numbers can be substituted in place of □ or △, and that if one of either □ or △ is determined, the other quantity is also determined.

Expressions with □ or △ can also be used to summarize or explain properties of the four arithmetic operations. In the case where two or more symbols such as □ and △ are used in one expression, it is important to have students understand that the same number is used for the same symbol. It is also important to take care that students appreciate the merit of using symbols such as □ and △ to express the quantitative relations and the computation rules simply, clearly, appropriately, and generally.

D (3) Properties of the four arithmetic operations

(3) To help pupils deepen their understanding of the properties of the four basic operations.

a. To summarize the properties of commutative, associative, and distributive laws.

By grade 3, students learned that commutative, associative, and distributive laws hold in concrete situations, while studying how to add or multiply, and checking the result of those calculations.

In grade 4, within the range of numbers and calculations learned so far, students are expected to summarize the properties of the four basic operations and to be able to utilize them when necessary.

a. Commutative, associative, and distributive laws

Commutative, associative, and distributive laws are represented by the following algebraic expressions:

(Commutative Law)

$\square + \triangle = \triangle + \square$

$\square \times \triangle = \triangle \times \square$

(Associate Law)

$\square + (\triangle + \bigcirc) = (\square + \triangle) + \bigcirc$

$\square \times (\triangle \times \bigcirc) = (\square \times \triangle) \times \bigcirc$

(Distributive Law)

$\square \times (\triangle + \bigcirc) = \square \times \triangle + \square \times \bigcirc$

$\square \times (\triangle - \bigcirc) = \square \times \triangle - \square \times \bigcirc$

$(\square + \triangle) \times \bigcirc = \square \times \bigcirc + \triangle \times \bigcirc$

$(\square - \triangle) \times \bigcirc = \square \times \bigcirc - \triangle \times \bigcirc$

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In grade 4, students are expected to deepen their understanding of the properties of the four basic arithmetic operations in calculating whole numbers by trying to simplify the calculation using the commutative, associative, and distributive laws, or by finding the distributive law in the algorithm of multiplication, and to become able to utilize these properties as necessary. Also, students are expected to check that the above properties that hold for whole numbers also hold for decimal numbers.

D (4) Collecting, sorting, and organizing data

(4) To help pupils gather and organize data accordin

a. To explore features of the data by organizing the data from two viewpoints.

b. To get to know how to interpret and draw broken-line graphs.

[Handling the Content]

(7) As for the content (4)-a in "D. Mathematical Relations," the examination of data without omissions and duplications should be dealt with.

The objective of grade 4 is to extend students' ability to collect, sort, and organize data purposefully and to grasp characteristics and tendencies of the data.

a. Sorting and organizing from two viewpoints

In grade 4, students are expected to become able to sort and organize data from two viewpoints simultaneously and to represent that data using tables and graphs.

When collecting and sorting data, it is necessary to group the data, depending on the purpose, by sorting the data according to some perspective. For example, suppose one is viewing data from two different points of view: from viewpoint A, it could be said, "The data have characteristic a" or "The data do not have characteristic a," and from viewpoint B, it could be said, "The data have characteristic b," or "The data do not have characteristic b." If these two perspectives are considered simultaneously, the data can be divided into four categories.

It is important to view things from two different viewpoints and to sort them or to logically investigate possible situations, and to be careful that no possible cases are missed and that no case is counted twice.

Note (7) of "Handling the Content" says, "the examination of data without omissions and duplications should be dealt with." As for ways to check for omissions and duplications, consideration should be taken to teach concrete ways to avoid omissions and duplications such as counting systematically so that no data will be skipped, organizing possible cases in advance, or marking items already counted to avoid duplicating the counting. It is necessary to foster the disposition to eliminate errors to get correct results.

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b. How to draw and interpret a broken-line graph

In grade 4, as is stated in D-(1), broken-line graph is introduced as a tool for representing functional relationships. The objective here is to use a broken-line graph to represent data according to a purpose, and to interpret and examine that data.

When teaching about broken-line graphs, it is important to help students decide on the overall size of the graph and interval of tick marks based on the size of the paper or the purpose. Help students notice the fact that the appearance of the graph changes even though they represent the same data if they change the scaling of the vertical axis, etc.

[Mathematical Activities]

(1) The content listed in "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometrical Figures" and "D. Mathematical Relations" should be taught through, for example, the following mathematical activities:

a. Activities to estimate the result of calculations according to one's purpose, and to make proper decisions about the way of the calculations and the results.

b. Activities to explore and explain ways to determine the area of geometrical figures composed of rectangles by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

c. Activities to actually measure the area of objects found in everyday life.

d. Activities to investigate features of geometrical figures, such as parallelograms, rhombuses and trapezoids, by tesselating them on a plane.

e. Activities to find two quantities in everyday life that vary in proportion to each other, and to represent and investigate the relationships of numbers/quantities in tables and graphs.

[Terms/symbols]
sum, difference, product, quotient, greater than or equal to, less than or equal to, less than, proper fraction, improper fraction, mixed fraction, parallel, perpendicular, diagonal line, plane

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5. The Content of Grade 5

[A. Numbers and Calculations]

A(1)Properties of Integers

(1) To help pupils deepen their understanding of the properties of integers.

a. To understand that, if a viewpoint is fixed, integers are classified into some subsets such as even numbers and odd numbers.

b. To get to know divisors and multiples.

[Terms/Symbols] greatest common divisor, least common multiple

[Handling the Content]

As for the content (1)-b in "A. Numbers and Calculations," the greatest common divisor and least common multiple should be dealt with in line with concrete situations without putting too much emphasis on the formality of obtaining them. Also, prime numbers should be touched upon in the process of studying divisors.

In Grade 5, teach even and odd numbers, and divisors and multiples, and help students deepen their understanding of the properties of integers.

a. Even numbers and odd numbers

When a whole number is divided by 2, the remainder is either 0 or 1. A whole number whose remainder is 0 when divided by 2 is called an even number and a whole number whose remainder is 1 is called an odd number.

In this way, integers can be divided into two sets, even numbers and odd numbers. That is, any integer belongs either to the set of even numbers or to the set of odd numbers.

There are situations in our daily life where we can utilize even and odd numbers. For example, if you want to divide students of the class into two groups, you can use students' ID numbers. When you look at the students' ID numbers as whether they are divisible by 2 or not, each student will belong to one group or the other.

b. Divisors and multiples

The objectives are to teach the meaning of *divisor* and *multiple*, and to help students think of divisors or multiples of a number as a set.

The set of the common factors or common multiples of two integers is composed of common elements of the set of divisors or the set of multiples of each integer. For example, divisors of 8 are $\{1, 2, 4, 8\}$. Divisors of 12 are $\{1, 2, 4, 6, 12\}$. From this, we see that the common divisors of 8 and 12 are $\{1, 2, 4\}$. The greatest common divisor is the largest of the common factors, which is 4 in this case. Also, multiples of 8 are $\{8, 16, 24, 32, \ldots\}$. Multiples of 12 are $\{12, 24, 36, \ldots\}$. From this, we can see that common multiples of 8 and 12 are $\{24, 48, 72, \ldots\}$. The least common multiple is the smallest of the common multiples, 24 in this case.

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Help students deepen their understanding of the properties of integers by using the idea of divisors and multiples in daily life.

"Handling the Content"-(1) talks about how to treat the greatest common divisor and the least common multiple. These should be taught in concrete situations so that students understand their meaning. Also, as for prime numbers, help students enrich their sense of how to see whole numbers by understanding that there are whole numbers that have only two divisors, 1 and the number itself.

A (2) Notation system of integers and decimal numbers

> (2) To help pupils deepen their understanding of integers and decimal numbers based on the concept of the numeration system, and to use the understanding efficiently in calculation and so on.
>
> a. To make numbers that are 10 or 100 times as much/many, or $\frac{1}{10}$ or $\frac{1}{100}$ of another number, and to investigate their relationships.

The main objectives of grade 5 are to summarize the properties of the base-10 numeration system and to understand them, and to be able to use them efficiently in calculations, based on the standpoint that both integers and decimal numbers are represented using the base-10 numeration system.

a. Numbers that are 10, 100, $\frac{1}{10}$, and $\frac{1}{100}$ times as much as the original number

When summarizing the properties of the base-10 numeration system, teach that by moving the decimal point, we can make numbers 10 times, 100 times, $\frac{1}{10}$ times or $\frac{1}{100}$ times as big as the original one. In the base-10 numeration system, if you move the decimal point one place to the right, the resulting number is 10 times as big as the original one. If you move the decimal point 1 place to the left, the resulting number is $\frac{1}{10}$ of the original one. That is, we can make numbers 10 times, 100 times, $\frac{1}{10}$ times or $\frac{1}{100}$ times as big as the original one by moving the decimal point. This idea is utilized in the calculation of multiplication and division of whole numbers and decimal numbers. It is important to teach in a way that students can substantially use this idea in estimation as well. As necessary, students will also make numbers 1000 times or $\frac{1}{1000}$ times as big as the original number.

When examining the relationship among sizes of numbers, do not teach only procedural manipulation. Consideration should be taken so that students can enrich their sense of the size of numbers and of their structure.

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A (3) Multiplication and division of decimal numbers

(3) To help pupils deepen their understanding of the meaning of multiplication and division in decimal numbers, and use the calculations.

a. Based on the understanding of calculations in cases when the multiplier and the divisor are integers, to understand the meaning of multiplication and division in cases where the multiplier and the divisor are decimal numbers.

b. To explore ways of multiplication and division of decimal numbers and to do the calculations. Also, to understand the meaning of the size of the remainder.

c. To understand that the same relationships and rules can be applied to the multiplication and division of decimal numbers as in the case of integers.

Mathematical Activities (1)

a. Activities to explore and explain the meaning and way of calculations of decimal numbers by using words, numbers, algebraic expressions, figures, diagrams and number lines.

The objective of grade 5 is to broaden the meaning of multiplication and division so they can be used when the multiplier or divisor is a decimal number. It is important for students to be able to develop a new method of calculation, based on the meaning and calculation methods for multiplication and division with whole numbers.

a. The meaning of multiplication and division of decimal numbers

The meaning of multiplication of decimal numbers

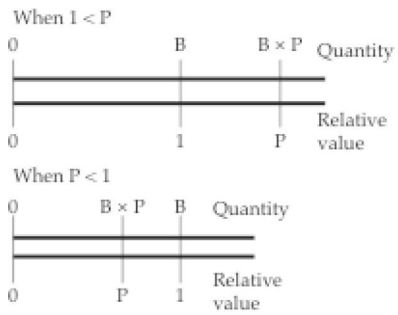
The multiplication of whole numbers is used "to determine the total amount for so many units when the amount for one unit is known," or "to determine the total quantity which is so many times as big as a given quantity."

In grade 5, multiplication is extended to cases where the multiplier is a decimal number. Also help students be able to use multiplication in a wider range of situations and meanings by thinking about the relationship with division, and to generalize them. We will use the same algebraic expression to represent a relationship between two quantities whether or not those quantities are whole numbers or decimal numbers, if the context of the problem involves the same relationship.

For example, when you buy 2 meters of cloth that cost 80 yen per meter, you can represent this using an algebraic expression, 80×2. In the same way, in the case of the following problem, "You bought 2.5 meters of cloth that cost 80 yen per meter. How much is the price?" since the length of the cloth is 2.5 times longer, the price will also be 2.5 times more, and we can represent this as 80×2.5.

As seen from the above, the meaning of the multiplication of integers and decimal numbers is represented as $B \times P = A$, where B is the base quantity, P is the relative value, and A is the corresponding quantity that has the specified relative size with respect to B.

The fact that the product becomes smaller than the multiplicand B when the multiplier P is smaller than 1, can be explained using a number line.



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The meaning of division of decimal numbers

As the inverse of multiplication, one of the meanings of division is to find the relative value. The other meaning of division is to find the base quantity.

When B is the base quantity, P is the relative value, and A is the corresponding quantity with the specified relative size with respect to B, the meanings of division can be seen in the following two cases:

(1) $P = A \div B$

In this case, the objective is to find how many times A is compared to B. If P is a whole number, this is the case of quotitive division. "A red ribbon nine meters long is how many times as long as 1.8 meters of blue ribbon?" is an example of such problems and the algebraic expression for this is $9 \div 1.8$.

(2) $B = A \div P$

In this case, the objective is to find the base quantity; if P is a whole number, this is "fair share" division. "Some cloth costs 200 yen for 2.5 meters. How much is 1 meter of this cloth?" is an example of such problems and the algebraic expression for this is $200 \div 2.5$.

It is important to understand that these algebraic expressions hold not only when B and P are whole numbers but also when they are decimal numbers. Case (2) seems harder to understand for students than case (1). In other words, with whole numbers, it could be easily understood that to determine the base amount means to determine the amount of one of P equal partitions. However, when the divisor is a decimal number, it is difficult to generalize this process of determining the per-unit quantity (the base quantity). Therefore it is important to teach this by using not only mathematical formula or expressions with words but number lines or diagrams, and by putting the problems in concrete situations so that students may find it easier to understand. Also, it is a good idea to write a multiplication expression first, then to use division to find the answer.

Many students have hard time understanding the relationship between the divisor and the quotient when the divisor is less than 1. It is important to utilize things such as number lines and help students to be able to explain why the quotient is greater than the divisor when the divisor is less than 1.

b. Calculation of multiplication and division of decimal numbers

Decimal numbers are represented using the same base-10 numeration system as whole numbers. Therefore, multiplication and division of decimal numbers can be carried out in the same manner as multiplication and division of whole numbers, if we pay attention to the place value of numbers and the position of the decimal points, and move the decimal points to see decimal numbers as whole numbers. Students should come to appreciate that the base-10 numeration system is useful this way in calculations with decimal numbers.

As for how to calculate multiplication and division of decimal numbers, we can think about the calculation process using the properties of calculations discussed in section c, below, to convert calculations with decimal numbers to calculations with whole numbers.

As for multiplication of decimal numbers, help students utilize the properties of multiplication — such as if the multiplier 10 times as large, the product also becomes 10 times as large — so that they can think through multiplication of decimal numbers by converting it to whole number multiplication which they have learned before. For example, 12×4.3 can be thought of as $12 \times (4.3 \times 10) \div 10 = 12 \times 43 \div 10$, and $12 \times (4.3 \times 10) \div 100 = 12 \times 43 \div 100$.

As for the division of decimal numbers, help students utilize the property of division that "if both the divisor and the dividend are multiplied by the same number, the quotient remains the same" in order to be able to think about how to calculate with decimals. For example, $7.2 \div 2.4$ can be thought of as $(7.2 \times 10) \div (2.4 \times 10)$, hence $72 \div 24$ and $(0.1 \times 100) \div (0.04 \times 100) = 10 \div 4$. While dividing a decimal number, students learn that they can continue dividing on even after they reach the smallest place in the dividend. For example, $0.5 \div 0.4 = 1.25$.

In the case of division of decimal numbers with remainders, many students make mistakes about where to put the decimal point in the remainder. When teaching, help students think about the meaning of the size of the remainder and the relationship between the quotient, divisor and remainder by using the fact that the remainder is smaller than the divisor and applying the formula (dividend) = (divisor) × (product) + (remainder).

c. Properties that hold for multiplication and division of decimal numbers

Help students understand that the properties that hold for the multiplication and division of whole numbers also hold for decimal numbers.

For example, if you calculate 30×2.5 and $30 \times 2+30 \times 0.5$ respectively, you get the same answer. Generally speaking, the distributive law $\square \times (\triangle + \bigcirc) = \square \times \triangle + \square \times \bigcirc$ holds for decimal numbers as well.

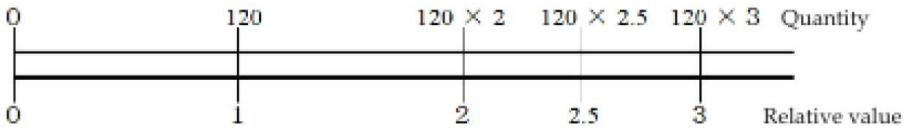
It is important to think about how to calculate or check the result of calculation by using these properties of calculation.

Mathematical Activities (1) a

Activities to explore and explain the meaning and way of calculations of decimal numbers by using words, numbers, algebraic expressions, figures, diagrams and number lines.

The objective of these activities are for students to express mathematically what they think based on what they have already learned, and to enhance their thinking to something better and easier to understand. That is, to explain the meaning of multiplication and division of decimal numbers in words or using a number line based on the ideas already learned about multiplication and division.

Broaden the meaning of the multiplication of decimal numbers so that it can be used when the multiplier is a decimal number. For example, when thinking about the meaning of 120×2.5, students are expected to explain the meaning of multiplication by using a number line like the one below, or by expressing it in words such as "the amount corresponding to 2.5 when 120 is considered to be 1," or by using a formula or expressions with words.



Also, when doing an activity to explain the method of calculation, students can use a number line or properties of calculation. For example, when thinking about how to calculate 120×2.5, if a property of calculation is used, students are to explain that idea with words such as "Multiply the multiplier 2.5 by 10 and make it 25. Calculate 120×25, then divide the result by 10 to get the real result."

It is important to utilize the same kind of activities when teaching the meaning of division of decimal numbers and methods for their calculation.

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A (4) Fractions

(4) To help pupils deepen their understanding of fractions, understand the meaning of addition and subtraction of fractions with different denominators, and to use the calculations.

a. To represent integers and decimals as fractions and to transform fractions into decimals.

b. To understand that the results of division of integers can always be represented as a number when using fractions.

c. To understand that a fraction obtained by multiplying or dividing the numerator and denominator of an existing fraction by the same number, has the same size as the existing fraction.

d. To explore equivalence and size of fractions and summarize the comparison of sizes.

e. To explore ways of addition and subtraction of fractions with different denominators, and to do the calculations.

f.

[Terms/Symbols] reduction to a common denominator, reduction

In grade 5, students are expected to deepen their understanding of the meaning of fractions and how to represent them, and to think about how to add and subtract fractions with unlike denominators and become able to actually calculate them. Also, teach multiplication and division of fractions where the multiplier or divisor is a whole number.

Teach not only calculations with proper fractions but also calculations with improper fractions and with mixed fractions. In this case, avoid unnecessarily complicated calculations; rather, teach so that students can utilize the calculation of fractions in their daily life or future study.

a. Relationships between fractions and integers or decimal numbers

In understanding the relationship among integers, decimal numbers, and fractions, it is important for students to realize that whole numbers and fractions, and decimal numbers and fractions, are not different things, but are all numbers, even though they use different notations.

In general, when you want to represent a whole number by a fraction, if a is a whole number, then $a = \frac{a}{1}$. But, the denominator of a fraction representing a whole number is not necessarily 1, and the value of the numerator is decided by the value of the denominator. For example, $2 = \frac{2}{1} = \frac{4}{2} = \cdots$.

When you want to convert decimal numbers into fractions, you can use 10, 100, 1000, and so on for denominators. For example, if you want to represent 0.13 using a fraction, since 0.13 is 13 if you use $\frac{1}{100}$ as a unit, you can represent it as $\frac{13}{100}$.

Also teach how to represent fractions using whole numbers or decimal numbers. For example, $\frac{1}{4}$ can be seen as $1 \div 4$ and can be represented as 0.25. It is important to help students understand that there are fractions that cannot be represented by whole numbers or terminating decimal numbers. For example, $\frac{1}{3} = 0.3333\cdots$.

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b. The results of division and fractions

It is always possible for the result of multiplication of two whole numbers to be represented by a whole number. In contrast, the result of division with two whole numbers cannot always be represented by a whole number or by a terminating decimal number because there are cases where the quotient is not divisible even if the calculation proceeds into decimal numbers ($4 \div 3 = 1.33\cdots$). By allowing the quotient of $a \div b$ (a and b are both whole numbers and b is not 0) to be represented by a fraction, $\frac{a}{b}$, the result of division can always be represented by one number.

It is desirable to help students pay attention to the fact that quotients that are whole numbers (e.g., $6 \div 3 = 2$) or decimal numbers (e.g., $2 \div 4 = 0.5$) can also be represented by fractions. Also, it is important to efficiently employ the idea that $a \div b$ can be represented by a/b, or that $\frac{a}{b}$ can be interpreted as $a \div b$ when representing fractions using decimal numbers or when multiplying or dividing fractions.

c. Fractions representing the same size

A fraction whose numerator and denominator are multiplied or divided by the same number represents the same size as the original fraction. For example, $\frac{2}{3} = \frac{2 \times 2}{3 \times 2}$, and $\frac{4}{6} = \frac{4 \div 2}{6 \div 2}$. As shown, fractions have the property that many fractions can represent the same size. This is the same property that holds for division, as can be seen from $a \div b = \frac{a}{b}$. Consideration should be taken so that students can be aware of these kinds of properties when thinking about how to calculate with fractions.

Simplifying a fraction means to divide the numerator and the denominator of a fraction by a common divisor to make its denominator smaller. A simplified fraction represents the same size as the original fraction. Even when teaching simplifying of fractions, it is important to utilize a number line or a diagram.

In this way, it is important to enrich the students' sense of numbers through teaching different fractions that represent the same size.

d. Size and equivalence of fractions

As is discussed in c, there are many different fractions representing the same size. For example ,$\frac{1}{2}$, $\frac{2}{4}$,$\frac{3}{6}$, $\frac{4}{8}$, $\cdots$ all represent the same size. Fractions with different denominators can be compared by converting those fractions to those with a common denominator. The process of converting fractions with different denominators to fractions with a common denominator is called finding a common denominator. The simplest way to convert two fractions with different denominators to a common denominator is to use the least common multiple as their common denominator.

When teaching "finding a common denominator," it is important that students understand its meaning and pay attention to the set of fractions of the same size. Teaching only the procedural aspect should be avoided.

e. Addition and subtraction of fractions with different denominators

If you convert fractions with different denominators to fractions with a common denominator, their addition and subtraction can be done in the same way as addition and subtraction of fractions with like denominators. For example, $\frac{1}{2} + \frac{1}{3} = \frac{1 \times 3}{2 \times 3} + \frac{1 \times 2}{3 \times 2} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$. Instead of simply teaching the procedure for finding a common denominator, it is important for students to think that by determining a common denominator they can see the calculation process as thinking about how many unit fractions there are. This idea of finding a common denominator is based on the fundamental principle of addition and subtraction: to add or subtract quantities, the unit of the quantities must be the same.

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f. Multiplication and division of fractions

Here, (fraction)×(whole number) and (fraction)÷(whole number) are taught. The meanings of (fraction)×(whole number) and (fraction)÷(whole number) can be explained in the same way as multiplication and division of whole numbers. The meaning of multiplication can be thought of as adding the same number several times (repeated addition), or finding the corresponding compared quantity for the relative value when the base quantity and relative value are known. The meaning of division is the inverse of that of multiplication, and can be explained as finding the relative value or the base quantity.

When teaching multiplication and division, it is necessary to help students so that they can devise ways to calculate by utilizing the calculation methods for whole numbers and decimal numbers, and one should avoid making students simply memorize procedures.

[B. Quantities and Measurements]

B (1) Area of geometrical figures

(1) To help pupils determine the area of geometrical figures by calculation.

a. To explore ways to determine the area of triangles, parallelograms, rhombuses, and trapezoids.

Mathematical Activities (1)

b. Activities to explore and explain the meaning and way of calculations of decimal numbers by using words, numbers, algebraic expressions, figures, diagrams and number lines.

The main objectives of grade 5 are for students to determine the area of fundamental geometrical figures that are composed of straight lines by utilizing the methods for determining the area of rectangles and squares that they have previously learned, and to develop and apply new formulas for determining the area of new geometrical figures. It is important to emphasize the process of thinking about how to determine areas or making formulas for finding areas based on previously-learned ideas and experiences.

a. Areas of triangles, parallelograms, rhombuses, and trapezoids

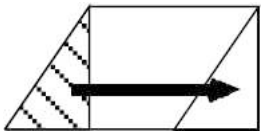
The main objectives here are to determine how to find the areas of triangles, parallelograms, rhombuses, and trapezoids based on the methods already learned for determining the areas of geometrical figures, to explain the methods, to create formulas, and to cultivate the ability to think logically in the process. Students are expected to explain how to determine area based on the ideas such as the following:

(1) Move part of the geometrical figure and transform it to an already-learned shape of equivalent area

(2) Think about the given shape as a half of the area of an already-learned geometrical figure

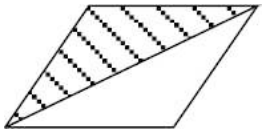
(3) Decompose a shape into several already-learned geometrical figures

Example of (1)

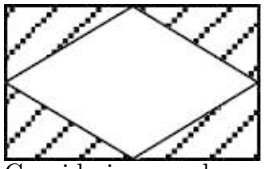


Transforming a parallelogram into a rectangle of equivalent area

Example of (2)

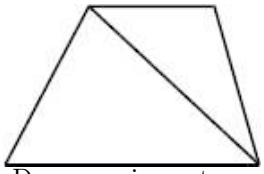


Considering a triangle as half of a parallelogram



Considering a rhombus as half of a rectangle

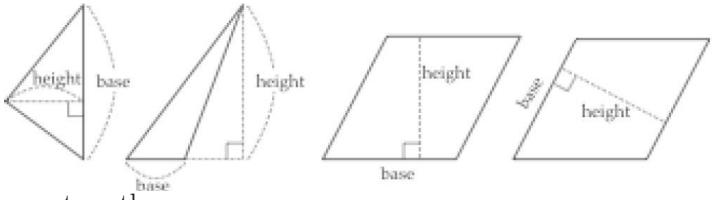
Example of (3)



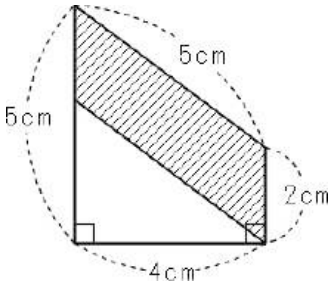
Decomposing a trapezoid into two triangles

By acquiring these ideas while learning how to determine the area of figures such as triangles, students will become able to exercise ingenuity in determining area on their own. Moreover, students are expected to be able to determine the area of composite figures made of basic figures such as triangles, and general quadrilaterals, by utilizing these ideas.

Also, when teaching how to find the area of triangles and parallelograms, it is important to solidify students' understanding of the height and base. That is, if a certain side is selected as a base, the height is automatically defined, and no matter which side becomes the base, the area stays the same.



When teaching, it is not enough to always give students all the necessary measurements and let them use the formula to determine the area. In order to deepen their understanding of formulas, it is necessary also to provide situations where students need to think about which lengths they should measure to determine the area. Also, for students to be able to utilize formulas for finding area, it is important that students be able to choose the necessary information when measurements of many sides are given, such as in the diagram at right.



Activities such as the following should be incorporated: to think about how the area changes when the base becomes two or three times as long as the original while the height remains fixed by utilizing both the formula and concrete drawings; to determine the area of a triangle necessary lengths are decimal numbers. It is important to connect the study of area formulas with the study of decimal number calcuation in "A. Numbers and Calculations" and the study of algebraic expressions and proportion in "D. Mathematical Relations," in order to deepen students' undersanding of area formulas.

Mathematical Activities (1) b

Activities to explore and explain the meaning and way of calculations of decimal numbers by using words, numbers, algebraic expressions, figures, diagrams and number lines.

An objective of these activities is to foster students' ability to think and represent mathematically through activities in which students think about and explain how to determine area. Another objective is for students to experience and understand that the study of determining area uses and expands the ideas that have already been learned creatively. Yet another objective is for students to really feel the importance of mathematical thinking and a rich sense of geometrical figures for creativity and expanding the study of area. These activities should be conducted for determining the area of triangles, parallelograms, rhombuses, and trapezoids. By repeating the activities, students can grow their ability to think and represent mathematically.

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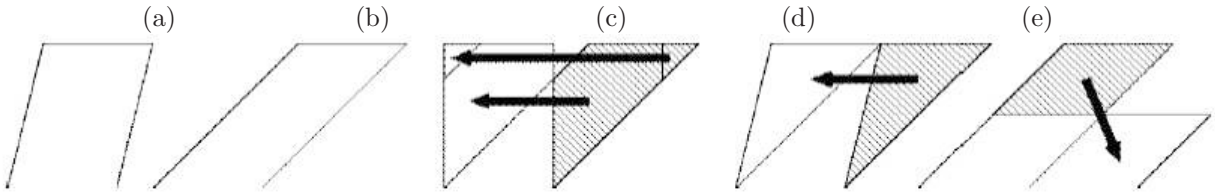
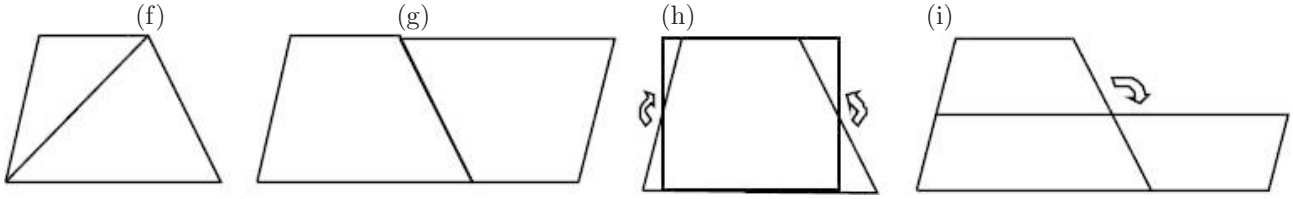
In the context of finding the area of special parallelogram (b) — after having found the area of parallelogram (a) using an equivalent-area transformation to transform it into a rectangle — "to think and to explain" means the following: explain that the area can be determined by transforming it into a rectangle (see (c)), or explain that the area can be determined based on a parallelogram (like (a)) whose area students already know how to find (see (d) and (e)).

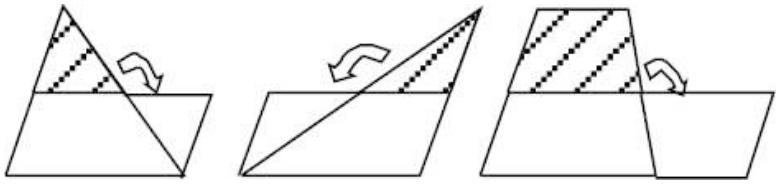


Students can utlize the previously-learned formulas for the area of a parallelogram or the area of triangle while determining the area of trapezoids, as shown in the diagrams below.



For (f), students can explain using concrete objects; e.g.: "Cut the trapezoid along the diagonal into two triangles. Then, determine the areas of those triangles." For (g), students can explain in the following way: "Use another trapezoid that is congruent to the original one, and combine those two trapezoids to make a parallelogram. Here, the base of the parallelogram is (upper base)+(lower base)." In (h) and (i), students can make an explanation such as: "When I moved a part of a trapezoid, I could make a shape we already learned. So I calculated using the formula for the area of that shape."

Through repeating these activities, students will understand, for example, that the idea for finding the area of a triangle using the equivalent-area transformation of the triangle into a parallelogram by bisecting the height can be used to determine the area of special triangles and trapezoids. By utilizing these ideas repeatedly, students will understand that these ideas are what is important in the study of area.



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B (2) Units and measurement of volume

(2) To help pupils understand the meaning of units and measurements of volume, and determine the volume by calculation.

a. To get to know the units of volume (cubic centimeter [cm^3], cubic meter [cm^3]).

b. To explore ways to determine the volume of cubes and rectangular parallelepiped.

By a grade 4, students have developed an understanding of measuring volume — mainly liquid volume — based on a unit. The objectives of grade 5 are for students to be able to determine the volume of solids; to understand the meaning of units of volume; and to understand the meaning of the measurement of the volume of solids, i.e. that the volume of a solid can be expressed with a number if we determine a unit and talk about the number of units, similar to the method used for measuring area. Also, students are expected to understand that the volume of cubes and rectangular parallelpipeds can be determined by computation based on the measurements of the lengths of the sides that define the geometrical figure instead of using a tool to measure the volume directly, just like they did for finding areas of squares and rectangles.

a. Units of volume (cm^3 , m^3) and its measurement

Analogous to determining area based on a unit size, here students are expected to understand that a unit of volume should be a solid figure with which the space can be filled without any gaps. Moreover, students should understand that it is convenient to use a cube — whose volume can be determined by the length of one of its sides — as that solid figure; and they should understand that it is convenient if the size of the cube is a unit length, such as 1 cm or 1 m, along the side.

Moreover, students are expected to become aware of the inconvenience of using cubic centimeters (cm^3) when measuring the volume of a large object, because the number can become too large to handle easily. In such cases, the bigger unit, cubic meter (m^3), should be introduced. Also, consideration should be taken so that students enrich their sense of volume through activities determining the volume of cubes and rectangular parallelpipeds in their surroundings, or observing a cube whose actual volume is $1m^3$. Moreover, the fact that the volume of a cube is 1ℓ if one of its sides is 10 cm should be touched upon.

b. Determining the volumes of cubes and rectangular parallelpipeds

Cubes and rectangular parallelpipeds can be made by piling up unit cubes with side length such as 1 cm or 1 m. Consequently, students may understand, from the analogy of determining the area of rectangles, that the volume of cubes or rectangular parallelpipeds can be determined by measuring their length, width, and height, and so obtain the formula (volume of rectangular parallelpiped) = (length) × (width) × (height). At the same time, it is important to help students understand that the first layer, where the unit volume is tiled without any gaps, can be represented by (length) × (width), and the number of layers is indicated by height.

Also, it is important to for students to foster a sense of volume through experiential activities where they actually determine the volume of cubes and rectangular parallelpipeds in their environment.

It is also important for students to deepen their understanding of the meaning of the formula for volume as they study algebraic expressions or proportional relationships in "D. Mathematical Relations," by thinking about how the volume of a rectangular parallelpiped changes when the height gets twice, three times, four times, ⋯ greater while the length and the width remain fixed. This concept is utilized in determining the volume of prisms and cylinders in grade 6.

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B (3) Measurement of quantities

(3) To help pupils understand the measured value of quantities.

a. To get to know the average of measured value.

By grade 4, students learned basic ideas of measurement, such as choosing appropriate measuring instruments and units depending on the purpose and the size of the object to be measured; how to read tick marks; and how to express measured quantities. The objective of grade 5 is to help students to become able to express the results of measurements with appropriate numerical values by utilizing means.

Knowing an approximate size through estimation when measuring is important for planning how to measure, as it reduces the chance for measuring errors, and enables one to select a suitable unit and measuring instrument. It is important to help students understand that when you determine a measured value by reading tick marks on a measurement instrument, the result of the measurement is an approximate value, because when you measure, you round the smallest digit to the nearest tick mark, or estimate it by dividing the space between the smallest intervals into ten equal parts.

a. The mean of measured values

It is necessary for students to be made aware of the fact that there are always errors in measurement, and teaching should take this fact into consideration. Generally speaking, when there are several values as the result of measuring one thing, we may be able to get an appropriate value if we even out those values. So here the idea of using the mean of measured values is introduced. For example, if you measure some distance by pacing it off, you usually walk the distance several times and take the mean. To be able to calculate the mean procedurally is not enough. It is necessary to help students understand the meaning of this. If there are extreme or unexpected values, ask students to investigate the reason, and help them to be able to think about omitting those values to find the mean. For example, we may consider omitting the data for failed jumps—e.g. because the last step was not lined up well on the takeoff plank—when getting the mean of long jump data. Activities for obtaining the mean of measured values can be utilized not only in daily life but also in science class.

As for the calculation of the measured values, help students to be aware that it is meaningless to get more digits than the number of digits in the original measurements. Teach students that unless there is a specific need, the number of the digits is usually kept to that of the original numbers.

B (4) Ratio of two different kinds of quantities

(4) To help pupils understand how to represent and compare quantities that are obtained as a ratio of two quantities of different types.

a. To get to know size of per-unit quantities.

In grade 5, students will learn that, besides the quantities they have learned so far, there are quantities that are represented by the ratio of two quantities. In order to compare the yield of rice, one might consider the yield of rice per 10 a; in order to compare the density of people, one might compare the population per one square kilometer, which is often referred to as population density. These are examples of such kinds of quantities. The main objective is for students to understand this method of comparing and representing quantities, and to become able to use them.

When comparing, since this is the first time for students to compare quantities that do not have the fundamental charactersitcis of quantity, teach carefully, as illustrated below, so that they understand the meaning of comparing quantities that can be considered as ratios of two different kinds of quantities.

First, like the population density example, help students understand that there are situations where they cannot make comparisons by paying attention to only one quantity, such as number of people, and that there are quantities that cannot be quantified by the idea of measurement, i.e. counting the number of a unit quantity. In this case, it is important for students to discover the existence of such quantities, so teachers need to provide concrete situations and have students compare these quantities with quantities like weight or length. For example, the teacher needs to help students notice that one cannot determine whether a playground is crowded or not without considering both the area of the playground and the number of students playing on it, and compare and contrast such a situation with comparisons of length or weight.

Next, help students think about how they can compare and quantify the situations. In general, since there are two kinds of quantities involved, we can usually use the method of fixing one of the quantities and compare the other quantity. When we use this way of thinking, we assume that two quantities are in proportional relationships. It also involves the idea of average. When teaching, it is necessary to help students pay attention to these points, and help them understand their meanings. For example, in the case of population density, two kinds of quantities are involved: number of people and area. Since the degree of crowdedness stays the same if an area gets twice, three times, or four times larger when the population gets twice, three times or four times bigger, help students notice that if we use this method and fix one quantity — for example the area — and compare the other quantity — the population — it is much easier to make comparisons. That is, when we want to compare the degree of crowdedness between 7 people in a 10 m^2 room and 10 people in a 15 m^2 room, if we make the area of the rooms 30 m^2, the number of people in each room becomes 21 and 20 people respectively, so that we can come up with the idea of comparing by equalizing the size of the areas.

a. Size of per-unit quantities

The objective of this content is for students to understand that it is more efficient to use per-unit quantities when comparing quantities that involve a ratio of two different kinds of quantities, particularly when we compare three or more such quantities or wish to be able to make comparisons at any time. Moreover students need to become able to utilize the method of per-unit quantity to compare things.

When comparing the population density, it does not matter whether we consider the area as the unit quantity and compare the population per km2 or consider the population as the unit quantity and compare the area per person. In general, however, in the case of population density, since it is more convenient if a big number corresponds to a high population density, we often use the area as the unit quantity and compare the population.

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[C. Geometrical Figures]

C (1) Properties of plane figures

(1) Through activities such as observing and composing geometrical figures, to help pupils deepen their understanding of plane geometrical figures.

a. To get to know polygons and regular polygons.

b. To understand congruence of geometrical figures.

c. To investigate and compose geometrical figures by finding the properties of geometrical figures.

d. To understand the ratio of the circumference of a circle to its diameter.

Mathematical Activities (1)

c. Activities to construct and make congruent figures.

d. Activities to inductively think and explain that the sum of the three angles of a triangle is equal to 180 degrees. Activities to deductively think and explain that the sum of the four angles of a quadrangle is equal to 360 degrees.

(Handling the Content)

(2) As for the content (1)-d in "C. Geometrical Figures," 3.14 should be used for the ratio of the circumference of a circle to its diameter.

In grade 5, teach polygons and regular polygons and help students deepen their understanding of plane figures.

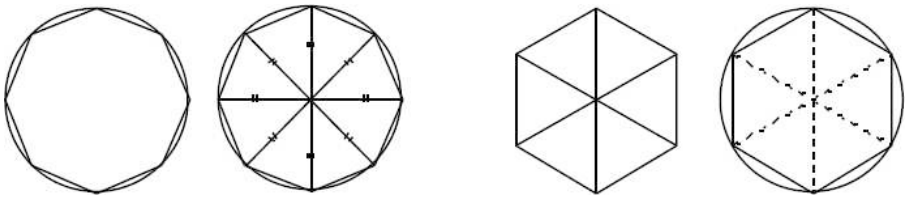
a. Polygons and regular polygons

Polygons are the geometrical figures surrounded by three or more straight lines. For example, a geometrical figure surrounded by six straight lines is called a hexagon. Geometrical figures whose sides are all equal length and whose angles have the same measure are called regular polygons. Equilateral triangles and squares are regular polygons.

Regular polygons have properties such as exactly fitting into a circle (they can be inscribed in a circle) and exactly fitting outside a circle (they can circumscribe a circle).

For example, when you connect the center of a circle and each vertex of a regular octagon inscribed in the circle by straight line, triangles are formed. Those triangles are isosceles triangles and they are all congruent. If you arrange six congruent equilateral triangles so that they share one vertex, you can form a regular hexagon. The geometrical figure formed by cutting the circumference of a circle by its radius is a regular hexagon whose side is the same length as the radius of the circle.

In this way, you can examine the properties of regular polygons in connection with circles and the drawing of circles.



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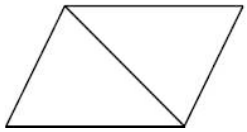
b. Congruency of geometrical figures

Students have experienced the fact that if they cut a square or an isosceles triangle in half, they can get two triangles of the same shape and size.

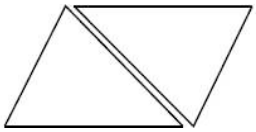
In grade 5, students are expected to understand the congruency of geometrical figures. When two geometrical figures overlap exactly, that is, when two shapes are the same shape and size, they are called congruent. When two geometrical figures are congruent, their corresponding side lengths and corresponding angle sizes are the same. It is important for students to be able to determine and verify the properties of geometrical figures through activities such as finding, drawing, and constructing congruent geometrical figures.

It is also important for students to be able to identify the corresponding sides and angles of two congruent geometrical figures regardless of their orientations, even when they are slid, rotated, or flipped.

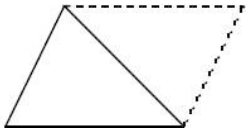
The diagrams below show that when dividing a parallelogram into two triangles along a diagonal, you get congruent geometrical figures. Building upon this observation, students can also investigate combining two congruent triangles to form a parallelogram.



Dividing into two congruent triangles



Combination of two congruent triangles



Drawing a congruent triangle

c. Properties of geometrical figures

In grade 5, help students to identify the properties of geometrical figures and teach them to use the properties to examine and compose geometrical figures.

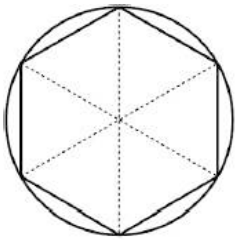
There are things that always hold for a certain geometrical figure. Those things are called properties of geometrical figures. For example, if you add the measures of three angles of any triangle, the result is 180 ° . This is a property of triangles. This property can be explained by thinking inductively. To think inductively means to find a general phenomenon based on common features of several concrete examples.

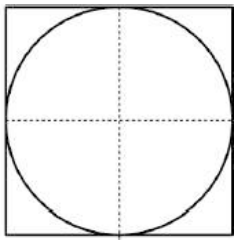
As for quadrilaterals, if you add the sizes of four angles of any quadrilateral, the result is 360°. This is a property of quadrilaterals. This property can be explained by thinking deductively, using a property of triangles (i.e. the property that if you add the sizes of three angles of any triangle, the result is 180°.)

In addition to these properties, students are expected to find properties of geometrical figures they have already learned, and be able to explore and construct shapes using those properties. In order to find the properties of geometrical figures, it is necessary to incorporate activities such as composing and decomposing geometrical figures. It is also important to help students be interested in thinking logically and to appreciate its value.

d. The ratio of the circumference of a circle to its diameter (π)

Students learned about the center, radius, and diameter of a circle in grade 3. In grade 5, they will learn the meaning of the ratio of the circumference of a circle to its diameter (π).





Help students become aware that there might be a relationship between the diameter and circumference of a circle, and to try to help them have foresight of how many times as long as the diameter is the circumference. For example, if students use a regular hexagon inscribed in a circle and a square circumscribed around it, they can see that the circumference is longer than three times the diameter (six times the radius) and shorter than four times the diameter. If students actually measure the diameters and circumferences of several circles and think inductively, they can find that the ratio of the circumference to the diameter of a circle remains the same regardless of the size of a circle. This ratio of the circumference to the diameter of a circle is called pi. By teaching this, help students understand the relationships between diameter, circumference, and pi, such as the students can calculate the circumference from the diameter and the diameter from the circumference.

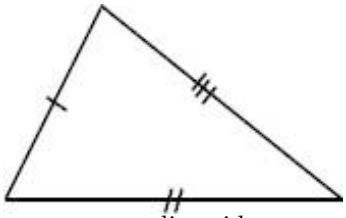
Also, note (2) of "Handling the Content" states that "3.14 should be used for the ratio of the circumference of a circle to its diameter."

[Mathematical Activities] (1) c

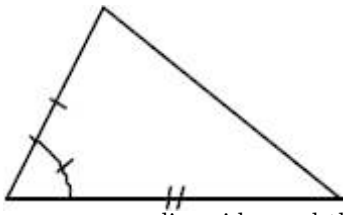
Activities such as drawing and constructing congruent shapes

The objective of these activities is to help students understand experientially congruency of geometrical figures. Through activities such as overlapping congruent shapes and tracing them, and drawing and constructing congruent shapes using the idea of correspondence, students will be able to focus on the conditions that are necessary for drawing or constructing congruent figures.

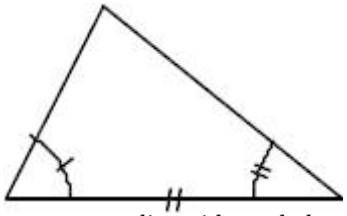
It is important to help students notice that the following conditions are necessary for congruency of triangles through activities such as drawing and constructing congruent figures.



These corresponding sides are equal length



Two corresponding sides and the angle between them are equal length and size



One corresponding side and the two adjacent angles are equal length sizes

Also, through activities where students verify that the figures they draw or compose are congruent, or that the figures satisfy certain conditions, students acquire the disposition to explain based on solid reasoning.

[Mathematical Activities](1) d

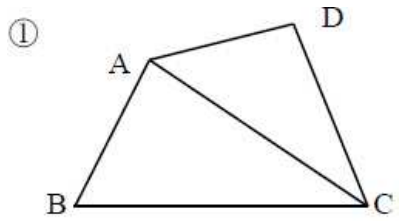
Activities to inductively think and explain that the sum of the three angles of a triangle is equal to 180 degrees. Activities to deductively think and explain that the sum of the four angles of a quadrangle is equal to 360 degrees.

The objective of these activities is for students to think about and explain the sum of the three angles in a triangle, and the four angles in a quadrilateral.

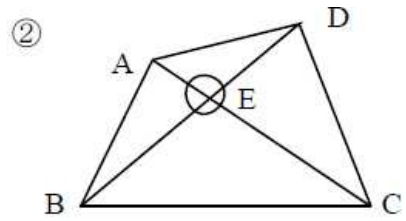
To think inductively is to find common and general facts among several concrete examples. The activity here is for students to think about the sum of the three angles in a triangle by examining various triangles and explain that the sum is 180°.

The methods for examining the sum of three angles of a triangle is 180 degrees are: tiling with the congruent triangles; measuring angles using a protractor; gathering together the three angles, etc. Through these activities, help students discover with surprise that the sum of the three angles of any triangle is 180 degrees and appreciate its beauty.

To think deductively is to explain that something new is also true based on something already proved to be true. The activity here is to think about and explain that the sum of the four angles of a quadrilateral is 360 degrees based on the fact that the sum of the three angles of a triangle is 180 degrees.



180 × 2



180 × 4 − 360

There are two typical methods of thinking and explaining this fact deductively: ① divide a quadrilateral into two triangles along a diagonal, and, based on the fact that the sum of three angles of a triangle is 180 degrees, deduce 360 degrees by multiplying 180 degrees by two; ② add point E to the inside of a quadrilateral, then divide the quadrilateral into four triangles by connecting point E with each vertex; determine the sum of all angles by multipling 180 degrees by four based on the fact that the sum of three angles of a triangle is 180 degrees, and then subtract 360 degrees which is the sum of the angles around point E, to get 360 degrees. By utilizing these ideas, help students become interested in thinking logically and notice its value through thinking and explaining deductively.

C (2) Properties of solid figures

(2) Through activities such as observing and composing geometrical figures, to help pupils understand solid geometrical figures.

a. To get to know prisms and cylinders.

[Terms/Symbols] base, side face

[Handling the Content]

(3) As for the content (2)-a in "C. Geometrical Figures," drawing of sketches and nets should be dealt with.

As for solid figures, in grade 4, students learned the properties of cubes and rectangular parallelpipeds by focusing on the number of their constituents such as edges, faces, and vertices; the shapes of faces; and parallel or perpendicular relations of faces or edges. In grade 5, prisms and cylinders are introduced as solid figures. The objectives here are for students to learn how to draw solid figures on a plane, to imagine the shape of solid figures from the drawings on a plane, and to enrich their sense of space.

Base and side faces are mentioned in "Terms/Symbols." It is important for students to be able to explain and express using these terms.

a. Prisms and cylinders

Prisms and *cylinders* are solid figures composed of *bases* and *side faces*[6]. Prisms are solid figures whose bases are polygons and whose side faces are rectangles. For example, a prism whose base is a triangle is called a triangular prism and a prism whose base is a quadrilateral is called a *quadrilateral prism*. Students are expected to understand that cubes and rectangular parallelpipeds are prisms. *Cylinders* are solids whose bases are circles. Through activities such as observing prisms and cylinders, students are expected to recognize the number or shape of their components — such as vertices, edges, and faces — and the parallel and perpendicular relationships between edges, between edges and faces, and between faces.

Note (3) of "Handling the Content" indicates that drawing sketches and nets of shapes should be taught. Through drawing sketches and nets of shapes, help students examine connections and positional relationships between edge and edge, edge and face, and face and face. Also, through the activity of drawing nets of solid figures and constructing those figures, students are expected to deepen their understanding of prisms and cylinders, and enrich their sense of space.

[6] "side face" in this document is a direct translation of the Japanese mathematical term that means lateral surface of prism, cylinder, pyramid, and cone.

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[D. Mathematical Relations]

D (1) Relationships between two quantities that vary simultaneously

1. To help pupils use tables to explore the relationships between two quantities as they vary simultaneously

    a. To get to know proportional relationships in simple cases.

[Terms/Symbols] proportion

By the end of grade 4, students learned the relationships between two quantities that vary simultaneously in situations involving addition, subtraction, multiplication, and division. The objectives of grade 5 are, on top of that, for students to enhance their ability to examine two quantities that vary simultaneously to understand proportional relationships, and to develop their functional thinking.

When teaching, focus on the use of tables. It is important for students to become able to utilize tables by fully carrying out activities of making and interpreting tables. For example, when teaching content related to multiplication and division of decimal numbers, formulas for the area of triangles and quadrilaterals, and percentage and ratio, help students investigate two co-varying quantities using tables.

It is important to deepen the way of viewing quantitative relations through interpreting the characteristics of how quantities correspond and how they vary by utilizing tables.

a. Proportional relationships in simple cases

In grade 5, proportional relationships of two quantities as they vary simultaneously in simple cases are taught. “Simple cases” means, for example, relationships between two quantities whereby if one becomes two, three, four, ⋯times as much as the original quantity, the other quantity also becomes two, three, four... times as much, and the pattern can be seen from a table representing those changes. A concrete example would be something like: “the relationship between the number of the steps and the total height of the staircase when the height of one step of the staircase is 15 centimeters”, or “the relationship between the width and the area of a rectangle when the length is fixed at 6 centimeters.” While teaching students to examine the changes in quantities while filling in the table through these activities, students will be able to identify the characteristics of how the two quantities correspond or how they change.

When teaching these points, help students become aware that they are closely related to multiplicative situations that they have seen previously. Also, help students verbalize the characteristics or rules they find — for example, “if the width becomes twice, three times, four times, ⋯ as long as the original, the area will also become twice, three times, four times, ⋯ as big as the original one” — so that they can capture these characteristics and rules appropriately.

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D (2) Algebraic expressions representing quantitative relations

(2) To help pupils deepen their understanding of algebraic expressions that represent relationships between numbers/quantities, and pay attention to the correspondence between two numbers/quantities and the aspect of variation in the relationships represented by a simple algebraic expression.

In grade 4, expressions that represent quantitative relations such as formulas or expressions with $\square$ and $\triangle$ were taught. Building on these understandings from prior grades, the objectives of grade 5 are for students to examine quantitative relations represented by expressions such as $\square = 2 + \triangle$, $\square = 2 \times \triangle$, or $\square = 3 \times \triangle + 1$, and to deepen their understanding of the algebraic expressions that represent quantitative relations.

To be able to interpret the meaning of algebraic expressions, teacher need to provide ample opportunities to examine the characteristics of how two quantities that vary simultaneously correspond or change by examining tables, and ample opportunities to represent the relationship between two quantities with algebraic expressions using words. Also, to deepen their understanding of the meaning of algebraic expressions, it is important to broaden students' functional thinking.

Teach also that the relations represented by formulas apply not only to whole numbers but also to decimal numbers, and deepen students' understanding of the role that algebraic expressions play in representing quantitative relations and rules concisely and generally.

D (3) Percentage

(3) To help pupils understand percentage.

[Terms/Symbols] %

[Hanlding the Content]

(4) As for the content (3) in "D. Mathematical Relations," the representation of buai (Japanese expression of proportion) should be touched upon.

When investigating data numerically, sometimes we use difference to compare two quantities and sometimes we use ratio. When we consider the relationship between the whole and part of the data, or among parts of data, we often use ratio.

In grade 4, students experienced how to set the size of a base quantity as 1 and to represent the relative value of a compared quantity using decimal numbers. The objective of grade 5 is to understand percentage and become able to use it.

Percentage is a way to represent the relative value of a compared quantity using whole numbers as much as possible by setting the size of the base quantity at 100. It is important to help students recognize the merits of representing ratio using whole numbers.

In daily life, percentage is used to describe definitive incidents such as "The ratio of absenteeism was 15%" or "I purchased it for 20% off the regular price." It is also used to describe an uncertain incident; for instance, the weather report might say, "Tomorrow's chance of precipitation is 20%." Consideration should be taken to accommodate activities such as finding occasions in daily life where percentage is used so that students become aware that mathematics is used in various parts of our daily lives.

Note (4) of "Handling the Content" of grade 5, states that, "the representation of buai (Japanese expression of proportion) should be touched upon." Concerning the meaning of buai, which we teach here, teachers should touch upon the idea that one can consider the compared quantity as 10, and express the relative value as wari as is done with percentage. Also help students recognize that buai, like percentage, is a useful way to represent ratios in everyday life.

D (4) Pie charts and percentage bar graphs

(4) To help pupils gather and organize data according to their purposes, and represent them by using pie graphs and band graphs, and investigate features of data.

[Mathematical Activities] (1)

e. Activities to select and use tables and graphs according to one's purpose.

The objectives of grade 5 are to help students understand that there are phenomena whose characteristics are easy to grasp when we compare the whole and the part or compare part to part, and to help them make and interpret pie charts or percentage bar graphs that represent these relationships.

When teaching pie charts and percentage bar graphs, one should teach not only how to draw them but also how to interpret them in relation to percentage. It is important to take advantage of the characteristics of the representation known as a graph, and to foster a statistical viewpoint through these activities. As for pie charts, circular graphing paper with 10 or 100 equally divided tick marks on the circle should be used.

[Mathematical Activities] (1) e

Activities to select and use tables and graphs according to one's purpose

The objective of these activities is for students to become able to utilize tables and graphs by selecting more suitable tables or graphs and using them to solve problems purposefully through representing, interpretating, and making decisions. Another goal of this activity is for students to enjoy learning mathematics, and for them to really feel and experience mathematics.

Through grade 5, students learn about two-dimensional tables for tables, and bar graphs, broken-line graphs, pie charts, and percentage bar graphs. It is important to provide opportunities for students to explain the characteristics of data or express what they want to say using the knowledge and skills with tables and graphs that they have acquired.

Students should learn to make their purposes clear such as: "show the size of quantities," "show the change of quantities," or "show the ratio of quantities." Help students select the appropriate table or graph for their purposes. Then, carefully consider the choice of table and graph, asking students to think about what categorization or items need to be indicated, how to place tick marks, or if multiple graphs should be combined. Also, plenty of activities such as interpreting these things in relation to one another or judging them should be carried out.

[**Mathematical Activities**]

(1) The content listed in "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometrical Figures" and "D. Mathematical Relations" should be taught through, for example, the following mathematical activities:

a. Activities to explore and explain the meaning and way of calculations of decimal numbers by using words, numbers, algebraic expressions, figures, diagrams and number lines.

b. Activities to explore and explain ways to determine the area of triangles, parallelograms, rhombuses and trapezoids by using concrete objects, words, numbers, algebraic expressions, figures and diagrams.

c. Activities to construct and make congruent figures.

d. Activities to inductively think and explain that the sum of the three angles of a triangle is equal to 180 degrees. Activities to deductively think and explain that the sum of the four angles of a quadrangle is equal to 360 degrees.

e. Activities to select and use tables and graphs according to one's purpose.

[Terms/Symbols] greatest common divisor, least common multiple, reduction to a common denominator, reduction, base, side face, proportion, %

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6. The Content of Grade 6

[A. Numbers and Calculations]

A (1) Multiplication and division of fractions

(1) To help students deepen their understanding of the meaning of multiplication and division of fractions, and use the calculations.

a. To understand the meaning of multiplication and division when multipliers and divisors are fractions, based on the concept of such calculations in cases where multipliers and divisors are integers.

b. To explore ways of multiplication and division of fractions, and to do the calculations.

c. With respect to multiplication and division of fractions, to understand that the same relationships and rules as integers can be applied.

Mathematical Activities (1)

a. Activities to explore and explain the meaning of calculation of fractions and ways of the calculation by using words, numbers, algebraic expressions, figures, diagrams and number lines.

[Handling the Content]

(1) As for the content (1) in "A. Numbers and Calculations," regarding division as multiplication by using reciprocals, and integrating multiplication and division of integers and decimal numbers into the calculations of fractions, should be dealt with.

The four arithmetic operations of whole numbers and decimal numbers have been taught by grade 5. As for fractions, addition and subtraction of fractions with like and unlike denominators and multiplication and division of fractions in which the multiplier or divisor is a whole number have been taught as well.

The main objective of grade 6 is for students to think about how to multiply and divide in cases where the multiplier or divisor is a fraction and actually be able to perform the calculations.

As for calculating fractions, teach by starting with proper fractions, and also include improper fractions and mixed fractions. Avoid teaching unnecessarily complicated calculations. It is important to teach in a way that students can value and utilize the calculations of fractions in their everyday life and future studies. Help students become aware of the fact that multiplication and division of fractions are much easier to carry out if you use improper fractions and not mixed fractions.

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a. The meaning of multiplication and division when the multiplier or divisor is a fraction

In grade 5, the meaning of multiplication with a whole number multiplier was taught. This was explained as adding the same number many times (repeated addition), or finding the total size from the unit size and the ratio on which it is based. Division of fractions with whole number divisors is considered to be the inverse of multiplication.

In grade 6, based on the idea of calculation of whole numbers and decimal numbers taught so far, students are expected to understand multiplication and division when the multiplier or divisor is a fraction. The meaning of multiplication can be expressed as such: If B is the base quantity, P is the relative value, and A is the corresponding quantity that has the specified relative size with respect to B, their relationship can be represented by B×P=A. The meaning of division can be thought of as the inverse of multiplication: finding the relative value or finding the base quantity.

b. Multiplication and division of fractions

Students are expected to think about and be able to carry out multiplication and division with fractions, based on the multiplication and division of whole numbers or decimal numbers, or multiplication and division of fractions where the multiplier or divisor is a whole number. A multiplication or division of fractions can be carried out by changing it to a multiplication or division that was discussed previously by applying the properties discussed in the next section (c). Students can use the number line or diagrams to think about how to perform those calculations. This is discussed in Mathematical Activities (1)-a.

For reciprocal discussed in note (1) of "Handling the Content," the fact that division can be converted to multiplication by the reciprocal of the divisor is taught. For example, since the reciprocal of the divisor in $\frac{2}{5} \div \frac{3}{4}$ is $\frac{4}{3}$, this division can be rewritten as $\frac{2}{5} \times \frac{4}{3}$. Representing multiplication and division of whole numbers and decimal numbers using fractions is also taught. For example, $5 \div 2 \times 0.3 = \frac{5}{1} \times \frac{1}{2} \times \frac{3}{10} = \frac{5 \times 1 \times 3}{1 \times 2 \times 10}$.

c. Properties of multiplication and division with fractions

Students will examine properties of multiplication and division that hold for whole numbers and decimal numbers also hold for fractions. The properties to be investigated are not limited to commutative, associative and distributive properties. In addition, students will examine the property of multiplication that when the multiplier becomes 2, 3, 4, $\cdots$ times as much, the product also becomes 2, 3, 4, $\cdots$ times as much, and the property of division, that when the dividend and the divisor are multiplied or divided by the same number, the quotient does not change. For example, when dividing fractions, we can utilize the properties of division and think of $\frac{2}{5} \div \frac{3}{4}$ as

$$\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times 4) \div (\frac{3}{4} \times 4) = (\frac{2}{5} \times 4) \div 3 = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}.$$

We can also think of it as:

$$\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times \frac{4}{3}) \div (\frac{3}{4} \times \frac{4}{3}) = \frac{2}{5} \times \frac{4}{3} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}.$$

It is important to cultivate mathematical thinking such as logical thinking through these teachings.

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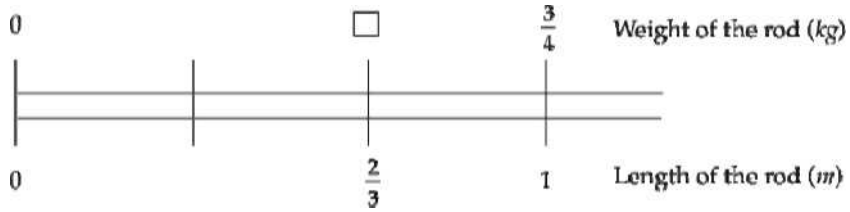
Mathematical Activities (1) a

Activities to explore and explain the meaning of calculation of fractions and ways of the calculation by using words, numbers, algebraic expressions, figures, diagrams and number lines

An objective of these activities is to enhance students' ability to represent ideas mathematically based on their prior learning. Another objective is to foster the attitude to think logically with clear rationale. This means to think about and explain the meaning and the procedures of multiplication and division of fractions using their previous learning about the meaning of multiplication and division of fractions by whole numbers.

For example, when solving a problem such as, "There is a 1 meter rod that weighs $\frac{3}{4}$ kilograms. How many kilograms is $\frac{2}{3}$ meter of this rod?" students are expected to express this problem — using words such as, "If we see $\frac{3}{4}$ kilogram as 1, and we want to find what corresponds to $\frac{2}{3}$"; using a algebraic expression with words such as "weight of 1 meter"×"length of the rod"="weight of the rod"; or using a number line — to see that the answer can be calculated by $\frac{3}{4} \times \frac{2}{3}$.

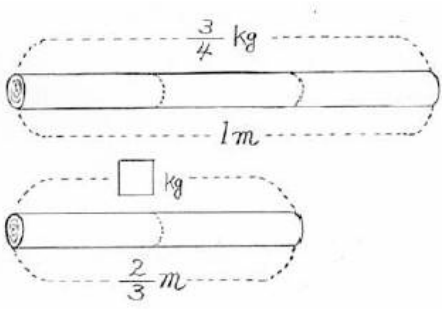


Students will express their thoughts and ideas using numbers, algebraic expressions, or diagrams, based on their prior learning.

Students can also explain how to perform calculations using a number line or the properties of the four arithmetic operations.

Here, students will explain that the answer is given by $\frac{3}{4} \times 2 \div 3$ or $\frac{3}{4} \div 3 \times 2$.

Students can also explain how to determine the weight of $\frac{2}{3}$ meter by seeing an actual rod. Looking at a rod that is 1 meter long and weighs $\frac{3}{4}$ kilograms, like the one in the diagram at left, students can explain that if the rod is cut into three equal parts, two of them will correspond to the weight of $\frac{2}{3}$ meter of the rod. This can be represented by the algebraic expression: $\frac{3}{4} \div 3 \times 2$. Here, again, students are further developing their attitude of thinking logically with clear rationale, as they base their reasoning on prior learning, i.e., (fraction)÷(whole number) and (fraction)×(whole number).



A (2) Consolidating computation skills with decimal numbers and fractions

(2) To help pupils consolidate their ability to calculate decimal numbers and fractions and extend their ability to use the calculations.

The four arithmetic operations of decimal numbers and fractions are taught from grade 3 through grade 6. In grade 6, students are expected to consolidate the computational skills of the four arithmetic operations of decimal numbers and fractions, and to extend their ability to use them.

Computational skills involve understanding the meaning of computation and thinking about how to perform calculations. Computational ability involves mastering the basic and fundamental computational skills and utilizing those skills in life and learning.

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[B. Quantities and Measurements]

B (1) Approximate shapes and estimated area

(1) To help pupils estimate the area of shapes in their surroundings by approximating them with familiar geometrical figures.

The objective of grade 6 is to help students master measurements and be able to measure efficiently according to a purpose, using estimation and other techniques, by actually measuring a variety of objects.

When measuring the area or volume of an object around them, students may notice that the object does not necessarily have a basic shape. In such a case, they may approximate it as a triangle or a rectangle, or divide it into several basic shapes. If the object they want to measure is a solid shape, they may approximate it as a cube or a rectangular parallelpiped, or divide it into those shapes. Approximation plays an important role here. Students need to enrich their experiences with these processes by thinking about not only regular shapes but also about objects around them.

When calculating with measured values, make sure students understand that using too many digits is meaningless and help students calculate appropriately according to the objectives.

B (2) Area of a circle

(2) To help pupils determine the area of geometrical figures by calculation.

a. To explore ways to determine the area of circles.

[Handling the Content]

(2) As for the content (2)-a in "B. Quantities and Measurements," 3.14 should be used for the ratio of the circumference of a circle to its diameter.

By the completion of grade 5, students have learned how to determine the area of shapes such as triangles and quadrilaterals that are surrounded by straight lines. Students also learned that the circumference of a circle is represented by (diameter)$\times\pi$.

In grade 6, students are taught how to think about determining the area of a circle — a geometrical figure surrounded by a curve.

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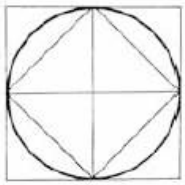
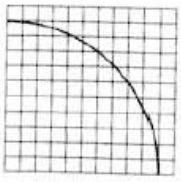
(a) Ways to determine the area of a circle

When thinking about how to determine the area of a circle, it is important to first estimate the area. Looking at the diagram at right, it is clear that the area of a circle is somewhere between 2 and 4 times as large as the area of a square whose side is the length of the radius of the circle. There are several ways of thinking about how to determine area. For example,:

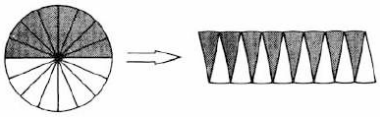
① Draw a circle on a grid paper, and determine the area by counting the squares in the circle.
The diagram at right shows a circle with a radius of 10 centimeters drawn on 1 centimeter grid paper. (The diagram shows only $\frac{1}{4}$ of the actual circle.)



② As shown at right, equally divide a circle and rearrange the resulting shapes to make a parallelogram-like shape, and determine the area of a circle. In this case, the more finely you divide a circle, the closer the shape will be to a parallelogram with the base that is half of the circumference and the height that will be the radius of the original circle. Hence, the area of a circle can be represented by the following expressions:



$$\begin{aligned} \text{(area of circle)} &= \text{(area of parallelogram)} = \text{(circumference)} \div 2 \times \text{(radius)} \\ \text{If you use (circumference)} &= \text{(diameter)} \times \pi, \text{ then :} \\ \text{(area of circle)} &= \text{(diameter)} \times \pi \div 2 \times \text{(radius)} \\ &= \text{(radius)} \times \text{(radius)} \times \pi \end{aligned}$$



Note (2) of "Handling the Content" says, "3.14 should be used for pi." This is because if one uses 3.14, it is sufficient for general use in daily life and studies.

B (3) Volume of prisms and cylinders

(3) To help pupils determine the volume of geometrical figures by calculation.

a. To explore ways to determine the volume of prisms and cylinders.

Students were previously taught about the volume of cubes and rectangular parallelpipeds. They also learned prisms and cylinders as basic solid shapes in grade 5. The base of prisms and cylinders are triangles, quadrilaterals, circles, etc. They learned how to determine area of triangles and quadrilateral in grade 5, and how to determine area of circles in grade 6. The objective of grade 6 is for students to be able to determine the volume of prisms and cylinders. It is also necessary to consider deepening students' understanding of prisms and cylinders as geometrical figures.

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a. Determining volume of prisms and cylinders

The main objective here is for the students to understand that the volume of prisms and cylinders can be determined by calculation based on the ways of determining the volume of cubes and rectangular parallelpipeds taught in grade 5.

When determining the volume of cubes and rectangular parallelpipeds, first we thought about the volume of solid shapes whose height is 1 centimeter, then we multiplied that volume by the number corresponding to the height, and thereby derived the formula for volume. We can use this method to determine the volume of prisms and cylinders.

By analogy from the formula for determining volume of rectangular parallelpipeds, we can derive the formula for determining the volume of prisms and cylinders. First, we note that (length) × (width) in the formula for rectangular parallelpipeds corresponds to (area of the base):

(volume of rectangular parallelepiped) = (length) × (width) × (height) = (area of the base) × (height)

Based of this, students are expected to understand that the formulas for determining the volume of prisms and cylinders can be generalized as:

Volume of Prisms and Cylinders = (area of the base) × (height)

B (4) Speed

(4) To help pupils understand and determine speed.

In grade 5, the ratio of two quantities of different types, such as crowdedness and population density, was taught.

In grade 6, *speed*, which is the ratio of two different quantities, is taught. For speed, students have heard people say fast or slow referring to the speed of someone running or a moving vehicle.

Two quantities — the distance traveled and the time needed — are necessary to express speed as a quantity. If we consider speed as an amount of distance traveled during a unit time, it can be expressed as (speed) = (distance) ÷ (time). For example, 60 kilometers per hour means the speed for traveling 60 kilometers in one hour. In our daily life, speed is sometimes thought of as the time required to travel a particular distance. For example, in a 100-meter sprint, speed is indicated by the time required to run 100 meters. The shorter the time, the faster the speed.

If we think of speed as a distance traveled in a unit time, the faster the speed, the larger the value. If we think of speed as time needed to travel a unit distance, the faster the speed, the smaller the value.

Since speed can be expressed by the algebraic expression (speed) = (distance) ÷ (time), speed can be determined by distance and time. Also, distance can be determined by speed and time, and time can be determined by speed and distance. It is important to connect speed with actual situations so that students can utilize it in their daily life and study.

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B (5) System of metric units

(5) To help pupils understand the system of the metric units.

[Mathematical Activities] (1)

b. Activities to find units of quantities used in everyday life and investigate how they are related to the units that pupils have learned before.

By the end of grade 5, students understood the meaning of quantities such as length, weight, area, volume, time, and angle, and became able to measure them. The objectives of grade 6 are to summarize the metric system and its units, for students to deepen their understanding of it, and for them to be able to use metric units effectively in measuring.

The characteristics of the metric system are that the units follow the structure of the base-10 numeration system, and that derived units are formed based on the basic units.

In the metric system, units are made by adding prefixes to the basic units, as shown in the following table:

Milli- (m)	Centi- (c)	Deci- (d)		Deca- (da)	Hecto- (h)	Kilo-(k)
$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10 tiems	100 times	1000 times

Though there are units in this chart that are seldom used in our country [Japan], the metric system can be summarized based on these prefixes.

For units of area and volume, derived units are based on the unit of length and they are summarized in the table below:

Unit of length	$1cm$	$(10cm)$	$1m$	$(10m)$	$(100m)$	$1km$
Unit of Area	$1cm^2$	$(100cm^2)$	$1m^2$	$(100m^2)$ 1a	(100a) 1ha	$1km^2$
Unit of volume	$1cm^3$ $1m\ell$	$(1000cm^3)$ 1ℓ	$1m^3$ $1k\ell$			

[Mathematical Activities](1) b

Activities to find units of quantities used in everyday life and investigate how they are related to the units that pupils have learned before

The objective of these activities is to experience and understand the value of the metric system in that it enables us to estimate the size of new units by analogy, utilizing what was learned about the workings of units of the metric system.

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In the daily life, many units can be seen in students' surroundings. Milligram and centiliter are used to represent the quantity of drinks. Kiloliter is used to represent the amount of tap water consumed or the capacity of a tanker truck. These units are made by adding prefixes m (*milli-*) c (*centi-*) and k (*kilo-*) to the basic unit. So students are expected to explore their surroundings to find these units, then investigate the type of units they find as well as the relationship between these new units and the ones they have already learned. For example, if a student finds "100 mg" in the ingredients section on a box of medicine, he will understand this is the unit of weight since g (gram) is used. Also, he will understand from *m (milli-)* that 1 *mg* (milligram) is $\frac{1}{1000}$ of 1 gram.

Here, it is important to foster the attitude in students to willingly examine the relationship between the units they find in their daily life and the units they learned previously based on their interests

[C. Geometrical Figures]

C (1) Reduced and enlarged figures, and symmetric figures

(1) Through activities such as observing and composing geometrical figures, to help pupils deepen their understanding of plane geometrical figures.

a. To understand reduced figures and enlarged figures.

b. To understand symmetric figures

[Mathematical Activities] (1)

c. Activities to find reduced figures, enlarged figures and symmetric figures in everyday life.

[Terms/symbols] line symmetry, point symmetry

The objectives of grade 6 are to learn new ways of viewing geometrical figures, and to use those new viewpoints to summarize plane figures students have already learned. The viewpoints to be used in summarizing are scaling and symmetry. Students are expected to deepen their understanding of geometrical figures from these viewpoints and to enrich their geometry sense.

Line symmetry and point symmetry are listed in Terms/Symbols. It is important for students to be able to explain or represent figures using these terms.

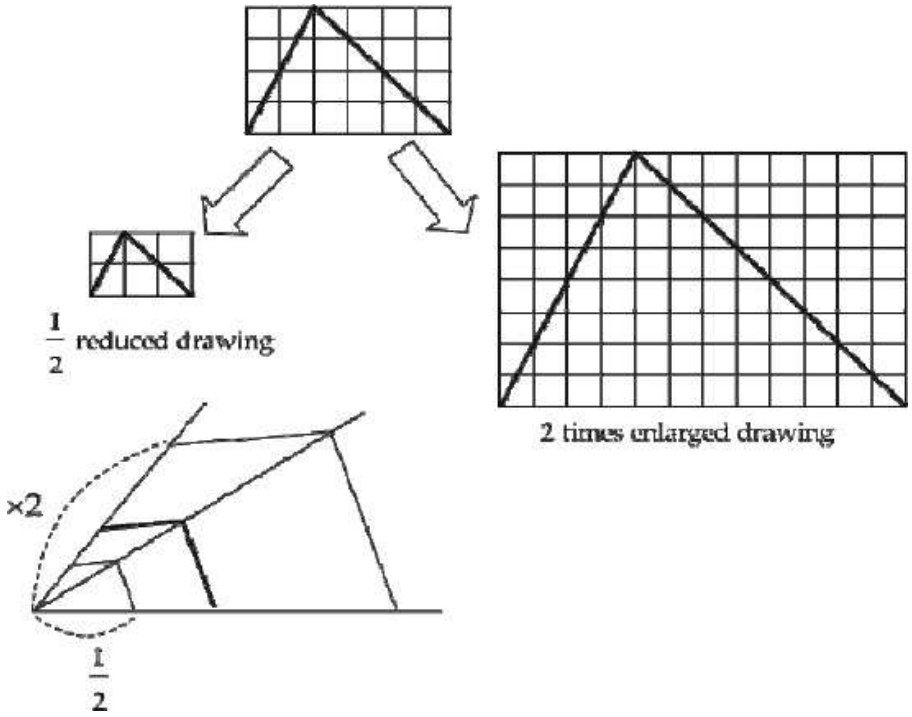
a. Reduced and enlarged drawing

s In grade 5, students learned about congruency and considered geometrical figures from this viewpoint. In grade 6, reduced and enlarged figures are taught. The objectives of this grade are to enrich students' experience so it will provide the foundation for understanding similarity, and to help them become able to draw and interpret scaled drawings appropriately, according to a purpose.

When the shape and the size of two figures are the same, these two figures are said to be congruent. *Reduced or enlarged figures* have the same shape regardless of their size. The measures of corresponding angles of scaled figures are equal, and the ratio of the lengths of the corresponding sides of scaled figures is constant.

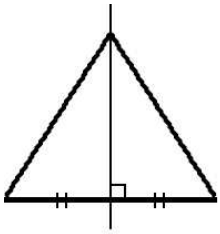
When actually drawing scaled figures, sometimes we scale vertically and horizontally using grid paper, and sometimes we scale the lengths of the sides or diagonals from one vertex, as shown in the figure below.

Students are expected to understand the meaning and characteristics of scaled figures by actually drawing these kinds of figures.

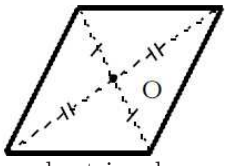


b. Symmetric figures

Students are expected to consider geometrical figures they previously learned from a new perspective of symmetry, and to deepen their understanding of geometric shapes.



For symmetry, we examine a shape from the viewpoints of line symmetry and point symmetry. *Line-symmetric* figures are shapes that overlap exactly when folded along a certain line. The folding line perpendicularly bisects all segments that connect corresponding points of the figure. *Point-symmetric* figures are figures that exactly overlap when rotated 180 degrees around a point. All segments that connect the corresponding points of a figure run through the center of symmetry and are bisected by this point.



It is important to understand the meaning of line symmetry and point symmetry through such activities as observation and drawing, and to deepen the ways of viewing geometrical figures by distinguishing line-symmetric figures, point-symmetric figures, and figures that have both line and point symmetry.

When looking at triangles from the viewpoint of line symmetry, one can grasp that isosceles triangles and equilateral triangles are line-symmetric figures. When quadrilaterals are examined from the viewpoint of point symmetry, parallelograms, rhombuses, rectangles, and squares are point-symmetric figures.

Students began to learn about symmetry in lower grades when they studied geometrical figures. For example, when they learned about isosceles triangles in grade 3, they learned that an isosceles triangle exactly overlaps when folded by the perpendicular bisector of the base. In this way, students focused on symmetry through concrete manipulation. In grade 6, students are taught to focus on the beauty of balance, stability, and general beauty of geometrical figures through activities such as observation and drawing.

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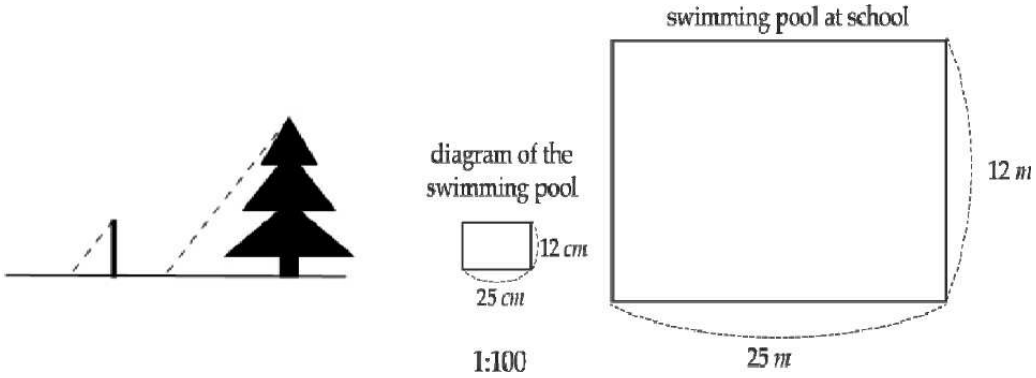
[Mathematical Activities](1) c

Activities to find reduced figures, enlarged figures and symmetric figures in everyday life

The objective of thiese activities is to have students become aware of the fact that reduced and enlarged figures are utilized in various aspects of our daily life and to foster a disposition to willingly utilize them.



For example, help students become aware that scaled figures are used in many aspects of our daily life — copy machines, maps, blueprints, microscope images, photos, and movies, etc. — and help them find scaled figures from the surroundings such as floors or walls tiled with congruent figures. Or have students determine the height of a tree by measuring not the actual tree, which is not feasible, but by measuring its shadow, which is easier to measure. Students can also determine actual distance from a map by calculation, utilizing its scale factor; for example, a ratio of 1 : 100 means 1 centimeter on a map represents 1 meter in reality. Another example is to have students calculate the actual length of the pool from a blueprint of a pool with a specific scale factor. Through these activities that illustrate their practical applications, foster in students a willingness to utilize scaled drawings in their daily life.

With respect to symmetry, the objective is for students to experience and understand that symmetric figures are used in their daily life by finding them in their surroundings. Symmetric figures like the ones in tiled figures and tiled patterns can be found in the beauty of balanced figures in everyday life. Symmetric figures can be found anywhere, such as in plants and animals, accessories, patterns, map symbols or prefecture marks. It is important to utilize activities involving finding symmetric figures.

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[D. Mathematical Relations]

D (1) Ratio

(1) To help pupils understand ratio.

[Terms/Symbols] :

When comparing the size of two quantities and representing their proportions, we sometimes use a pair of simple whole numbers to represent this without using a unit quantity. This is called ratio. Students learned the foundation of ratio when they learned multiplication, fractions, and proportional relationships.

In grade 6, the representation of ratio in the form of a:b is introduced, and students will understand ratio.

When teaching, consideration should be taken so that students understand when ratios are equal and what the equality of ratios means by examining concrete situations. For example, suppose you mixed two different kinds of liquid, three cups for one and five cups for the other. If you want to mix these two kinds of liquid to get a liquid of the same concentration, the ratio of the two kinds of liquid should stay the same — such as 6 cups for one and 10 cups for the other or 9 cups for one and 15 cups for the other. From this fact, help students to understand that 3:5 is equal to 6 : 10, 9 : 15 or 1.5 : 2.5..

Ratio is used in various aspects of our daily life. When teaching, use ingenuity and incorporate activities such as finding places where ratio is used in daily life or utilizing ratio to handle things. Also, since ratio is related to proportional relationships, inverse proportional relationships, and scaled drawing, it is necessary to endeavor that the study of each of these topics deepens students' understanding of the others.

D (2) Proportional Relationships

(2) To help pupils explore the relationships of two numbers/quantities as they vary simultaneously.

a. To understand proportional relationships, and to explore their features by using algebraic expressions, tables and graphs.

b. To solve problems by using proportional relationships.

c. To get to know inversely proportional relationships.

[Mathematical Activities](1)

d. Activities to find two numbers/quantities that are in proportional relationships in everyday life and solve problems by using the proportional relationships.

The objective of grade 6 is to enhance students' ability to think about and reason with functions as a way to summarize the viewpoints of quantitative relations taught so far, through examinations of co-varying quantities with a focus on proportional relationships.

a. Proportional relationships in algebraic expressions, tables, and graphs

Proportional relationships have the following characteristics:

(a) Suppose there are two quantities, A and B; if one of them becomes 2, 3, 4, ⋯ times as much, or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, ⋯ times as much, the other changes in the same way and becomes 2, 3, 4, ⋯ times as much, or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, ⋯ times as much.

(b) Generalizing (a), when one of two quantities becomes m times as much, the corresponding quantity of the other also becomes m times as much.

(c) The quotient of two corresponding quantities remains constant.

Through grade 5, in regard to the relationship between two quantities that vary simultaneously, teaching focused on examining how quantities corresponded and changed by using tools such as tables. Particularly, in grade 5, proportional relationships in simple cases were taught.

In grade 6, in order to summarize the relationship between two quantities, a relationship with the characteristic described in (a) is defined as a proportional relationship. It is important to incorporate activities such as representing two quantities that vary simultaneously in everyday life using tables, etc. and identify proportional relationships by examining the characteristics of change.

For students, it is not necessarily easy to generalize (a) in the form of (b). Therefore, it is important to incorporate activities such as actually examining various numerical values from tables so that students can become aware of the viewpoint in (b).

The characteristic described in (c), focusing on the quotient of two corresponding values, is effective when examining whether two quantities are in a proportional relationship from the viewpoint of functional relationships. It is necessary to teach students so that they understand this way of examining correspondence.

An algebraic expression that represents the proportional relationship, if the quotient in (c) is k, is $y = k \times x$.

Generally speaking, a graph that represents a proportional relationship is a straight line passing through the origin. This is an important characteristic used to distinguish proportional relationships. Here, it is important to teach students using concrete numbers, and through the activities such as representing on a graph two quantities that vary simultaneously so that they understand that if two quantities are in a proportional relationship, the graph representing this relationship is a straight line.

b. Solving problems using proportional relationships

In grade 6, multiplication, fractions, ratio, and proportional relationships are summarized from the perspective of proportional relationships, and students will deepen their ability to think about and reason with functions by using proportional relationships to solve problems.

Provide situations where proportional relationships are used effectively, and help students appreciate the value of using proportional relationships to solve problems efficiently, and foster their attitude to willingly use proportional relationships when solving everyday problems.

c. Inverse proportional relationships

The objectives are to know about inverse proportional relationships and to deepen one's understanding of proportional relationships. Inverse proportional relationships have the following characteristics:

a. Suppose there are two quantities A and B; if one of them becomes 2, 3, 4, ⋯ times as much, the other becomes $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, ... times as much, and if one of them becomes $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, ... as much, the other becomes 2, 3, 4, ⋯ times as much.

b. Generalizing the characteristic in (a), when one of the two quantities becomes m times as much, the corresponding quantity of the other becomes $\frac{1}{m}$ times as much.

c. The product of the two corresponding quantities remains constant.

When teaching, it is important to compare and contrast proportional relationships and inverse proportional relationships. For inverse proportional relationship graphs, have students plot several points that satisfy the inverse proportion relationships or show teacher-made graphs so that students can examine the change.

[Mathematical Activities] (1) d

Activities to find two numbers/quantities that are in proportional relationships in everyday life and solve problems by using the proportional relationships

The objective of these activities is for students to be able to use proportional relationships, such as finding phenomena that show a proportional relationship in daily life or solving problems efficiently. Feeling and experiencing the joy of studying mathematics and appreciating its significance are also important objectives.

The following are examples of problem situations that can be solved efficiently using proportional relationships:

"You have to count a lot of sheets of paper. You become aware that the number of sheets and the weight of the paper are in a proportional relationship. Therefore, if you weigh several sheets of paper and note that the total weight is □ times as much as the weight of those sheets, then the total number of sheets of paper is also □ times as many as the number of the sheets you weighed." Or: "You need to find the length of a coil of wire. You notice that you can use the fact that the length and the weight of wire are in a proportional relationship. You take out a certain length of wire and weigh it. If you note that the total weight of the coil of wire is △ times as much, then the total length of the whole wire is also △ times as long as that of wire you weighed." Many other situations can be found.

It is important to enable students to find these types of problems by themselves, and to foster the attitude to willingly utilize proportional relationships to deal with various situations in daily life and in the study of mathematics.

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D (3) Algebraic expressions with letters

(3) To help pupils deepen their understanding of algebraic expressions that represent the relationships of two numbers/quantities, and to use the algebraic expressions.

a. To represent relationships in algebraic expressions by using letters such as "a" and "x" instead of words, □, △, and so on, and to explore the relationships by substituting numbers for the letters.

The objectives of grade 6 are to deepen the understanding of algebraic expressions that represent quantitative relations, and to be able to use algebraic expressions to represent such relationships, and to interpret them.

a. Algebraic expressions with letters such as a and x

In grade 4, students learned to use symbols such as □, △, in algebraic expressions to represent relationships among numbers and quantities, and to investigate their relationships by substituting numbers for the symbols. In grade 6, based on what was taught in grade 4, letters such as x and a are used instead of words or symbols like □, △, etc., and students are expected to gradually become accustomed to using letters.

In the lower secondary school, letters are used extensively. For a smooth transition to lower secondary school mathematics, it is important to cultivate a foundation for appreciating the merits of using literal symbols like a and x, such as conciseness.

When teaching, first make sure that students understand the use of □ and △, and then help students use a or x in place of □ or △. Consideration should be taken so that students understand, through activities such as substituting numbers in letters, that not only whole numbers but also decimal numbers and fractions can be substituted, and that they can think in terms of an expanded range of numbers.

D (4) Analyzing data

(4) To help pupils determine the average of data and the distribution of data, and to explore and represent the data statistically.

a. To get to know the average of data.

b. To get to know the tables and graphs that represent frequency distribution.

The objective of grade 6 is to improve the ability to analyze and represent data statistically through discussion of topics such as: the mean as a representative value; frequency tables; and histograms.

a. Mean

Students learned about the mean of measured values in "B. Quantities and Measurement"-(3) in grade 5. Based on this, in grade 6 students are expected to understand the mean as a representative value of data, and deepen their understanding of mean.

When data are scattered over a range, the mean is often used to represent the entire data in order to grasp the tendency of the data. In grade 6, based on the idea of evening out data when there are several values, the mean is taught as a representative value of a group when there are several values.

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Also, teach that distribution is another characteristic of data sets. It is necessary for students to understand that, even if the mean is the same, the characteristics of data differ if the data are concentrated or dispersed. To help students understand this, it is important to incorporate activities where students devise different ways to represent data such as by recording values on a number line.

It is important to consider enhancing students' ability to examine and represent things around them statistically using the mean.

b. Tables and graphs representing frequency distribution

Teach students that frequency tables and histograms are good methods for representing the overall distribution and characteristics of the data when they are scattered over a range, and help them to draw and interpret them.

A *frequency table* is a table in which the range of a variable is divided into several intervals (called *categories*), and each interval and its frequency is displayed, so that investigation of how the data are distributed becomes easy.

For the histogram, it is enough to understand it as comprising rectangles for each category with the horizontal dimension as the interval and the vertical dimension as the frequency.

An important idea of statistical manipulation is the organization of data according to a purpose. How well data are sorted and organized — for example, how wide an interval is — greatly influences how easily the tendencies and characteristics of the data can be grasped; therefore, consideration should be taken in these areas as well.

D (5) Possible outcomes

(5) To help pupils analyze all the possible outcomes systematically for actual events.

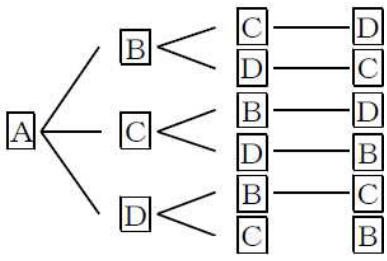
The objectives of grade 6 are to observe all possible cases from appropriate points of view, to sort and organize them, and to list them in order, based on grade 5 activities of classification and organization.

"Analyze all the possible outcomes systematically" means to organize them in order — rather than randomly, which may easily cause omissions or duplications — so that all possible outcomes are clearly and correctly shown.

When teaching, an emphasis should be on systematic analysis rather than obtaining the total number of outcomes. It is necessary to take consideration to foster the attitude to systematically examine the events, avoiding omissions or duplications, by representing them using diagrams and tables.

For example, think about arranging 4 people in a line. First, think about a case where A is at the head of the line. If B lines up second, then the third is either C or D. Then think about a case where C comes second, and then think about a case where D comes second. It becomes clear from the following diagram that there are 6 cases in which A comes first. Since there are also cases where B, C, and D come first, one can conclude from the diagram that there are 24 possible arrangements.

A-B-C-D
A-B-D-C
A-C-B-D
A-C-D-B
A-D-B-C
A-D-C-B



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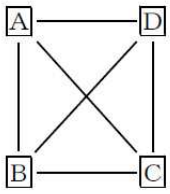
When thinking about all possible matches among four teams, a diagram or a table such as the ones below permits one to investigate all the possible combinations without omissions or duplications.

A - B
A - C
A - D
B - C
B - D
C - D



	A	B	C	D
A				
B				
C				
D				

In this way, it is important for students to become able to use tables and diagrams appropriately and to think logically according to the conditions. It is also important to experience and understand that representing names concisely by using symbols is effective for systematic sorting and organizing.

[Mathematical Activities]

(1) The content listed in "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometrical Figures" and "D. Mathematical Relations" should be taught through, for example, the following mathematical activities:

a. Activities to explore and explain the meaning of calculation of fractions and ways of the calculation by using words, numbers, algebraic expressions, figures, diagrams and number lines.

b. Activities to find units of quantities used in everyday life and investigate how they are related to the units that pupils have learned before.

c. Activities to find reduced figures, enlarged figures and symmetric figures in everyday life.

d. Activities to find two numbers/quantities that are in proportional relationships in everyday life and solve problems by using the proportional relationships.

[Terms/Symbols] Line symmetry, point symmetry, :

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Chapter 4: The Construction of Teaching Plans and Handling the Content

In " III. THE CONSTRUCTION OF TEACHING PLANS AND HANDLING THE CONTENT" of the *Course of Study*, "The Construction of Teaching Plans" and "Handling the Content" are stated separately.

1. Remarks concerning the construction of teaching plans

(1) Continuous teaching and smooth transition between grades

> (1) The content of each grade listed in Subsection II should, if necessary, continue to be taught in the following grades. In order to acquire and maintain basic proficiency in numbers, quantities and geometrical figures, instructions should be given in a planned manner by offering practices when necessary. Also, in order to smoothly link up the content of instructions of different grades, instructions should be repeated on an appropriate basis.

In teaching mathematics, it is necessary for students to securely acquire basic and fundamental knowledge and skills about quantities and geometrical figures, and it is also equally important to become able to apply the knowledge and skills in appropriate situations. In order to achieve this, it is necessary to assess that students are acquiring the knowledge and skills taught in each grade, and if it is found to be necessary, it is important to continue teaching the contents in the next grade and beyond.

Mathematics is a subject whose content is systematically and coherently organized, and whose organization is relatively clearly defined. So new contents are often taught on the basis of previously-learned contents, like adding a new layer on previous layers. Even in cases where content was already taught in the previous year, if it is needed for learning new content in the following grades, it should be taught as necessary and appropriate to the level of the students.

For basic abilities that affect knowledge, skills, thinking, making decisions, and expressing, it is necessary to provide opportunities to practice in order to master and maintain these skills. It is important to give the appropriate opportunities for repeated practice by monitoring students' learning progress.

Applying knowledge and skills to problem-solving and daily life can also provide good opportunities to master and maintain these skills, and it should be taken into consideration when constructing teaching plans.

In this revision, a major emphasis was placed on developing a spiral curriculum in which the level of complexity is gradually increased across grade level by incorporating repetitions of some topics while considering the systematic path of content between grades. In order to create a smooth connection between grades, it is necessary to teach through appropriate repetition as more advanced materials are taught gradually.

(2) Coordination of teaching between domains

(2) It is necessary to ensure coordination between the teaching of the "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometrical Figures," and "D. Mathematical Relations" in content of Subsection II for each grade.

In the Course of Study, domains are defined for each content, such as numbers, quantities and geometrical figures. The content in grades 1 through grade 6 is defined by four domains: A. Numbers and Calculations, B. Quantities and Measurements, C. Geometrical Figures, and D. Mathematical relations. But in mathematics, the content of two or more domains is often interrelated. For example, if we take a look at teaching ways to determine area, the meaning and representation of area is in "B. Quantities and Measurements" while the geometrical figures that are the subjects for determining area are in "C. Geometrical Figures." Meanwhile, actual calculation of area relates to "A. Numbers and Calculations," and creating formulas and analyzing them is in "D. Mathematical relations." Therefore, while developing teaching plans, it is necessary to consider the relationships between the domains and to incorporate content learned in other domains effectively.

(3) Teaching through mathematical activities

(3) Since mathematical activities play important roles in consolidating the acquisition of basic and fundamental knowledge and skills, in improving the ability to think, judge and express themselves, and in finding the pleasure and significance of learning mathematics, mathematical activities should be taught throughout all grades with respect to the corresponding content listed in "A. Numbers and Calculations," "B. Quantity and Measurements," "C. Geometrical Figures," and "D. Mathematical Relations."

The "Objectives" of mathematics begins with the phrase "through mathematical activities," and this phrase applies to the entire statement. And it describes the fundamental idea of how teaching should proceed, which is by having students engage in mathematical activities and teachers giving proper guidance, to achieve the objectives listed.

Concretely speaking, this means that teaching through mathematical activities is needed for all content in each domain. At the same time, this does not deny the need for explanation given by teachers or carrying out practice sessions as necessary.

"Mathematical activities" mean various activities where students willingly engage in mathematics with purpose.

What is meant by "willingly engage in mathematics with purpose" is trying to find new properties or to create new ways of thinking or to solve concrete problems. It is necessary to teach students to have a clear purpose and to willingly engage in learning in order for them to understand the meaning of quantities and geometrical figures, to enhance their ability to think, to make decisions, and to express, and for them to feel the joy and meaning of learning mathematics. For this reason, activities where students just listen to teachers' explanations or complete practice problems are not included in mathematical activities.

Mathematical activities can include various activities. Hands-on activities, experimental activities, physical activities, and activities that use concrete objects are often considered to be typical examples of mathematical activities. But there are others. Thinking about mathematical problems, building on mathematical knowledge and applying that knowledge, representing and explaining what students think – these do not deal with concrete objects but are included in mathematical activities. Generally, in grades 1 and 2, the main mathematical activities are hands-on and experimental. Later grades include more activities that involve thinking and representation.

In this revision, in order to enrich mathematical activities and to clarify their role in mathematics lessons, actual examples are included in the content of each grade. As implied by the word "examples," the mathematical activities stated there can be implemented as they are, or similar activities can be added. Furthermore, activities not mentioned be created and introduced by schools and teachers.

If we consider the characteristics of mathematical activities, they can be categorized in the following way:

Hands-on activities such as making things by hand

Experimental activities where students actually try out something or confirm reported phenomena inside or outside the classroom

Activities that use things in the students' environment

Activities in which students investigate actual situations or quantities

Exploratory activities where students can find the meaning and properties of quantities or geometrical figures, or create new ways of solving problems

Activities where students can further develop material already learned

Activities where students apply what they have learned to various situation

Activities where students integrate what they have learned in mathematics and other subjects.

(4) Relationship to moral education

(4) Based on the objectives of moral education listed in Subsections I-2 of Chapter 1 "General Provisions" and in Subsection I of Chapter 3 "Moral Education", instructions concerning the content listed in Subsection II of Chapter 3 "Moral Education" should be given appropriately. The instructions should be in accordance with the characteristics of arithmetic and should be related to the period for moral education.

The Course of Study, Chapter 1, I (2) states the following: Moral education in school is carried out throughout all educational activities, with the moral education class playing a central function. Moral education should be taught appropriately, according to the developmental stages of students, with consideration to the special characteristics of each class: moral education class, general subjects, foreign language learning activities, general study, or other special activities.

Below, we state how we need to teach moral education in mathematics, considering properties of the subject of mathematics.

Moral education in mathematics should be taught appropriately, while considering learning activities, students' attitude toward learning, the influence of teachers' attitude and behavior, and with awareness of the relationships of the objectives of mathematics with moral education as described below.

In mathematics education, the objectives are stated as follows: Through mathematical activities, to help pupils acquire basic and fundamental knowledge and skills regarding numbers, quantities and geometrical figures, to foster their ability to think and express with good perspectives and logically on matters of everyday life, to help pupils find pleasure in mathematical activities and appreciate the value of mathematical approaches, and to foster an attitude to willingly make use of mathematics in their daily lives as well as in their learning.

The ability to use foresight to generate and organize logical thinking and to represent ideas can equally be applied to the cultivation of abilities in making moral judgments. Also, attempting to apply mathematical thinking to problems in daily life and study can be considered as fostering the attitude to willingly apply one's own ideas to make life and learning better.

Next, the relationship to moral education lessons, the centerpiece in moral education, should be considered. Sometimes it is effective to use the instructional materials used in mathematics in the moral education lessons. When applying the materials used in moral education lessons in mathematics, it should be done in a way that utilizes what was achieved in moral education class. For this reason, when making annual teaching plans, it is important to consider the content and schedule in relation to the annual plan of moral education so that the two subjects can mutually enhance one another.

2. Considerations of Handling the Content

(1) Grasp approximate size and shape and judge adequately

> (1) To help pupils develop rich sense of numbers, quantities and geometrical figures, estimate approximate size and shape, make proper judgments based on estimates, and devise effective ways of performing tasks.

It is important to foster a rich sense of numbers, quantities, and geometrical figures; it affects the contents in each grade. In mathematics class, it is important to teach by giving consideration to helping students grasp approximate numbers, quantities, and shapes, and make appropriate judgments based on this understanding, and helping them think of new and efficient procedures.

By grasping approximate size and shape, students can have foresight about how to progress and can prevent major errors. In problem solving, it is necessary to emphasize the clarification of purposes for which an estimate or approximate numbers are to be used, and carry out the process with foresight of approximate results and methods.

For this reason, beginning in grades 1 and 2, it is necessary to make connections to approximation such as estimating the results of computation so that students will develop a basis for understanding approximate size and shape.

When students make estimations, they tend to use what they have already learned, exhibiting various ways of thinking and expressing, so teaching should respond to these appropriately.

(2) Activities to communicate with others by expressing ideas

> (2) In order to develop the ability to think, judge and express themselves, instructions of the content in each grade should actively incorporate such learning activities as thinking by using words, numbers, algebraic expressions, figures, diagrams, tables and graphs, and explaining and communicating their thoughts among themselves.

The ability to think, reason, and express plays an important role in thinking logically and efficiently as well as in communicating intelligently. The results of the national survey indicate that students tend to have problems expressing their ideas. To foster these abilities in teaching, it is necessary to actively implement learning activities that involve thinking and explaining using words, numbers, algebraic expressions, diagrams, tables, and graphs, and that help students express and exchange their own ideas with others.

Mathematics has the special characteristic that in addition to words, students can use numbers, algebraic expressions, diagrams, tables, and graphs. It is important to teach in ways where students can learn mathematical representations and how to apply them..

(3) Teaching terms and symbols

(3) Terms and symbols indicated under the content for each grade have the purpose of clarifying the range and the extent of the content dealt with in each grade, so they must be included in teaching the content of other items as occasion demands, and must be taught to help pupils appreciate the value of thinking by using and representing them

This statement clarifies the intent of the items in "Terms and Symbols" in each grade and points of consideration for teaching them. It is important that students use the terms and symbols. Furthermore, consideration must be given to help students understand the value of using them.

(4) Skill in calculating by using standard algorithms and estimation of results

(4) Emphasis should be placed on solidifying the skill of paper-and-pencil calculation and pupils should be able to estimate the result of calculation in accordance with one's purpose and to appropriately make decisions on the method of calculation and the result. In teaching "A. Numbers and Calculations" of the Content for the lower grades, care must be taken to deepen pupils' understanding of the meaning of numbers and calculations through appropriate utilization of soroban or specific teaching aids, etc.

This statement shows that it is important to acquire skills in calculation with standard algorithms as a foundational ability for further study of mathematics. In teaching algorithms, one should not just teach students the standard algorithms mechanically; rather, algorithms should be derived by thinking about the structure of numbers and the meaning of operations. It is also necessary that students estimate the results of calculation so that they can make appropriate judgments about the results and the procedures of calculation. Moreover, especially in lower grades, since it is necessary to deepen students' understanding of the meaning of numbers and calculation by using concrete objects, or to help them express their own ideas, it is suggested here that appropriate use of an abacus and other concrete objects can be effective.

(5) The use of computers

(5) It is necessary to appropriately use computers and so on, when necessary, in order to enrich pupils' sense of numbers, quantities and geometrical figures and to improve their ability to represent data by using tables and graphs.

This statement indicates that it is important to use tools such as computers for students to utilize their knowledge and skills, and to further develop their skills in creative ways. Computers should be used effectively, utilizing their special features—for example, data sorting, tables, graphs, animation of geometrical figures, or mathematical experiments. Teachers should devise effective use of computers to enrich students' sense of quantities and geometrical figures, and enhance their ability to express their own ideas.

小学校学習指導要領解説算数編作成協力者（五十音順）

Committee

（職名は平成20年６月末日現在）

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