## **INNOVATION OF MATHEMATICS TEACHING WITH ICT**

- The case of dynamic geometry software: Geometric Constructor -

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Geometric Constructor is one of the dynamic geometry softwares used in Japan. There are three versions (DOS, Windows, Java), some web applications using GC/Java. One of the features of Geometric Constructor is the existence of powerful users. By the collaboration with teachers, this software has been able to be developed. One of the core of collaboration is lesson study. Three lessons are described in this paper. The whole-class discussion is suitable to the lesson with Geometric Constructor.

#### **INTRODUCTION**

*Geometric Constructor* (GC : in short) is one of the dynamic geometry software (DGS : in short) like *cabri*, *Geometer's Sketch Pad* etc., which is developed by the author. Since 1990, it has been used in many schools (mainly junior high schools) in Japan. And we had made lesson study with many teachers. In this paper, I would like to overview the outline of the innovation with GC, focusing the features of software and examples of lessons.

## FEATURES OF GEOMETRIC CONSTRUCTOR

#### Dynamic geometry software used in Japan

*GC* is one of the dynamic geometry software. We can construct a geometrical figure. Under dragging some points, we can find some invariants or functionality in the figure. The first version of *GC* was MS-DOS version (*GC/DOS*, 1989 - ). According to the development of computer and network, we have developed the windows version (*GC/Win*, 1997 - ) and make the web site *Forum of Geometric Constructor* to provide software , manuals and other educational resources for practices. Since 2000, I and *Zeta* corporation have developed the Java version (*GC/Java*. 2000 -) to make many content according to the junior high school textbook.

From this project, we make two another approaches. I make some web application (*GC\_BBS*, *PukiWiki with GC*) using *GC/Java*, by which we can construct figures and save it at everywhere and every time, without installing specific software. *Zeta* is developing *GCL*(which is one of the dynamic geometry software developed based on macromedia Flash) and *dbook* to make commercial contents according to the textbooks, which will be available in this April. These approaches make a distinct feature of GC from commercial softwares like *cabri* and *Geometer's SketchPad*.

GC has developed as a tool of mathematical investigation for us and as a tool of mathematics teaching for teachers and as a tool for content providers.



Figure 2: GC/Java

## Interactive theorem finding through variation of geometrical figure

Schumann (1991) mentioned about interactive theorem finding through variation of geometrical configuration with *cabri*. We can do it similarly with GC. We can use a mouse or keyboard to drag. The most typical example of GC is shown in Figure 1,2. We can drag points A, B, C, D to deform ABCD and investigate the relation of ABCD and EFGH.

We can make various special cases and make many findings from comparison of them.



Figure 3:continuous variation by dragging point A

In Figure 4, red line is a bisector half line of angle BAC. By dragging point A, we get Figure 5, by which only few students can find an invariant.



Figure 4: a bisector half line of angle ABC



Figure 5:variation of bisector

But, if we make the trace of half line as Figure 6, many students find the fact that line and circle is crossed at the same point.



Figure 6:trace of the bisector

We have many examples like these.

# Three styles of *GC*: Standalone software, static web contents and web application

We can use GC in three different styles, which are connected seamlessly.

According to e-Japan strategy, we will use a computer with projector in an ordinary classroom. We have not enough time to use computers in mathematics teaching (In fact, we have only three hours for mathematics per week in junior high school). So, the basic use of ICT in mathematics is presentation of contents in the CD/DVD or in the Internet. We can use many web contents using *GC* from them. In this case, we use *GC/Java* as a viewer of the static web contents. We can make a presentations, discussions and investigations in mathematics teaching, but we cannot save it (it means static).

If we want to make students experience mathematical investigation, we need to write report and to save figure(it means dynamic). To do so, we have two solutions. One of them is the use of GC/Win or GC/DOS as a standalone software. Another is the use of  $GC_BBS$  or PukiWiki with GC as web applications.



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Figure 8: PukiWiki with GC

ABCD

EFGH

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## Three modes of GC/Java

GC/Java can be used as a viewer for beginners. For beginners, simple is best. But occasionally, we hope to investigate with same contents. In this case, we want to use GC/Java as a tool for investigation as same as GC/Win. For the sake of this purpose, GC/Java has three modes, which can be changed with clicking icon. This is a different feature of GC/Java from cabrijava and JavaSketchPad, which can be used as a viewer only.

**Viewer mode** : We can use only some functions ; drag, locus, zoom, etc.

Applet mode : We can use some icon for construction.

Window mode : GC/Java has the own window. We can use full menu of GC/Java and change the size of the window.



Figure 9: Three modes of GC/Java

## Web contents with GC

We have developed many web contents which can be used in mathematics teaching. The portal site of *GC* is *Forum of Geometric Constructor*. There are some kinds of samples about mathematics topics, questionings, records of mathematical investigations etc.



Figure 10: Forum of Geometric Constructor

Uehara provides many web contents suitable for mathematics teaching in junior high school in the following site.



Figure 11: Mow<sup>3</sup>'s Room of GC

## Instant web content making with GC

It is easy to make a web content using GC/Java. To make a web content we can use the on-line saving function of GC/Win; (1) we make a figure, (2) make a title and a question and (3) save it on the server (iijima.aeumath.aichi-edu.ac.jp). We can use them immediately and globally.



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list of contents instant web content

Figure 12: instant web content with GC/Java made by GC/Win

## COMMUNITY OF POWERFUL USERS OF GC

One of the features of GC is the existence of the community or powerful users (mainly, teachers in junior high schools), which has made many discussions about software, web contents and lesson study.

For discussion, we use our mailing list, from which we get about 1,000 e-mails for year. They have made many requests about new functions of GC. If I implement such a new function of software or a new prototype of content, I upload them on the server, and propose to discuss in the list. In a week, we have some cycles of check and re-making. In this way, we have developed many web contents to enjoy and discuss in this community.

This collaboration between researchers and teachers is very satisfactory for us. And it will be more important in the future. I think that we cannot realize it without ICT.

## LESSON STUDY AND DIGITAL VIDEO LIBRARY

One of the cores of discussion in this community is lesson study, which is the theme of this conference. In many case, about 10 teachers attend to observe and discuss the lesson. But, many other teachers cannot attend, because of their own job at own schools. So we send them copies of video tape (or a video file in CD/DVD, Figure 13) and discuss about the lesson in the mailing list.



Figure 13: DVD contained video files and resources of the lesson

Since 2002, we archive them to a digital video library. It contains full-video files, lesson plans, transcripts and video-clips of the lesson (if possible). Members of our community who has ID can access the library at their schools and can discuss about lessons archived in it. And if possible, we use it in our undergraduate and in-service teacher training. If our university and schools will be connected with broader network, the importance of such video library of lessons will be increased.

Now, most comprehensive library about video clips of lessons with IT is <u>http://www.nicer.go.jp/itnavi/</u>, which contains 430 examples (about all subjects). In which, three lessons with *GC/Java* by our members are contained.





transcript of lessen



a list of the clips(left) and the first video clip and transcript(right) Figure 14: lesson archived in the video library



Figure 15: lesson archived in library of NICER

## INTERNAL STANDARDS TO USE GC IN MATHEMATICS TEACHING

If we can make softwares and contents to be used easier in schools, it is not easy to make a good practice for many teachers, who has no experience of lesson with ICT. We have to share the key concepts or standards appropriate to lesson with ICT.

Hershkowitz(2002) shows the standards of CompuMath team as follows;

- 1. Inquiry (observing, hypothesizing, generalizing, and checking) is a desirable mathematical activity.
- 2. Mathematical activity should be driven by the goals of understanding and convincing.
- 3. Proving is not only the central tool for providing evidence that a statement is true but should also support understanding why it is true.
- 4. Mathematical activity should take place in situations that are meaningful for the students.
- 5. Mathematical activity must stem from previous knowledge (including intuitive knowledge).
- 6. Mathematical activity should be largely reflective.
- 7. Mathematical language (notion systems) fosters the consolidation of mathematical knowledge; it should be introduced to students when they feel the need for it.
- 8. Technical manipulation is not a goal in itself but a means to do mathematics.
- 9. Computer tools support and foster the above and beyond.

These standards are important for us, and we have some internal standards to have a lesson with GC, which may be implicit or explicit. I will sketch them as following section.

## Use ordinary know-how for the whole-class discussion effectively.

In Japan, many teachers emphasize the whole-class discussion, which is effective in the lesson with GC. It is important to emphasize ordinary know-how for the whole-class discussion to make a good practice and to make relax teachers and students.

For example, in many case, we project *GC/Java* on a blackboard as Figure 16, not on a screen. In a whole-class discussion, *Bansho* is important. Teachers want to write several mark and keyword and whole proof in some case. They can write them easily in the case of blackboard.



Figure 16: GC/Java projected on a blackboard

## Do not use technology excessively.

We don't spend much time to manipulate GC. We spend about 5 minutes in a presentation with GC in usual case. We spend about 10-20 minutes for individual/group investigations in the case of 90 minutes lesson. We spend more time to understand, formulate, hypothesize, and discussion. More excessively, less mathematically, we think.

## We use open approach in many case.

With GC/Java, we can pose problematic situation without words. Inevitably, we use open questions. There are a variety of formulations of the situation. The process of formulation from the situation is important in the lesson with GC.

## Elegant use of GC does not always make a good practice.

We want to make a good problematic situation, not nice presentation of the computer. In many case, with elegant use of GC, students feel no problem. According to the problem, more primitive and unskilled use of GC is better to make a good practice,

## Not only objectively, but also affectively, subjectively.

Our problem solving should be objectively. But we emphasize students' affection and involvement to the problem. From one situation, we can make a variety of problems, a variety of processes. According to the students' findings, ideas, suppositions, utterances, awareness, teacher have to navigate a nice process of problem solving.

## SOME LESSONS WITH GC

## Angle of circumference: construction and group investigation with GC(2001)

In this lesson, we could use many but old computers. So teacher used GC/Win and students used GC/DOS. Teacher talked how to construct and measure the angle of circumference.

Problem : investigate about the angle ACB.



Figure 17: angle of the circumference





Figure 18: problem posing and first investigation

Students found many things. Some students dragged points A and B, and found that if the center O is on the segment AB(which means AB is a diameter of circle), the angle BAC is 90 degrees. Some students found that if they drag point A, then angle BAC is constant, but if they drag point B and C, then angle is changed. Following conversation was interesting;

- 1 S1: (She is dragging point A) The size of this angle does not change. It is not interesting.
- 2 S2: Yes, it is not interesting.
- 3 S3: Teacher, this does not change. It is not interesting.
- 4 T1: What does not change?
- 5 S4: This angle.

- 6 T2: Why you feel not interesting?
- 7 S5: I hope the size of this angle to change.
- 8 S6: Why does not the size of this angle change?

They investigated some other special cases, and they were involved in the situation.

Some minutes later, teacher projected a student's display to the blackboard with the use of video camera and projector (Figure 19).

- 1 T1: Drag the point (A).
- 2 S1: (He is dragging the point A)
- 3 T2: What do you find from this?
- 4 S2: The size of this angle does not change.
- 5 T3: (To the classroom) Do you find same thing?
- 6 T4: Other group found other thing. Talk about it, dragging it.
- 7 S4: If point A pass the segment BC, the size of this angle changes.
- 8 T5: Yes. Let's observe this on your computer, and find what happens.



Figure 19 projection to the blackboard and second investigation

Following is the conversation of same group who talked that this is not interesting.

- 1 S1: This angle is 54 degrees.
- 2 S2: Yes, it is just.
- 3 S3: Very comprehensible! (The sum of two angles is 180 degrees.)

After these investigations (it took 19 minutes), teacher asked to make the formulation of the theorem and the proof of it (33 minutes remained).

## **Relation of Angles : presentation and group investigation with paper(2004)**

In this lesson, teacher use a computer with projector. Teacher showed the relation of the angles which was learned at previous lesson. And he dragged the vertex and show some other relations.



Figure 20: Presentation of some relations by dragging

And he asked to make new problems by the change of some condition of original figure. A student said that he wanted to make more vertices and angles. Teacher made requested figure, but it was not interesting. So, teacher asked other problem. Other student said what if, lines are not parallel. And teacher showed it. And he showed today's main problem: What is the relation of these angles in this figure?

(Teacher spent 5 minutes in this presentation.)



Figure 21: teacher changes figure according to student's request

Students started their investigation with worksheet individually. And next, they had group discussion. Teacher walked around into groups, and sometimes he gave a advice (Figure 22).



Figure 22: individual, group investigation and teacher's advice

Each group summed up each idea on the board, and explained it on the blackboard, and they discussed the comparison and relation of ideas.



Lastly, teacher used GC/Java to show extension in two minutes.

Figure 23: Whole class discussion

## Quadrilateral composed by four angle bisectors of a Quadrilateral: group investigation and whole class discussion (1992)

First lesson was done in the computer room. Teacher asked to remember the problem about quadrilateral composed by four middle points of four segments of a quadrilateral, which was learned last year with other software. He talked that if we change the shape of ABCD, then the shape of EFGH change. He manipulated GC to present students how to investigate and how to manipulate GC. He changed the shape of ABCD to rectangle, rhombus, square, parallelogram, trapezoid and so on. He checked the relation (showed in table 1) only verbally.



Figure 24: presentation of how to investigate with problem learned previously

ABCD	EFGH	
Square	Square	
Rectangle	Rhombus	A E C
Rhombus	Rectangle	P P P P
Parallelogram	Parallelogram	
Trapezoid	Parallelogram	
Kite	Rectangle	<b>REAT</b>
Quadrilateral	Parallelogram	↓ ↓ ↓ ↓ ↓
Wedge	Parallelogram	

Table 1:relation of quadrilaterals (1)

He showed today's problem.

There is ABCD. We draw angle bisectors of four angles, and name the intersections of bisectors E, F, G, H. We want to investigate this figure like that investigation. We want to investigate the relation of ABCD and EFGH.



Figure 25: quadrilateral composed by intersections of angle bisectors

He asked what shapes of ABCD do we investigate, and sum up students' answers to the table on the blackboard. And he said. Firstly, suppose the result, and investigate the figure with GC, and write results and sketches on the worksheet.



Figure 26: group investigation with GC/DOS

Students investigated in pair for 10 minutes.

And teacher and students made up following table in whole class discussion.

ABCD	EFGH	EFGH	Sketch
		(supposition)	
Square	Point		
Rectangle	Square		

Rhombus	Point	
Parallelogram	Rectangle	
Trapezoid	∠HEF=∠HGF	
	=90°	
Kite	Point	
Quadrilateral	Quadrilateral	
Wedge	Two triangles	L.

 Table 2:relation of quadrilaterals (2)



Figure 27:Whole-class discussion

- 1 T1: What is a difference between your supposition and observation.
- 2 S1: Parallelogram is ...
- 3 T2: Parallelogram is ?
- 4 S2: Parallelogram is missed. (Students laughed.)

This student pointed out that parallelogram was supposed in some column but he could not observe it. Other student pointed out rhombus and trapezoid too. They got problems.

- Is this table valid?
- Can we make EFGH parallelogram, rhombus, trapezoid?
- If we can not, why?

In next lesson in the ordinary classroom (several days later), to investigate these problems, the teacher picked up these cases, and asked to make proofs.



Figure 28:making proofs about the special cases

Secondly, they think about the general case.



Figure 29: whole-class discussion about the general case

They found EFGH was a cyclic quadrilateral in any case, therefore EFGH can not be parallelogram, rhombus. (Some students pointed out that EFGH could be a trapezoid. This finding was not expected by the teacher.)

Teacher presented the circle always inscribed to EFGH with *GC*. In fact, this was true in the case of wedge. There was relation about two triangles, which was not expected.



Figure 30: EFGH is always inscribed by a circle

## DISCUSSION

## Mathematical activity can be supported and fostered with ICT

With computer, we can investigate mathematical problems deeper and wider. We have experienced this since 1990. But, how many teachers have experienced and enjoyed deeper and wider investigations with computer? Not so many, I suppose. This is a serious problem for the expansion of the use of ICT in mathematics teaching in Japan.

## Know-how of the whole-class discussion is appropriate with ICT

According to the e-Japan strategy, we will use a computer with a projector in a ordinary room. But, one way presentation is more tedious for students than traditional chalk-and-talk style. To be more attractive, we should use the presentation interactively. Is it difficult? No, I think. Because, many teachers have the know-how about the whole-class discussion in Japan. I hope it make the breakthrough of the Japanese style lesson with ICT.

## How to make the *de facto* standards for mathematics teaching with ICT in Japan

In fact, many teachers do not use ICT in mathematics teaching in Japan. I think that one of the reasons is the lack of the standards for mathematics teaching with ICT in Japan. It is not good for us, of course. But, what can we do in this condition?

I think the answer is to make *de facto* standards, which may be possible with ICT as an infrastructure for teachers.

Of course, it is not so easy. But we can provide softwares and contents from our servers. We can make a community to discuss and collaborate.

#### Lesson study is important to collaborate with teachers in Japan

Many teachers want to make a good practice. Using ICT is not a goal in itself. It is a means to do a good practice. To collaborate with teachers, lesson study is important.

For example, we had a lesson study at Hikarigaoka junior high school at Komaki on 22 November, where we had a comparison between lessons with GC and without GC. Over 100 persons came to observe and discuss the lessons. They discussed eagerly. We made resources (transcriptions, video-clips, etc.) for the discussion, and had a meeting on 23 December at Komaki. Over 60 persons attended and discussed.

## CONCLUSION

The history of GC is the history of collaboration with teachers. Designing and implementation of the software is my task. But to device user-friendly interface, the monitoring by teachers is necessary. I have experienced my own mathematical investigations with GC, and proposed problems to teachers. They have accepted them and revised them suitable to their students. The core of collaboration has been lesson study. To make a good practice, we have had many discussions before and after the lesson. That produced many mathematical problems, know-how, lesson plans and hints for improvement of software and contents. In this process, ICT has been useful for us. We can provide software and contents with our server in the Internet, and we can discuss almost everyday with mailing list. Now, we are trying to make and use the digital video library of lessons. It is not easy to make a good lesson study, but it is challenging for us.

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#### Web Sites

Forum of Geometric Constructor : http://www.auemath.aichi-edu.ac.jp/teacher/iijima/

Mow<sup>3</sup>'s Room of GC : <u>http://www.mowmow.com/math/gc/</u>

Navi for lesson with IT http://www.nicer.go.jp/itnavi/