THE POTENTIAL OF LESSON STUDY IN ENABLING TEACHERS TO IMPLEMENT IN THEIR CLASSES WHAT THEY HAVE LEARNED FROM A TRAINING PROGRAM

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In the Philippines, there are different activities intended to help mathematics teachers grow professionally. Several of them have some of the characteristics of a lesson study but none has its full essence. This paper describes the possible ways by which lesson study in its pioneering stage in the Philippines enabled teachers to plan how good mathematics teaching practices to develop mathematical proficiency among students that they have learned from a teacher training program could be implemented in their own classrooms.

FORMS OF PROFESSIONAL DEVELOPMENT IN THE PHILIPPINES

According to Bentillo, et al (2003), the cascading model of teacher training is often used to implement changes on a nationwide scale such as the curriculum reform in the late 80's and the promotion of the practical work approach in the mid 90's. The training content is decided at the central level. The training moves from the national, regional, division, then school level with decreasing duration at each lower level. There is much dilution in using this top-down one-shot model. Another model called cluster-based training, involves teachers from several schools attending the same training program conducted by invited subject specialists as trainers. The content is determined by the master teachers of the schools in consultation with the teachers. While dilution may be avoided, the trainers may not be fully aware of the school situations so as to address training relevance. Recently, there has been an increase in the incidence of school-based training. This can be because of the recognition of the following: schools have specific teaching-and-learning needs that can be best addressed by the teachers of the schools working together, there are teachers in the schools who are capable of providing the training, the training can be done on a regular and continuing basis, and such training does not require much financial resources which the school can provide.

Besides training, curriculum materials development such as the ones under the Philippines-Australia Science and Mathematics Education Project (PASMEP) in the early 90's also helped teachers grow professionally. In groups, selected teachers from all the regions in the country who previously underwent training under PASMEP developed together daily lesson plans at UP NISMED with guidance from Australian and UP NISMED consultants. To try out for improvement, they demonstrated the lessons to other groups. They then went back to their classes to try them out with their students after which they came back to UP NISMED to revise and finalize the lessons accordingly. The process was done in three one-month curriculum-writing workshops with trying out in between. The outputs were two volumes each of daily lesson plans that were endorsed by the Department of Education for use by grades 8 and 9 mathematics teachers.

INTRODUCING MATHEMATICS LESSON STUDY IN THE PHILIPPINES

The researcher conceptualised introducing lesson study in order to enable teachers to use good mathematics teaching practices in their classes that would result to their students' mathematics proficiency. Several factors were considered to achieve this. First, the teachers who would comprise the lesson study group need to see good mathematics teaching practices "in action" so that they would get a very clear idea of what they are. As earlier studies reveal, teachers mainly teach by exposition (Department of Education, et al 2000). They first provide the definitions of terms then present the rules/ procedures and apply them using several examples. After which, they ask students to practice the skills that they have learned by doing several exercises. They present problems that are often worded at the end of the treatment of the topic when the students already know the procedures to deal with them. Thus, these problems are just routine ones and are often with only one method of solving. So the teachers who will be in the lesson study group need to know that good teaching practices involve among others: raising questions that give opportunities for all students to contribute an answer, making students think, providing problems/questions that may have many different ways of solving and/or may have many different correct answers, using real-life situations whenever possible and relevant, developing mathematics concepts, ideas, and skills based on problems (that is, teaching mathematics through problem solving), building on students' previous knowledge and experiences, requiring students to argue clearly and convincingly about the correctness of their answers, and making available follow up tasks to reinforce what students have learned. The above list of good teaching practices is based on the outputs of the workshops of the specialists' sessions of the APEC Conference on Innovations in Teaching and Learning Mathematics held in Tokyo on January 15 – 20, 2006.
Second, the teachers need to be willing to perform the tasks involved in the lesson study. This willingness might stem from their open-mindedness and desire to develop professionally. Moreover, their administrators have to provide the needed support to make lesson study work. "Which school and who among the teachers in the school would be willing to venture into lesson study?" was the big question that confronted the researcher. What she considered very important all along was that there has to be a context for introducing lesson study. Thirdly, lesson study should fit naturally into the teachers' overall school activity so that they could do it easily. Lastly, the researcher realized that conducting a lesson study without exposing teachers to good teaching practices would yield a lesson study that has no substance and modelling good teaching practices without conducting a lesson study would not promote continuous professional growth.

The Training for Pasig City Secondary Schools Mathematics Teachers

The researcher had to look for the appropriate opportunity and time to introduce lesson study. The months of February and March 2006 were inappropriate since these were the last months of the school year. Teachers were very busy finishing their lessons. However in March, there was a request for the Mathematics Group of UP NISMED where the researcher is a member, to conduct trainings for the secondary school mathematics teachers of Pasig City in all the four-year levels. The trainings would be in April when it was already school vacation time. The trainings that were cluster-based and included 10 schools were held at Rizal High School, a school with about 9000 students, which was in the cluster. The researcher realized that the trainings could take into account the factors that she considered. They could provide the context for showing to the teachers what good mathematics teaching practices are so that they can reconceptualize what it means to be good mathematics teachers. They could also naturally provide the rationale for engaging in lesson study.

A request to allow four grade 8 mathematics teachers to work with the researcher to conduct a lesson study was sought from the division schools superintendent and the principal of Rizal High School. There were 11 teachers from the school. Since the school had previously participated in the international research Learner's Perspective Study (LPS) in which the researcher was also involved, the principal granted the request and assigned the department head to coordinate with the researcher. The department head's involvement was beneficial because she provided the needed support. The researcher chose one of the teachers previously considered for the LPS while the department head chose the other three teachers. These were the better teachers in the school. The researcher thought that if they could be exposed to the process of lesson study, they could comprise the core group that can introduce it later to their fellow teachers.

The training for grade 8 teachers was conducted on April 24 to 28, 2006 from 8:00 a.m. to 12:00 noon and 1:00 p.m. to 5:00 p.m. There were sessions on sample teaching that were problem-based, emphasized connecting concepts and procedures, and making sense of mathematics, highlighted mathematical habits and higher order thinking skills, and exemplified assessment as an integral part of teaching. To some extent, the sessions also attempted to address teachers' beliefs and practices.

Orientations About the Lesson Study

All meetings with the researcher related to lesson study were done after the sessions ended at 5:00 p.m. On the first day, the department head, two teachers, and the researcher met. The researcher asked them what kind of students they envision to have as a result of having gone through grade 8 mathematics. She also asked what a teacher's role is to achieve such a vision. She then described briefly what a lesson study is. According to the teachers, they envisioned that their students would know how and where to use or apply what they have learned and that they would develop logical thinking and discipline. They claimed that students' retention depended on how teachers presented the lesson. A teacher said that she encouraged students to solve a problem in different ways and she was surprised that at times they preferred their peers' solutions than what she offered because the former were easier for them to understand.

She required students to explain their solutions/answers and not just to read them. From these accounts, it can be inferred that the teachers' ideas of good mathematics teaching though limited, were aligned with the framework of this project.

On the third day, the department head, the four teachers, and the researcher now a complete group, met. There was further discussion on the lesson study. The researcher lent the CD on lesson study developed by Global Education Resources (2002). On the fourth day, the lesson study group under the leadership of the department head and without the researcher, met to clarify the teachers' involvement in the professional development activity. During their break on the fifth day, the group without the researcher listened to the CD. After the training session that day, the researcher asked the kinds of professional development activities that the teachers engage in. According to them, they have a monthly in-service "trainings" that are planned a week before the school year begins. Each training which is done from 2:30 p.m. to 5:30 p.m. every third Thursday of the month when all classes are over, is of two types: demonstration teaching and reporting or sharing. If there is something new to be shared such an innovative strategy for teaching a topic, a teacher is assigned by the department head to prepare the lesson plan for it which she checks. The teacher may consult other teachers in preparing the lesson. During the demonstration teaching involving the teacher's actual class, the teachers who observe may come from all year levels. They see and get a copy of the lesson plan only on that day that the lesson will be carried out. After the demonstration teaching, a discussion follows in which the teachers discuss the results of the observation checklist that they accomplish while they observe the class. However, there is no documentation of the improved plan if at all it is revised and copies are not given to teachers. In short, there is no systematic and comprehensive collaboration among teachers in the development of lessons. Actual classroom results when the lessons are carried out are not documented. Apparently, there is no intention to document the suggestions for improvement and incorporate them in the plan and have the modified plan accessible to other teachers.

The teachers who attend a training/seminar are asked to report to the other teachers what they have learned and to share with them the handouts they obtained from it. Based on the results of national and regional student achievement tests, the teachers identify the least learned competencies. The department head would then assign some teachers to discuss the problematic topics so that other teachers can teach them well to their students. Such is another form of sharing.

According to the members of the lesson study group, the teachers prepare their lessons individually seeking help from others only as they need it. The lesson plans are skimpy. They do not provide the necessary details on the questions that the teacher will raise to develop concepts and the anticipated variety of responses from the students. As such they do not make it easy and natural for the teacher to develop students' thinking based on the kind of responses that they give. Oftentimes, questions are not also those that call for many different correct answers.

Hence, it may be said that the teachers to some extent help one another in preparing lessons giving the activity some form of collaboration. However, they do not come up with collaboratively and carefully developed lessons that are well-documented and which detail exactly the activities that the teacher and students will engage in as well as the questions that the teacher will ask and the answers that the students are expected to give. They also do not include other remarks that will guide the teacher to teach effectively. None of those that the teachers have done before has the real essence of a lesson study.

USING LESSON STUDY TO PREPARE FOR THE CLASSROOM IMPLEMENTATION OF LEARNINGS FROM A TEACHER TRAINING PROGRAM

During the last training day, the group met to decide on what topic to do lesson study. The first mathematics topic in grade 8 for the school year is systems of linear equations and inequalities (Department of Education 2002). The teachers admitted that it is difficult for many students. In the training, there was a sample teaching on "Linking Concepts and Procedures: Systems of Linear Equations." The teachers agreed to collaboratively develop a lesson plan about systems of linear equations in two variables based on how they understood and experienced the way it was presented to them in the training.

The researcher first asked the teachers how they teach the topic. One teacher said that first, she defines what a system of linear equations is. Then each day for several days, she teaches the procedures for solving systems using the substitution method, graphical method, and elimination method each time highlighting the disadvantage of a method to provide the need for other methods. Lastly, she gives word problems that involve solving systems for which students can use any method. She reasoned out that she teaches this way because she follows the sequence of the competencies listed in the Basic Education Curriculum (BEC).

Since the training was short, there was very little provision for teachers to reflectively discuss about their current teaching practices, about how they view what they learned in light of what they have been doing, and about how they intend to make use of what they learned in their own classroom teaching. In the training, they have been introduced to new ideas and have been made to experience teaching approaches that were learner-centred such as actively engaging learners in constructing mathematical knowledge. They wanted to find out if these would work in their actual classroom contexts.

The teacher who was chosen to carry out the lesson that the group prepared together, expressed that she appreciated how "systems of linear equations in two variables" was developed in the training. Starting from a single simple real-life situation, many mathematical ideas emerged towards the end of the lesson such as the meaning of systems of linear equations. The graphical and substitution methods of solving systems were naturally put to the fore from considering the situation. Such a reaction which the others in the group shared implies that the teachers realized that the way they teach the topic may still be improved; that for as long as the topics are covered, the sequence of presentation does not have to be as they are ordered in the BEC; and that it does not mean that only a single competency needs to be taken up each day. However, they raised concerns on where they can break the lesson as it was presented in the training, in order to give place to the practice exercises that will reinforce the new concepts and skills that the students will learn. They were also concerned on how they can give daily end-of-the-lesson-evaluation that they have traditionally been doing to find out if students have mastered the lesson for the day if they adopt the approach they encountered in the training. They asked if there is really a need for them to continue administering this evaluation on a daily basis.

The implementation of the lesson plan that the teachers developed together will be in the first week of June 2006 when classes resume. So its description and that of the discussion after the lesson is taught cannot be included anymore for the purposes of this paper. Nevertheless, during the paper presentation, a video of the classroom teaching and a discussion relating to it will be taken up.

Some Comments on the Lesson Plan

The teachers will meet again the week before classes start on June 5 to improve the plan shown on the Appendix. As it is, the lesson encourages maximum participation from the students right at the very start. Question 1 is easy enough for everyone to be able to contribute an answer. In Question 2, the students can draw upon their experiences for the answer because the situation is based on real life. However, after asking Question 3, students should be asked to compare their estimated answer to Question 1 and their answer to Question 3 to determine how good their estimates are. Question 4 might have been intended to make students realize that there can be many different correct answers. It provides a concrete meaning to the mathematical concept that an infinite number of ordered pairs can satisfy a linear equation in two variables. Given the real-life context, it means that several discrete values of the two quantities satisfy the given condition. In the classroom, it may be anticipated that different students may give different pairs of values. What the teachers had done was to summarize those possible answers sequentially using a table and labelled the two quantities x and y. What they missed was to explicitly write in the plan a question that will require the students to explain how they would arrive at those pairs of values. In Questions 5 and 6, the teachers apparently had included both the correct and incorrect answers that students may give. In their discussion on Question 5 while planning the lesson, they pointed out that there are equivalent correct equations. What they need to explicitly state is how they would process the wrong responses and what they would do so that students would recognize the equivalence of the different forms of correct equations that they have given. It was only after Question 6 where the teachers would introduce the meaning of a system of linear equations (although they had not written the formal definition) and it came out very logically and naturally in the flow of the lesson. The teachers appreciated this approach. It is definitely in contrast with what they had always done.

Further along the lesson, the teachers must have wanted the students to understand what the common values that will satisfy both of the two linear equations in the system would mean graphically. However, they should have provided the expected interpretations. Lastly, substitution as one of the methods of solving a system of linear equations in two variables was naturally introduced and practice exercises were provided later. Another real-life situation was presented as a context for the application of what students have learned. Again, the teachers need to give the correct answers to the questions raised.

To sum up, by continuously raising appropriate questions, the teachers aimed at actively involving students in generating mathematical ideas. In particular, they would teach mathematics through and for problem solving. Apparently, the way the teachers planned to carry out the lesson with their students showed that they had deliberate attempts to try out what they had learned from the training program.

CONCLUSIONS

The purpose of conducting a lesson study after the teachers had participated in a training program was to ensure that they are adequately prepared to implement the good teaching practices that were modelled in which they have experienced learner-centred teaching strategies. So far, only the planning stage in the lesson study cycle was reached as of this writing. Even so, it had already provided them the important opportunity to collectively and systematically reflect on their classroom practices. It was during this stage that they verbalized their realization that their teaching of a specific topic can still be improved, that there are concerns that they need to address in the process of making changes for improvement, and that the bases and reasons for their long-held practices have to be examined. It was also then when they substantially shared to each other their experiences in teaching the topic and worked collaboratively in preparing the lesson from start to end. As Bell and Gilbert (1996) note, when teachers have focused interactions about what they have learned and planned together on how they could adapt them in their own classes, the learning becomes clearer to them. Lastly, it was also then when they have initially put to action their willingness to try out in their classes the new ideas and approaches that they have encountered for the first time in the training. Although they may not be aware of it, the teachers have somehow grown personally, socially, and professionally in the process (Bell and Gilbert 1996). Moreover, they have begun to engage in the study of their own practices which is one characteristic of successful professional development programs (Glickman, et al 2001). Studies show that after undergoing training, teachers often revert to their usual classroom practices such that innovations sometimes do not get implemented (Talisayon, et al 2000). However, in the case presented here, there are good indications that lesson study had enabled the teachers to be prepared to implement the innovations that they had learned.

RECOMMENDATIONS

The next lesson study meeting will be held during the mathematics department's planning meeting for the whole school year. The teachers will have to improve the lesson plan to fill in some gaps and systematically address their concerns. Also they need to plan for which other topics they have to develop lessons together. They can adapt those that were covered in the training. Then they can attempt to develop their own original lessons. If they have already internalised what good mathematics teaching practices are, then they should be able to exhibit them in their classroom teaching. They can involve the other teachers in the department. They can also learn how to make extensive documentations of the accomplishments of their lesson study group and make them accessible to other teachers through publications or presentations in workshops and conferences.

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LESSON PLAN

(as of April 28, 2006)

Topic: **System of Linear Equations in Two Variables**

(to be covered for several days)

Objectives: At the end of the lesson, the students should be able to:

1. formulate equations representing mathematical situations
2. state the meaning of a system of a linear equation in two variables
3. solve problems involving systems of linear equations in two variables using different methods

Materials: mangoes, oranges, graphing board

Prerequisite Knowledge and Skills: linear equations in two variables, graphing on the Cartesian plane

Instructional Procedures:

1. Show to the class 1 piece of mango and 1 piece of orange taken from a plastic bag of mangoes and oranges.

Ask: Which do you think is heavier? (**Question 1**)

Expected answers:

1. *The mango because it is bigger.*
2. *Students will heft the fruits first before answering.*

Ask: What do you think is the weight of this mango? this orange? (**Question 2**)

Expected answer:
Based on their experience, students can estimate the weight of each fruit.

2. Present the following information: One kilogram of mangoes consists of 4 pieces of mangoes and 1 kilogram of oranges consists of 5 pieces of oranges provided each fruit of the same kind weighs the same. (**Information 1**)

Ask: What is the weight of each mango and each orange? (**Question 3**)

Expected answer:

The weight of each mango is 250 g and each orange is 200 g.

3. Ask: If this bag contains 6 kg of fruits (mangoes and oranges), how many of each kind are there? (**Information 2, Question 4**)

Expected answer: (Ordered pairs given will be organized into a table like the one below later and the two quantities will be represented using variables)

Number of mangoes (x)		*20*	*16*	*12*	*8*	*4*
Number of oranges (y)	*5*	*10*	*15*	*20*	*25*	

4. Ask: Can you make an equation out of this table? (**Question 5**)

Expected answers:

$4x + 5y = 6$

$250x + 200y = 6\ 000$

$y = \frac{-5}{4}x + 30$

$y = \frac{5}{4}x + 20$

$\frac{x}{4} + \frac{y}{5} = 6$

$5x + 4y = 120$

5. Present the following information: Suppose the number of mangoes is 4 times the number of oranges. Can you write an equation for this? (**Information 3**)

Expected answers:

$x = 4y$

$y = 4x$

6. Say: So the two equations we have based on the given information are:

$5x + 4y = 120$ Equation 1

$x = 4y$ Equation 2

Since x represents the number of mangoes and y represents the number of oranges in both equations, then we should have the same value for x and the same value for y in both equations. So we will solve these equations simultaneously. Together, the two equations that we are solving simultaneously are called a system of linear equations in two variables. The solution satisfies both equations.

7. Let the students graph the two given equations. Let them describe/interpret the

graphs.

Expected answer:

The lines representing the two equations intersect or they have a common point. The coordinates of this intersection point are the values of x and y that are common to the two equations. They satisfy both equations.

8. Say: Examine the two equations:

$$5x + 4y = 120 \quad \text{Equation 1}$$

$$x = 4y \quad \text{Equation 2}$$

Since the value of x and y are the same in both equations, then we can replace 4y by x in the first equation. This gives:

$$5x + x = 120$$

$$6x = 120$$

$$x = 20$$

Solving for y using equation 2 since it is simpler, we get

$$20 = 4y$$

$$5 = y$$

So the solution of the system is (20,5). The method that we used to solve the system is known as the substitution method. This is one of the methods used in solving systems of linear equations in 2 variables.

9. To ensure that the students understand the substitution method of solving systems of linear equation in 2 variables, let them solve the following systems.

a. $x + y = -12$ b. $3x + 2y = 8$

$y = 3x$ $x = 2y$

Expected answers:

Solution: (-3, -9) *Solution: (2, 1)*

10. For further application of what they have learned, give them the following problem:

Michael left his home one morning to jog. At the same time, Sara whose home is 1 km away from Michael's, also left for brisk walking. Suppose Michael jogged at 6 km per hour and Sara walked at 3 km per hour, both at about a constant speed.

a. Use equations or graphs to show the distance-time relationship for each person.

b. What information can we get from the graphs/equations?

Expected Responses:

1. *The students might ask the following questions:*
    a. *Are they heading on the same direction?*
    b. *Are they heading on opposite directions?*
    c. *Are they heading towards each other?*

2. *The resulting graphs are 2 lines that intersect. As such, the students might assume that Sara and Michael will meet.*

Note: If time permits, give the above problem to be answered in groups. If not, it will serve as their assignment.

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DESCRIPTIONS OF THE VTR ON THE LESSON "DEVELOPING THE MEANING OF A SYSTEM OF LINEAR EQUATIONS IN TWO VARIABLES"

(Based on the actual teaching of the lesson on June 8, 2006)

Summary

"Developing the Meaning of a System of Linear Equations in Two Variables" is the topic of this second year (grade 8) mathematics lesson. The objective is for the students to formulate equations representing mathematical situations, and to determine how the solutions of one equation may be related to those of the other.

Components of the Lesson

1. The teacher showed to the class a mango and an orange taken from an opaque bag. She asked the students which is heavier (**Question 1**) and to estimate their weights (**Question 2**). They were able to answer the first question correctly. However, their estimates revealed that they were not good at estimating weights. This valuable finding would not be obtained if she used exposition. Using learner-centered teaching strategies made it possible for students' weaknesses to surface. After this introductory activity, she presented the first information shown below.

One kilogram of mango consists of 4 pieces of mangoes and one kilogram of oranges consists of 5 pieces of oranges provided each fruit of the same kind weighs the same. (**Information 1**) What is the weight of each mango? each orange? (**Question 3**)

2. After they correctly answered 250g (or $\frac{1}{4}$ kg) and 200g (or $\frac{1}{5}$ kg), respectively, she presented the second information that follows.

If this bag contains 6 kg of fruits consisting of mangoes and oranges, how many of each kind are there? (**Information 2**, **Question 4**)

She asked them to work in small groups and to come up with possible values. She called on individuals, each time writing the phrases "number of mangoes" and "number of oranges" as she got a value for each quantity. Later, she asked how their data could be organized and how the quantities could be represented. They correctly answered "make a table of values" and "use variables" (x for the number of mangoes and y for the number of oranges), respectively. After the table was set up, she asked if it was possible that there was only one kind of fruit in the bag. A student responded that it could not be because of the given information.

Through the question, she made them realize that many different values satisfied a given condition but there were also values that did not because of the given context.

Also, in order to communicate information concisely and efficiently, she led them to organize data using a table and to represent changing quantities using variables.

3. Later, she asked them if they could make an equation based on the table of values. They gave

the following equations which she wrote on the board: $\frac{1}{4}x + \frac{1}{5}y = 6$, $4x + 5y = 120$, $4x + 5y -$

$120 = 0$, $y = -\frac{4}{5}x + 24$, and $x = -\frac{5}{4}y + 30$. She later asked if all of them were correct and how

one could know which ones were correct. A student answered that if the equation became true when ordered pairs from the table are substituted then it is correct. Some students who verified their answer asked her to disregard it specifically, $4x + 5y = 120$, $4x + 5y - 120 = 0$, and $y =$

$-\frac{4}{5}x + 24$, even before the class verified it. She asked them to explain why they considered it

wrong.

While verifying if $x = -\frac{5}{4}y + 30$ was correct, a student asked if she could give another equation. The teacher said that she would call her later. After the answer was determined, she called the student who gave $5x + 4y = 120$ which the class accepted as correct. So eventually only the two equations $\frac{x}{4} + \frac{y}{5} = 6$ and $5x + 4y = 120$ were left. She asked how it was possible that there were two equations that represented the same situation and if they were related. A student answered that their solutions are the same and so they are equivalent. Another student explained how $5x + 4y = 120$ could be obtained from $\frac{x}{4} + \frac{y}{5} = 6$ by applying the properties of equality. She later asked them which equation they preferred and why.

With the pleasant manner that she handled the wrong equations that they gave, she gave the impression that it was alright to make mistakes. It was also good that she called on the student who volunteered to give an additional equation. More importantly, she used it as an opportunity to call the students' attention regarding the relationship of the two different-looking correct equations given. Asking them which equation they preferred made them aware that while different answers may all be correct and acceptable, one may be preferable because it is easier to use.

4. Then she presented the third information shown below.

Suppose the number of mangoes is 4 times the number of oranges. (**Information 3**) Can you write an equation for this? (**Question 5**)

They gave the following equations: x = 4y, y = 4x, y = $-\frac{x}{4}$, and 4x = y where x represents the number of mangoes and y represents the number of oranges. She asked if all the equations were correct and how they could tell. A student said that values from the table should be substituted to the equations. She asked if any ordered pair from the table could be used. Then there was an exchange of ideas among a few students while the rest of the class keenly followed. Whenever she directed a question to the entire class, the students immediately responded in chorus. In particular, she asked them to analyze if the ordered pair (8, 20) that is found in the first table satisfied the third information.

According to Christian they should make another table. He gave the following ordered pairs: (20,5), (16,4), (24,6), and (32,8). Interestingly, these values corresponded to the equation x = 4y and not to the equation y = 4x that he gave earlier. The teacher did not catch this. When she asked if what he gave were possible values for the information, Carol disagreed. She said that (16, 4) was not equal to (16, 10), apparently thinking that the values that satisfy the third information should also satisfy the second information. The teacher noted that Carol was already relating the two tables. But Christian said that the two tables should not be related. According to him, if there was a new problem, there would be a new equation and so there should be a new table of values. The teacher asked the class if they agreed. They did not. When she asked for more observations and reactions, Mutya asked “Magkarugtong ba ito?” “Is this (referring to the third information) a continuation of the one before it?” (referring to the second information). The class answered “Yes.” The teacher recounted how the presentation of information about the fruits in the bag progressed. So when she asked if the new table was related to the previous table, the class said “yes” and Christian said that he was changing his answer.

Christian’s point seemed to be that the first table corresponded to the equation 5x + 4y = 120 which was based on the second information. The second table which he gave corresponded to the equation x = 4y which was based on the third information. But instead of using the word “information”, he used the word “problem” so he considered the two information as two different “problems.” Meanwhile the teacher and the rest of the class seemed to consider the entire situation that included all the information as one problem. Up to the part that he thought that there should be a separate table of values that corresponds to each equation that is based on a "problem", he was correct. What he needed to see was that afterwards, the relationship between these tables of values had to be determined. He later realized that the first table was related to the second table. He said that all the values of the second table satisfied the second equation but there were values in the first table that also satisfied the second equation. He must be referring to (20,5) which is only one ordered pair.

The teacher brought the class back to checking which of the equations they gave for the third information was correct. Aries said that his equation y = $-\frac{x}{4}$ was incorrect. It should be y = $\frac{x}{4}$. Yves said that only the first equation, x = 4y, was correct based on the third information. The teacher asked if x = 4y is related to y = $\frac{x}{4}$. The students recognized that they were equivalent but that they preferred x = 4y because it was not in fraction form.

In the process of establishing that the second information was related to the third information, and so their corresponding tables of values were also related, and thus, their associated equations were likewise related, the teacher welcomed students' viewpoints. She gave them the opportunity to discuss them. Carol had noticed that (16,10) and (16,4) were ordered pairs that satisfied only one but not both equations. It was possible that just like Christian she noticed that (20,5) satisfied both. Though she could not elaborate on her answer, he tried to argue convincingly.

5. The teacher wrote on the board the two equations that they finally have: 5x + 4y = 120 and x = 4y. By asking what the variables in each equation mean, she led the class to realize that their values are the same for both equations. So, she explained that they will solve these equations simultaneously. She added that the two equations that they will solve simultaneously illustrate a system of linear equations in two variables and its solution satisfies both equations. Hence, it was only at this point that she introduced the meaning of a system of linear equations.

Possible Issues for Discussion and Reflection with Teachers Observing the Lesson

- What good teaching practices did the teacher exhibit in the lesson?
  - She raised a question that gave opportunity for all students, regardless of ability to contribute an answer.
  - She used a real-life situation as a basis for introducing a mathematical problem
  - She asked students to estimate.
  - She developed mathematics concepts and skills through problem solving. In short, she taught mathematics through problem solving.

- She asked students to discuss in groups to determine the possible answers to a question.
- She built on students' previous knowledge, skills, and experiences.
- She challenged students how they knew if their answers were correct and made them evaluate which correct answer they preferred and give reasons.
- She wrote all the student responses to questions, both correct and incorrect, and provided them the opportunity to discover and explain the reasons for their incorrect ones.
- She required the students to argue clearly and convincingly about the correctness of their answers and did not interfere with what they explained.
- She consolidated important parts of the lesson for students to realize and appreciate connections or relationships.
- She accommodated students' questions and additional answers even when the lesson had already progressed beyond the question for which the answer was given.
- She asked questions that had many different ways of finding the answers and many different correct answers.

- She made students think.
- She made students realize or consider connections among their seemingly different responses.
- She fostered a very friendly classroom atmosphere that encouraged students to answer her questions and even volunteer them, and also to raise questions without fear that their answers may be wrong or their questions may not be appropriate.

• What else could the teacher have done to make her teaching effective?

  - She could have asked the students how many pieces of mangoes (or oranges) of the size that she had shown there are when they or their mother buy a kilogram of these fruits. Based on their answer, they could determine how good their estimated weights of the fruits were.
  - She did well to accommodate the seemingly different or conflicting answers of the students. However, she needed to be more careful in analyzing their responses. For example, she should have asked Christian to identify or write the second equation that he referred to when he said that all the values in the second table that he gave satisfied the second equation. This she should have done to make sure that everyone correctly understood what he explained. His table of values was correct. However, the equation that he gave for the third information was incorrect. Nonetheless, his idea that to a given information there is a corresponding equation and table of values, remained correct. It was possible that based on the table that he gave, the second equation that he meant was not the second one listed for information 3 but the first one listed for information 3. This is if he already realized that the equation that he gave was wrong. It was second in the sense that $5x + 4y = 120$ was the first and $x = 4y$ was the second. Moreover, she should have asked him to specify the values in both tables which he claimed satisfied both equations.

- What was the significance of the question of Mutya "Magkadugtong ba sila?" [meaning "Is this (referring to the third information) a continuation of the one before it?" (referring to the second information)] . Explain your answer.
    - This question made the class focus on a very important matter – that ultimately they have to find an ordered pair that would satisfy the two equations $5x + 4y = 120$ and $x = 4y$. This is what the lesson is all about.
- According to Christian, given a "problem" (meaning information), there is an equation and a table of values associated with it. All the values satisfy that equation. However, there are values from another table associated with another "problem" (meaning another information) with its respective equation that also satisfy the other equation. How could the teacher have used this comment as an opportunity to build the meaning of a system of linear equations in two variables?
    - The teacher could have picked up the idea that each information can be considered separately and each can be represented by a linear equation in two variables with a corresponding table of values all of which satisfy the equation. So this linear equation and its solutions (or the values that satisfy it) can also be considered separately from the other linear equation and its own solutions. But the moments these two equations are considered together or simultaneously, then they comprise a system of linear equations in two variables. So from a discussion similar to the foregoing one, she could have introduced the meaning of a system of linear equations in two variables. Moreover, if their solutions are also considered together or simultaneously, that is the values that satisfy both of them, then this process is known as solving a system of linear equations. So if she had asked either Carol or Christian to specify the ordered pair [which must be (20,5)] that they discovered satisfied both equations, then she could have naturally introduced what the solution of the system means.