

Mathematics Lessons In Korea: Teaching with Systematic Variation

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Introduction

- In the past decade or so, there has been increasing interest in the study of mathematics classrooms in East Asian countries.
- However, relatively little has been published in the international literature on classroom practices of Korea
- First, characteristics of the Korean mathematics classroom are identified through an analysis of the Korean data of the **Learners' Perspective Study (LPS)**.
- Then the classroom characteristics identified are interpreted in terms of the underlying cultural values that they share with other East Asian countries.

Learner's Perspective Study (LPS)

- An international video study of the mathematics classroom
- One teacher from each of three schools in each participating country was sampled, and a series of 10 to 15 **consecutive lessons** taught by the teacher were videotaped.
- The teachers chosen were judged to be **competent teachers** in their respective countries
- Three cameras were employed in the videotaping: a "teacher camera," a "student camera" and a "whole class camera."

Learner's Perspective Study (LPS)

- To get students' **reconstructive account** of the teaching and learning, the integrated images of student camera and teacher camera were used for **stimulated recall in interviews** conducted immediately after the lessons
- In-depth documentation of **student perspective** over several lessons in the same classroom
- The methodology of the LPS offers an informative complement to the survey-style approach of the TIMSS video study

Theory of Variation (by Marton)

Learning: process in which learners develop certain capability through **discerning** some features of the learning object

Experiencing **variation** is essential for discernment

Important to attend to **what varies** and **what is invariant** in a learning situation

Enacted space of learning is constructed through creating certain dimensions of variation for the experience of students

Teaching with Variation (by Gu)

Meaningful learning enables learners to establish substantial and non-arbitrary **connection** between new knowledge and previous knowledge

Classroom activities can be developed to help students establish this kind of connection by experiencing certain **dimensions of variation**

2 types of variation are helpful for meaningful learning: **conceptual variation** and **procedural variation**

Conceptual Variation

Varying the connotation of a concept
standard variation & non-standard variation

Highlighting the substantial features of the concept by contrasting with counterexamples or non-examples

Function : to provide learners with multiple experiences from different perspectives

Procedural Variation

The process of forming a concept logically and chronologically (scaffolding, transformation), arriving at solutions to problems, and forming knowledge structure (relationship among different concepts)

Function: to help learners acquire knowledge step by step, develop learners' experience in problem solving progressively, and form well-structured knowledge

Rationale to coin the new terms

While the two kinds of variations suggested by Gu are potentially powerful tools for analyzing classroom events, the terms "conceptual variation" and "procedural variation" may be misleading

"Conceptual understanding" and "procedural understanding" by Hiebert (1986)

Procedural variation is not related to the formation of concept

Renaming



enhancing conceptual understanding through multiple representation and varied examples of a given concept



this variation helps the learners to construct knowledge structures through progressively acquiring the knowledge

Sample

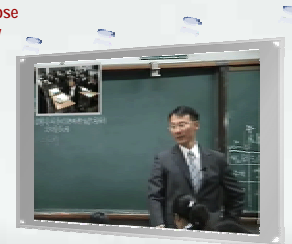
	School H	School K	School W
Type of schools	Girls' school	Co-educational	Co-educational
SES of parents	Mostly middle class	Mostly middle class	Mostly middle class
Teacher Gender, age	Male (47)	Female (32)	Female (33)
Teaching experience	18 years	6 years	7 years
Class size	36	34	37

Korea: Learner's Perspective Study (LPS)

Dominance of teacher talk and reticence of students

The ratios of number of words spoken by the teacher to those spoken by the students vary between 18 and 40, with an average of 28

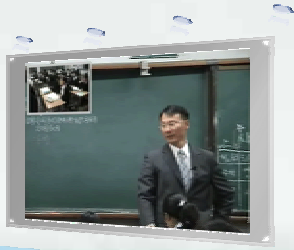
Mathematics content delivered efficiently, with mathematics concepts often stated directly



Korea: Learner's Perspective Study (LPS)

Focus of the lessons on the **final product** rather than the process of arriving at the product

Much use of **formal mathematical language** rather than everyday life language such as metaphors



Fine grained analysis

A lot of variations in concepts and practicing exercise

- 1 Introducing a new topic based on a review of the content covered in previous lessons (HD1, WD1)
- 2 Consolidation through summary (HM4)
- 3 Learning concepts through comparison and contrast (HM1)
- 4 Linkage between mathematics and concrete examples (KM1)
- 5 Multiple representation of a concept (HM2, KM2)
- 6 Generalization through abstraction (WD2)

Systematic and continuous variation: HM3

Basic equation, $x + y = 5$: coefficients of the equation and domain of the unknowns are natural numbers

Domain being natural numbers unchanged, coefficients changed systematically, first to another set of natural numbers ($3x + y = 15$), and then to include a negative integers ($-3x + y = 12$), i.e., the coefficients are now integers

HM3 (cont'd)

Next, domains of the unknowns are extended to include negative integers

Then the domains are further extended to real numbers

At this point, the concept that the graph of a linear equation with two unknowns is a straight line in the coordinate plane is well expounded

HM4

Finally, the general form of a linear equation with two unknowns ($ax + by + c = 0$) where the domains for the unknowns (x and y) are real numbers is introduced

In the analysis above, we can see that there are **systematic variations** starting with the basic equation $x + y = 5$ and moving step by step to the general form $ax + by + c = 0$.

In each variation, all but one of the components of the equation concerned are kept constant, so that the effect of the varied component is elucidated

KM3 and KM4

Basic diagram: pair of **rectangles** with ratio of similarity **1 : 2**

First, ratio of similarity kept **constant (1 : 2)**, and basic diagram changes to a pair of **triangles**

Next, geometric figures (**pair of rectangles**) unchanged, and ratio of similarity changes to **1 : 3**

Then both figures (**from rectangles to triangles**) and ratio of similarity (**from 1 : 2 to 2 : 3**) change

KM3 and KM4 (cont'd)

Finally, keeping ratio of similarity (2 : 3) constant, figure further changed to a pair of pentagons

With these systematic variations, students are guided to understand the concept that for a pair of any similar polygons, if the ratio of similarity is $m : n$, then the ratio of area is $m^2 : n^2$

WM1

Basic situation : pouch with three red stones

Situation varies systematically : each time a red stone being replaced by a blue one until eventually there are three blue stones

These systematic variations bring out the facts that:

1. probability of an impossible event is 0
2. probability of a certain event is 1
3. probability is a value lying between 0 and 1

In the three examples above :

All start with a certain simple **basic situation**

Then only one of the different aspects of the basic situation is varied at a time

Variations follow a systematic pattern until the situation reaches a target form

These systematic variations constitute a kind of **exploration** on the part of the students

Systematic variations

Variations carefully designed by the textbook and teacher, leading students to discern attributes of the object of learning or the concepts involving the final situation

In addition, in all the lessons analyzed above, there are also systematic variations in the exercise given to students

Discussion: Systematic variation

Results of the analysis of the LPS data

there are actually well designed and **systematic variations** in both the classroom activities and the practicing exercises in the Korean classroom

Such **systematic variations** will create the **necessary condition** for different features or critical attributes of the object of learning to be experienced by the students

Discussion: Systematic variation

According to the theory of variation, experience of variations on the part of the student will lead to **understanding**

Leung (2001) pointed out, understanding is "not a yes or no matter, but a continuous process or a continuum."

The process of learning often starts with gaining competence in the procedure, and then through "**continuous practice with increasing variation**", students gradually gain understanding.

Discussion: Exploration

The fine-grained analysis of the data shows that in the seemingly **teacher-directed** Korean classroom, students still had the opportunity of **exploring** mathematics ideas under the close guidance of the teacher.

Exploration in the Western context

- Students given open-ended tasks and engaged in free exploration activities
- Usually conducted in a small group or individualized setting

Discussion: Exploration

Teacher dominance and whole-class teaching however do not necessarily mean that students are not actively engaged in the lesson

Active student engagement is still possible in a classroom where the class size is large and the activities are dominated by the teacher

Discussion: East Asian culture

Teacher dominance and whole-class teaching accord well with the traditional East Asian philosophy, which emphasizes **integration** and **harmony**, in contrast to the Western culture which stresses **independence** and **individualism**

East Asians are known to have a tendency of complying with orders more than Westerners, giving rise to a tendency for **uniformity** and **conformity**

In such a cultural environment, it is not surprising that classrooms are found to be teacher dominated, with whole-class teaching being commonplace

Discussion: East Asian culture

The learning styles in East Asia are often portrayed in the literature as "**learning by rote**" or "**passive learning**", and the teaching strategies characterized as "**procedural**"

Behind the seemingly procedural teaching and passive learning, the Korean students are actually involved in **exploration** when following the prescribed classroom activities designed by the teacher

Discussion: East Asian culture

In the East Asian culture, **practice** has always played an important role in the learning process

The term in Chinese for "**learning**" consists of two characters (学习), and the second character (习) conveys the meaning of **practice**

So in the CHC tradition, practice is an inherent part of the learning process. The well known saying (熟能生巧) which is often translated as "**practice makes perfect**", reflects this philosophy of learning well

**Thank you very much
for your attention**

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