2007 TOKYO–SAPPORO APEC CONFERENCE CONCLUSION

We introduce Lesson Study in mathematics classrooms as an innovative method to improve the quality of education in all APEC economies within a collaborative framework and also to improve science and mathematics education for the benefit of all APEC economies. Because mathematical thinking is a necessary foundation for science, technology, economic growth and sustainable development in a knowledge-based society, we selected mathematical thinking as the Lesson Study topic for the 2007 conference.

Aims of the Project

1) Collaboratively share the ideas and ways of cultivating mathematical thinking, which is necessary for science, technology, economic growth and development of the APEC member economies, and
2) Collaboratively develop among the APEC economies the teaching approaches on mathematical thinking through Lesson Study.

Developing new teaching approaches and providing good examples are the methods for improving the quality of education in mathematical thinking.

What we have already done?

- Four keynote speakers presented lectures on mathematical thinking.
- Specialists observed four research lessons in Japanese classrooms (Two lessons in Tokyo and two lessons in Sapporo).
- Specialists shared their ideas on mathematical thinking based on the keynote lectures. They then tried to use those ideas to analyse the four lessons they observed in Tokyo and Sapporo. During Phase II, specialists are expected to implement their understanding to develop mathematical thinking of students in classrooms in their economies.

Opening Questions for Working Group Discussion

- What are the features of tasks that have a high potential to produce mathematical thinking?
- What are the teacher activities that can promote mathematical thinking (including the questions used)?
- What are the anticipated pupil responses? Do these indicate mathematical thinking? At what level?
- What is / are the global goal(s) of the lesson?

What we have been doing within the Working Groups

The participants were divided into four working groups, with approximately ten members in each group.

- Before beginning their discussion, each specialist summarized his/her paper related to the three questions posed by the organizing committee and shared the ideas in the papers within the group.
- A moderator facilitated the discussion of the ideas presented in each paper.
- Specialists analysed other views on mathematical thinking and how to teach it, including anticipated challenges.
- Working Groups synthesized an understanding of mathematical thinking and how to teach it.

Outcomes of Discussions within each Working Group

(See appendix)

Recommendations from the Working Group Discussions

For developing teaching approaches, the following activities are necessary:

1. Specify the mathematical thinking
2. Select a task for the lesson that develops the specified mathematical thinking
3. Consider teachers' activities and anticipated pupil responses
4. Identify indicators of mathematical thinking

PREPARATION FOR KHON KAEN SESSION

During Phase II, in preparation for Phase III in Khon Kaen, each specialist is expected to work on the following issues:

1. Consider shared mathematical thinking based on the four keynote lectures
2. Consider the results of then working group discussions (see appendix)
3. Develop teaching approaches for cultivating mathematical thinking of students at elementary school levels[1] in the context of each economy.
4. Report the results of Lesson Study with last year's format (For the report format, please refer to pp. 308-309 of the 2006 APEC Tokyo conference proceedings: http://www.criced.tsukuba.ac.jp/math/apec2006/Tsukuba_Journal_25.pdf)

[1] The elementary school level differs depending on each economy's education system. In this case, it does not include high school level.

5. Develop an analysis document about the video of one lesson you were assigned from those we observed in Tokyo and Sapporo:

   - Members of Group 1 (Patsy Wang-Iverson is group moderator.) comment on Hosomizu sensei's 5th grade class on "Area of the circle."
   - Members of Group 2 (Yeap Ban har is group moderator.) comment on Seiyama sensei's 2nd grade class on "Placing plates."
   - Members of Group 3 (Takahashi Akihiko is group moderator.) comment on Morii sensei's 6th grade class on "Thinking systematically."
   - Members of Group 4 (Lim Chap-Sam is a group moderator.) comment on Muramoto sensei's 3rd grade class on "The Multiplication Algorithm (1)."

   Comments on the videos should be made according to the format on page 193-195 of
   http://www.criced.tsukuba.ac.jp/math/apec2006/Tsukuba_Journal_25.pdf

The APEC Khon Kaen University International Symposium will be held on August 16-20, 2007 and include the following four activities:

- Keynote lectures on mathematical thinking for professional development
- Research reports on the result of Lesson Study conducted by specialists in each economy
- Panel discussion on "how to use each video for teacher education" in relation to the commentary documents
- Observe Lesson Study classes for learning teaching strategies to develop mathematical thinking

Appendix

Summary on Mathematical Thinking Framework in Working Group Discussion

Outcomes of Working Group 1

Work Group 1

Masami Isoda
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Anchalee Tananone
Shangzhi Wang
Patsy Wang-Iverson
Makoto Yoshida

*Importance of curriculum revealed by data from TIMSS**

**Third/Trends in International Mathematics and Science Study*

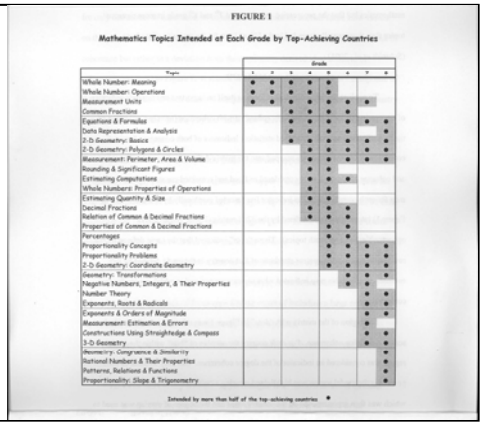


FIGURE 1

Mathematics Topics Intended at Each Grade by Top-Achieving Countries

A+ Countries:

Belgium (Flemish)
Czech Republic
Hong Kong
Japan
Korea
Singapore

Features of tasks with high potential to produce mathematical thinking

- Teachers must develop mathematical thinking in order to encourage mathematical thinking in students
- Encourage students to think, to discuss their ideas, and to pose their own problems
- Not teaching concepts in isolation; guiding students to see/understand connections between concepts
- Identify "met-befores" that create problems for students' later learning (e.g. adding two numbers produces a larger number)
- Using non-routine tasks to develop mathematical thinking in routine tasks

Not teaching concepts in isolation: Comparing and contrasting

2.3	2.3
+ 1.25	x 1.25

Outcomes of Working Group 1

Teacher activities that promote student thinking (including questions used)

- **Focusing on gap between procedure and meaning (Isoda, 1996, p.9)**
- **Helping students see the connectedness between concepts**
- **Helping students "compress knowledge" to use mathematics fluently (Tall, 2006, p. 8)**
- **Questioning: student responses; promotes student deep thinking**
- **Orchestrating student synthesis and reconciliation of different ideas through discussion --> students appreciate value of mathematical thinking**

Questioning to promote students' deep thinking

# Pencils	1	2
# Pens	9	8
Total price	670	640

Promoting mathematical thinking while students are creating the table.

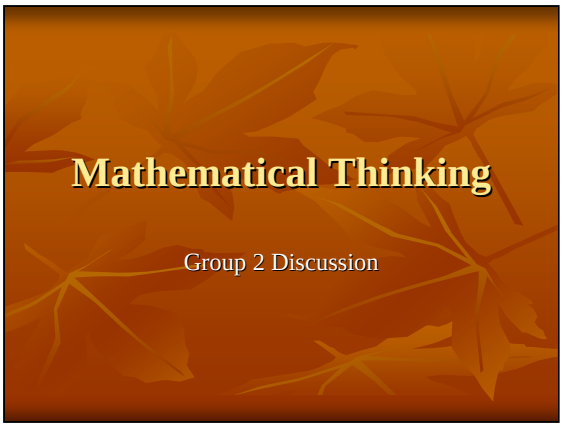
Anticipated student responses

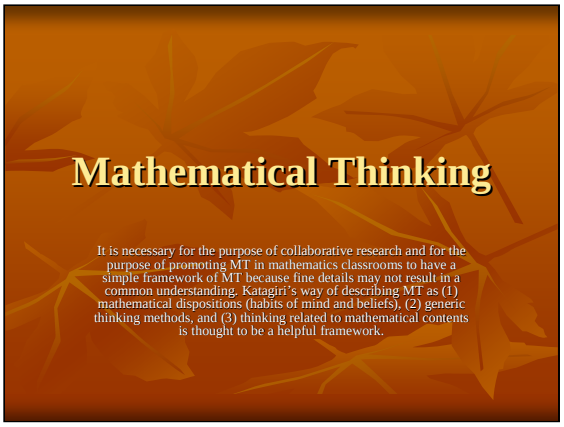
- **The heart of anticipating student responses is to develop teachers' eyes to look at students' learning process and understanding**
- **Anticipating different ranges of student responses (Isoda, 1996, pp. 24-25) in order to plan classroom discussion that is understandable for students**

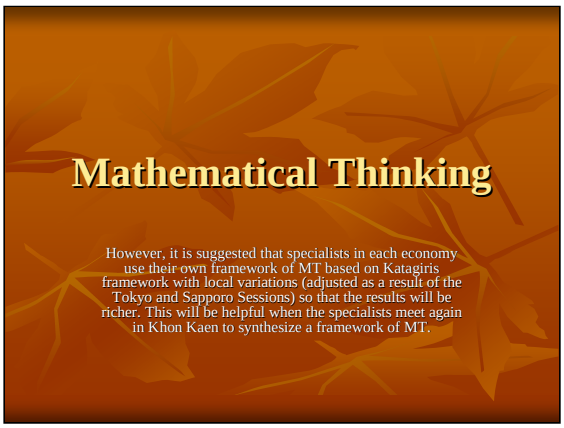
Teachers must possess mathematical thinking in order to anticipate/assess student learning
Have you read Katagiri, 2006; Isoda, 1996?

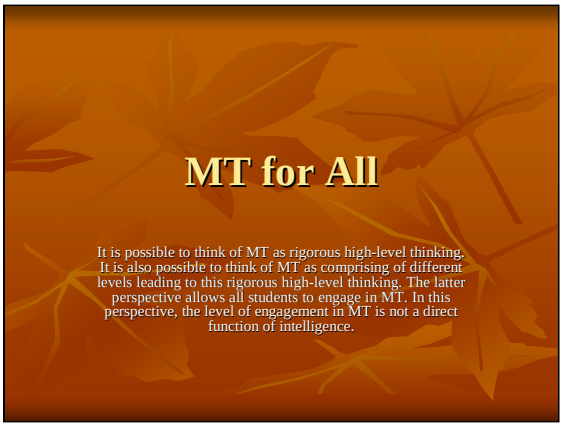
Therein lie all the answers!

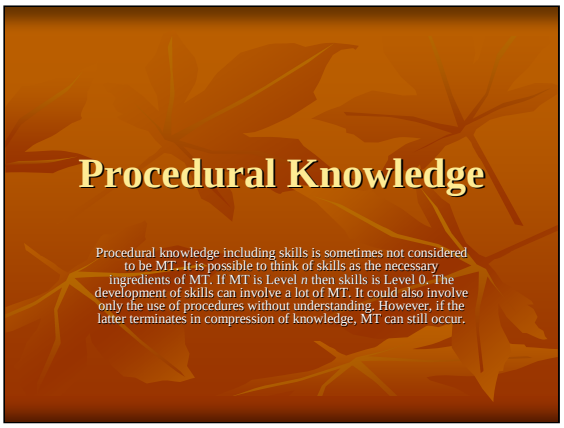
Outcomes of Working Group 2

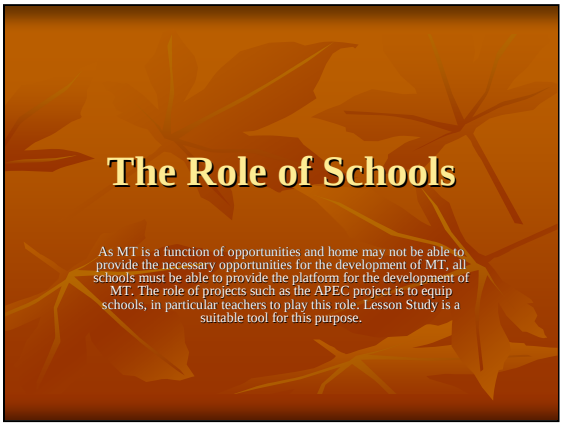




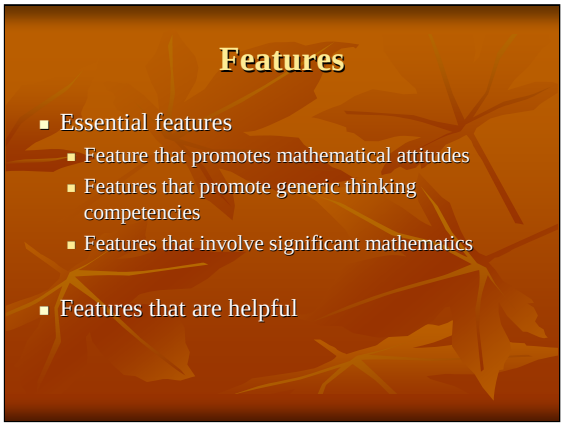


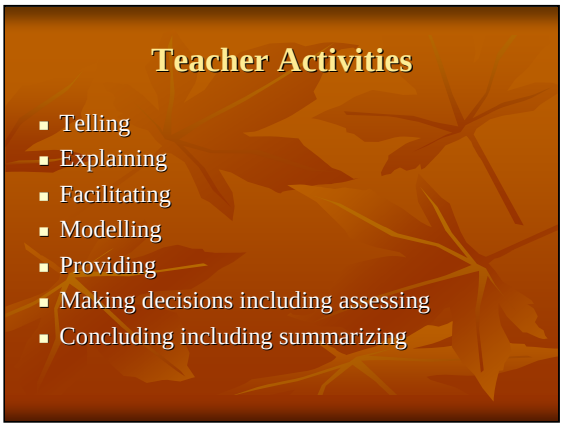




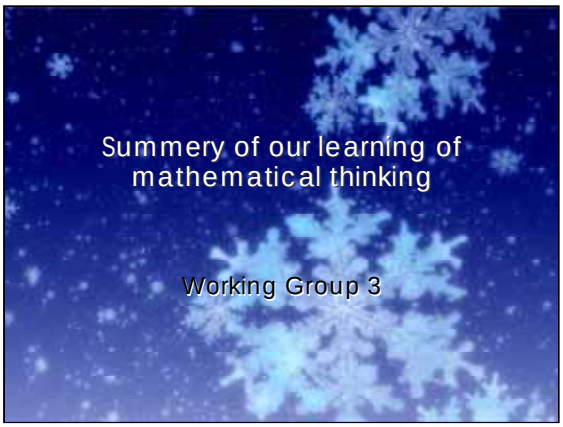


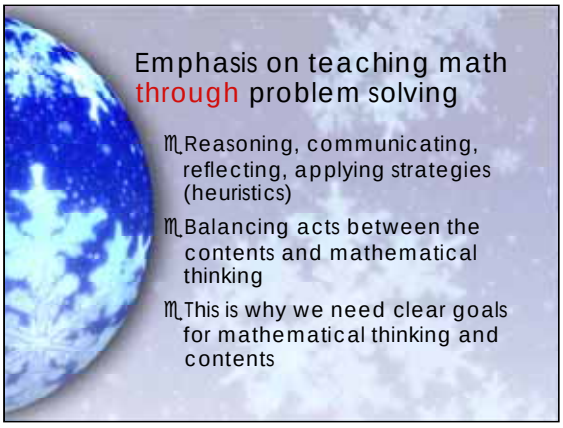
Outcomes of Working Group 2

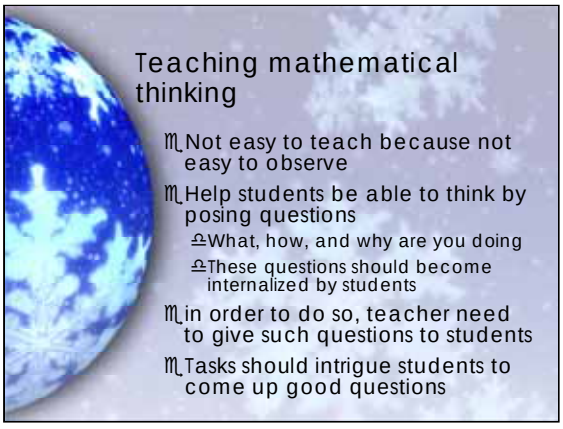


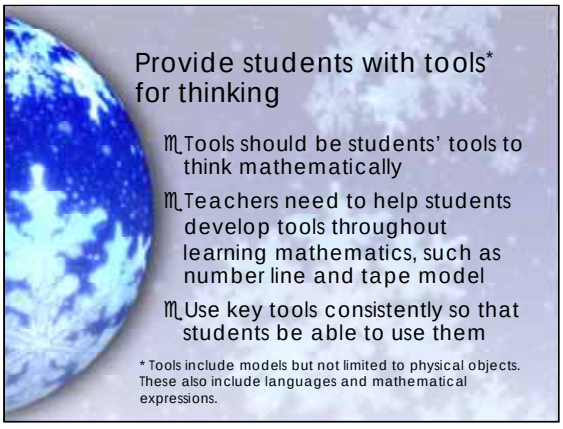


Outcomes of Working Group 3









Outcomes of Working Group 3

Assessing students' mathematical thinking

- Teachers should be able to assess students thinking throughout lessons,
- Teacher should help students to develop their communication including writing skill to enable them to express their habits of thinking, process of thinking, and making decisions.

Blackboard writing

- Three purposes
  - to be able for students to visualize the class discussion
  - to help students be able to review what and how they learned during the lesson
  - to help students be able to develop skill to express their mathematical thinking

Teaching Mathematical Thinking

-using the CRYSTAL BALL problem as an example-

- Identify previous knowledge
  - Basic addition and subtraction
  - Idea of multiple

$$53-(5+3)=45$$
$$23-(2+3)=18$$
$$99-(9+9)=81$$
$$(10a+b)-(a+b)=10a-a+b-b=9a$$

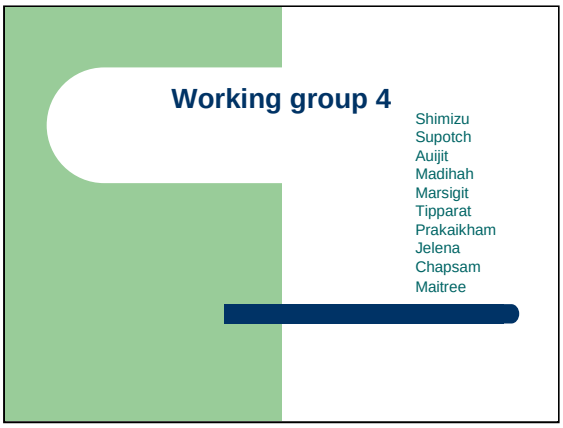
- If the goal is to simply see the pattern from several examples to solve the problem, it might be for younger grade.

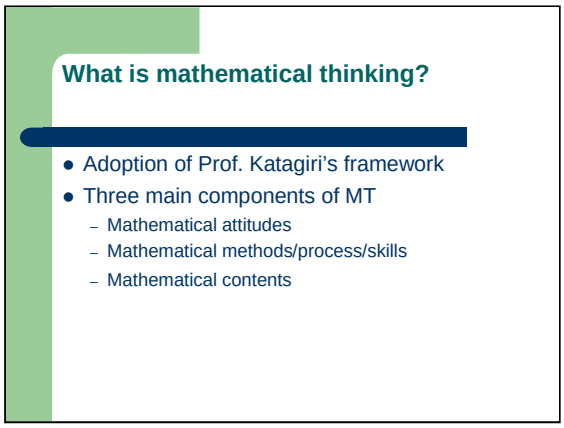
Goals of the lesson

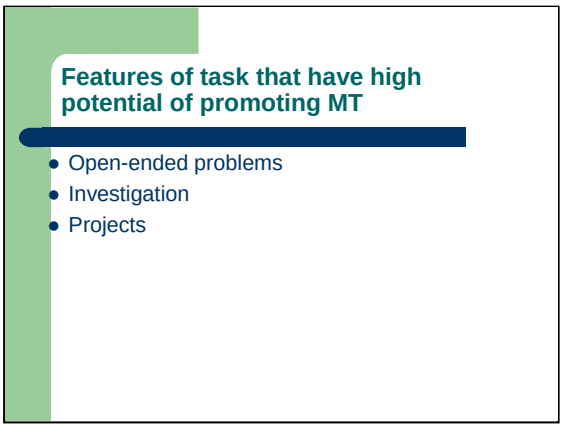
- What mathematical thinking we expect students develop
  - Recognizing the pattern
  - Inductive and deductive reasoning
  - Mathematizing the game (Generalizing pattern to develop the mathematical expression)
  - Extending the idea to develop another problem (transfer)
  - Conjecturing

The lesson would be for 6th grade and above

Outcomes of Working Group 4







Outcomes of Working Group 4

Analyzing MT in a lesson (Grade 6, Atsutomo Morii)			
Teaching activities	**Math attitudes**	**Math method**	**Math content**
Set introduction Given pen and pencil and ask for the price	Attempting to discover mathematical problem in daily life		
Giving the problem and solved by using the table	Willingness to attempt	Observing Trial and error Inductive thinking	Idea of units
Evaluating which method is faster easier and accurate	Attempt to solve better Economize thought and effort	Comparing and evaluating	
Transforming from table method to mathematical sentence	From concrete do abstract	Inductive thinking	Idea of formula Functional thinking
Generalizing to "if…then…"	From concrete do abstract	Generalizing	
Give another similar problem to solve	Attempting to think based on previous knowledge	Applying knowledge to solve similar problem Analogical thinking	Functional thinking