INTRODUCTION

At the third APEC Education Ministerial Meeting held on 29-30 April 2004 in Santiago, the ministers defined the four priority areas for future network activities. “Stimulating Learning in Mathematics and Science” is one of the four priority area. Based on this priority, the APEC project “A Collaborative study on innovations for teaching and learning mathematics in different cultures among the APEC Member Economies” was approved by APEC Member Economies in August 2005. The project held two meetings in Tokyo, Japan (January, 2006) and Khon Kaen, Thailand (June, 2006). The project was managed by the Center for Research in Mathematics Education (CRME) at Khon Kaen University and the Center for Research on International Cooperation in Educational Development (CRICED) at University of Tsukuba. The presentations and papers will be made available at: http://www.criced.tsukuba.ac.jp/math/apec2006/

Based on the success of these meetings, the specialists from the APEC economies decided to continue the project for the next four years with a focus on the following topics: mathematical thinking (2007), communication (2008), evaluation (2009), and generalization (2010). The first three topics were selected based on the three phases of the Lesson Study process: plan (for mathematical thinking), do (for communication) and see (for evaluation). Each year’s results will become the basis for the following year’s project. In the final year, generalization will be the theme, which will extend the implementation of Lesson Study to all subject areas.

For 2007, the APEC project “Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) - Lesson Study focusing on Mathematical Thinking” was accepted in May 2006 and was approved at the APEC Ministerial Meeting held in Hanoi, Vietnam, on 15-16 November, 2006 under the project number HRD 02/2007; it was ranked second of all five approved projects. Keeping in mind the emphasis on the third APEC Education Ministerial Meeting priority, this year’s project focuses on mathematical thinking, which is a necessary prerequisite for science, technology, economic growth and development.

Using Lesson Study, the project aims to collaboratively:

1) Share the ideas and ways of mathematical thinking which are necessary for science, technology, economic growth and development, and
2) Develop the teaching approaches on mathematical thinking through Lesson Study among the APEC member economies.

As part of the project, we will publish a report and possibly a book with CD-ROMs including videos of effective teaching practices for developing mathematical thinking for teacher education in APEC economies and other countries. In order to achieve the goals, the project is conducted in four phases, as was done the previous year.

Phase I: A workshop and a Lesson Study meeting (workshop for specialists) among key mathematics educators from APEC member economies hosted by the Center for Research on International Cooperation in Educational Development (CRICED), University of Tsukuba, Japan was organized to share the ideas and ways of mathematical thinking (in Tokyo and Sapporo, December 2006).

Phase II: Each APEC member economy specialist will engage in the Lesson Study project for developing some topics of mathematical thinking (January-July 2007).

Phase III: An International Symposium and a Lesson Study meeting (a workshop for general teachers) will be organized to share teaching approaches by specialists for developing mathematical thinking. The symposium will be hosted by the Center for Research in Mathematics Education (CRME), Faculty of Education, Khon Kaen University, in Khon Kaen, Thailand (August 2007).

Phase IV: Professional development for mathematics teachers will be conducted based on the results obtained (August-September 2007).

Specialists from member economies

The project and the meetings are planned for all APEC economies' welfare. The project has been carried out by specialists from member economies who participated in the past two meetings in Tokyo (January, 2006) and Khon Kaen (June, 2006), and is open to new delegates recommended by any APEC economy. From the project side, the specialists of the last meetings will be recommended to participate in future project meetings.

For economies recommending new specialists, please consider the following criteria for selecting representatives.

A specialist:

- is expected to be a researcher of mathematics education
- is working in the ministry of education or academic institutions including universities and teachers' colleges
- has research experience in classrooms at the elementary (including students up to age 13) school level
- knows and is interested in Lesson Study
- is interested in the conference
- will engage in Lesson Study and develop teacher education programs in each economy in the project
- will participate in both meetings and present his/her reports in Japan and Thailand

For sharing the welfare to improve the quality of education through the Lesson Study, it is very welcome the member economies will support travel grants for a number of the delegates. The organizing committee will consider ways to support one specialist from each economy through the APEC grant or a grant from Japan. However, there are financial constraints. According to APEC policy, the travel cost of one specialist from

each eligible APEC member economy (Chile, China, Indonesia, Malaysia, Mexico, Papua New Guinea, Peru, Philippines, Russian Federation, Thailand, and Vietnam) will be supported by an APEC grant. Other APEC economies are asked to provide financial support for their own delegates.

Publication of project results

Phase I and Phase III results will be published as progress reports after the meetings. To facilitate easy access, all reports and videos from the meeting will be made available at the following website:

http://www.criced.tsukuba.ac.jp/math/apec/apec2007/

In addition, according to APEC EDNET policy, these above-mentioned resources will also appear at the Knowledge Bank website of APEC EDNET:

http://www.apecknowledgebank.org/

Furthermore, based on the results of the meetings, we are planning to develop a teacher education textbook, which can be used by teacher educators to help teachers include mathematical thinking in their classrooms through the implementation of Lesson Study.

QUESTIONS FOR DISCUSSION IN THE MEETING OF PHASE I IN JAPAN

We focus on mathematical thinking in elementary (including students up to age 13) school classrooms. Mathematical thinking is a necessary for analysing subject matter and planning the lesson with the specified aim. Here, we pose three questions which were discussed at the meeting in Japan. Each specialist is expected to present his/her report with some examples in relation to these three questions.

Question 1: How is mathematical thinking defined in your curriculum documents and your lesson?

From the view point of Lesson Study, mathematical thinking should be developed through lessons. Usually, mathematical thinking is defined by the curriculum and embedded in the aim of each lesson. Thus, curriculum documents of each economy would be the clearest resources for analysing what mathematical thinking is in each economy.

In the Japanese curriculum, mathematical thinking has been defined for clarifying the quality of activity since 1951 for secondary school and since 1953 for elementary and middle school.

In Japanese curriculum documents, mathematical thinking is defined with mathematizing activity, and it has three components to be taught: the ability of ‘see as’, ‘ways of thinking’, and ‘appreciation of its significance’. In Japan, there are four categories of evaluation standards: attitude, mathematical thinking, representation, and understanding. Each category is related to the others. Mathematical thinking is based

on mathematical attitude, is carried out with mathematical representation and is necessary for understanding. The order of these four categories resembles the process of thinking, but it is not specific to mathematics because similar conditions exist in other academic subjects.

The Japanese Ministry of Education recommended that teachers have decision making authority for teaching a lesson based on the observation conditions developed from these four categories. In lesson planning during the first part of Lesson Study, teachers analyse subject matter and anticipate students' responses. In this process, teachers plan the lesson keeping in mind the four categories. Thus, the Ministry recommended that teachers describe these four categories with specific mathematical conceptions which should appear in a specific lesson.

OECD PISA Frameworks and NCTM's Principles and Standards for School Mathematics (PSSM)

At the Tokyo meeting (January 2006) Jan de Lange described the OECD's PISA frameworks' meaning of mathematical literacy with necessary competency for living:

http://www.criced.tsukuba.ac.jp/math/apec2006/Tsukuba_Journal_25.pdf

In the PISA frameworks (2003), mathematical literacy and key competencies of mathematics are defined as reasoning, argumentation, communication, modelling, problem-solving, reproduction and connection:

http://www.pisa.oecd.org/dataoecd/1/60/34002216.pdf

In the NCTM Principles and Standards of School Mathematics (PSSM, 2000), there are five content standards and five process standards which are described with illustrative examples. Process standards consist of: problem solving, proof and reasoning, communication, connection and representation.

http://standards.nctm.org/

Unlike the Japanese course of study, which specify teaching contents at each grade and the specific sequence, the PISA frameworks and PSSM are not meant to be curriculum documents. The PISA frameworks were used for assessment purposes, while PSSM serve as guidelines for what should be taught by grade bands.

Question 2: What is your key window for considering mathematical thinking?

Mathematical thinking has meant many things for many people. Thus, it is very difficult to discuss its development without having a window in which to discuss it.

When we focus on each lesson, we usually focus first on specific knowledge and skills (understanding), and may forget about attitude, mathematical thinking and representation. Japanese middle-school curriculum documents support a dynamic learning activity with the following three features of mathematics. The first feature is reorganization through mathematization by reflective thinking. The second feature is acquisition and using mathematical concept on ideal world (existing expected harmonized world) according to Platonism. The third feature is learning how to learn,

develop and use mathematics in the previous two types of learning. All three features are necessary perspectives for planning a lesson and learning how to learn, develop and use mathematics is a kind of mathematical thinking specially recommended in the classroom. Thus, learning how to learn itself is an important key window.

John Dewey (1910) described a very basic process of thinking, that of reflection. Reflective thinking is a key window to develop mathematics such as mathematization (e.g., Hans Freudenthal, 1973)

The methods of developing mathematics were popularized through the work of George Polya (1957) in *How to solve it.* It showed the strategies of mathematics, which help us distinguish among a variety of mathematical thinking presented from his perspective. His idea is a key window currently shared in the mathematics education community. *Thinking mathematically* by John Mason, Leone Burton and Kaye Stacey (1982) is another resource for teacher education. Letting people know the ways to develop mathematics itself is necessary for teaching mathematical thinking.

Mathematical thinking has been described in the context of problem solving. For example, from the viewpoint of representation, the permanence of the equivalence of form is a way of extending mathematical form which is trying to keep the mathematical (or algebraic) structure. Representation is also a key window.

Alan H. Schoenfeld is well known by his cognitive research on problem solving. He suggested the Vygotskian perspective to develop mathematical thinking as internalized communication and the importance of belief (value) systems for thinking mathematically. From his perspective, the ways of communication is necessary for developing mathematical reasoning. In this case, communication is a key window. Belief, value and attitude (including the affective domain) are also studied in cognitive (Douglas B. McLeod) or cultural contexts (Alan Bishop). These are driving forces of mathematical thinking. Thus, these domains can also be key windows.

There is much research focusing on special ways of reasoning. George Polya studied induction and analogy. Analogy was discussed with the knowledge development on the embodied cognition by George Lakoff and Rafael Nunez (2000) that is a framework to develop knowledge with hands-on activity such as dragging on the computer. Charles Sanders Peirce studied abduction is a kind of reasoning. Willi Dörfler described the generalization process. Each way of reasoning can be a key window.

There are a number of windows, each with a specific range for describing mathematical thinking. Please consider our target of the meetings is sharing the framework to develop student's mathematical thinking by teachers in classrooms.

Question 3: How can we develop mathematical thinking through the lesson?

In the problem solving approach for mathematics teaching such as the Open-ended Approach, students meet an unknown problem which can be solved with previously learned mathematics. Students represent their own ideas, which they then discuss with

each other. A solvable or approachable unknown problem for students is an important condition of problems in a lesson. Even if most of the mathematics problems that mathematicians encounter may take years to solve, they are solved based on what is already known.

In the process of communication, it is necessary to share the norms, so such ideas are based upon what students already learned. This approach helps students learn how to learn in mathematics, because it follows the deductive ways of reasoning that are fundamental in mathematics. Communication in mathematics sometimes resembles the debate in society but in mathematics everyone cannot decide the validity of ideas without sharing presupposition in community and cannot decide it by the majority. But sometimes, authorized teachers teach everything without communication.

There are a number of didactical suppositions which may be useful for Lesson Study for developing mathematical thinking. Clarifying these suppositions based on authentic mathematical activity with examples is useful for teachers in developing the lesson. Without examples, every teacher may agree on the importance of these suppositions but may find it difficult to understand the real meaning in his/her teaching process and may not be able to develop his/her lesson to implementing these ideas effectively.

In the case of Japan, these didactical suppositions are integrated into various teachers' theories of mathematics education through Lesson Study. The problem solving approach and Open Approach are applied models of Japanese teachers' theories.

Workshop report format

The report is expected to include your claims for mathematical thinking with examples. Your paper will be used as discussion documents in answer to the three questions. Specialists are expected to read all reports before the meeting. We will upload your report to the web site, which can be seen only by the specialists. After the workshop you may have a chance to rewrite your manuscript for the progress report. The progress report will be available to the public.

Please consider that the final goal of the project is to develop teacher education materials. Teachers do not have a chance to share your original academic ideas but will have a chance to develop good lessons based on your ideas.

It is necessary to respond to the three questions and to include some examples.

You are welcome to include video clips from the lesson. The format of writing with video is explained in following:

> http://www.criced.tsukuba.ac.jp/math/apec2006/progress_report/General/Conclusion.pdf
>
> http://www.criced.tsukuba.ac.jp/math/apec2006/progress_report/Specialist_Session/Isoda.pdf

You are expected to report your result of Lesson Study with your video during the Thailand session (Phase III) in August, 2007.

The format of the lectures in the open symposium

Papers should integrate issues of research trends, curriculum development in relation to mathematical thinking, or teachers' theories for planning lessons to develop mathematical thinking.

MEETING STRUCTURE IN TOKYO AND SAPPORO, DECEMBER 2-7, 2006

The aims of Phase I meetings are to share the ideas based upon the three questions and to know the lessons which develop mathematical thinking. The APEC–Tsukuba meetings consisted of three components:

1. Open symposium on December 3 and 4

   Lectures and a panel for sharing ideas of mathematical thinking to help develop lessons by teachers.

2. Workshop on December 5-7

   A workshop to develop a collaborative framework for using Lesson Study to develop mathematical thinking.

3. Lesson Study meeting on December 2, 5 and 6.

   Sharing examples of Lesson Study to develop mathematical thinking

Schedule of APEC - Tsukuba meetings

Following is the schedule of the APEC–Tsukuba meetings in Tokyo and Sapporo.

December 1 FRI - 2 SAT Morning: Arrival of participants

Evening of December 1 and morning of December 2: "Symposium to develop excellent students in mathematics"

December 2 SAT. Afternoon: Day 1 of the APEC Program

APEC Lesson Study Meeting: Sharing examples to develop mathematical thinking

at the Elementary School of University of Tsukuba.

December 3 SUN: Day 2 of the APEC Program

APEC-Tsukuba International Conference: Tokyo Open Symposium: Opening remarks, keynote presentations and lectures.

December 4 MON Morning: moving to Sapporo

December 4 MON Afternoon: Day 3 of APEC Program

Open symposium: Panel and Lecture

December 5 TUE and 6 WED: Days 4 and 5 of the APEC Program

Workshop (morning) and Lesson Study (afternoon) at Hokkaido Elementary School

December 7 THU: Day 6 of the APEC Program

Workshop and Closing,

Return to Tokyo in the evening.

December 8 FRI: Departure

Venues of APEC - Tsukuba Conference in Tokyo and Sapporo

The meetings will be held at following places:

Dec. 1 FRI: JICA INSTITUTE FOR INTERNATIONAL COOPERATION
http://www.jica.go.jp/english/contact/ific/index.html
(Accommodation in Tokyo)

Dec. 2 SAT: Attached Schools, University of Tsukuba at Tokyo
http://www.gakko.otsuka.tsukuba.ac.jp/map.jpg

Dec.3 SUN: JICA INSTITUTE FOR INTERNATIONAL COOPERATION
http://www.jica.go.jp/english/contact/ific/index.html

Dec. 4-8 MON-FRI: JICA SAPPORO INTERNATIONAL CENTER
http://www.jica.go.jp/branch/hics/jimusho/hics.html#map
(Including accommodation in Sapporo)

IMPORTANT INFORMATION FOR PARTICIPANTS AT APEC-TSUKUBA CONFERENCE

First Announcement will send:

October 14, 2006

Contact URL: http://www.criced.tsukuba.ac.jp/math/apec/apec2007/

Dead line of the nomination of specialists from member economies:

October 30, 2006

Contact address: apec@criced.tsukuba.ac.jp

Invitation letter, Information of Trip and Second Announcement will be sent:

October 31, 2006

Contact address: apec@criced.tsukuba.ac.jp

Contact URL: http://www.criced.tsukuba.ac.jp/math/apec/apec2007/

Dead line of the submission of paper:

November 24, 2006

Contact address: apec@criced.tsukuba.ac.jp

All Papers for Discussion will be on the website:

November 27, 2006

Contact URL (to be announced to specialists)

Final Announcement will be on the website:

November 30, 2006

Contact URL: http://www.criced.tsukuba.ac.jp/math/apec/apec2007/

Arrival days of Participants

December 1 FRI - 2 SAT Morning

CONFERENCE ORGANIZATION

Host Organization:	University of Tsukuba
Co-Sponsors:	Ministry of Education, Japan
	Hokkaido University of Education
Supporters' Organization:	Japan International Cooperation Agency (JICA)
	Japan Society of Mathematical Education (JSME)
	Japan Society of Science Education (JSSE)
	Sapporo City Board of Education

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