Study Lessons
in
Attached Elementary School
of the University of Tsukuba,
Hokuto Elementary School,
&
Maruyama Elementary School

Related documents

2nd Grade Mathematics Lesson Plan

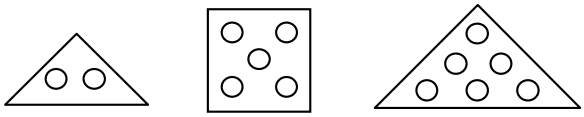
Place: Tsukuba University Attached Elementary School
Students: 2nd grade, 40 students
Instructor: Takao Seiyama

Research Theme	Thinking about a lesson that helps foster students' mathematical thinking by connecting numbers and geometry

1. Title: Placing Plates (Unit: triangles and quadrilaterals)

2. About the research theme and the learning materials

There are candies placed on small plates that are shaped like triangles and a quadrilateral, just like those shown below:



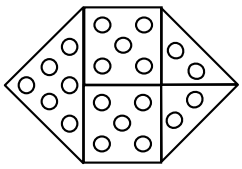
One of the tasks of this lesson is to make a large hexagonal plate by fitting together small plates like those shown above. Rules for making a large plate are as follows:

> You must fit together the small plates and make a shape that matches the large plate exactly.

Below are some examples. After you complete the task, count the number of candies.

①

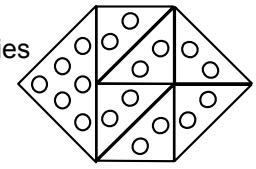
20 candies



$5 \times 2 = 10 \quad 2 \times 2 = 4$
$10 + 6 + 4 = 20$

②

18 candies

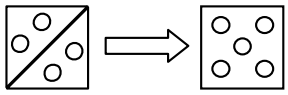


$2 \times 6 = 12 \quad 12 + 6 = 18$

Students will notice the difference between the number of candies on the various small plates by using multiplication which the students learned before to find out the number of candies. After students present various solutions to this problem, I would like to expand the lesson by paying attention to students' awareness of the problems involved.

- ○ Organize the total number of candies → e.g., 16, 17, 18, 20, 21 candies → "I wonder if we can make a large plate that has 19 candies." "I wonder if we can make a large plate that has more than 21 candies." → exploration activity
- ○ Another point that I would like my students to think about is how to make a large plate that has 19 candies. For example, if you think about increasing 1 candy from the case

of 18 candies, you can think about replacing two small triangular plates each containing 2 candies with a small square plate that contains 5 candies. (4 candies → 5 candies.)



There are two objectives for this lesson. The first one is to foster students' geometric sense through composition of geometric shapes. And the second one is to foster students' ability to think logically and understand mathematical expressions by asking them to think about the composition of geometric shapes and their matching mathematical expressions.

3. Goals of the Unit

- ○ To foster student understanding of triangles and quadrilaterals through concrete manipulative activities.
- ○ To enrich the basic learning experiences of students by composing and drawing triangles and quadrilaterals.

4. Instructional Plan (Total: 6 periods)

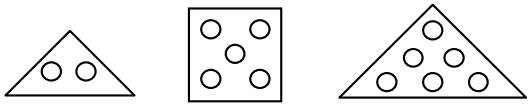
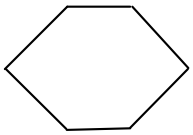
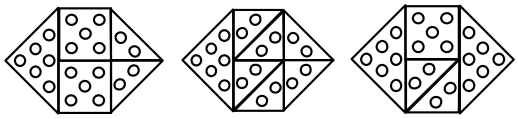
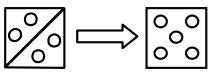
Phase 1: Meaning of triangles and quadrilaterals ----- 2 periods
Phase 2: Composition and construction of triangles and quadrilaterals ----- 2 periods
Phase 3: Summary and practice ----- 1 period

5. Instruction of This Lesson

(1) Goals:

- To foster students' geometric sense through composition of geometric shapes
- To foster students' ability to logically think and understand mathematical expressions by asking them to think about the composition of geometric shapes and their matching mathematical expressions.

(2) Process of the lesson

Main Learning Activity	Important Points for Instruction
1. Understanding the problem situation ◇ There are candies on the plates 	◆ Confirming the shapes of the plates and the numbers of candies on those plates. ◆ Pass out multiple copies of small plates for each shape.
2. Posing the problem ◇ Let's fit together the small plates to make a large plate that matches the shape of a large plate. 	◆ Pass out a worksheet that has a hexagonal shape of the large plate on it ◆ Ask the students to record their arrangement of the small plates ◆ Count the total number of candies on the large plates that are presented by the students and write down the mathematical expressions the board. ◆ Depending on how the student presentation unfolds, the teacher will ask the students why the total number of candies increased or decreased by 1.
3. Presentation e.g., 20 candies 18 candies 21 candies 	
4. Try to organize the total number of candies e.g., $16 \rightarrow 17 \rightarrow 18 \rightarrow 20$ ○ "Oh no, we don't have 19." ○ "I wonder if we can find an arrangement of small plates that gives us a total of 19 candies. ○ "I wonder what is the largest number of candies and how can we arrange the small plates?"	◆ Record the different arrangements of small plates by drawing on paper and pasting them on the blackboard. → Make it easier to rearrange the small plates.
5. (e.g.) Let's think about the case of 19 candies! ◇ "I wonder if we can use the case of 18 candies as a hint to think about this?" ○ "If we can see it like  what's shown on the right, the number of candies increases by 1,	◆ Pay attention to what students have noticed when the total numbers of candies were organized. ◆ If I can, I would like to set up the question indicated in number 5 (on the left) based on students' questions.

Grade 5 Mathematics Instruction Plan

Instructor: Yasuhiro Hosomizu, University of Tsukuba Elementary School

Research Theme	How to develop lessons designed to foster students' secure academic ability through relishing the joy of thinking

1. The topic of the lesson: Area of the circle

2. Relationship between the research theme and the lesson

Through teaching mathematics, I would like my students to develop 'secure ability' for finding problems on their own, studying by themselves, thinking, making decisions, and executing those decisions. Moreover, I would like to help my students like mathematics as well as enjoy thinking.

I chose this theme so that I can find a way to design lessons that help my students to cultivate these feelings.

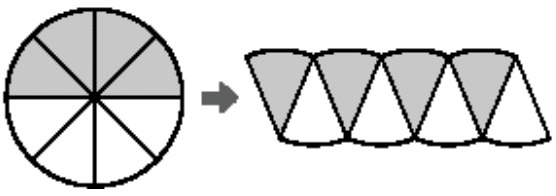
Specifically, the following are my ideas to pursue the theme.

(1) Think about multiple ways to find the area of a circle

When finding the area of a circle, I would like my students to experience multiple approaches to do so. These approaches include ways such as counting the number of the unit squares with the area of 1 cm^2 on grid paper (idea of the unit square), approximating the area of the circle by using an inscribed regular polygon and a circumscribed regular polygon (successive approximation), and segmenting a circle into basic shapes so the area can be found using the previously learned formulas for finding the area of shapes (composing and decomposing, idea of limit).

This lesson is designed for students to develop the formula for finding the area of a circle by using the approach of segmenting a circle into basic shapes. Prior to this lesson, the students already experienced the two other approaches to finding the area of a circle: approximating from the number of 1 cm^2 unit squares in the circle; and from the area of an inscribed square and a circumscribed square.

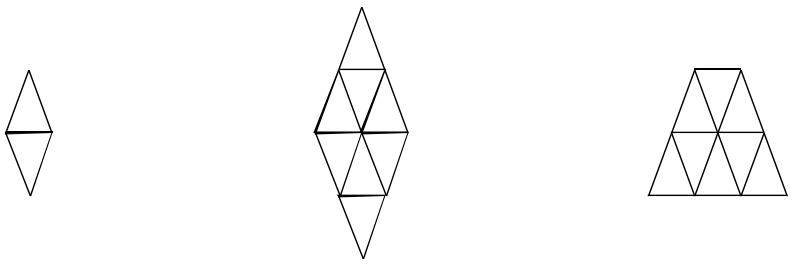
The students' textbook shows the diagram of segmenting a circle into sectors and rearranging them to make a parallelogram in order to use the formula for finding the area of parallelogram. It also shows the diagrams of the circle segmented by 16 congruent sectors, 32 congruent sectors, and 64 congruent sectors. By showing that the rearranged shape



almost becomes a parallelogram, the textbook shows that the area of circle can be found by using the formula for finding the area of the parallelogram and deriving the following formula, Area of circle = $Radius \times Radius \times \pi$.

(2) Deriving the formula for finding the area of a circle through activities such as expressing and interpreting ideas by using mathematical expressions

In this lesson, students will have the opportunity to find the area of a circle by rearranging the sectors. These sectors are made from a circle by segmenting it into 8 congruent parts, so that the previously learned formula for finding the area can be used. Then, the students will derive the formula for finding the area of the circle from them. Because manipulating a mathematical expression is very useful during this process, I will emphasize the activities of expressing and interpreting mathematical expressions.



1/8 of circumference ×(radius ×2) ÷2 ×4 (1/8 of circanference+3/8 of circumference)×(radius ×2) ÷2

1/4 of circumference ×(radius ×4) ÷2

3. Unit plan （Circle, total 10 lesson periods）

1st section Circle and regular polygons (2 lesson periods)

2nd section Length of circumference (3 lesson periods)

3rd section Area of circle (3 lesson periods) today's lesson 2/3

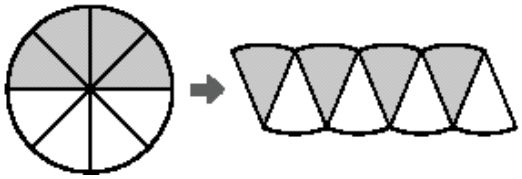
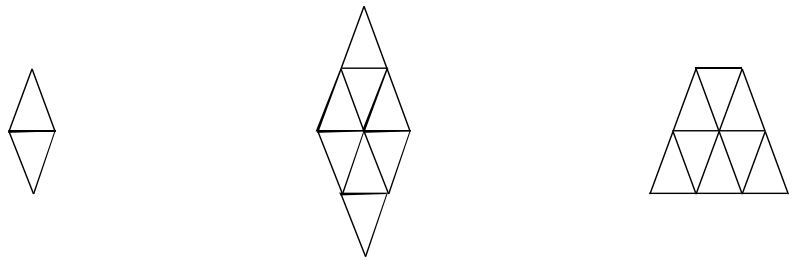
4th section Summary and application (2 lesson periods)

4. Goal of the lesson

1) Goal

Students will be able to come up with ways to find the area of a circle by rearranging the shape of the circle so that they can use previously learned formulas, and be able to derive the formula for finding the area of a circle.

2) Process of the lesson

Learning activities	Points of Consideration
1. Present the problem Come up with ways to find the area of the circle by using the sectors that are made by segmenting the circle into eight  The area of parallelogram=base×height	○ Make sure that the students are aware that the area of a circle can be found by rearranging the shapes using the eight sectors. ○ First, by find the way to rearrange the shape into parallelogram. Next, find the area of the circle through a whole class discussion. Then derive the formula for finding the area of a circle, $Radius \times Radius \times \pi$ from them. ○ Set up the situation for thinking of other methods that use other previously learned formulas for finding the areas of basic shapes. ○ Once each student comes up with a way, facilitate opportunities to exchange their ideas. ○ Using colored chalk to emphasize which sides need to find the area ○ Derive the formula for finding the area of a circle by manipulating mathematical expressions.
2. Think about different ways to rearrange the shape so that other formulas for finding the areas of basic shapes can be used Rearrange the shape and find different formulas to find the area 1/8 of circumference ×(radius ×2) ÷2 ×4 (1/8 of circanference+3/8 of circumference) ×(radius ×2) ÷2 	
3. Derive the formula for finding the area of a circle Area of a circle = $Radius \times Radius \times \pi$	Summarize that the area of a circle is π of the area of a square that has sides with the same length as the circle.

Grade 6 Mathematics Lesson Plan

Date: December 5, 2006 (Tuesday)
Place: Sapporo City Hokuto Elementary School
Class: 6B, 27 students (14 boys and 13 girls)
Instructor: Atsutomo Morii

1. Name of the lesson: "Thinking Systematically"

2. Intention of this lesson and mathematical thinking would like to foster among the students

During 4th grade, students learned how two quantities change in the unit called "investigating changes in quantities." In the 4th grade, they also learned how to express the relationship between two quantities using tables and math sentences. In addition, the students had experience reading the changes of the quantities and their characteristics.

In the 5th grade, based on their experience in 4th grade, students learned to solve problems by finding the relationship between two quantities and their regularity using tables.

The aim of this lesson is to use knowledge from prior grade levels to solve problems using tables that have more items. This lesson is included in the mathematics textbook. This lesson is not included as a part of a unit but it is set up as individual lesson. Title of the next unit is "proportional relationships." In the unit, students will construct tables, finding regularity, and expressing the relationship using math sentences. I believe this lesson is included here to help students prepare to learn about proportional relationships.

In this lesson, I anticipate that the students might solve this problem by coming up with an appropriate value and then calculating or by constructing a table. I believe that constructing a table is not a difficult task for the students because of their prior learning experiences. Moreover, I believe that many of the students will use a table to solve the problem.

The table in the textbook shows the number of pencils and ballpoint pens from 1 to 9, but in this lesson I decided to use the number from 0 to 10. This is decision relates to my hope for a certain kind of mathematical thinking that I want my students to acquire.

I would like to focus on a kind of mathematical thinking, i.e. hypothetical thinking.

Something like, "If it is …. then …."

By changing the quantities of the items in the problem on their own, the students can come up with better solution methods. In order to do that I think it is important for the students to see an extreme case in the table such as " I bought 10 items of one kind and 0 items of the other kind."

Moreover, in order for students to find better ideas to solve the problem, it is important for the students to have an opportunity to feel that they really want to do so.

Starting in April (beginning of the school year), I taught the students to look at something from a particular point of view such as "faster, easier, and accurate' when they think about something or when they compare something.

If you think about the method that uses the table from this point of view, students might notice that "it is accurate but it takes a long time to figure out" or " it is accurate but it is complicated."

In order to solve a problem in a short time and with less complexity, it is important for the students to notice that calculation using a math sentence is necessary.

To do so, how to find regularity from the table becomes a key to finding a better solution. I would like to make sure that all students understand that "the price increases or decreases by 30 yen."

Lastly, by applying the idea of "if it is … then …" to a similarly structured problem that has a different situation, I believe the students will understand the merit of doing so and build the students' desire to want to use the idea.

3. Goals of the lesson

(1) For students to notice that using a table helps them understand easily and try to use a table to solve the problem. (Interest/desire/attitude)
(2) For the students to be able to use the table to organize and categorize and investigate in logical order (expression/manipulation)
(3) For the students to find a regularity (pattern) from the table and be able to use it. Moreover, I hope the students will understand that you can solve the problem using calculations instead of using the table (mathematical thinking)

4. Process of the lesson

Students' activities and thinking process	Teacher's support
We bought pencils and ballpoint pens and the total number of items were 10 and the price was 460 yen. The price of each pencil was 40 yen and the ballpoint pen was 70 yen. How many pencils and how many ballpoint pens did we buy? If we calculate it… If we make a table…	○ Listening to the students' muttering (or voices) and pick up the idea to use a table to solve this problem. Then ask the students to fill in the table on the worksheet.

# of pencils	0	1	2	3	4	5	6	7	8	9	10
# of ballpoint pens	10	9	8	7	6	5	4	3	2	1	0
Total price (yen)	700	670	640	610	580	550	520	490	460	430	400

The answer is 8 pencils and 2 ballpoint pens!

○ I wonder if there are any rules?

If the number of pencils increases one the total price decreases 30 yen.

If the number of ballpoint pen increases one the total price increases 30 yen.

It is increase or decrease by 30 yen.

○ The price difference between the pen and the ballpoint pen is 30 yen. Therefore:
- o If the number of ballpoint pens increases by one, the total price increases 30 yen.
- o If the number of pencils increases by one, the total price decreases 30 yen.

Do we need to draw the table to find the answer?
Let's find the solution methods that are "faster," "easier," and "accurate!"

○ Ask the students to look at various solution methods from the point of view of "faster," "easier," and "accurate" and ask them to think about calculation methods that do not require the table.

If we buy only ballpoint pens the total price would be 700 yen.
$(700 – 460) \div 30 = 8$
Thus, the answer is 8 pencils and 2 ballpoint pens.

If we buy only pencils the total price would be 400 yen.
$(460 – 400) \div 30 = 2$
Thus, the answer is 2 ballpoint pens and 8 pencils.

If we buy them the same number (5 pencils and 5 ball point pens) then price would be 550 yen.
The total price that you actually paid was 460 yen and it is:
$460 < 550$
Therefore the number of pencils is more.
$(550 – 460) \div 30 = 3$
Thus the answer if 8 pencils (5 + 3) and 2 ballpoint pens (5 – 3).

If we use the idea of "If it is … then …" then we can find the answer without using the table.

○ Help students to see the value of idea for thinking the following: "If we buy only ballpoint pens…"

○ Let's solve another problem using the idea we used!

We bought colored pencils and markers and the total number of items were 12 and the price was 820 yen. The price of each colored pencil was 60 yen and the marker was 80 yen. How many colored pencils and how many markers did we buy?

○ Asking the students to solve another problem that helps the teacher and the students to evaluate student learning. Also, providing another opportunity for the students to experience the merit of solving the problem with out creating a table.

This time the total price increase or decrease by 20 yen.

$(960 – 820) \div 20 = 7$ $(820 – 720) \div 20 = 5$

We bought 7 colored pencils and 5 markers.

3rd Grade Mathematics Lesson Plan

"Multiplication Algorithm (1)"

December 6, 2006 (Wednesday)
1:35 p.m. ~ 2:20 p.m.

Sapporo City Maruyama Elementary School
Hideyuki Muramoto

- ○ The goal of the Mathematics Group at Maruyama is to develop students' ability to use what they learned before to solve problems in the new learning situations by making connections
- ○ In addition, we want to provide 3rd grade students with experiences in mathematics that enable them to use what they learned before to solve problems in new learning situations by making connections.
- ○ This lesson, "The Multiplication Algorithm (1)," is designed to utilize prior learning to make connections and solve problems in new learning situations.

Goals of the Unit:

- Lessons that enable students to consciously think about the connection between what they learned before and what they are learning now
- Lessons in which students learn from each other and that help them consciously think about their own solution processes
- An evaluation method that helps foster students' logical thinking abilities.
- Unit plan
- This lesson (goals, process of lesson)

Mathematics Group at Maruyama

Developing students who can use what they learned before to solve problems in new learning situations by making connections

1. Goals of mathematics group

In the course of study, the objectives of arithmetic (elementary mathematics grades 1 to 6) are indicated below:

"To help children acquire basic knowledge and technical skills with respect to numbers, quantities, and geometric figures through mathematical activities; to foster the ability to think insightfully and logically; and to foster the attitude to enjoy the activity and appreciate the merit of mathematical manipulation, and to willingly make use of it in day life."

2. The actual state of students at Maruyama

When we analyzed the achievement test scores of the students at our school, we found that our students are way above the scores of the national average in every domain of arithmetic, although the drop in student achievement is becoming a topic of discussion in Japan.

3. The key issue at Maruyama

Although the students at Maruyama seem to be doing well, as reported above, we feel that there are many students who are just waiting to receive instructions on how to solve problems instead of attacking new and challenging problems. We think that there are not many students who show a strong desire to tackle very challenging problems by saying, "I want to solve this problem on my own, even if it takes a long time." Also, there are not many students who enjoy solving problems by trial and error.

We believe that this is a result of lessons that have not been providing enjoyable experiences where students reach solutions on their own, see interesting regularities or patterns in the investigations, and think about and share the questions that come up from the learning with their classmates.

In order to develop students who can feel the enjoyment of mathematical learning and acquire the logical thinking skills that are stated in the course of study, we decided to develop lessons with the following point of view.

4. What kinds of lessons develop students who can use what they learned before to solve problems in new learning situations by making connections?

(1) Contents of the subject

We believe that promoting problem solving through mathematical activities will help us to achieve this objective.

Developing learning with clear systematic content connections

We believe that teachers need to be more clear about how the topics that students study are connected to each other. We need to think about "how students can use previously learned content to solve problems in new situations" or "how different problem solving situations require different kinds of prior learning," and incorporate these ideas into the development of units and lessons.

Helping students to become more conscious about the process of their own solution

We believe that students should be more aware of their own solution processes, be able to articulate how they made a connection between prior learning, and how they used the idea to solve problems in new situations.

Students acquire the ability to think on their own by drawing diagrams and number lines, reflecting on their own solution processes, determining what they do understand and what they don't understand, and comparing their own solution with those of their classmates.

We believe that students should not only focus on the rightness or wrongness of their answers, but should also reflect on their own solution processes and understand that it is important to feel the real enjoyment of mathematical learning in addition to getting right or wrong answers.

Thinking about evaluations that help students to become particular about the solution process

We need to think about what the points we need to pay attention to are to evaluate student learning in order to help students to develop the "**mathematical thinking**" necessary to conduct meaningful and effective problem solving activities.

(2) Important points of view of mathematics group

Considering the actual state of learning of the students at Maruyama and the content of the above subject, we think that it is important to develop units and lessons with the following three points of view in order to achieve the objective of "developing students who can use what they learned before to solve problems in new learning situations by making connections."

[Point of view #1]
"Develop instructional materials that pay attention to the connections between previously learned content and new content"

"It is important to clarify the mathematical thinking that students learn in 6 elementary school years by investigating the instructional materials and developmental processes of students. To do so, we need to understand how previously learned content is necessary to learn new content, and how useful it is."

(1) Connections between grade levels
(2) Connections within the domain
(3) Connection between self and problems

What students learned about multiplication in 2nd grade is not just useful for calculation. The idea they learned about "how many times a quantity is as a unit quantity", rather, is a fundamental idea in mathematics.

Furthermore, in 2nd grade students learn "length" by direct comparison, indirect comparison, and measurement with arbitrary units. Afterwards, the students who recognize the necessity of measuring with a universal unit can also learn "weight" in 3rd grade by using similar thinking.

Students who think about the "why" in the process of problem solving can begin making connections between the problem and what they need to think about, as well as how they need to think.

[Point of view 2]
"Students can learn from each other and this will help them to consciously think about their own solution processes."

There are many of new things that students can learn from each other by learning during lessons in classroom when they actually feel the merit of the mathematics and beauty and value of it.

- Students who can clarify their own solution processes and participate in the discussion to learn from each other
- Students who can learn through discovery by comparing their own thinking and that of others.
- Students who can reflect and evaluate what they do and don't understand
- Students who are particular about how they solve problems

Classroom learning experiences that foster learning from each other as explained above not only enhance student learning but also develop a strong tie among the students.

[Point of view 3]
"Evaluation that foster students' logical thinking ability"

In order for the students to be able to "think logically," we believe that they need to be particular about their own solution processes when engaged in problem solving activities.

Teacher's support for enhancing students' learning from each other

First of all, in order for students to enhance their learning by learning from each other, we thought that it very important to provide teacher support by organizing the blackboard and highlighting important points of the lesson to enhance student thinking.

Secondly, we wanted to plan appropriate supports so that students need feel like they need to think about what prior knowledge they need to recall and can make connections to the new problem situation. In addition, we wanted to include supporting questions to encourage students to think about their solution processes deeply, understand each other' ideas, including the similarity and differences in these ideas, to deepen the understanding they can gain from one another. Lastly, we prepared a second problem that helps us to understand students' learning during the lesson to support the understanding of the effect of students learning from each other in the lesson.

Teacher's support for enhancing students' learning from each other

We will administer tests in order to understand students' actual state of learning. Testing is not the only way to understand students' actual state of learning but it can be useful if teachers can use it to reflect and improve their own teaching and classroom lessons conducted.

Fostering students' expressive mathematical ability

It is important to foster students' expressive mathematical abilities by learning from each other in the classroom. Some of the examples of the abilities that we want to foster are:

- Being able to describe ideas using number lines and diagrams
- Being able to manipulate concrete materials and explain ideas to others
- Being able to think about and understand the meaning of numbers and operations as expressed in math sentences
- Being able to take notes that reflect the students' thinking and points of view

Students who acquire this kind of ability can participate in classroom learning in following ways:

At the end of 2nd grade, students begin using expressions such as "because…" to describe their reasons and support their ideas.

In 3rd grade, they begin comparing their own ideas with others and use expressions such as "my idea is similar to so-and-so's idea…"

In 4th grade, students use expression such as "for example…" and "because…," more frequently Moreover, they begin to use hypothetical statements such as "if it is… then…"

In 5th grade, they can become more sophisticated and make statements such as, "If it is … then it will be □, but if it is ◆ then I think we can say ✶" by looking at different conditions.

Finally, in 6th grade, students can start describing in ways such as, "It can be said when it is … but in this situation □ is much better," and starting to make decisions about how to choose a better idea.

We are hoping to see this expressive mathematical ability more often in the classroom, therefore, we would like to examine students' actual state of learning more carefully.

(3) What is important to the Mathematics group?

Making a better communication with other subject area groups

It is important to for the Mathematics group to establish good communication with other subject groups at the school.

Our value and view of classroom teaching and learning described before should be understood by all staff members so we can provide a consistent and systematic approach to educating our students in the school as a whole.

In order to foster "assured abilities" and "enriched hearts and minds"

We believe that "feelings and emotions" need to be involved in student learning. The "feelings and emotions" that we are talking about here are the "hopes," "desires" and "particularity" that are necessary for students to autonomously and actively be involved in their own learning. This includes the feelings and emotions expressed through words such as "I wonder why?" "If it is … then …" "I wonder if it is always true…," and, "I found this discovery…"

These are some of the things we hope to and are striving to achieve. We believe that "**the knowledge gained though feelings and emotions**" will truly help the students to acquire "assured abilities" and "enriched hearts and minds."

3rd Grade Mathematics Lesson Plan "The Multiplication Algorithm (1)"

Date: December 6, 2006
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Classroom: 3rd grade, class B (39 students)

1. Theme: Third grade mathematics lessons that foster students' ability to use what they learned before to solve problems and make connections in order to solve problems in new learning situations

We conducted a survey about mathematics learning with 3rd grade students at Maruyama elementary school.

Do you like mathematics?
Yes ---------- 50%
Somewhat yes ---------- 44 %
Somewhat no ---------- 5%
No ---------- 1%

Students that said "yes"
- I like calculations and enjoy them.
- If I understand, it is fun.
- Because answers are clear.
- Because I can listen to everybody's different ideas.

Students that said "somewhat yes"
- I like calculations, but I don't like story problems.
- It is very difficult to construct math sentences for story problems.

Students that said "somewhat no" and "no"
- I don't like story problems.
- Exams are difficult.
- It is very difficult to construct math sentences.

The 3rd grade students at Maruyama like calculations but many of them feel that they are not good at being able to construct math sentences for story problems. Therefore, the following ideas were used to develop units and lessons.

We want to increase the number of students who like thinking logically and equip them with the skills they need to understand the structures of story problems by using diagrams and number lines.

We want to increase the number of students who are interested in listening to other students' solution processes, think about "whether the solution process is similar or different", and be able to communicate.

The topics students learn in the 3rd grade are as follows:
(1) Addition & subtraction (3-digit numbers)
(2) Multiplication (2-digit & 3 digit by 1-digit by using the algorithm)
(3) Division (meaning, remainder)
(4) Large numbers (up to 10 million)
(5) Time and elapsed time (meaning)
(6) Capacity, length, weight
(7) Characteristics of rectangles and squares
(8) Shapes of boxes (characteristics & nets)
(9) Tables and bar graphs (categorizing data and constructing tables and bar graphs)

One of the most important mathematical ideas that students learn in almost all of the domains in 3rd grade mathematics is thinking about quantities in terms of "how many times as much it is as a unit quantity." For example,

- In addition, subtraction, and large numbers, we look at 1, 10, 100, etc, as unit
- In multiplication and division, we look at "how many times as much as a unit" and "dividing something into a number of units"
- In time and elapsed time, capacity, length, and weight, we look at "how many times as much as a unit"

By using the big mathematical idea of "how many times as much one quantity is as a unit quantity" as an umbrella, we can develop lessons that help emphasize this idea as well as think about everyday lessons that will help nurture this idea by, for example:
- Developing lessons that help students to become aware of the connection between what they learned before and what they are learning now and use previously learned knowledge to overcome obstacles in a new situation. (Connections between previously learned knowledge and new learning)
- Representing a problem situation with diagrams based on the idea of "how many times as much as a unit quantity" consistently and helping students to understand the situation and solution of the problem more clearly; and developing lessons that incorporate this idea and help students to use the diagram to think

about the solution of a problem logically. (Logical thinking)
- Developing lessons that help students to understand what they need to compare with. This understanding will be enhanced with lessons that pay attention to the process of solution methods that utilize previously learned knowledge. How to express the process as part of the solution and student presentation are also important. (Solution process)

We would like to help the students to be aware of the importance of the idea of "how many times as much as a unit quantity"

2. Teaching "the Multiplication algorithm (1)" in a way that develops students who can use what they learned before to solve problems in new learning situations by making connections

Goals of the Unit
- To try to think about how to calculate 2- or 3-digit x 1digit numbers by using the ideas about multiplication that they learned previously (2- or 3-digit x 1-digit number calculation by utilizing the idea of decomposing numbers or the base 10 decimal system)
- Be able to do 2- or 3-digit x 1digit number calculation using algorithm

About the Instructional Material:

The list below shows the content that students learned prior to this unit:
- 1-digit x 1-digit multiplication (grade 2)
- Multiplication involving 0, multiplication of 10's (grade 3)
- Using the idea of the distributive property of multiplication to create the multiplication table (e.g., the multiplication table of 7 can be developed by using the tables of 5 and 2.)

The purpose of this lesson is to help students think about how to multiply 2-digit numbers by 1-digit numbers. By just looking at the math sentence (e.g., 23 x 3), many students feel that the problem cannot be solved by using the multiplication tables. If students can see the structure of the problem with an array diagram, however, they can notice that they can calculate this problem by using the multiplication facts that they already learned. I want to make sure that students can see that they can utilize the idea of "how many times as much as a unit quantity" in this case also.

In this lesson, students will decompose the 2-digit number into numbers that are easy to use the multiplication tables with. Through this investigation, students will realize the merit of decomposing the 2-digit number into tens and ones (e.g., 23 → 20 and 3) to do the calculation. In addition, they will learn that this idea is the basis for the process of the multiplication algorithm (pencil and paper calculation method).

A main point of this lesson is for students to investigate how to decompose the number 23 so they can use their previously learned knowledge. It is important that students understand the merit of decomposing 23 into 20 and 3 to in order to understand the mechanism of the algorithm. In this lesson, however, I would like students to look at an array diagram to recognize the merit of decomposing the 2-digit number so they can use the multiplication table to calculate.

Learning from each other by paying attention to the solution process

Since the beginning of the school year (April 2006), I have been teaching students to draw a diagram of the problem situation in order to think about the solution. In addition, I have been emphasizing the importance of mathematical learning in class so students are able to use the diagram to explain their thinking processes logically.

There are some students in the class who already know how to do multiplication using the algorithm. Although, many of them know the calculation procedure, it is not clear if they really understand the mechanism of the calculation. The students can understand this, however, by looking at an array diagram, recognizing the meaning and merit of decomposing the 2-digit number to calculate, and generalizing the idea of "how many times as much as a unit quantity."

The solution to the problem 23 x 3 is always 69 regardless how you decompose the number 23 to do the calculation. Students will realize how different ideas for the calculation can be used by learning from each other in the classroom.

Students that are hesitant or experience difficulty with 2-digit multiplication may not be able to come up with the idea of decomposing the 2-digit number and instead may think about using addition (23 + 23 + 23 = 69). By learning from each other in the classroom and being exposed to different ideas, however, they may start thinking, "If I decompose 23, I wonder if the calculation will become easier…"

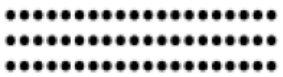
An array diagram that shows how the number 23 was decomposed differently and the math sentences that go with each different method will help these students to compare ideas and think about a better method.

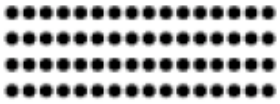
Evaluation points for fostering students' logical thinking

In order to foster students' logical thinking ability, I would like to pay attention to the following points and help students to recognize them individually and as whole class.

- Are the students using diagrams to understand the problem situation?
- Can the students show their own thinking using the diagrams?
- Can they reflect, justify, or analyze their thinking by using the diagrams?
- Can they express their thinking or thinking process by using words like "because…"therefore…" "for example…" "if it is… then…" and "since… then…"
- What point of view do the students use to compare the different ideas?
    - How the answers are different
    - How the expressions are different
    - The reasons behind their thinking
    - How previously learned knowledge is used
- Can they recognize the point and merit of comparing different thinking and the new questions that result from the comparison?
- Can the questions that evolve in the classroom students to their own ideas?

Unit Plan
(Total 13 lessons, this lesson is the 4th lesson)

	Learning Activities
1	 How many ● are there? Let's find out by calculation! ○ Because we have 3 groups of 20 dots… ○ I wonder if we can use the multiplication facts to do the calculation… ○ 20 x 3 20 + 20 + 20 ○ 20 is two 10s. ○ We can find out how many 10's are there by using 2 x 3.
2	Let's think about story problems that show the math sentence 20 x 3 ○ "Each chocolate costs 20 yen. We bought 3 of them. What is the total price?"
3	If the price of one item is 300 yen, then what is the math sentence? ○ 300 x 3 ○ This time we can think about how many groups of 100 there are. ○ We can find out how many 100's are there by using 3 x 5.
4	 How many ● are there? Let's find out by calculation! ○ This time one group is 23 dots. ○ It is about 60 dots. ○ The math sentence would be 23 x 3. ○ We can't easily calculate this using multiplication facts. ○ If we divide 23 into smaller parts then it looks like we can use multiplication facts. ○ We can use an algorithm (paper and pencil calculation menthod) to calculate. ○ 9 x 3, 9 x 3, 5 x 3 Altogether, it becomes 69. ○ 10 x 3, 10 x 3, 5 x 3 Altogether, it becomes 69. ○ 20 x 3, 3 x 3 Altogether, it becomes 69. ○ Which one of these ideas is easier to calculate? ○ All of the ideas use the idea of dividing 23 into smaller parts.
5	Let's think about how to calculate using the algorithm (paper and pencil calculation method)! 23 x 3 69 ○ Think about 23 as 20 and 3. ○ Putting together 3 x 3 and 20 x 3 ○ We can do the calculation using multiplication facts

6	 How many ● are there? Let's find out by calculation! ○ The math sentence is 16 x 4. ○ It should be more than 40. It looks like it's a lot more than 40. ○ We can do this calculation by dividing 16 into 10 and 6 just like we did before. Let's do this calculation by using the algorithm. ○ 6 x 4 = 24 We can't write 24 in the ones place. I wonder how I should write the number... ○ We can write the 2 of the 24 in the tens place.
7	Let's make many problems of □□ X □! Let's think about everybody's problems using the algorithm. ○ Some of the answers become 3-digit numbers. ○ There are answers where the tens place becomes 0. ○ There are problems that involve regrouping twice.
8	The price of 1 *m* ribbon is 312 yen. We bought 3 *m* of ribbon. How much did the ribbon cost? ○ What would the estimated answer be? ○ It should be more than 900 yen. ○ The math sentence is 312 x 3. I wonder if we can use the algorithm again for this one... ○ If we divide 312 into smaller parts, we can calculate this. ○ 300 x 3 10 x 3 2 x 3 Altogether, it becomes 936.
9	Let's make many problems of □□□ X □! ○ I made a problem where the answer becomes a 4-digit numer. ○ I made a problem that involves regrouping.
10	Let's practice calculating with the algorithm!
11	The price of a piece of cake is 60 yen. There are 4 pieces of cake in each box. If we buy 2 boxes, how much will the total price be? ○ I think we need two math sentences to solve this problem. ○ First, find out the price for 1 box. 60 x 4 = 240 We have 2 boxes of 240 yen. 240 x 2 = 480 ○ First, find out total number of cakes. 4 x 2 = 8 One piece of cake is 60 yen, so 60 x 8 = 480 ○ We can begin calculating either way.
12	Let's pracitice!
13	Let's review what we learned in this unit!

3. The learning of this lesson

Goals:

- To be able to think about how to carry out the calculation of a 2-digit number x a 1-digit number by using what was previously learned about multiplication (mathematical thinking)

The process of the lesson:

Learning activities and students' anticipated reactions and thinking	Points to remember
 How many ● are there? Let's find out by calculation!	In order to understand the problem task, help students to see the dots as "how many in a group" and "how many groups."
• There are 23 dots in a row. • There are 3 groups of 23 dots. • There are more than 60 dots. • We can find out the number of dots by counting or by addition. • I wonder if we can use what we learned before about multiplication. • The math sentence would be 23 x 3.	Before doing the actual calculation, encourage students to estimate the answer.
We can't simply use the multiplication table to do the calculation. What should do we do?	Praise the idea of recalling what the students learned before.
 Divide 23 into 9, 9, and 5 9 x 3 = 27 9 x 3 = 27 5 x 3 = 15 27 + 27 + 15 = 69 69 dots	Try to understand students' different ideas by walking around the classroom.
 Divide 23 into 10, 10, and 3 10 x 3 = 30 10 x 3 = 30 3 x 3 = 9 30 + 30 + 9 = 69 69 dots	When you find students that are solving the problem with addition, ask them, "Can you use multiplication for this calculation?"
 Divide 23 into 20 and 3 20 x 3 = 60 3 x 3 = 9 60 + 9 = 69 69 dots	Make sure to use the diagrams to represent how the calculations were done.
If we divide 23 into smaller parts, we can use many different multiplication facts from the multiplication table to do the calculation.	When students are learning each other's calculation methods through the students' presentations, make sure that they consider how the methods are related.
Which one of them do you think is a good idea? What are the similarities or differences among the different solutions?	Make sure to encourage the students to compare the different ideas and help them to make a conscious effort to make their own value judgments about the different ideas.
• All of the methods decided to divide 23 into smaller parts. • There are methods that involve dividing 23 into 3 parts and 2 parts. • The numbers used in the math sentences are different. • If we use the multiplication 20 x 3 that we learned previously, we have two math sentences.	If I find a child is using an algorithm to do the calculation, I would like to ask the child to think about how the calculation

• I used an algorithm (paper and pencil calculation) to do the calculation 23 x 3 69 If we compare this method and the array diagram… • This method also divides 23 into 20 and 3. If we divide a number into some small parts so we can use the multiplication facts, we can do the calculation in today's problem. The idea we used in the algorithm (paper and pencil calculation) is similar to the idea of dividing 23 into 20 (2 in the tens place) and 3 (3 in the ones place).	method is related to the array diagram. Make sure to highlight the idea of "make the calculation easier by using the multiplication table and the other kinds of multiplication the students learned before." Also, if a child shows the algorithm calculation method, then help him/her to consciously connect the idea with this idea.