SOME REMARKS ON MATHEMATICAL THINKING IN EDUCATION

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For the past two decades, there is a new movement in the mathematical education of elementary school. The new pedagogy particularly emphasizes the importance of constructing elementary mathematics by students themselves. However, this new movement recently has been criticized as, "has crippled students with its de-emphasizing of basic drills and memorization in favor of allowing children to find their own ways to solve problems", recently reported by New York Time in the article *As Math scores lag, a new push for the basics*.

The difficulty that mathematics constructivism would ever meet with is not unexpected. After all, any success of the new pedagogy completely depends on the core of the mathematical education: the mathematical ability of teachers. The required ability is not only to ask teachers should have ability for solving problems, but also should understand the whole structure of mathematics and the developing process of mathematical cognition for the period of elementary school. In fact, as far as I know, there exists no such comprehensive reference about this subject which is helpful to

mathematical teaching.

The traditional way of training teachers is to provide more advanced mathematics course for teachers. For examples: course for linear algebra, calculus…etc. But, this kind of training course never works to help teachers improving their mathematical teaching ability. In this short article, we are not going to discuss this. But, if possible, I should take an example of the article *Mathematical literacy for living from OECD-PISA perspective* by Professor Jan de Lange to depict the minimum of the mathematical ability for a mathematical teacher to carry out such a task. Also see the article: *Reaching for common ground in K-12 mathematics education,* which appeared in the Notices of AMS, October 2005.

Recently, there is a trend in mathematical education to emphasize the mathematical thinking, more than to ask students mastering computational skills. As a mathematician, I certainly understand how important the mathematical thinking is. However, in many instances, the mathematical thinking is often misunderstood as "creative pedagogy". If you carefully exam those cases of creative pedagogy, you find most of them are kind of improvisations for creative pedagogy, full of creative improvisation but no mathematics at all! This is one of the key window we are considering: **Improvisation for creative pedagogy vs. Mathematical thinking.**

Alone the reasoning with the above paragraph, I would like to make some remarks to stress the importance of assessing quality of mathematical thinking. We should keep in mind that developing mathematical thinking is not only for fun, but with the definite purpose for helping students to learn mathematics. Thus, the question whether a program of mathematical thinking fits or not becomes essentially important when we come to **asset** the mathematical thinking. The assessment should be a key window of mathematical thinking.

While designing the curriculum in mathematics, we should know mathematics is not only a local stream of logic sequence, but should be designed according to student's cognitive ability at different stages. In my opinion, it is better that mathematical thinking should be developed alone with curriculum. Neither any mathematical thinking in the program should be picked up randomly, nor a mathematical-thinking-developing curriculum is a collection of isolated ideas. Instead ,it should be interwoven together as a whole, so that mathematical ideas could be developed further at the next stage. This assessment we call it as: **Fragmentary vs. Continuation.**

There are several goals for each level (elementary, junior high, senior high) of mathematical curriculum. Mathematical courses of each year are building blocks of the whole mathematical curriculum for each level. Indeed, the developing mathematical thinking for earlier grades should be mathematically useful to stages of upper grades. As we know, mathematics becomes more and more sophisticated and solving a problem in upper grades often requires many different type of mathematics.

Therefore, each type of mathematical thinking should be suitably developed at an earlier stage. Mathematical thinking at different grades should be connected in the whole curriculum. This kind of quality assessment we call it as: **Localization vs. Globalization.**

From the view of developing mathematical thinking, mathematics reading is an important key window. Only through reading mathematics, students can develop their own logic sequence, develop connection of several different concepts, and develop deeper thinking.

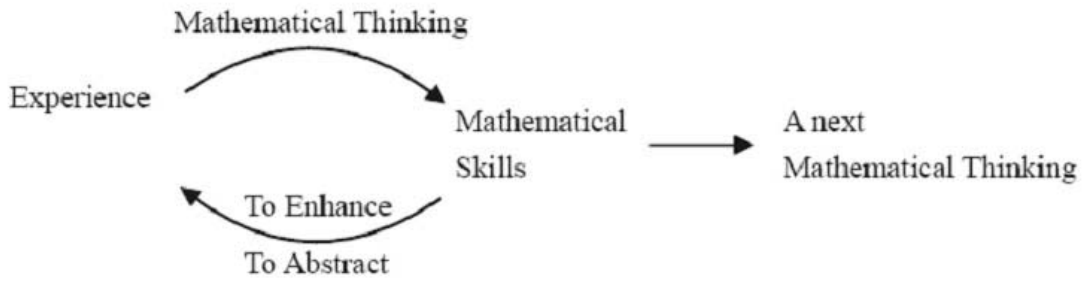
Also, reading mathematics is what students could do after class. After mathematical classes, students have rare chance to encounter mathematics in daily life. This is not the same as other curriculums. While students watch T.V. at home, they have chances to encounter subjects such as: language, history, geography, biology, even nature science but not mathematics.

We should know mathematics is a language of nature. Like learning language, students should be given enough time to be familiar with many different ways of the mathematical thinking. Reading mathematics could base on what he knows and construct his (her) own mathematical thinking. In this way, it certainly can help students to feel more comfortable with mathematics. Hence, it is important to have mathematics books (in addition to textbook) which students can read.

While reading some articles from sources of this conference, I find the description on the mathematical thinking are very abstract, more abstract than mathematics. I can imagine there are many different ways to see what mathematical thinking is by different people. Certainly, this is a good opportunity in this conference to discuss how to define the mathematical thinking and to clarify what is mathematical thinking. I think it is important because it can avoid a lot of misunderstanding while globally carrying out the mathematical thinking in the classroom.

For me, the mathematical thinking is a certain process of performing mathematics. To begin with, a mathematical thinking is inspired or is motivated by something related to daily-life experiences. Through the guiding of student's experiences or imagination, this mathematical thinking can be developed into some mathematical skills (not only mathematical algorithms) or certain mathematical concepts.

In turn, the mathematical skills or the mathematical concepts could enhance and abstract student's experience. The fruit of the whole process would be the base of a next mathematical thinking. In short, we have the diagram:



The above is a very simple diagram. In the conference, if I have time, I will give some examples.

Of course, to develop mathematical thinking is very complex. Even for a course in one semester, the content of mathematical thinking should be like a story book containing many different styles of stories.