PERSPECTIVES ON MATHEMATICAL THINKING IN KOREA

Inchul Jung

Chonnam National University, Republic of Korea

QUESTION 1 : HOW MATHEMATICAL THINKING IS DEFINED IN YOUR CURRICULUM DOCUMENTS AND YOUR LESSON?

To begin with the posed question for the definition of mathematical thinking in Korea, I would like to introduce the brief history of curriculum of Korea so that you can understand better. Korea has a long history of emphasizing theoretical mathematics for a selective examination since the "Koryeo" Kingdom (918-1392AD) introduced mathematics into its examination system to select government officials (Needham, 1954, p. 139). There were 7 stages of mathematics curriculum in the late 20th century. The first three stages are mainly about theoretical perspective of mathematics, next three stages are considered to be the transition, and finally the last stage is about ‘Practical mathematics’, which has mainly focused on the learner’s perspectives.

Curriculum	Period	Main Focus	Remarks
1st Curriculum	1955-1963	Real life Centered	Theoretical Mathematics
2nd Curriculum	1964-1972	Mathematics Structure Centered	
3rd Curriculum	1973-1981	“New Math” Oriented	
4th Curriculum	1982-1988	“Back to Basics” Oriented	Transition
5th Curriculum	1989-1994	“Problem Solving” Oriented	
6th Curriculum	1995-1999	Problem Solving & Information Society Oriented	
7th Curriculum	2000-	Learner Centered	Practical Mathematics

The 1st mathematics curriculum can be characterized as real life experience centered curriculum, which was influenced by Progressivism in the U.S. which valued learner's experience in real life. Because this curriculum regards the school subject mathematics as a tool for the betterment of living, the structure or the system of mathematics was ignored. Thus, the contents of the mathematics curriculum were in low level and mainly life-problem oriented. Lenience and ignorance in the mathematics structure of the 1st mathematics curriculum caused the decline of students' mathematics achievement, which necessitated the 2nd curriculum revision. The focus of the 2nd curriculum was systematic learning and placed great value on the logical and theoretical aspects of mathematics, and pursued the improvement of students' mathematical abilities. The 3rd mathematics curriculum was influenced by New Math, which occurred as the result of the discipline centered curriculum and mathematics modernization movement. The 3rd curriculum attempted to introduce abstract but fundamental ideas (for example, sets) early in the curriculum and to

[1] The answers to three questions posed for this conference are based on ‘The Report on Mathematics Education in Korea’ (2004) presented at Copenhagen in Denmark, and doctoral dissertation of Inchul Jung(2002).

continually return to these ideas in subsequent lessons, relating, elaborating, and extending them. Bruner's discovery learning was also crucial element in the 3rd curriculum. The 4th mathematics curriculum started from the failure of New Math and the emergence of the Back to Basics Movement in the U. S. Students' basic computation skills were weakened due to the structural approach to mathematics of the 3rd curriculum. Thus the 4th curriculum reduced contents, lowered the level of difficulty, and emphasized obtaining of minimal competencies in mathematics. The 5th mathematics curriculum basically maintained the tradition of the 4th curriculum. The main direction of revision was to emphasize students' mathematical activities in mathematics class, and to consider affective aspects of learning mathematics. From this period, keeping in step with the current social trends, the mathematics curriculum started to take the informative society into account. The 6th mathematics curriculum is not so much different from the previous one. The 6^{th} curriculum increasingly stresses mathematical thinking abilities by the way of fostering mathematical problem-solving abilities. This curriculum period especially emphasized the necessity of discrete mathematics in school mathematics.

Whereas, the 7th curriculum emphasizes various types of instruction to improve efficiency and significance of students' mathematical learning. It recommends that students should be able to experience the joy of discovery and maintain their interest in mathematics by pursuing the following instructional methods in their classrooms: to emphasize concrete operational activities in order to help students to discover principles and rules and solve problems embedded in such a discovery; to have students practice basic skills to help students be familiar with them and problem-solving abilities in order to use mathematics in their everyday life; to present concepts and principles in the direction from the concrete to the abstract in order to activate self-discovery and creative thinking; to induce students to recognize and formulate problems from situations both within and outside mathematics; to select appropriate questions and subsequently provide feedback in a constructive way in order to consider the stages of students' cognitive development and experiences; to use open-ended questions in order to stimulate students' creativity and divergent thinking; to value the application of mathematics in order to foster a positive attitude toward mathematics; to help students understand the problem-solving process and use basic problem-solving strategies in order to enhance students' problem-solving abilities.

The 7th curriculum encourages that mathematical power should be evaluated by realizing the following evaluation methods in their classrooms: to emphasize processes more than products in order to foster students' thinking abilities; to focus on students' understanding of a problem and the problem-solving process as well as its results in order to evaluate students' problem- solving abilities; to focus on students' interests, curiosity and attitudes toward mathematics in order to evaluate students' mathematical aptitudes; to focus on student's abilities to think and solve problems in a flexible, diverse and creative fashion in order to evaluate mathematical learning; to use a variety of evaluation techniques such as extended-response questions, observations, interviews as well as multiple-choices in order to evaluate students' mathematical learning.

Mathematical thinking, in conjunction with problem solving, has been consistently emphasized as the most important part of students' mathematical experience throughout their school years. Since thinking mathematically is a conscious habit, it should be developed through consistent use in many contexts. Being able to reason is essential in making mathematics meaningful for students. In all content areas and at all grade levels, consequently, students need to develop ideas or arguments, make mathematical conjectures, and justify results.

The question of "why do you think so?" is the most salient feature in Korean elementary textbooks that are designed to elicit students' account of how they accomplish a given task. In fact, nearly all activities of textbooks include such a question at the end. Simply knowing the answer or solving a given problem itself is not enough. Thanks to questions such as "why do you think it is always true?", "why do you think so?", or "how do you know?", students come to realize that statements need to be supported or refuted by evidence, or something that is mathematically acceptable as an adequate argument. As students experience mathematical reasoning over and over, they come to know that mathematical reasoning is based on specific assumptions and rules.

QUESTION 2 : WHAT IS A KEY WINDOW FOR CONSIDERING MATHEMATICAL THINKING?

Mathematical thinking is deeply related with learning with understanding which has been recognized as one of the most important areas in mathematics education research. We have a reasonable description of mathematics understanding although there is not an agreed upon precise definition. I would like to start with the general concepts and essential characteristics of understanding addressed by Brownell and Sims (1946, pp. 28-43). First, we may say that a pupil understands when he is able to act, feel, or think intelligently with respect to a situation. The term, 'situation,' is used to mean any set of circumstances to call for an adjustment, in other words there is "no obvious way" (Mayer, 1985, p. 123) or "the direct route" (Kilpatrick, 1985, p. 3) in resolving the circumstances. A situation may be equated with 'problem,' which "occurs when you are confronted with a given situation - let's call that the *given state* - and you want another situation - let's call that the *goal state* - but there is no obvious way of accomplishing your goal" (Mayer, 1985, p. 123). Second, rather than being all-or-none affairs, understandings vary in degree of definiteness and completeness. It is generally assumed that the completeness and definiteness of our understandings may vary directly with the amounts and kinds of experiences we have had. And the degrees, qualities, and kinds of understandings manifested by the several members of a group of children may vary for the given situation. Third, the completeness of understanding to be sought varies from situation to situation and varies in any learning situation with a number of factors. In order to understand anything in the sense of completeness, one is required to have a thorough grasp of its function, structure, and incidence. Fourth, typically, the pupil must develop worth-while understandings of the world in which we live as well as of the symbols associated with this world. Fifth, most understandings should be verbalized, but verbalizations may be relatively devoid of meaning. Sixth, understandings develop as the pupil engages in a variety of experiences rather than through doing the same thing over and over again. Seventh, successful understanding comes in large part as a result of the methods employed by the teacher. Eighth, the kind and degree of the pupil's understanding is inferred from observing what he says and does with respect to his needs.

Haylock (1982) defines understanding something as "to make (cognitive) connections" (p. 54). He also claims that the more connections a learner can make between the new experience and previous experiences, the deeper the understanding is. The new experience sometimes connects previously unconnected experience. If this happens, drastic advance in understanding is expected. However, if students fail to make connections between the new experience and previous experiences, the new experience will be presented as an isolated chunk and float around in an unstable situation, which is called 'rote learning.' If that is the case, it is doubtful whether concepts learned through rote learning can be fully used whenever they are needed. Probably, it is hard to keep the value of new experience, and furthermore it will be lost forever at some point.

The delicate issue of Haylock's model is how we recognize the evidence of student's understanding, i.e. something that indicates that students made connections. Haylock regards this task as one of the most important tasks for the teacher because the teacher can reward and therefore encourage students. Haylock suggests four components should be considered for identifying important connections in mathematics: words, pictures, concrete situations, and symbols (Figure 1). It is assumed that students can demonstrate some degree of understanding and that they can make a suitable connection between categories. Thus any arrow in the figure may

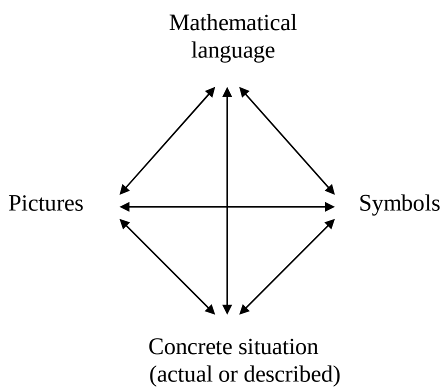

Figure 1. Four components of identifying connections

suggest means of assessing an aspect of understanding for the given mathematical ideas.

For example, "*Write in figures four hundred thousand and seventy-three*" (p. 55) can be accepted as an item to assess the connection from mathematical language to symbols. The next picture problem (Figure 2) can be regarded for the connection from pictures to symbols. The problem 'to write a story' for a calculation like 84 ÷ 28 will be the item for the connection from symbols to a concrete situation.

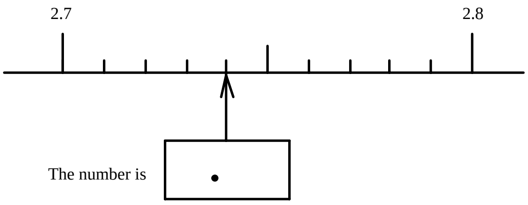

Figure 2. Connection form pictures to symbols

Wiggins (1993) defines understanding as the ability to use knowledge "wisely, fluently, flexibly, and aptly in particular and diverse contexts" (p. 207). Hiebert and Carpenter (1992) define understanding based on the way information is represented and structured. "A mathematical idea or procedure or fact is understood if it is part of an internal network. More specifically, the mathematics is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and the strength of connections. A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections" (p. 67). They claim that understanding grows as the networks within students' mind become larger and better organized.

> Understanding increases as networks grow and as relationships become strengthened with reinforcing experiences and tighter network structuring .... Growth can be characterized as changes in networks as well as additions to networks. ... Ultimately understanding increases as the reorganizations yield more richly connected, cohesive networks." (p. 69)

It is argued that understanding is "neither inherently hierarchical nor the product of incremental teaching methods, but a natural consequence of curiosity, experience, reflection, insight, and personal construction" (Hannafin & Land, 1997, p. 181) and "involves continually modifying, updating, and assimilating new with existing knowledge. It requires evaluation, not simply accumulation" (p. 189). Pirie and Kieren (1989) summarized understanding as follows:

> Mathematical understanding can be characterized as levelled but non-linear. It is a recursive phenomenon and recursion is seen to occur when thinking moves between levels of sophistication .... Indeed each level of understanding is contained within succeeding levels. Any particular level is dependent on the forms and processes within and, further, is constrained by those without. (p. 8)

Thus, mathematical thinking is very closely somehow related with understanding.

QUESTION 3 : HOW CAN WE DEVELOP MATHEMATICAL THINKING THROUGH THE LESSON?

I would like to mention the answer for this question relating this issue with the curriculum. Compared to Western countries, Korea has a very short history of modern mathematics education and a short curriculum revision term as well. This supposedly could be a main reason why we had not have invested enough times and efforts in curriculum revision and construction study for a new mathematics curriculum. Thus we cannot escape from the blame that the background philosophy of the mathematics curricula usually follows those of foreign curricula, lacking our own educational philosophy. However, nowadays one fortunate thing is that Korean students have become to show extraordinary abilities in mathematical exploration and high achievement levels in mathematics learning in international mathematics achievement competitions. This might symbolize that Korean mathematics education and the mathematics curriculum have not been so much slapdash. Finally, reflecting the past mathematics curriculum revision processes we are going to discuss further about the future directions of the mathematics curriculum.

1. Construction of Our Own Curricular Philosophy

In Korea the main blaming that is targeted towards the policy of mathematics curriculum revision has been that our mathematics curriculum has not contained our own philosophy in terms of the mathematics curriculum and usually followed those of foreign curricula. Although western countries' philosophy of mathematics education is introduced so well in Korea that does not mean that it can be directly imported as our own philosophy of mathematics education (Park, 2003). For example, in Korea attempts are being made to introduce the social process of creating knowledge into mathematics classes. Class activities using cooperative small group learning are also being encouraged in an effort to let more students participate in discussion and the social negotiation process. The widely-discussed method of small group cooperative learning doesn't seem to sit well with Korean students. This is because they are traditionally taught not to doubt the teachings of their ancestors or the great men of past generations, let alone argue against it. However, students of the West have been trained at an earlier age to actively engage in and take advantage of small group cooperative learning. In contrast, small group activities do not make much sense to students in Korea, as they have never received this type of training. This could be a typical example that demonstrates the fact that western philosophy cannot be immediately transplanted to Korea.

On the other hand, one interesting observation with relation to this fact is that, while the East makes efforts to follow its Western counterpart, the West makes endeavors to take after the East. For example, educational experts in the U.S. are very much interested in Singapore's mathematics textbooks and have tried to discover what is securing Singapore in a top position in the TIMSS(Third International Mathematics and Science Study) and TIMSS-R in the category of mathematics textbook.

Otherwise, Western scholars are amazed about how Japanese mathematics textbooks are so small and thin and yet display core ideas so economically. In such ways, both the East and the West are benchmarking each other in the mathematics education field. That is, we need to strive to find out what is the most optimal philosophy of mathematics education, that incorporates our own way of thinking and circumstances rather than indiscreetly following the western mathematics education.

2. Optimization of Mathematics Contents

One of the main objectives of a curriculum revision is to determine the appropriate amount and the level of depth and difficulty of educational contents. Since the 4th curriculum, curricula have been revised under the basic principle of reduction of the amount and lowering the difficulty level in order to accomplish the optimum amount and difficulty of educational contents. Furthermore, the 7th curriculum policy specifically instructed 30% reduction, which the mathematics curriculum was unable to fully comply with. Accordingly, the next mathematics curriculum revision should be more proactive in reducing the amount and lowering the level of difficulty of mathematics contents. At the same time, the topics to be omitted should be determined based on more comprehensive perspective and systematic consideration rather than considering simply educational conveniences.

However, in pursuit of the optimization of educational contents, we are bound to encounter some kind of educational dilemma (Park, 2003). That is, how can an optimal level of school mathematics be decided? By how much should we reduce the amount and to what extent should we lower the level of difficulty? Even if we agree with the fact that the majority of students find mathematics difficult and we reduce the amount and lower the difficulty level, there will likely still be complaints that mathematics requires much work and is difficult. This is because of the abstract and deductive nature of mathematics. We cannot lower the level of difficulty too significantly because we have to consider mathematically superior students as well. In order to satisfy two different types of students, we must move away from inflexible practices such as imposing the same amount and level of difficulty of mathematics to the entire group of students.

Therefore, instead of indiscreetly reducing the amount and lowering the level of difficulty, it is recommended to divide the contents into two core contents and optional contents. In fact, the 7th curriculum attempts to divide the contents into a core section and an optional one, and these are explicitly stated in the curriculum document. However, optional contents tend to function as core contents for all the students because in Korea, the topics in curriculum are considered as a minimum essential. Hence, it is necessary for the next revised curriculum to clearly mention that optional contents are for mathematically superior students and strictly differentiate optional contents from core contents (Park, 2003).

3. Complement of the Differentiated Curriculum

Ever since the introduction of the differentiated curriculum in the 7th curriculum, the drawbacks of implementing the differentiated curricula providing differentiated educational contents depending on the different levels of students have long been confidentially talked about. However, judging from the current tendency of educational philosophy, such a differentiated curriculum system is likely to be continued to the next curriculum revision along other complementary measures. According to Park(2003), the idea of adopting differentiated curricula for different levels of students has been criticized for not adhering to the East Asian tradition portrayed in Collective We-ness. The East Asian culture believes in orthodoxy, and students are expected to adhere to a uniform curriculum despite their individual differences. In the Western culture however, the individual is of paramount importance. Hence the curriculum must be adjusted to the needs of the individual rather than the individual adjusting to an orthodox curriculum(Leung 2001; Park & Leung 2002).

Nevertheless, if the differentiated curriculum that is first applied in the 7th curriculum with much expectation ends with no tangible results and does not continue later, it may cause even more confusion (Park 2003). Therefore, it seems to be reasonable to maintain the differentiated curriculum by complementing the drawbacks of the curriculum in the next curriculum and attempt to gradually stabilize it.

4. What and How to Teach Mathematics in the Next Curriculum

Prospective students must be provided with experiences that will cause them to become active, flexible thinkers and users of mathematics. It is critical that all students regardless of ability be involved in and see themselves reflected in the mathematics curriculum. It is essential that possibly all students are engaged in a program that contains appropriate mathematical content and learn the content to form a knowledge base. The program must be one which can be expanded in the future to broaden their career and economic horizons and allow the students to adapt with the changing times.

The primary focus of the mathematics curriculum is to help students become good problem solvers. Learning experiences must cycle between using problems to motivate knowledge base development and using the knowledge base to solve problems. To accomplish this, problem solving should include the processing of information, thinking analytically, coping with changes, and making decisions by using mathematics with varying degrees of sophistication. Classroom instruction should provide for a natural learning sequence with allows for the transition from concrete to semi-concrete to semi-abstract to abstract learning experiences.

The instructional climate must allow students to communicate their mathematical ideas freely and to turn mathematical errors into positive learning experiences. Discussions should occur using the language of mathematics to verbalize the processes used to develop concepts and solve problems. Experiences should be designed to allow students to interact with each other and the teacher while attaining a knowledge base and solving problems.

Since students comprehend mathematics through various learning strategies, a variety of evaluation techniques must be employed. Evaluation of the students' knowledge base and problem-solving ability must not be limited to only paper-and-pencil testing. More minutely saying, future mathematics curriculum should be able to guide mathematics education to the following goals. Students will be provided experiences which: (1) emphasize problem solving and thinking skills; (2) give a broad perspective to the mathematics content structure, and the interrelationships among the various structural branches; (3) consider different learning styles by using a variety of instructional strategies and materials; (4) emphasize a participatory role for learning by using mathematical language, oral discussion, writing, listening skills, and observing skills; (5) create mutual respect and equal treatment regardless of ability; (6) expand career and economic horizons; (7) incorporate technology as a thinking and learning tool; (8) assess performance through a variety of evaluation techniques.

The basic principle in developing mathematics textbooks is to follow and specify what the curriculum intends. The most recently developed seventh curriculum has a level-based differentiated structure and emphasizes students' active learning activities in order to promote their mathematical power, which encompasses problem solving ability, reasoning ability, communication skills, connections, and dispositions. This curriculum resulted from the repeated reflection that previous curricula were rather skill-oriented and fragmentary in conjunction with the expository method of instruction, and that previous curricula did not consider various differences among individual students with regard to mathematical abilities, needs, and interests (Lew, 1999). The main motivations to the current curriculum include increasing concern for individual differences and the desire to provide maximum growth of individual students on the basis of their abilities and needs. Given the curriculum, mathematics textbooks intend to provide students with a lot of opportunities to nurture their own self-directed learning and to improve their creativity. To accomplish this purpose, several directions are established in developing mathematics textbooks.

First, textbooks should consist of mathematical contents with which individual students can improve their own creative thinking and reasoning ability. At some point in a learning sequence, instructional resources are presented differently on the basis of individual differences of mathematical attainment. Whereas high-achieving students confront with advanced tasks including real-life complex situations, low-achieving counterparts solve basic problems involving the fundamental understanding of important mathematical concepts and principles. Most Korean mathematics elementary textbooks developed under the previous curriculum have been translated into English and analyzed through Truman Faculty Research grants, Eisenhower Foundation funds and the National Science Foundation Award (Grow-Maienza, Beal, Randolph, 2003; see also http://eisenhowermathematics.truman.edu)

Second, textbooks should consist of mathematical contents which contribute to improving the process of teaching and learning. Most of all, textbooks have to underline a learning process by which students solve problems for themselves through individual exploration, small-group cooperation activities, or discussion.

Third, textbooks are to be easy, interesting, and convenient to follow on the part of students. For instance, instructions of games or activities in the textbooks should be specific enough for students to initiate them without a teacher's further explanation and demonstration. Textbooks should take into account students' various interest and stimulate their learning motivation. Textbooks should also consider various editing, design, and readability for students. Textbooks should also deliberate the appropriate use of different multimedia learning resources.

Fourth, textbooks are to be flexible in a way that teachers refine or even revise them reflecting on the characteristics of their schools or provinces. A textbook should be recognized not as the sole material to be followed but as an illustration of embodying the idea of the curriculum.