DEVELOPING STUDENT'S MATHEMATICAL THINKING THROUGH LESSON STUDY IN THAILAND

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INTRODUCTION

In Thailand, one of the major goals of organizing school mathematics teaching activities is to help the learners to acquire basic mathematical knowledge as well as being good at mathematical thinking. However, the current practice of mathematics teaching activities in Thailand does not seem to suffice to help develop the students' mathematical thinking. This is due to the fact that in the traditional Thai classroom culture most of the mathematics teachers either are incapable of encouraging their students to express their mathematical thinking freely or the teachers themselves lack a clear understanding on mathematical thinking. To organize mathematics teaching activities for the development of mathematical thinking the teachers who play the most significant role in organizing the activities will need first to alter their own mathematical thinking. They will also need to change their instructional culture from emphasizing rote learning of mathematics content, laws, formular or theories for the development of mathematical thinking to the type of activities that will allow the students to develop mathematical thinking for themselves. This is not an easy task.

To be able to efficiently organize teaching activities for the development of mathematical thinking the teachers will need to rely on some kind of innovation that will enable them to comprehend the significance of change in instructional culture. Lesson Study is one of the most promising innovations that will help the teachers appreciate old and the new mathematics instructional cultures. This paper will describe how mathematical thinking is defined in Thai curriculum documents. Then, it also describes how Lesson Study has been introduced into Thai school culture in order to develop mathematical thinking.

HOW IS MATHEMATICAL THINKING DEFINED IN THAI CURRICULUM DOCUMENTS?

In Thailand, curriculum documents which include mathematics textbooks provide some ideas on what it means to 'mathematical thinking'. Below is how mathematical thinking has been described in curriculum documents in the last 20 years.

1) Mathematical Thinking in Thai mathematics curriculum and textbooks in the Pre-Education Reform Era (1980s – 1990s)

According to the 1979 mathematics curriculum (revised in 1990), parts of curriculum can be considered to emphasis on mathematical thinking are as follows:

(1) Being having knowledge and understanding of basic mathematics and skills in Computation
(2) Be able to think rationally and express their thinking systematically and precisely
(3) Valuing mathematics and positive attitudes toward mathematics
(4) Be capable of applying knowledge and skills from classroom experiences into daily life

As just described, the ideas concerning defining mathematical thinking in the Thai mathematics curriculum in the pre-education reform era. were not clear as to the aims and directions of the definition. In fact, there was not even any explicit definition of mathematical thinking, although there was a stated aim for teaching mathematics to the extent that mathematical thinking was tied up with computation skills as stipulated in the textbooks. To illustrate it, Inprasitha (1997) analyzes the characteristics of Thai mathematics textbooks. For textbooks of elementary school, it is found that 1) the first part of each unit contains routine exercises for drilling computational skills, 2) word problems appeared at the end of each unit and these word problems are routine exercise like that of Milne's textbooks, 3) most of the word problems in the first and second grades require the students to write symbolic sentence before solving the problems, 4) almost all of exercises and word problems have one and only one correct answer, and are well formulated. Similarly, for textbooks of secondary school, it is found that 1) most of the classroom exercises are designed for leading to conclusion of some rules or principles, 2) the exercises at the end of each section are designed to drill some rules or principles which have been learnt, 3) word problems at the end of each section are designed for students to apply some rules or principles, 4) tests at the end of each unit are designed to revise the rules or principles. An example of mathematical exercises can be seen in an appendix A.

To sum up, the nature of mathematical thinking as appeared in the mathematics curriculum and what the teachers actually do in their classrooms when they use textbooks can be clearly comprehended in the light of how a student is viewed as being capable of doing mathematical thinking. For instance, if a student is to be considered capable of doing mathematical thinking he must be able to do fast calculations, to solve problems correctly and in the styles prescribed by the teachers, to have some special computation techniques, and to make good scores on objective achievement tests. Furthermore, the curriculum usually emphasizes mainly on the content than mathematical thinking process of the students. Priority was given to the students' ability to solve mathematics problem and less to anything else.

2) Mathematical Thinking in Thai mathematics curriculum and textbooks in the Pre-Education Reform Era (1980s – 1990s)

The mathematics curriculum in the current education reform era (1999-present) was based on the principles of curricular development of the NCTM of the U.S.A, which emphasizes the integration of five content areas with five mathematical processes. It

has also made some cognitive aspects of mathematics learning and the desirable attributes more distinct. Mathematical thinking is emphasized in the 2001 new Basic Education Curriculum as follows:

Basic Education Curriculum A.D. 2001

(1) Numbers and numerical works

- Understanding of number exposition diversity and application in real life.
- Understanding of outputs of numerical works and their relationship, ability to apply numerical works in problems solving.
- Application of estimation in computation and problem solving.
- Understanding numerical systems and ability to apply numerical property.

(2) Measurement

- Understanding measurement foundation.
- Measurement and estimation of size of objects to be measured.
- Solving measurement problems.

(3) Geometry

- Capacity to explain and analyze two and three dimensions geometric figures.
- Application of visualization, spatial reasoning and geometric model for problem solving.

(4) Algebra

- Capacity to explain and analyze algebraic pattern, relationship and various functions.
- Capacity to apply algebraic expression, equation, non-equation, graph and other Mathematics models to represent various situations; interpretation and problem solving.

(5) Data analysis and probability

- Understanding and capacity to apply statistic procedures for data analysis.
- Capacity to apply statistic procedures and probability, knowledge in reasonable estimation.
- Capacity to apply statistical knowledge and probability in decision making and problem solving.

(6) Mathematics skills and procedures

- Problem solving capacity.
- Reasoning and justification ability.
- Communication ability, presentation and interpretation of mathematics knowledge.
- Capacity to relate various disciplines within mathematics and outside.
- Creative thinking.

Even though the mathematics curriculum in the reform era puts greater emphasis on mathematical skills, the process and its content, the definition of mathematical thinking remain a perplexed matter that the teachers found it difficult to comprehend. Worse still, most of the school mathematics teachers regard mathematical thinking something essentially related to computation and emphasize the ability to solve problems in the styles they prefer. Therefore, according to these teachers, the students who are capable of doing mathematical thinking are the same as those who make high scores on mathematics learning achievement tests. Their judgments are clearly based on the way the students do on the close-ended tests as fast and correctly as possible.

It can be seen that in the first period, there was not a clear definition of mathematical thinking in Thai curriculum for either elementary or secondary education levels. Generally, ideas on mathematical thinking are related to ability to computation and mental computation, and memorization on rule, formula, and theorem. In the second period, there is an attempt to emphasize on 'Mathematical Thinking' as described in the 6^{th} category. Even though, practically there are a lot of struggle to implement those emphasis on the real classrooms. Unfortunately, little can be seen as successful.

KEY WINDOWS FOR CONSIDERING MATHEMATICAL THINKING

From the researcher point of view, mathematical thinking can be seen through many theoretical frameworks. Based on Schoenfeld (1985, 1992), Inprasitha and other (2003) conducted a research to investigate elementary and secondary students' mathematical processes which emphasizing their mathematical thinking during solving open-ended problems. It revealed that the major obstacle to the students' successful participation in the mathematical problem solving activities was that almost all of the mathematics problems used in Thai schools are the exercises designed to drill the students in what they have been taught only. In addition, the exercises usually provide only one correct answer. This has essentially inhibited students from entering into varying ways of thinking and to use different methods for working together to solve problems. Quite contrarily, the teachers should have used the open-ended problems instead of the exercises because through such approach, problems can yield various answers and offer various processes for solving the problems. The problems also can develop into other problems for solving. Such characteristics of the open-ended problems make them look like situational problems from which students can create problems for themselves. This is a crucial condition in which the students can work together to solve the problems, and to participate in the problem-solving activities for a longer period than doing the old-pattern exercises. Furthermore, as the students engage in the solving of problems they have created, the teachers can observe their students' processes of learning and student's mathematical thinking.

The research findings also pointed to the fact that the Thai social and cultural context has greatly influenced the students' mathematical thinking, especially the role of

mathematics teacher which seems to inhibit free expression of mathematical thinking by the students. Therefore, a change in the way the teachers administer their classroom activities from the one emphasizing presenting new subject content, giving examples and making summaries at the end of each period, to a new approach of learning activities through open-ended activities; and to change their role as providers of answers or transmitters of knowledge to those of encouraging the students to appreciate the significance of thinking. They can do this by switching from the type of questions aiming at making certain that the students make correct answers to that of questioning for the purpose of stimulating the students to reflect on or to review their own thinking. In addition to Schoenfeld's framework, the frameworks for mathematical thinking which are used as a guideline for the development of undergraduate as well as graduate mathematics curricula are based on the ideas of some major mathematicians and mathematics educators and researchers like G. Polya, Edward A. Silver, Frank Lester, Alan J. Bishop, Becker and Shimada, and Nohda, etc. Their ideas are attested to the findings through mathematical-thinking-related research works done by Center for Research in Mathematics Education (CRME) and graduate students.

The graduate school in mathematics education at Khon Kaen University with the support by CRME has been collaborated in order to put the ideas on mathematical thinking into real classroom. This idea has getting started on 2002. The next session will briefly describe the outline of the project and some continuing works.

HOW MATHEMATICAL THINKING CAN BE DEVELOPED THROUGH LESSON STUDY?

The CRME's vision is that to organize activities for the development of mathematical thinking in the students it is necessary to rely on some innovations that would be capable of changing the teachers' traditional ways of teaching mathematics in the context of Thai culture which emphasizes comprehension and memorizing and the content and the process skills to a more innovative way in which students are allowed a greater opportunity to explore various thinking methods. An innovation that can be used as a major means to the development of mathematical thinking in students is an integration of open-approach method and Lesson Study.

As widely known, Lesson Study is a comprehensive and well-articulated process for examining practice that many Japanese teachers are engaging in (Fernandez, Cannon & Chokshi, 2003). In fact, recently a number of American researchers and educators have suggested that Lesson Study might be an incredibly beneficial approach to examining practices for US teachers (Lewis, C., 2002; Fernandez et al., 2003).

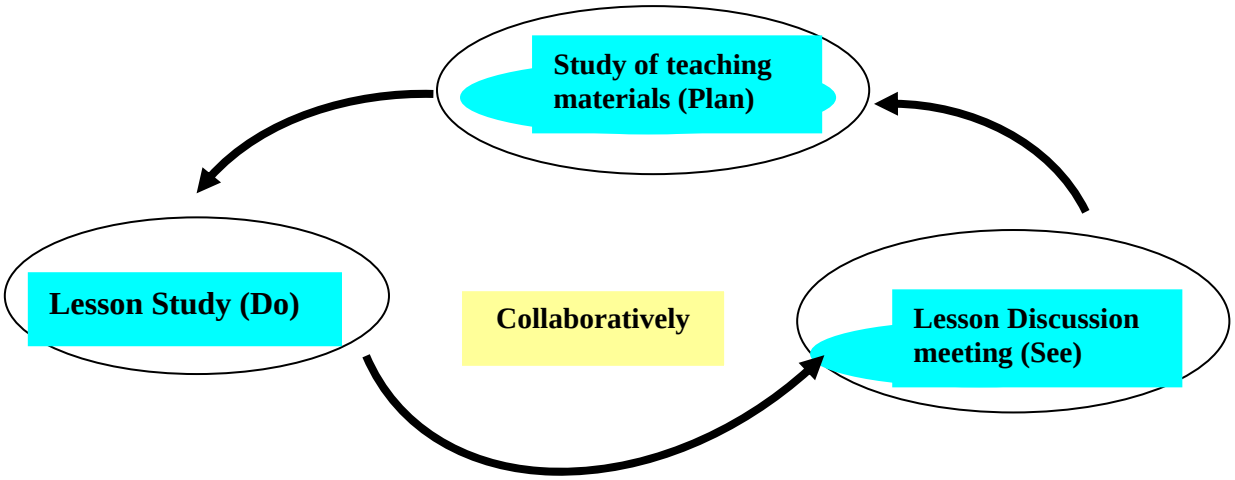
In Thailand, there have been many attempts to use Lesson Study to improve teacher education. In 2002, the Faculty of Education at Khon Kaen University, in an attempt to improve the teacher education program, conducted a project to investigate how student teachers develop their own pedagogy. In addition, to investigate how students

in the classroom are responding to the open-approach teaching method and whether or not they recognize their learning experiences.

It is found that during the first half of the semester all student teachers in the project experienced some difficulty adjusting to their new teaching roles and classroom organization. Participation in the weekly seminar facilitated the student teachers gradual change of the teachers' role. The most critical point of change was encountered while sharing their differing teaching experiences among friends and colleagues. Sharing experiences with their friends during the weekly seminar not only resolved their common concerns but also developed and expanded their own pedagogy, teaching practices and professional development. The greatest paradigm shift for student teachers' was that teaching mathematics does not mean focusing on the coverage of content but emphasizing the students' learning processes, original ideas, attitudes towards learning mathematics and satisfying one's own competence.

The survey of school students experienced on learning through this Lesson Study project revealed that most of the school students have positive attitudes towards learning through the open-approach method used in this project. In all areas of the survey, the school students indicated a marked improvement in their learning environment and capabilities in comparison to their traditional classroom. Regarding the classroom activity, the school students responded that they have more opportunities to act, think, play an active role, do something original, and conclude things by oneself. Regarding the change of their own learning process, they show some interesting responses as follows: more reasonable, more skillful in observation, more cool-hearted, know how to work cooperatively and more confidence in asking "why?" and "how come?" type questions.

From this promising result, in 2003 the Faculty of Education attempted to introduce Lesson Study into Thai school culture. A simple three phases of Lesson Study as in the below chart has been implemented.



Three phases of Lesson Study

To develop ideas for stimulating mathematical thinking and to apply them in the school setting successfully is quite problematic. It takes time and a lot of effort to identify a more suitable strategy for such purpose. Center for Research in Mathematics Education (CRME) of the Faculty of Education, Khon Kaen University has introduced an integration of Open- Approach method and Lesson Study into schools with the aim of developing mathematics teaching profession in cooperation with the Office of Development and Knowledge Management. The project carries on its activities in two Lab schools for a period of three years starting from April 2006 by following a model for the development of teaching profession as prescribed in the Lesson Study. While the project is being continued, some interesting findings can be described.

CONCLUSION REMARKS

During the initial period of trying out the learning activities in accordance with the Lesson Study and Open Approach, the teachers seemed to be unsure of their role in the new approach and were not confident about constructing open-ended lesson plans, while the students themselves were unfamiliar with the new approach and tended to stick with their old learning culture. That has inevitably kept them quiet and was not as expressive as they should be. They dared not ask questions when they encountered the matters that had been presented to them which they did not fully understand, nor they were able to explain in writing the methods or reasons they used. They were still in the habit of summing up the results and their class presentation was not expressive.

Nevertheless, after nine months of applying Lesson Study and Open Approach to the development of teaching profession which, in turn, would eventually result in the enhancement of mathematical thinking of the students, some significant changes had taken place as follows:

1) Changes on the part of the teachers

The greatest paradigm shift for mathematics teachers was that teaching mathematics does not mean focusing on the coverage of content but emphasizing the students' learning processes, original ideas, attitudes towards learning mathematics and satisfying one's own competence.

The teachers had acquired a better knowledge and become more confident about organizing learning activities for their own classes. They had altered their approaches and methods of teaching to be in accordance with the Lesson Study and Open Approach. This is especially true in the case of the mathematics teachers of grade 1, 4, and 7.

The teachers' attitude towards the organization of learning activities, the class and the students had been changed while the teachers themselves gave more weight to the studying of the students' thinking process;

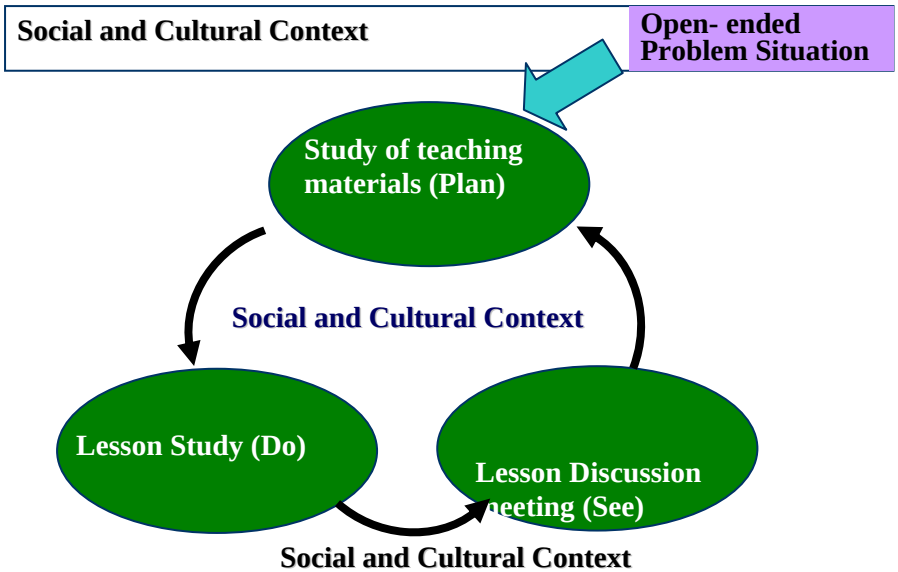
The teachers had an opportunity to work together to prepare lesson plans, to observe classroom activities and to reflect on the outcome of the plans at the end of each teaching period. More importantly**,** they were able to exchange their ideas which essentially represent a new teaching culture in the context of Thai education.

2) Changes on the part of the students

Most of the school students have positive attitudes towards learning through

the open-approach method. They have more opportunities to act, think, play an active role, do something original, and conclude things by oneself. They show some interesting responses as follows; more reasonable, more skillful in observation, more cool-hearted, know how to work cooperatively and more confidence in asking "***why?*** **"** and ***"how come?"*** type questions. These findings are consistent with what has been revealed in the teacher education project in 2002.

The students had realized that they were able to apply their daily-life encounters to their learning activities in classroom. They became more eager to learn and were able to work cooperatively. They had become more expressive. They spoke louder and were able to do class presentation very nicely. They liked to participate in class activities and to present ideas and methods of their own group (especially the grade-1 students). They dared to think differently and in various ways, enjoyed talking and expressing their own opinions. More significantly, they stuck fast with the activities or problems they encountered for a longer period of time and wouldn't give up easily. They had developed a more positive attitude towards group work or working with other people. Even though it is far from to say about the sustainability of this innovation in Thai school culture, it may be useful to summarize the ideas on introducing Lesson Study in to Thai school culture in order to develop mathematical thinking as below:



Adapted from: The history of Japan's Educational Development, 2004

Hypothetical Model of Integration between Open-Approach Method and Lesson Study

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APPENDIX A

Example of word problems used for developing students' mathematical thinking (cited from an IPST Mathematics textbooks)

1. **Multiplication and division problems from a grade-7 textbook.**
   Directions: Find the sums of the following items:
   1) [(-16+40)] x (-12) = …….
   2) [0 – (-15)] x 20 = ……..
   3) [18 + (-12)] x (-17) = ……
   4) [6 + 10] x (-20) = ……..
   5) [12 x (-19)]+ 8 = ……
   6) 10 x [32 + (-45)] =.....
   7) (-8) x [56 - 8] =.....
   8) [12 + (-33)] x 10 =.....
   9) 32 - (-56)] x (-19) =.....
   10) 18 x [14 + 6] =.....

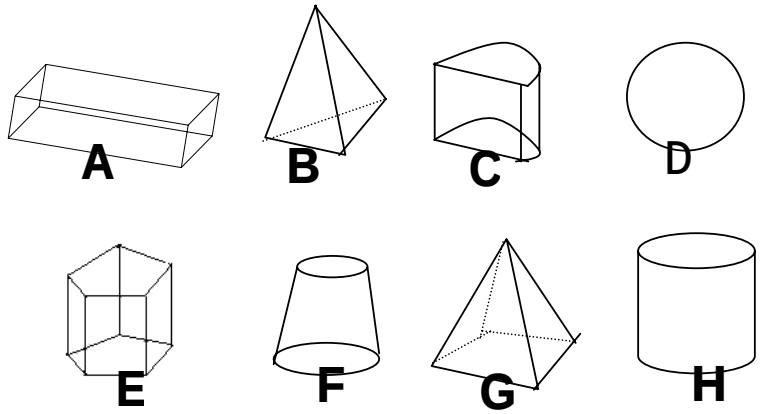
2. **A geometry problem from a grade-8 textbook**

A rectangle with an area of 108 cm^2 and the ratio of its length to width is 4:3. Find the length and width of the figure.

APPENDIX B

Examples of open-ended problem in geometry used in the research project

Solid Geometry



Consider the solid figures as shown. Choose one or more figures that share the same characteristics with figure B and write down those characteristics. Next, choose one or more figures that share characteristics with figure H and write down those characteristics