USING LESSON STUDY TO IMPLEMENT MORE EFFECTIVE LESSONS: FOCUSING ON MATHEMATICAL THINKING

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The purpose of the reform curriculum in Vietnam is to activate the learning mathematics of students. The curriculum tries to lessen the training of basic skills and procedures in mathematics but increases more hands-on activities to help students grasp the mathematics ideas and develop mathematical thinking. The mathematics teachers have learnt the innovative teaching strategies to implement more effective lessons focusing on mathematical thinking. In this article we will discuss some background from Vietnamese curriculum documents and try to answer three questions related to the development of students' mathematical thinking. Firstly, how mathematical thinking is defined in our curriculum documents and lessons? Secondly, what is our key window for considering mathematical thinking? And thirdly, how can we develop mathematical thinking through the lesson?

1. SOME BACKGROUND FROM VIETNAMESE CURRICULUM DOCUMENTS

The position of mathematics education

The school mathematics equips students with mathematics that is basic and modern. It refines computational skills and mathematical thinking that contributes to the development of students ability in solving problems and general thinking, especially, the ability of analysis, synthesis, abstraction and generalization.

The knowledge, skills and mathematical methods are the foundation to achieve the knowledge on science, information and other learning areas where mathematical concepts are central; and apply mathematics in the real-life situations.

General aims of mathematics education

School mathematics education aims to help students achieve four following objectives: knowledge, skills, thinking and attitudes.

a. Knowledge

Mathematics education aims to help students achieve basic knowledge on:

- Numbers and operations on number sets (from natural to complex numbers); algebraic expressions and transcendent expressions (exponential, logarithm, trigonometric); equations (linear, quadratic, trigonometric, exponential, logarithm); system of linear equations; inequations (linear, quadratic, exponential, logarithm) and system of linear inequations.

- Functions, limits, derivatives, primitive functions, integral and their applications.
- Geometric relationships and popular figures (points, lines, planes, polygons, circles, ellipses, polyhedrons, rotational surfaces; transformations on the planes; vectors and coordinates.
- Measurements.
- Getting started on statistics, combinations and probability.

b. Skills

Mathematics education aims to develop basic skills at:

- Computational operations: addition, subtraction, multiplication, division, exponential, roots, and logarithm.
- Changing the algebraic and trigonometric expressions to simple forms; solving equations, system of equations, inequations, and system of inequations.
- Calculating limits, derivatives, primitive functions, integral; considering the continuity of functions, and investigating and drawing the graphs of basic functions.
- Drawing mathematical figures, charts. Calculating lengths, angles, areas, and volumes. Writing the equations of straight lines, circles, ellipses, planes, and spheres.
- Collecting and processing data; calculating statistics and probability.
- Estimating measurement and calculation results.
- Using tools on measurement, drawing, and calculation.
- Reasoning and proving.
- Solving and applying mathematical knowledge in learning areas and real life situations.

c. Thinking

The mathematics curriculum provides opportunities for students to develop:

- The ability of observing, predicting, rational reasoning and logical reasoning;
- The ability of expressing precisely and clearly their own ideas and understanding the ideas of others;
- Spatial imagination;
- The characteristics of thinking, especially the flexible, independent and creative thinking.
- Thinking operations: comparison, analogy, generalization, and specialization.

d. Attitudes

- Self-study, active and confident in learning mathematics;
- Having characteristics of being: honest, hard working, overcoming difficulty, careful, accurate, disciplinal, and creative;

- Having a sense of cooperation, appreciate their own works and others;
- Recognizing and valuing the beauty of mathematics and like mathematics.

Teaching strategies

- Promote active, initiative and self-conscious learning of the learners;
- Form and develop the ability of self-study;
- Cultivate the characteristics of flexible, independent, and creative thinking;
- Develop and practice the logical thinking;
- Apply problem solving approaches;
- Apply mathematics to real life situations.

2. MATHEMATICAL THINKING IN VIETNAMESE CURRICULUM DOCUMENTS AND LESSONS

In Vietnam, primary students' ages range from 6 to 11 years old. There are 5 grades in the primary education. The process of teaching primary mathematics is divided into two stages. The first stage includes grades 1, 2 and 3. The second stage includes grades 4 and 5. In the first stage, students recognize basic and simple mathematical concepts visually through concrete materials, real models, and pictures. In this stage, pupils mainly know the "whole object" or "separate object"; they have not clarified yet the relationships, properties of objects and phenomena. In the second stage students need to learn mathematics deeply, generally and precisely. The level of abstraction and generalization of mathematics have been lifted in the second stage.

In the teachers' guidebook for primary mathematics teachers at each grade there are four main activities in a lesson that teachers should follow to develop mathematical thinking:

Activity 1. Teacher manages students to work and achieve the following aims:

- Examine the students previous knowledge;
- Consolidate the previous knowledge involved with new lesson;
- Introduction to the new lesson.

Activity 2. Teacher facilitates students explore mathematical knowledge and construct new knowledge by themselves.

Activity 3. Students practice the new knowledge by solving exercises and problems in the textbook.

Activity 4. Teacher concludes what students have learnt from new lesson and assigns the homework.

Engaging to the lesson, the pupils will have opportunities to show their mathematical thinking through:

- The ability of observing, predicting, rational reasoning and logical reasoning;

- Knowing how to express procedures, properties by language at specific levels of generalization (by words, word formulas);
- Knowing how to investigate facts, situations, relationships in the process of learning and practicing mathematics;
- Developing ability on analyzing, synthesis, generalization, specifying; and starting to think critically and creatively.

In the national standard mathematics curriculum (2006) for primary level, we emphasize more in word problems that considered being good situations for pupils to explore and solve mathematical problems. Students' mathematical thinking will be enhanced when they solve word problems. Most of these problems are rooted from the real life situations. The achievement objectives for each grade is illustrated in the following table.

Grades	***Achievement Objectives***	***Samples***
Grade 1 6-7 years old	Know how to solve problems involving with single addition or subtraction and express the solution including answer sentence, operation, and the answer.	Tan has 9 oranges. He gives to his friend 3 oranges. How many oranges are left for Tan? *Solution*: The number of oranges left: 9 – 3 = 6 (oranges) *Answer*: 6 oranges.
Grade 2 7-8 years old	Know how to solve and express the solution of problems involving with single multiplication or division.	30 liters of oils are poured into cans of 5 liters. How many cans are there?
Grade 3 8-9 years old	1. *Problems require conducting one operation: multiplication or division.*	
	a. Problems require applying knowledge on multiplication or division directly.	28 oranges are given equally to 4 friends? How many oranges does each friend get?
	b. Problems involving increasing or decreasing in a number of times.	Lan has 8 stamps. The number of Hue's stamps is 6 times more than Lan. How many stamps does Hue have?
	c. Find one part in many equal parts of a number.	Dung folded 24 paper boats. Hung folded $\frac{1}{3}$ of the number of Dung's boats. How many boats did Hung fold?
	d. Compare the proportion of two given numbers.	In a garden, there are 5 palms and 20 orange trees. Find the proportion of palms to orange trees.
	2. *Know how to solve and express the solutions of problems having two operations.*	Lan has 8 stamps. The number of stamps of Hue is 6 times of Lan. How many stamps do they have?

	Problems related to the deduction to units, and have geometric contents.	A rectangle with the length of 19cm, its width is 10cm less than the length. Find the area of that rectangle?
Grade 4 9-10 years old	*Know how to solve and express solutions of problems having three operations of natural numbers, fractions.*	
	a. Find the average	Classes 4A, 4B, 4C have 33, 34, 35 students respectively. What is the average number of students of each class?
	b. Find two numbers that their sum and difference are known.	Class 4A has 35 students. The number of girls is 3 less than boys. Find the number of girls and number of boys in that class?
	c. Find the fraction of a number.	There are 12 oranges in a basket. We take away 1/3 of oranges. How many oranges are left?
	d. Find two numbers that we know their sum and ratio.	There are 35 students in a class; the number of boys is three quarters of girls. Find the number of boys and girls of that class.
	e. Find two numbers that we know their difference and ratio.	Mother age is 25 years more than her son age. The son age is 2/7 of the mother age. Find the age of each person?
Grade 5 10-11 years old	*Know how to solve and express solutions of problems having four operations.*	
	a. Ratios	Two workers dig 6 meters of land in one hour to install the water pipes. With the same rate, how many meters of land that 6 workers can dig in one hour?
	b. Percentages	The number of girls in a school is 420 and its percentage to the number of students of whole school is 52.5%. How many students are there in that school?
	c. Linear motions	At 7 o'clock, one cyclist departs from city *A* to city *B* with speed of 12km per hour. After three hours, a motorist departs also from *A* to *B* with speed of 30 km per hour. At what time the motorist will catch the cyclist?
	d. Geometry and shapes	A trapezoidal rice field with the large base of 120 meters, the small base is 2/3 of the larger. The height is 5 meters less than the small base. In average, 64.5 kg of rice are harvested from 100-meter squares. Find the number of kg of rice that is harvested from that field?

3. KEY WINDOW FOR CONSIDERING MATHEMATICAL THINKING

Students' mathematical thinking can be observed, tested, evaluated and reported through the students' works, talks and representations when they solve mathematical problems. To help primary classroom teachers feel comfortable in preparing lesson plans emphasizing on mathematical thinking, we consider mathematical thinking is a

process of mind that occurs when students solve "*difficult exercises*", where the term "*difficult exercises*" has the same meaning with "*problems*" in problem solving. The level of difficulty and complexity of a problem is defined by the achievement objectives in the standard curriculum for each strands of mathematics.

The exercises in the practice lessons are ranked:

- From easy to difficult,
- From simple to complicated,
- From direct practice to flexible and combined application.

The mathematics curriculum provides opportunities for students to develop and enhance their mathematical thinking through posing and solving problems such as:

- Applying correctly algorithm or procedure to solve specific mathematical problems.
- Devising and using appropriate problem-solving strategies to explore problematic situations mathematically.
- Completing the cognitive tasks by themselves to develop their own mathematics under the managing and facilitating of the teacher.
- Expressing their understanding, mathematical ideas, and solutions logically.
- Using logical reasoning to solve problem.
- Knowing how to practice exercises and solve mathematical problems.
- Interpreting information and results in mathematical contexts;
- Observing, predicting, reasoning and logical reasoning;
- Expressing precisely and clearly their own ideas and understanding the ideas of others;
- Developing spatial imagination;
- Developing the characteristics of thinking, especially the flexible, independent and creative thinking.
- Using thinking operations in solving problems.
- Developing the understanding and skills to interpret written representations of formulas.

From the standard curriculum, the textbook writers tried to create activities, situations, exercises and problems that stimulate students' mathematical thinking. These activities aim to help students develop the following types of thinking:

- Visual thinking;
- Conceptual thinking;
- Intuitive thinking;
- Reasoning (inductive thinking, deductive thinking);
- Logical thinking;
- Critical thinking (e.g. students critically follow a chain of reasoning);
- Creative thinking (e.g. finding patterns, making conjectures);
- Problem solving thinking.

In our point of view, the key windows for considering mathematical thinking are as follows:

- Students learn mathematical concepts with *meaningful understanding*;
- Students construct *individual algorithm* and *techniques* themselves with understanding to solve some specific problems;
- Students use learnt mathematics to *solve* mathematical problems effectively;
- Students show mathematical thinking by *communicating* (talking, writing, arguing, discussing, and representing);
- Students *reflect critically* their mathematical thinking in order to improve their learning;
- Mathematical thinking is *social* and *relative* to each individual student;
- Students apply *logical* and *systematic* thinking in mathematical and other contexts;
- Students use *thinking operations* in solving problems: *comparison*, *analogy*, *generalization*, and *specialization*;

4. THE DEVELOPMENT OF MATHEMATICAL THINKING THROUGH THE LESSON

The lesson helps students to pose and solve mathematical problems; to make that kind of lesson happen, teacher should utilize the following factors appropriately:

- Effective manipulative materials;
- Pictures;
- Mathematical models, geometric shapes;
- Presentations of mathematics concepts;
- Exercises;
- Algorithm;
- Logical reasoning;
- Problematic situations;
- Problem solving strategies;
- Inquiry learning;
- Discovery learning;
- Open-ended questions;
- Investigations;
- Meaningful problems.

From the curriculum and textbooks we can categorize lessons into two main types as follows.

1. New lesson

a. Help students pose, explore and solve problems

Teacher facilitates students to explore and pose questions, problems of the new lesson when they engage in a problematic situation. Teacher then facilitates students

to mobilize what they have experienced and learnt to recognize and seek for the relationships between the posed problem and their known knowledge to find the appropriate strategy to solve problem.

Example: Teaching new lesson on "Comparison two fractions with different denominators", teacher helps students explore the problem for the new lesson. Teacher gives an example: "Compare two fractions $\frac{2}{3}$ and $\frac{3}{4}$". Teacher facilitates students to give some commends on the characteristics of two fractions $\frac{2}{3}$ and $\frac{3}{4}$ to recognize those two fractions having different denominators. After engaging in this activity, students will pose the problem "How to compare two fractions with different denominator". This is the problem that students need to solve in the new lesson? The teacher can conduct two following activities.

Activity 1

We have two equal paper bands. Divide the first one into 3 equal parts, and then take 2 parts. Divide the second one into 4 equal parts, and then take 3 parts. Compare $\frac{2}{3}$ and $\frac{3}{4}$ of the paper bands?

S1: By using the real paper bands or drawing, I see that $\frac{3}{4} > \frac{2}{3}$.

S2: This method is intuitive but we cannot implement for arbitrary pairs of fractions. I think we need another method?

$\frac{2}{3} = \frac{2\times 4}{3\times 4} = \frac{8}{12}$ and $\frac{3}{4} = \frac{3\times 3}{4\times 3} = \frac{9}{12}$. So $\frac{3}{4} > \frac{2}{3}$.

T: First method is visual but it has not contributed to the solving general problem. The second method requires students relate to the learnt knowledge of comparing two fractions with the same denominator.

b. Give students opportunities to consolidate and apply knowledge constructed after learning new lesson such that students start to achieve the new knowledge

In the mathematics textbook at primary level, after new lesson usually there are three exercises for students to consolidate and practice what they have learnt in the lesson. The first two exercises require students know how to apply and practice directly the new knowledge. The third usually is a problem requires an indirect application of new knowledge.

Example:

Mai ate 3/8 of a cake. Hoa ate 2/5 of the same cake. Who ate more than the other?

2. Consolidation, practice and general practice lessons

a. Help students recognize learnt and new knowledge in various problems

Example 1 (grade 4)

Calculate 3167 × 204?

Example 2

A rectangular rice field has perimeter of 360 m. Its width equals one half of the length. Find the area of the rice field?

b. Help students self practice on their pace of ability

Teacher asks students to solve sequentially exercises and problems in the arranged order of the textbooks under the teacher instruction. In a period of time, some students can do more exercises than others. Students can practice exercises on their own pace, teachers manage good students to help slower students to solve problems but teacher do not show the solutions to students.

c. Give students opportunities to help each other in small group with effective interactions by using worksheets.

d. Practice students having habits of checking, evaluating their works.

e. Practice students having habits of finding various strategies and choosing most appropriate strategy to solve problem.

Example 1 (Grade 5): Calculate 4.2 + 3.5 + 4.5 + 4.8 by using your most convenient way?

S1: 4.2 + 3.5 + 4.5 + 6.8 = 4.2 + (3.5 + 4.5) + 6.8

= 4.2 + 8 + 6.8 = (4.2 + 6.8) + 8

= 11 + 8 = 19

S2: 4.2 + 3.5 + 4.5 + 6.8 = (4.2 + 6.8) + (3.5 + 4.5)

= 11 + 8 = 19

Both ways of calculating are convenient and derive to correct answer. Each way may be convenient to a number of students. The teacher should not claim which one is more convenient.

Example 2 (Grade 5): Find an appropriate decimal and fill in the blank such that: 0.1 < . . .< 0.2?

S1: 0.1 < 0.11 < 0.2

T: Can you explain the reason why you choose 0.11?

S2: The appropriate decimals may be 0.11, 0.12, and 0.19.

T: Can you find another appropriate decimals?

S3: 0.101, 0.102, …, 0.199.

T: How many appropriate decimals can you find between two decimal numbers?

S4: Many. For example, 0.1001, 0.1002, …

With this kind of teaching, teacher helps students dig deeply into a textbook problem and build up a habit of unsatisfying with achieved results, encourage students to be interested in seeking for another solutions, and creative in learning mathematics.

CONCLUSION

Teaching school mathematics aims to equip students with basic mathematics and develop their mathematical thinking to solve problems. Some senior classroom teachers have experienced to foster and develop students' mathematical thinking without theoretical background. Most of teachers in Vietnam really need a practical framework to develop students' mathematical thinking in their actual classrooms. Of course, each economy has slightly different approach in teaching mathematics, but we still have a lot of thing in common to share and build up together a realistic framework that helps classroom teachers in each APEC economy.

Reference

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