COMMUNICATION AS A PROCESS FOR STUDENTS TO LEARN MATHEMATICAL

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The Principals and Standards for School Mathematics (NCTM, 2000) suggest that communication is one of the important processes for teachers to facilitate during mathematics lessons. Although the Principals and Standards emphasize that the process of learning mathematical contents is as important as the acquisition of concepts and procedures, we still do not see many mathematics classrooms that emphasize the process of learning mathematics in the U.S. In order to provide students with opportunities to learn mathematics through communication, this paper will discuss the use of the "open-ended approach" to encourage students to communicate with each other as they learn mathematics.

INSTRUCTION AS INTERACTION

Rather than passively listening to teachers talk, students are expected to be actively involved in mathematics (Brown, 1994). It is expected that teaching and learning mathematics be viewed as the product of interaction among teachers, students, and the mathematics (National Research Council, 2001). In other words, students should be able to learn the contents of mathematics actively through interaction with teachers and peers. Thus, facilitating communication should be an integral part of supporting students in their learning of mathematics through interaction.

In order for students to be able to communicate each other, it is important to have a learning structure that stimulates communication among the students. If all the students are told to use the same approach to finding an answer they may not need to communicate with each other. They would focus on listening to what the teacher says and to the mathematical contents presented by the teacher. On the other side, if several different ideas are presented by the students during the class, they are encouraged to explain each idea and to justify each idea as reasonable. Especially, if several different approaches/solutions are presented and the teacher expects students to figure out which approach/solution is reasonable, students are required to understand these ideas and to make the decision through communication.

Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 1989, 2000) includes a discussion of the necessity for learning mathematical content through the processes of problem solving, reasoning and proof, communication, connections, and representation. This means that it is important for teachers to carefully plan their focus on not only what the students need to learn but also how the students acquire the mathematical contents such as knowledge and procedures during the class.

USE OF THE OPEN-ENDED APPROACH

In order to have meaningful communication during mathematics lessons, teachers need to provide rich mathematical situations for their students. It is expected that the mathematical situation will encourage students to learn something new by communicating with teachers and peers. One of the ways to provide students with such opportunities is to employ the open-ended approach (Becker & Shimada, 1997). The open-ended approach is an instructional approach using an open-ended problem, which has multiple solutions or multiple approaches to a solution. The lesson proceeds by using several students' responses to the given problem to provide experience in finding something new in the process. The open-ended approach is an outcome of the developmental research project on methods of evaluating higher-order-thinking skills in mathematics education between 1971 and 1976. Although the project was aimed at improving the evaluation of students' thinking skills, the researchers became aware that lessons based on solving open-ended problems have a rich potential for improving teaching and learning. The researchers summarize the advantages of using the open-ended problem in mathematics lessons as follows.

- Students participate more actively in the lesson and express their ideas more frequently.
- Students have more opportunities to make comprehensive used of their mathematical knowledge and skills.
- Even low-achieving students can respond to the problem in some significant ways of their own.
- Students are intrinsically motivated to give proofs.
- Students have rich experiences in the pleasure of discovery and receive the approval of fellow students.

(Becker & Shimada, 1997)

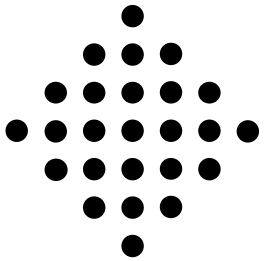
Since the open-ended approach was introduced, US teachers have started using the open-ended approach in their classrooms. Teachers who are practicing lesson study, especially, often use open-ended problems when they plan lessons for lesson study. Although having open-ended problems is a new idea and it may not easy for teachers to use them in their lesson, these teachers have been working hard to encourage students to interact each other to learn mathematic actively. Several lesson plans have been developed for US students based on popular open-ended problems in Japan. The attached lesson plan shows an example of the lesson plan for a research lesson that was conducted in Fresno, California (Appendix I).

By employing the open-ended approach, the lesson was designed to provide students an opportunity to develop their competence in using mathematical expressions and

equations. The lesson began with an introductory activity to let students understand how to use an equation to describe an arrangement of dots. The teachers then presented an open-ended problem where students compared and discussed a variety of ways to describe the arrangement of 25 dots by using equations. During the process of comparing and discussing a variety of mathematical expressions and equations, students communicated with each other to understand how other students described the same arrangement of 25 dots.

Find the number of dots and describe your way to find the number of dots by using a mathematical expression/equation.

If you can, find and describe several different ways to count the number of dots.



RETHINKING DEVELOPING LESSON PLAN

When lesson design focuses not only on *what* students need to learn but also on *how* students acquire mathematical knowledge, experience, and skills, teachers are required to develop lesson plans differently. Commonly used lesson plans mainly focus on what students need to learn and do not focus much on the process of student learning during the lesson. Lesson Study provides teachers the opportunity to think deeply about the students' learning process. Because the major purpose of observing a research lesson is to examine students' learning process for the sake of the post-lesson discussion, lesson planning requires teachers to anticipate students' variety of approaches to the solutions and to plan for facilitating discussion with the students. These processes of planning lessons help teachers think carefully to design the lesson by including students learning process.

Since lesson study provide teachers such learning opportunity, designing lesson through lesson study is an ideal way to help teachers provide learning opportunity for their students through communication.

References

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Mathematics Lesson Plan for Fifth Grade

For the lesson on Friday, July 22
At the Homan Elementary School, Fresno, CA
Instructor: Akihiko Takahashi

1) Title of the Lesson: How do you find the number of the dots?
2) Goal of the Lesson:
   a) Students will begin to recognize equations as a way to represent quantitative relations mathematically.
   b) Students will begin to be able to communicate with other students by:
      i) expressing their thinking processes using equations.
      ii) inferring possible thinking processes of other students from the equations they present.
3) Instruction of the Lesson

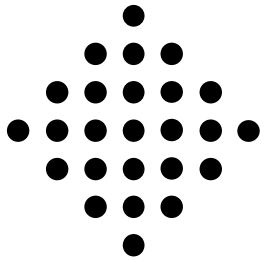
Learning algebra is a key for students to further their proficiency that they have developed by learning number and operations. The NCTM standers expect students to be able to 1) understand patterns, relations, and functions, 2) represent and analyze mathematical situations and structures using algebraic symbols, 3) use mathematical models to represent and understand quantitative relationships, 4) analyze change in various contexts. Among those expectations, this lesson is designed to provide students with opportunity to experience representational activities, which is the focus of the second expectation.

Representational activities of algebra in elementary and early middle school grades involve translating verbal expressions into mathematical (symbolic) expressions and equations. These activities are the essentials for students to be able to represent mathematical situations and structures by using mathematical expressions and equations in order to analyzing those situations and structures to understand quantitative relationships. These representational activities should focus not only on representing mathematical situations and structures but also interpreting mathematical expressions and equations. However, teaching algebra in elementary grades has been mainly focused on recognizing patterns and relations and not much on representational activities. As a result many students are struggling to express patterns and relations by using mathematical expressions and equations, although they are able to describe them by using verbal expression, tables, and models such as diagram and figures. Recognizing patterns and relations is important but we should not underrate the importance of the meaningful acquisition of algebraic notation because it is one of the most difficult aspects of algebra for students (Edwards, 2000).

In order for students to be able to translate their verbal expressions and other informal expressions such as using diagrams and tables into mathematical expressions and equations, it is important for students to recognize that mathematical expressions and equations as the representation of relations. Unfortunately many students view mathematical expressions and equations as a signal to compute because elementary school mathematics tends to be heavily answer oriented (Thompson, Philipp, Thompson, and Boyd 1994). Thus, students should have experiences to adjust their view of mathematical expressions and equations in order to use them to analyze mathematical situations and structures. One of the ways to provide students such an experience might be posing an open-ended problem that requires each student express the mathematical situation in various ways by using equations and communicate with other students by interpreting their equations.

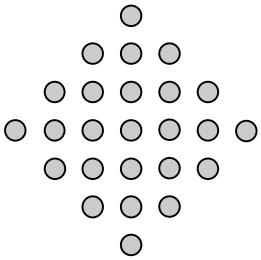
By employing open-ended approach, this lesson is designed to provide students an opportunity to develop their competence to use mathematical expressions and equations. The lesson begins with an introductory activity to let students understand how to use an equation to describe an arrangement of dots, then presents following open-ended problem to compare and discuss variety of ways to describe the arrangement of 25 dots by using equations. This type of open-ended problem is known as a problem with multiple solutions, because the major focus of this problem solving activities is to represent the arrangement of the dots by using an equation and not to find the number of the dots.

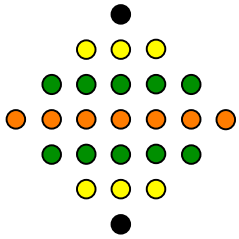
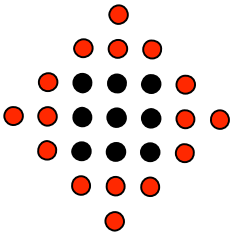
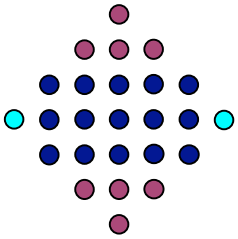
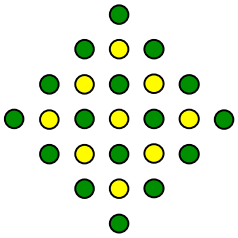
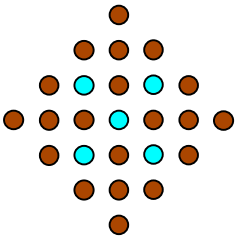
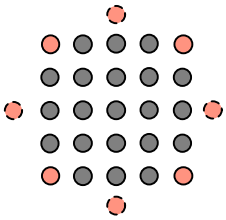
Find the number of dots and describe your way to find the number of dots by using a mathematical expression/equation.
If you can, find and describe several different ways to count the number of dots.



Because of the nature of open-ended approach, the main concern for the teacher during this lesson is to facilitate discussion meaningfully by including all the students in the class. In order to do so, the instructor will carefully examine students' equations during students' individual/group work and plan the discussion immediately following students' individual/group work.

4) Flow of the lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
a. Introduction **How many dots are there? Tell me what you see.** Anticipated students' responses i. one in the center and four in the outside ii. two, one, and two iii. three and two Anticipated students' responses three and three two, two, and two	Let students see the major focus of today's activities Help students realize that they can use equations to describe the arrangement of dots by translating students' verbal expression into an equation. (a) i) 1+4=5 ii) 2+1+2 =5, 2x2+1=5 iii) 3+2=5 (b) i) 3+3=6, 2x3=6 ii) 2+2+2=6, 3x2=6	• Do students realize that there are multiple ways to describe the arrangement of dots? • Do students understand that they can describe an arrangement of dots by using equations? • Are students motivated to find different ways to describe the arrangement of dots by using different equations?
2. Posing Problem Find the number of dots and describe your way to find the number of dots by using a mathematical expression/equation. If you can, find and describe several different ways to count the number of dots. 		
Students should use worksheets to write their equations so that they can use them during the whole class discussion to share and describe their equations.	Provide students worksheets with a large picture of the dots for the problem. Let students write an equation that describes their counting method on each worksheet. Students will be able to have as many worksheets as they want. Students might work with their partners.	Do students understand how to use the worksheet? Do students work comfortably with their partners?

3. **Solving Problem** Anticipated students' responses  1+3+5+7+5+3+1=25 2x1+2x3+2x5+7=25 7+2(1+3+5)=25  3x3+4x4=25 4x4+9=25  3x5+2x4+2x1=25 3x5+2x3+4x1=25  4x4 +3x3=25 4+4+4+4+3+3+3=25  5x5=25 4x5+5=25  5x5=25		Does each student find the number of dots, and is he/she able to describe his/her way by using the diagram on the worksheet? Does each student write an equation to describe his/her counting methods.
4. Sharing their equations Share your equations to the class but do not tell other where the equation comes from. <Show but don't tell> Other students: Let's figure out where the equation comes from.	Facilitate opportunities for students to interpret an equation. Encourage all the students to be engaged in the representing and interpreting activity because it is possible that some equations might come from different ways of representing the arrangement of the dots. The unique solutions among the students' solutions will be discussed later.	Do students see relationships between a diagram and an equation?
5. **Summing up** Reflect what we learned by looking at the board writing and recognize mathematical expression/equation can be a way to express mathematical situation.	Let students write their learning experience as a journal reflection.	

5) Evaluation

a) Did the students represent quantitative relations by using equations?

b) Did the students communicate with other students by expressing their thinking processes using equations, and inferring possible thinking processes of other students from the equations they present?