Communication and the development of mathematical thinking

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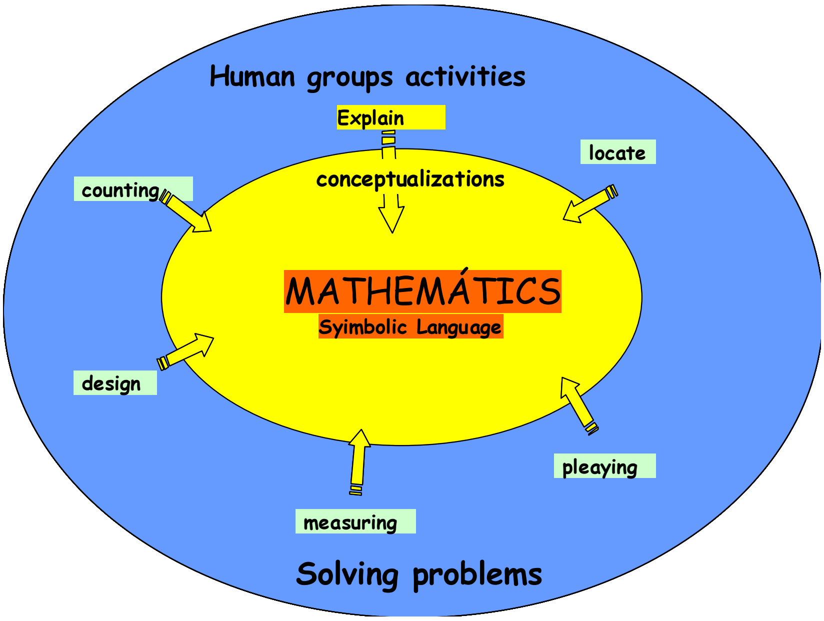
The need of communication led humans to create languages
It is well established that all human groups communicate and that all cultures have a language. (Bishop, A. 1999) Communication with others requires a language; this is oral, written or representational.

Geoglifos[1], Representational communication

Creating and developing mathematical language. Processes **trigged off by certain kinds of activities.**

Following Bishop, (1999, pg. 42 and 78.) We can say that it has been found that in all cultures human groups develop certain activities that lead to a cultural product we know as **mathematics**. These activities rise from relationships with the physical environment (counting, measuring, locate and design) or with the social environment (playing and explain). Likewise, all cultures generate and structure language at the service of communication, all classify objects and phenomena, all have explanatory links, they all have ways of connecting ideas through speech and all have a fundamental reference for validating explanations. Accordingly, to **explain** is as universal as language and certainly has a crucial for mathematical development.

[1] Geoglifo Across northern Chile there exist gigantic and stylized manifestations of prehistoric art that adorn the high slopes of the hills, known as geoglifos According to the archaeologists these geoglifos represent a mysterious and enigmatic past, those relating to artistic expressions, signs or sanctuaries.



In a characterization of Mathematics, proposed by Godino (2000) we can recognize the elements listed above and even to complement them:

(A) The mathematics are a human endeavour, produced in response to some kind of problem situations on the real world, social or mathematical itself. In response or solution to these external and internal problems, mathematical objects (concepts, theories, etc.), raise and evolve gradually. Actions of individuals should be considered, therefore, as the source of genetic mathematical concepts, according to piagetan constructivist theories.

(B) The mathematical problems and their solutions are shared within specific groups or institutions involved in the study of certain classes problems. Accordingly, the mathematical objects are socially shared cultural entities.

(C) The Mathematics creates a symbolic language in which they express the situations problems and their solutions. The systems of symbols, given by the culture, not only have a communicative function, but an instrumental role, which change at the same subject that uses them as mediators.

(D) The mathematical activity pursues, among other aims, the construction of a conceptual system logically organized. Therefore, when adding new knowledge to the existing structure, not only increases the structure, but all relationships are altered.

The communication in the process of construction of mathematical knowledge in the classroom

One of the central assumptions of the current constructivist said that the sociocognitive conflicts between members of the same social group may facilitate the acquisition of knowledge as they allow students aware of other different responses to yours, which force they to consider another point of sight of their initial response. The different answer from others carries information and draws attention on the subject about mathematical aspects of the task that had not considered until then.
Thus, sociocognitive conflicts provoke a double imbalance: interindividual imbalance due to the different responses of subjects; intraindividual imbalance due to the awareness of different responses, which invites the subject to doubt his own response. (Chamorro, M.2005)

In a scenario in which the teacher has proposed solving certain types of problems, an important moment of the "pooling" of the developed mathematical work in the classroom. At least for that group, pooling is to make public, the solutions, procedures and the reasoning. In this process the language as a means of social communication, is crucial. The language will allow students to structure what has been done, appropriating the new meanings, identifying concepts and procedures, and prepares the way for the demonstration. The demonstration, from the point of view of mathematical communication, it is a social act, as it is directed to other individuals whom it is necessary to convince and demands a verbal, written or representative record.

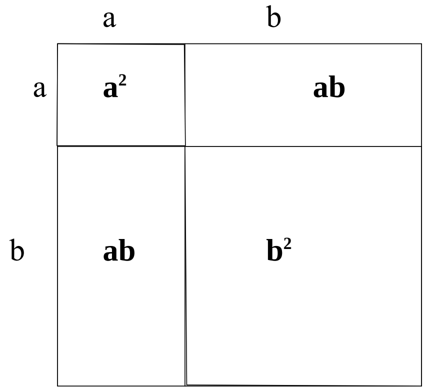
Other authors as Nunez, M. (2007) speak of the co-construction of knowledge. This concept is described as a combination of individual and social processes that occur during the process of teaching and learning science. In particular, the authors refer to the processes of interaction (dialogue) that occur between the teacher and students or between students. Through the study of the conversation in classes or in small groups they have attempted to characterize the reasoning of the students in terms of processes or the flow of argument propositions that an individual performs with it which allows the identification of specific forms of argument.
It is alleged that the conversation between children, which seeks to explain their methods contributes to their own understanding. Obviously, talk about their own actions leads to children having to think about it, which is not always attained otherwise, and therefore allows become more aware of them.
Other authors as Nesher (2000) distinguish between talking about mathematics in the classroom and talk mathematically. the author refers to "Mathematically speaking," to using mathematical ideas freely, as function, equal, proportions, to manipulate them according the syntax of mathematical language and to be able to apply them in various contexts. In contrast, to "talk about math" carry out other actions. We use natural language as a metalanguage to express all kinds of thoughts about math. It is different, because the vocabulary, grammar, syntax and semantics, belong to natural language.
Under the constructivist approach, it is very important that teachers listen to the explanations his students gave about his mathematical work. This requires to open spaces and to give opportunities to talk about mathematics in the classroom.

Recall that on the Cockroft Report (1985) it was stated that the students successful in mathematics came from those classrooms in which it was usual to participate in discussions of mathematical type, between teacher and students, as well as between students. This endorses Nesher (2000) noting that students develop learning math when discussing among themselves about valid arguments in mathematics or explain their strategies. For this purpose they use it in mixed-mode: Natural Language ideas partially expressed in mathematical terms.

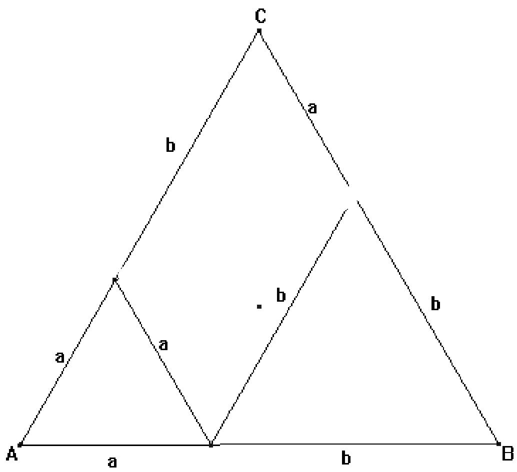
Example of a communication experience focused on mathematical thinking[2].

Students know the schematic representation of the binomial square:

$$(a + b)^2 = a^2 + 2ab + b^2$$

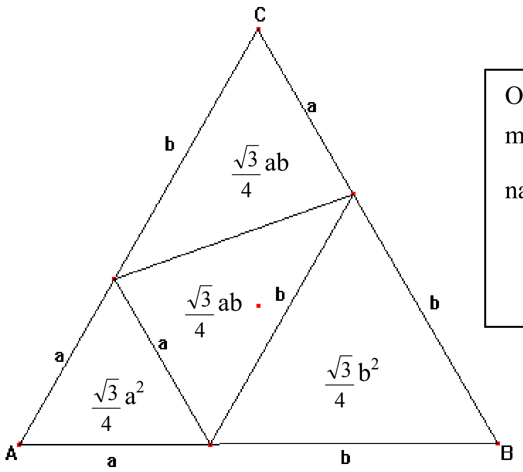


They are asked to look at this other representation based on an equilateral triangle, and comparing it with the previous, setting differences and similarities between the two. They must justify in writing their conclusions.



[2] Set of activities created and tested by Francisco Cerda B, on 8º y 9º grade.

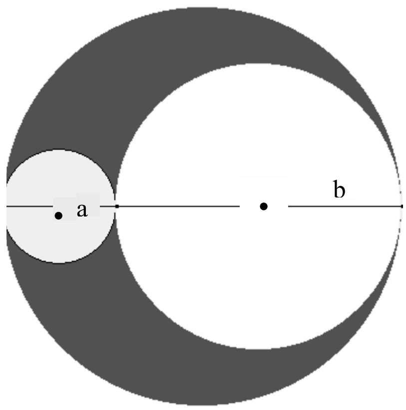
Once the work in pairs is ended, a pooling of some achieved answers was made, generating an interesting discussion among students.



On the equilateral triangle we can recognize a multiplication of the binomial square by a factor $\frac{\sqrt{3}}{4}$, namely:

$$\frac{\sqrt{3}}{4}\mathbf{(a+b)^2} = \frac{\sqrt{3}}{4}\mathbf{a^2} + \mathbf{2\bullet}\frac{\sqrt{3}}{4}\mathbf{ab} + \frac{\sqrt{3}}{4}\mathbf{b^2}$$

Then the students were challenged to work with other regular polygons as rhombus, hexagons to conclude for the circle:



$$\pi(a+b)^2 = \pi a^2 + 2\pi ab \quad \pi b^2$$

This activity continued searching for similarities in the different representations with regular polygons, such as:

$$(a^2 - b^2) = (a + b)(a - b)$$

Communication an mathematical communication in the Chilean curricular framework

In the Chilean educational curriculum are described the general purpose of education, i.e. the knowledge, skills, attitudes, values and behaviors that students are expected to develop at a personal level, intellectual, moral and social development. In this curriculum framework, it is defined as a central objective, the development of communication skills, which are linked to the ability to present ideas, opinions, beliefs, feelings and experiences in a coherent and informed manner, making use of many and varied ways of expression.
Spanish Language and Communication is a subject proposed to develop the most of the communicative abilities of students, so that they can function properly and effectively in the various situations they have to face. This subject highlights oral and written communication:

Oral Communication: It refers to promote student participation in oral communicative interaction situations in which they have the opportunity to analyze communicative situations interaction oral of argumentative type (debates, controversial, taped discussions radio or television) in order to distinguish: the structure of speeches (assumptions, arguments, conclusions), procedures (types of arguments, validity of them, and so on.), and the results and effects.
Written Communication: Reading and production of written argumentative texts produced in public situations, to perceive the overall structure of the text, the internal organization of their parts and components, with special emphasis on types of arguments used and validity of them, incorporation in the text of illustrations and statistical tables.

Mathematical communication
In the subject of Mathematics, among other things, it is stated that his learning contributes to the development of communication skills, which makes more precise and rigorous the expression of ideas and arguments, incorporating on the language and usual arguments forms of mathematical expression (numerically, graphically, symbolic logic, probability and statistics) and understanding quantitative and qualitative mathematical elements (data, statistics, graphics, drawings, etc.) present at the news, views, or advertising, and analyzed them independently. It relieves teamwork, communication and the exchange of ideas, the foundation of views and arguments, examining its logical connections and with the support technological elements. Moreover, education must contribute to a better performance of people in everyday life, through the use of mathematical concepts and skills that will enable them to reinterpret reality and solving everyday problems of family, social and professional circle, contributing to establish a language for understanding the phenomena of science and technology.

In an explicit way, there are some fundamental objectives which are related to mathematical communication. For instance, students will be able to:

• Identify and interpret the information provided by numbers present in the environment and used numbers to communicate information in both oral and written form.

• Communicate and interpret information relating to the place where objects or persons are placed (positions) and giving and following instructions to get from one place to another (trajectory).

• Manage basics features of problem solving, such as formulating the problem in their own words, take initiatives to resolve it and communicating the solution.
• Collect and analyze data in local, regional and national levels situations, and communicate results; selecting ways to present information and results according to the situation.
• Use systematically arranged and communicable reasoning in problem solving

Communication in the National Assessment System (SIMCE)
Since this year (2007) the national tests SIMCE (levels 4th, 8th and 10th), include open-ended questions, whose focus is to assess aspects that are more difficult to measure with alternatives items
In mathematics, students are requested to explain by words, the reasoning they make to obtain the solution of a problem.

Sample question (SIMCE 1st cycle)

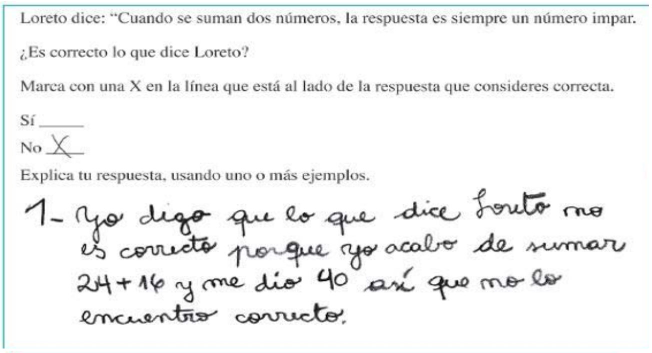
Loreto dice:
"Cuando se suman dos números, la respuesta es siempre un número impar"
¿Es correcto lo que dice Loreto?

SI **NO**

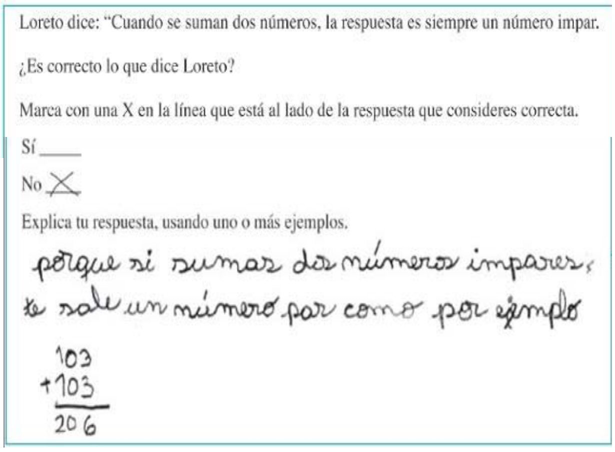
Explica tu respuesta, usando uno o más ejemplos.

Loreto says:
"When two numbers are added, the answer is always an odd number"
Is it correct what Loreto says?
YES **NO**
Explain your answer, giving one or more examples.

Below are two answers given by children to that question:


Loreto dice: "Cuando se suman dos números, la respuesta es siempre un número impar.
¿Es correcto lo que dice Loreto?
Marca con una X en la línea que está al lado de la respuesta que consideres correcta.
Sí ____
No X
Explica tu respuesta, usando uno o más ejemplos.
1- Yo digo que lo que dice Loreto no es correcto porque yo acabo de sumar 24 + 16 y me dio 40 así que no lo encuentro correcto.

YES--
NO X
1- I say that Loreto's answer is wrong, because, recently I added 24 + 16 and I obtained 40, and then it is not ok.


Loreto dice: "Cuando se suman dos números, la respuesta es siempre un número impar.
¿Es correcto lo que dice Loreto?
Marca con una X en la línea que está al lado de la respuesta que consideres correcta.
Sí ____
No X
Explica tu respuesta, usando uno o más ejemplos.
porque si sumas dos números impares, te sale un número par como por ejemplo
103
+103
206

YES--
NO X
Because if you add two odd numbers, you obtain an even one, for instance:
103 + 103 = 206

These answers, give a first approach about mathematical reasoning level that those children employed.

Maps of progress

In the framework of contents standards, the maps of learning progress (MPA) is a new curricular instrument that is in validation and development process. They described the typical sequence in which learning is progressing in certain areas or domains considered essential in the training of students in various subjects. The maps described 7 learning levels, ranging from primary school to secondary school.,

As an example we can noted that the learning map described in Numbers and Operations axis progress considering three dimensions that are developed in interrelated way:

A. Understanding and using the numbers. .
B. Understanding and using operations.
C. Mathematical Thinking (this dimension involves skills related to the selection, implementation and evaluation of strategies of problem solving; argumentation and communication strategies and results)

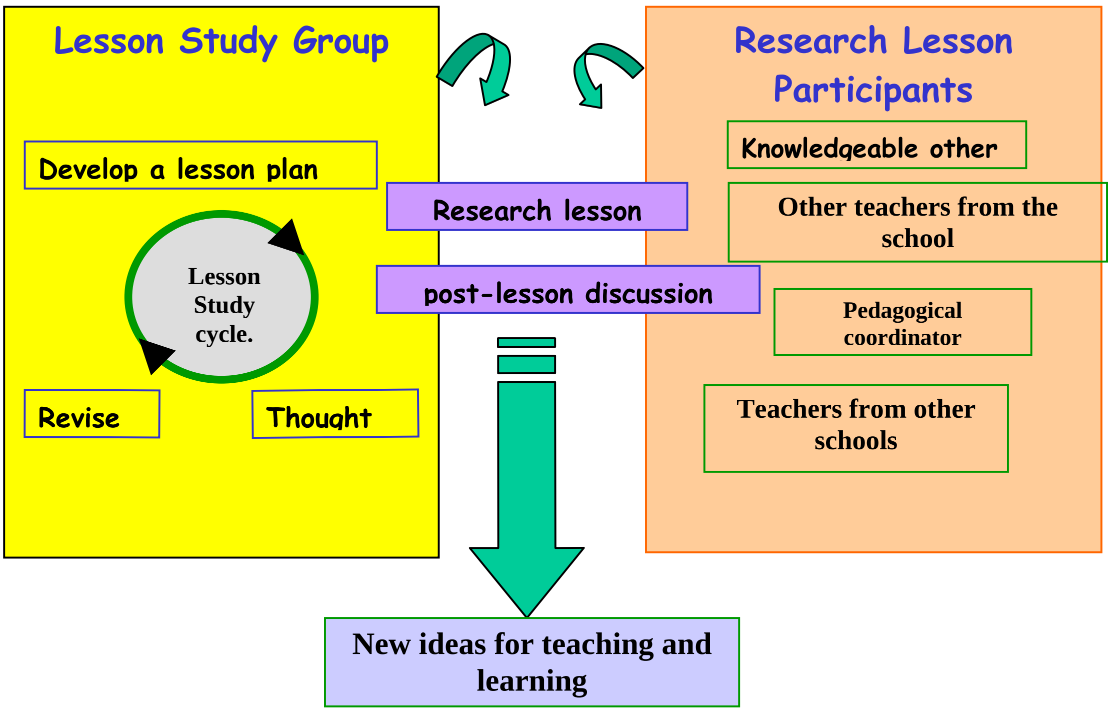
From Numbers and Operations Axis we extracted the progression of communication and argumentation in their 7 levels. (From low to high complexity)

Level of achievement	grade	Description of the communication
1	1 ° and 2º	Describes and explains the strategy used in problem solving
2	3 y 4	Justify the strategy, explaining his reasoning or verifying conjecture through examples.
3	5 y 6	Argues the validity of a process, strategy or conjecture raised.
4	7 y 8	Justify the strategy, the guesses made and the results obtained using concepts, procedures and mathematical relationships.
5	9 y 10	Argues its strategies or procedures and uses examples and counterexamples to verify the validity or falsity of conjecture.
6	11 y 12	Make conjecture that assume generalizations or predictions and argues the validity of the procedures or conjecture
7	exc.	Use mathematical language to present arguments demonstrating mathematical situations.

- **Enhancing mathematical communication in the lesson study**

  It has become apparent close relationship between mathematical thinking and mathematical communication. We had talked about lesson study focused on the development of mathematical thinking, now we are going to add a special regard to the mathematical communication of this process. The question is how teachers can help their students to become **competent math communicators** in order they can describe his thought process in a clear way. Teachers can help them make their thinking visible encouraging others to speak and write about the process being carried out to solve a problem.

  On the below diagram (adapted from Wang-Iverson and Makoto, 2005), we have the full process of lesson study. The proposal is to enhance the mathematical communication on each and every one of its stages:



At each stage, the approach to the topic will be specific and may require some instruments. The following questions can guide the work at every stage:

Stage I when the lesson study group meets to design the research lesson

- What is the language and reasoning level of students that they are going to live in the lesson?
- How or when to give space to students for sharing procedures and reasoning they use to solve the proposals mathematical tasks?
- When we can foster discussions among students and between students and teacher?
- On which specific points, we which to focus mathematical thinking and therefore mathematical communication?
- What we understand by "mathematics competition"?
- Can we draw up a list of mathematical communicative skills that includes sub-competencies?

- **Stage II. When lesson is applied and it is observed by other teachers**

  - How the teacher promotes the dialogue in the classroom?
  - How are the discussions and the *flow of propositions,* when students argue or justify the techniques they employ?
  - How is the teacher's role in promoting math communication?
  - How teacher balance the students communication, (oral, written and representational)?

- **Stage III When they examine and evaluate the implementation of the lesson.**
  - Did thinking and mathematical communication have an outstanding role in the development of the lesson observed?
  - What milestones can be stressed on the mathematical thinking and communication developed in that lesson?
  - What kinds of changes to the lesson plan are suggested to improve the mathematical thought and communication?

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