ENHANCING CLASSROOM COMMUNICATION TO DEVELOP STUDENTS' MATHEMATICAL THINKING

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Introduction

The reform curriculum in Vietnam tries to lessen the training of basic skills and procedures in mathematics but increases more hands-on activities to help students grasp the mathematics ideas and develop mathematical thinking. The mathematics teachers have learnt the generative teaching strategies that encourage mathematical communication to implement more effective lessons focusing on mathematical thinking. Vietnamese mathematics teachers believe that classroom oral communication is of outmost importance for students learning mathematics. In particular, teachers' questioning, listening, and responding approaches in the classroom have been suggested to characterize their pedagogical practices, and to reflect their beliefs about mathematics and its teaching and learning. In this article we will discuss and try to answer three questions related to the enhancing classroom mathematical communication to develop students' mathematical thinking. Firstly, how does our reform curriculum enhance mathematical communication for students? Secondly, what are our kinds of components of communication for developing the mathematical thinking? And thirdly, what is our appropriate teaching model to enhance classroom communication in mathematics?

1. Reform curriculum and students' mathematical communication

In Vietnam, because the emphasis of old curriculum was on procedural knowledge and memorization of algorithms, students often worked independently completing exercises in the textbooks and workbooks. Students practiced mathematics independently to learn the mathematical concepts. When asking students questions, most teachers seek one "right" answer to the math problem and will explain why the answer is correct.

School reform mathematics education aims to help students achieve four following objectives: knowledge, skills, thinking and attitudes. Especially, with thinking objective, the mathematics curriculum provides opportunities for students to develop:

- The ability of observing, verifying, predicting, rational reasoning and logical reasoning;
- The ability of expressing precisely and clearly their own ideas and understanding the ideas of others;
- Spatial imagination;
- The characteristics of thinking, especially the flexible, independent and creative thinking.
- Thinking operations: comparison, analogy, generalization, and specialization.

The assessment of students' ability to communicate mathematics should provide evidence that they can express mathematical ideas by speaking, writing, demonstrating and depicting them visually.

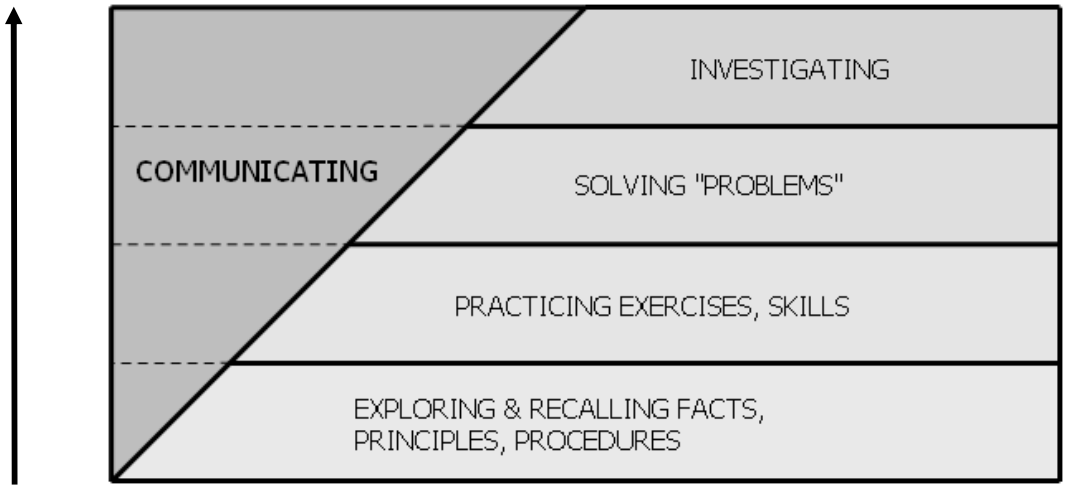


Figure 1. *Levels of thinking and opportunities for communicating at each level*

The reform curriculum supports active learning that creates opportunities for students' mathematical communication.

Traditional Curriculum vs. Reform Curriculum	
Traditional Curriculum Supports	*Reform Curriculum Supports*
Remembering formulas, facts.	Understanding mathematics concepts, deriving formulas.
Focus on procedural knowledge of algorithms	Communicate thoughts, explanations, focus on problem solving process and investigation.
Training of basic skills and procedures in mathematics.	More hands-on activities to help students grasp the mathematics ideas and develop mathematical thinking
Students work independently completing workbooks and home works. Students are not accustomed to "talk" about mathematical concepts.	Students think about, discuss, extend, elaborate, verbalize, write, listen, read, and inquire about mathematical concepts.
Textbook-driven, teachers' clear explanations and classroom control.	Communication is about participating, interpreting, and negotiating meanings, and it involves all classroom members alike.
Disseminate knowledge, facts, algorithms and generally expect students to identify and replicate the fields of knowledge disseminated	Students learn "naturally" by reading, writing, discussing, elaborating, thinking, and inquiring mathematics.
Students' independent practice.	Students should be able to work cooperatively in groups and independently to

	make the necessary mental constructions about a particular mathematical concept.
When asking students questions, most teachers seek one "right" answer to the math problem and will explain why the answer is correct	The teachers facilitates and organizes students to explore new knowledge and solve non-routine problems.

2. Main components of communication for developing the mathematical thinking

From the reform curriculum we can define five main components of mathematical process that may promote students' communication in the classrooms at each level.

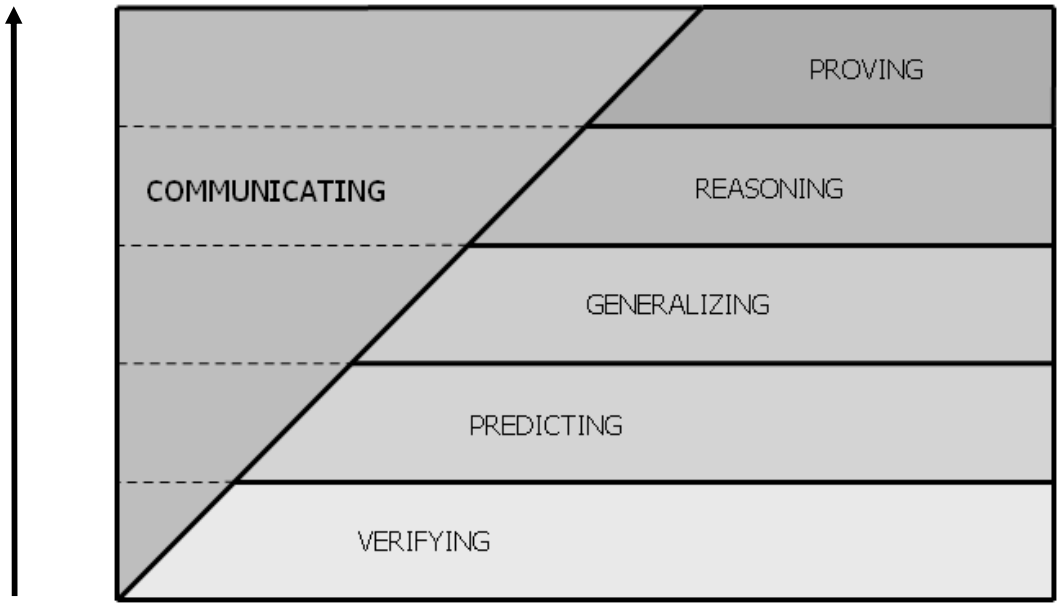
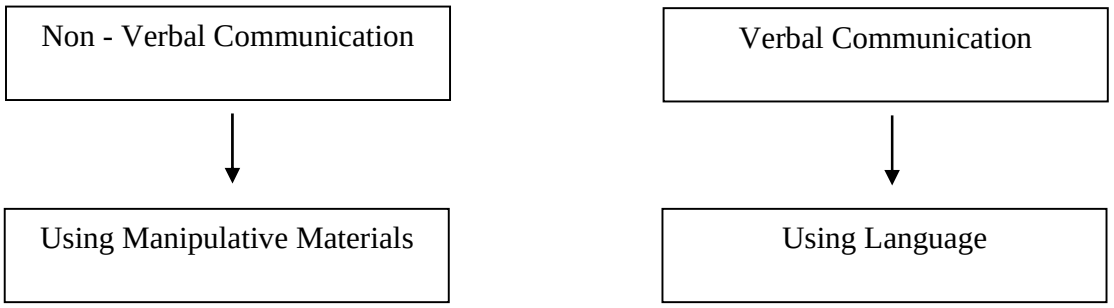


Figure 2. *Components of mathematical process that may promote communication at each level of thinking*

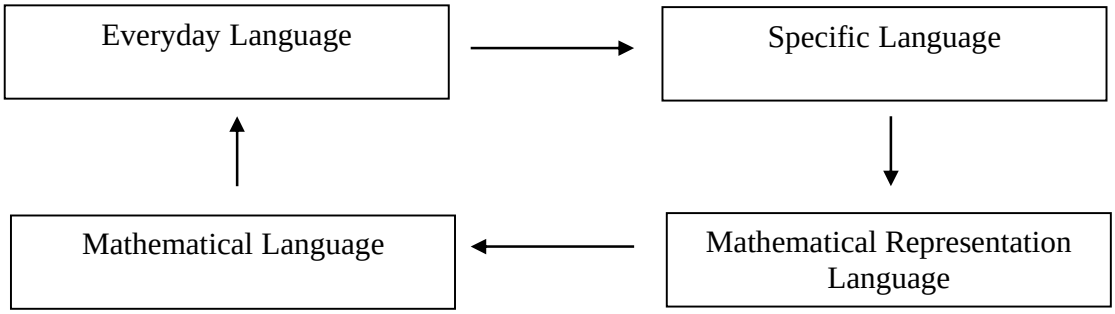
Sometime we categorize the features of communication in the classrooms as follows. These categories and their features will help teachers define what to do in the classroom to promote students' communication.

Categories	**Features**
Tools: WHAT	Language, mathematical language, mathematical representations: concepts, symbols, rules, formulas, real materials, virtual materials (computer-based), proofs, examples, counterexamples.
Ways: HOW	Graph, pictures, models, talking, writing solutions, ways of explanation, reasoning, reflecting, discussing, amending, negotiating, ways of proving.
Needs: WHY	To inquire, to understand, to explore, to adjust, to solve, and to check.
Contexts: WHEN	Attitudes, get stuck, meet false propositions, failure, and need help.

For very young children, non-verbal communication, such as the use of materials to demonstrate their thinking is the earliest form of communication.



Communication Skills in Mathematics



3. Teaching model to enhance classroom communication in mathematics

When students construct their own knowledge of mathematical concepts, they need to have the opportunity to think about, discuss, extend, elaborate, verbalize, write, listen, and read in the mathematics classroom. Normally in a mathematics class, students are not accustomed to talk about mathematical concepts. They are usually taught the concept by the teacher. Therefore, teachers need to use a number of approaches to probe students' thinking in mathematics. Students are not natural talkers in the mathematics classrooms. Older students will encounter more complex concepts in higher levels of math, discussing, talking, elaborating, writing, reading, and thinking about complex themes and concepts will help students to obtain deeper understanding in mathematics.

3.1. Types of lessons that support the curriculum

From the curriculum and textbooks we can categorize lessons into two main types as follows.

New lessons

a. *Help students pose, explore and solve problems*

   Teacher facilitates students to explore and pose questions, problems of the new lesson when they engage in a problematic situation. Teacher then facilitates students to mobilize what they have experienced and learnt to recognize and seek for the relationships between the posed problem and their known knowledge to find the appropriate strategy to solve problem.

b. *Give students opportunities to consolidate and apply knowledge constructed after learning new lesson such that students start to achieve the new knowledge*

In the mathematics textbook at primary level, after new lesson usually there are three exercises for students to consolidate and practice what they have learnt in the lesson. The first two exercises require students know how to apply and practice directly the new knowledge. The third usually is a problem requires an indirect application of new knowledge.

Consolidation, practice and general practice lessons

a. Help students recognize learnt and new knowledge in various problems
b. Help students self practice on their pace of ability
c. Give students opportunities to help each other in small group with effective interactions by using worksheets.
d. Practice students having habits of checking, evaluating their works.
e. Practice students having habits of finding various strategies and choosing most appropriate strategy to solve problem.

3.2. Four main activities in a lesson

Four main activities in a lesson that teachers should follow to develop students' mathematical thinking (MOET 2006):

Activity 1. Examine and Consolidate the previous knowledge involved with new lesson;

Activity 2. Teacher facilitates students explore mathematical knowledge and construct new knowledge by themselves.

Activity 3. Students practice the new knowledge by solving exercises and problems in the textbooks and exercise books.

Activity 4. Teacher concludes what students have learnt from new lesson and assigns the homework.

Each teaching and learning activity in classrooms gives opportunities for students' communication.

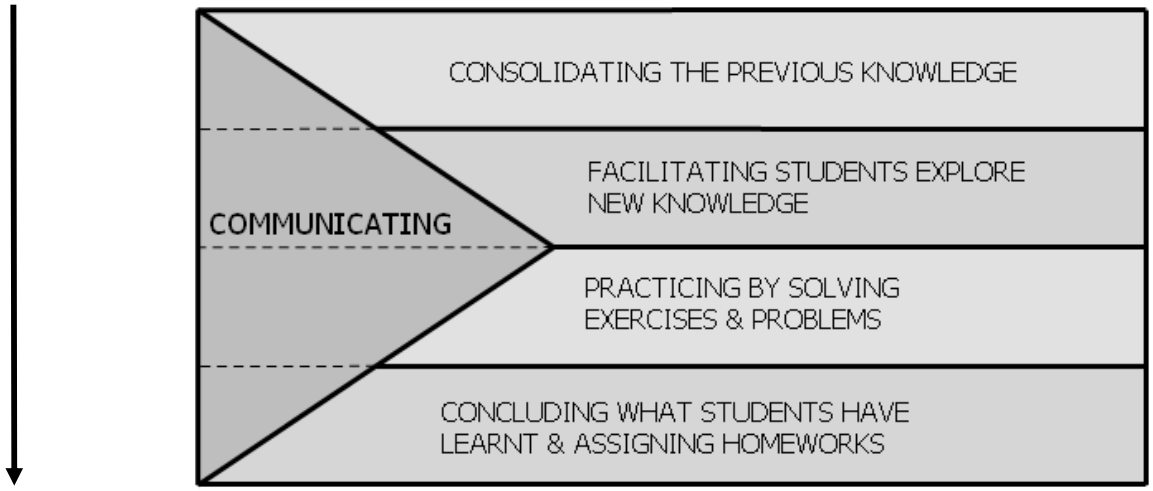


Figure 3. *Teaching activities and opportunities for communicating at each activity*

Engaging to the lesson, the students will have opportunities to show their mathematical thinking through:

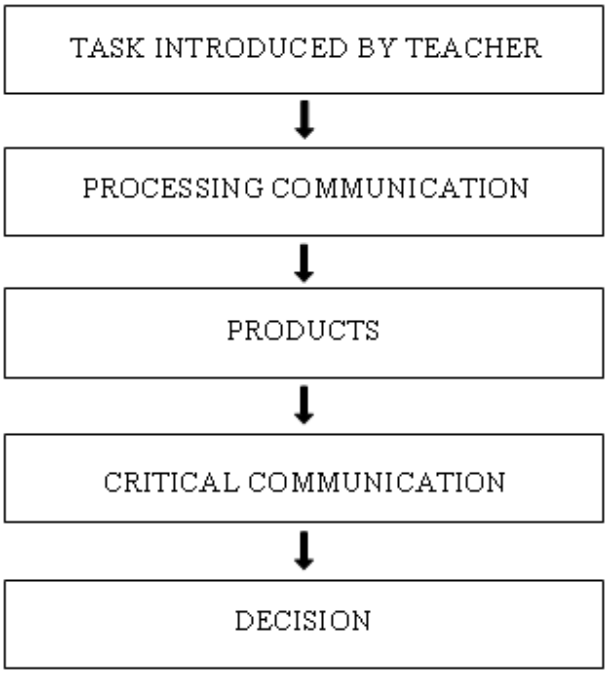
- The ability of observing, predicting, rational reasoning and logical reasoning;
- Knowing how to express procedures, properties by language at specific levels of generalization by words, word formulas;
- Knowing how to investigate facts, situations, relationships in the process of learning and practicing mathematics;
- Developing ability on analyzing, synthesis, generalization, specifying; and starting to think critically and creatively.

The level of difficulty and complexity of a problem is defined by the achievement objectives in the standard curriculum for each strand of mathematics. The exercises in the practice lessons are ranked: from easy to difficult, from simple to complicated, and from direct practice to flexible and combined applications. The communication increasing depends on the level of difficulty and complexity of a problem.

3.3. Teaching strategies

- Promote active, initiative and self-conscious learning of the learners;
- Form and develop the ability of self-study;
- Cultivate the characteristics of flexible, independent, and creative thinking;
- Develop and practice the logical thinking;
- Apply problem solving approaches;
- Apply mathematics to real life situations.

3.4. Teaching flows



3.5. The Teaching Modes

Interpretive teaching is focused on the establishment of a common language inside the classroom, valuing the social aspect of learning. Teachers teaching in an interpretive mode ask fewer testing questions and more genuine and provoking questions than if they were teaching in an evaluative mode. The interpretive listening mode is characteristic of an interpretive teaching mode; therefore, there are increasing opportunities for classroom interaction and discussion. However, responding to students still tends to be evaluative, and typical instructional approaches are mainly textbook-driven, though enriched with problems and activities. Within a classroom environment based on interpretive teaching, students' contributions to the classroom discourse still do not have a significant impact on lesson unfolding, since considerable concern for following lesson plans and avoiding ambiguity are still evident.

Regarding the *generative teaching* mode, communication is about participating, interpreting, and negotiating meanings, and it involves all classroom members alike. Genuine and provoking questions dominate the typical discourse of these teachers, although there is room for testing questions as well. It is the hermeneutic listening mode that characterizes generative teaching, and teachers respond to students by stimulating further discussion (probing, giving feedback, redirecting questions or comments). Instruction is inquiry-driven and not clearly structured. Generative teaching necessarily implies teachers' questioning of their own practices and beliefs, and revising of their own mathematical knowledge while exploring and constructing mathematical ideas with their students.

Most likely, an evaluative teaching mode is associated with an instrumentalist perspective of mathematics teaching and learning, an interpretive teaching mode is related to teachers' Platonist views of mathematics, and a generative teaching mode is connected to a problem-solving perception of what it means to teach and learn mathematics. It is important to notice, though, that *exact* correspondences between teachers' dominant teaching modes and their key beliefs were not anticipated beforehand.

3.6. Investigation-centered learning model

The new paradigm of teaching is to help students construct their knowledge in an active way while working cooperatively with classmates so that their talents and competencies are developed. The assumption is that teaching requires training and skill implicitly and explicitly.

We try to provide a teaching model based on student-centered learning style that should be relevant to the socio-culture of our region. Looking ahead into the future, the learning model should be balanced between behaviorism and constructivism. It means that by then routine learning shall have been lessened and the process of doing and understanding mathematics shall have been increased.

The investigation - centered learning model has been developed to create a rich educational activity for classroom use.

A rich educational activity should (Wheatley, 1991):

- be accessible to everyone at the start;
- invite students to make decisions;
- encourage "*what if*" questions;

- encourage students to use their own methods;
- promote discussion and communication;
- be replete with patterns,
- lead somewhere;
- have an element of surprise,
- be enjoyable,
- be extendable.

Stages in doing investigation

There are five steps in doing mathematics investigation that can be implemented in the classroom.

Stage 1: *Introduce the problem*. The teacher is to capture the students' interest making use of good motivation or having puzzles.

Stage 2: *Clarify the problem*. Use questions to draw out the underlying mathematical questions within the problem.

Stage 3: *Design the investigation*. The teacher led the students to choose the most satisfactory problem solving strategies.

Stage 4: *Carry out the investigation*. Students make and test hypotheses; make generalizations and the teacher must ask questions to guide the students to reach a satisfactory solution to the problem.

Stage 5: *Summarize the learning*. Students need time to present their findings and explain any theories they may have about the mathematical task given. Class questions may tie these discoveries together and reveal the processes used during investigation.

3.7. Group investigation model

Inquiries

- Confrontation with a problem;
- Knowledge results from group process;
- Learner - not teacher - driven.

Knowledge

- Different people see problems differently
- Negotiating the differences creates social and personal understanding.

The four Critical Components of Group Investigation

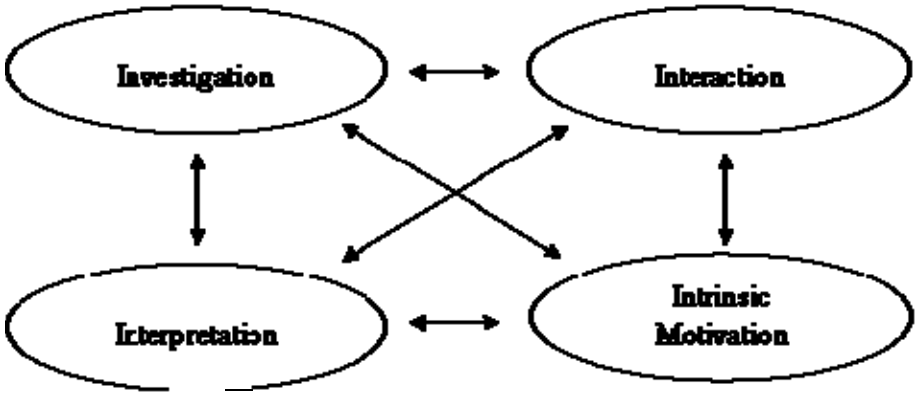


Figure 4. *The interrelation of four critical components in group investigation*

In investigating classrooms these four components are interrelated and occur simultaneously.

1. Investigation refers to the organization and procedures for directing the conduct of classroom learning as a process of inquiry.
2. Interaction identifies the interpersonal, or social dimension of the learning process as it unfolds in the communication among members of small groups in the classroom.
3. Interpretation occurs both at the interpersonal as well as at the individual cognitive level.
4. Intrinsic motivation refers to the nature of students' emotional involvement in the topic they are studying and in the pursuit of the knowledge they seek to acquire.

Stages of Implementation of Group Investigation

Stage 1: *A class determines the subtopics and organizes them into research groups*. Students scan sources, propose questions, and sort them into categories. The categories become subtopics. Students join the group studying the subtopic of their choice.

Stage 2: *Groups plan their investigation*. Group members plan their investigation cooperatively; they decide what they will investigate, how they will go about it and how they will divide the work among themselves.

Stage 3: *Groups carry out their investigations*. Group members gather, organize, and analyze information from several sources. They pool their findings and form conclusions. Group members discuss their work in progress in order to exchange ideas and information, and to expand, clarify, and integrate them.

Stage 4: *Groups plan their presentations*. Group members determine the main idea of their investigation. They plan how to present their findings. Group representatives meet as steering committee to coordinate plans for final presentation to class.

Stage 5: *Groups make their presentations*. Presentations are made to the class in a variety of forms. The audience evaluates the clarity and appeal of each presentation.

Stage 6: *Teacher and students evaluate their projects*. Students share feedback received about their investigations and about their affective experiences. Teacher and students collaborate to evaluate individual, group, and class wide learning. Evaluation includes assessment of higher level thinking processes.

CONCLUSION

Teaching elementary school mathematics aims to equip students with basic mathematics and develop their mathematical thinking to solve problems. Communication is an essential part of the students' learning in mathematical classrooms, it helps students to achieve basic knowledge and develop mathematical thinking. With the reform curriculum, communications are required in the mathematics classrooms. Students may use verbal language to communicate their thoughts, extend thinking, and understand mathematical concepts. They may also use written language to explain, reason, and process their thinking of mathematical concepts. Communication is a tool which can help students to form questions or ideas about concepts. Most of teachers in Vietnam really need a practical framework to develop students' mathematical communication in their actual classrooms. Of course, each economy has slightly different approach in teaching mathematics to promote the mathematical communication, but we still have a lot of thing in common to share and build up together a realistic framework that helps classroom teachers in each APEC economy.

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