

**Cultivating Mathematical
Thinking through Representation**
—Utilizing the Representational System—

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1. Introduction

(1) Importance Roles of Representation

- a) a method of **thinking**
- b) a method of **recording**
- c) a method of **communication**

(2) Important Three Term

- a) Representational **Method**
- b) Representational **Mode**
- c) Representational **System**

2. Previous Study

(1) Bruner's EIS Principle

(E) Enactive Representaion

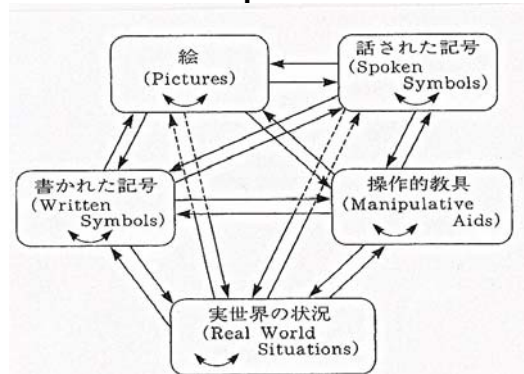
(I) Iconic Represntaion

(S) Symbolic Representation

○ Early Instruction

○ Remedial Instruction

(2) Lesh's Representational System



① Bruner's enactive rep. is divided into two categories.

② **Mutural transformation** are considered

3. Nakahara's Research

(1) **Five Categories** of Representational Mode

S2: Symbolic Rep.

S1: Linguistic Rep.

I : Illustrative Rep.

E2: Manipulative Rep.

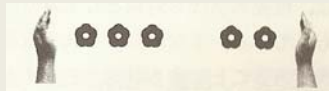
E1: Realistic Rep.

S2: $3 + 2 = 5$

S1: Adding 3 and 2 gives you 5.

I : 

E2:



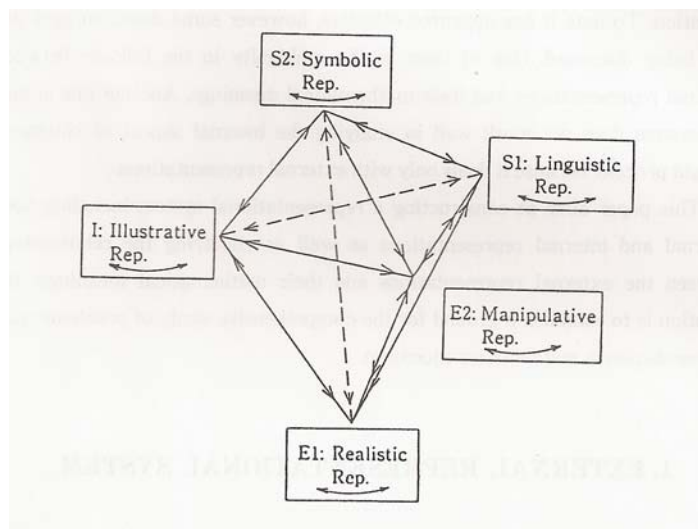
E1:



(2) Characteristics of Rep. Mode

- Symbolic Rep.: **rules**, succinct,
unambiguous
- Linguistic Rep. : rules, **familiarity**
- Illustrative Rep. : analogical, **visually**,
intuitively
- Manipulative Rep. : **dynamic**,
somewhat concrete
- Realistic Rep. : dynamic, **natural**

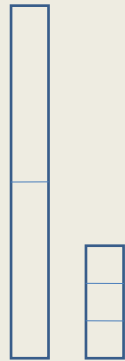
(3) Nakahara's Representational System



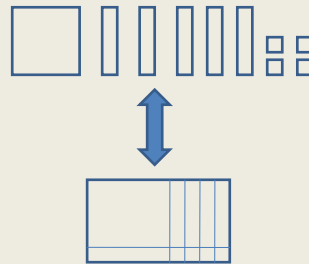
4. Characteristics and Utilization of Representational Mode

(1) Manipulative Representation

① $13 - 8$:



② $X^2 + 5X + 4$



○ Characteristics of Manipulative Rep.

① Dynamic operation and problem-solving

② Intermediateness and mediation

○ Utilization of Manipulative Rep.

① Children themselves carry out dynamic operation

② To generalize concepts or method through dynamic operation

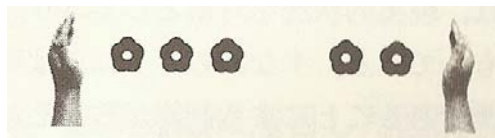
(2) Illustrative Representation

○ Types of Illustrative Rep.

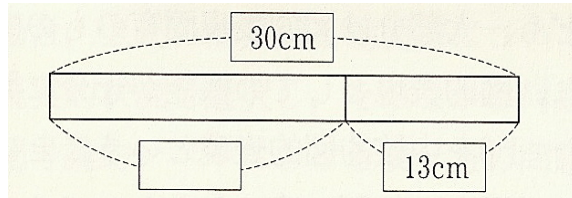
- ① Situation Diagram
- ② Scene Diagram
- ③ Procedural Diagram
- ④ Structure Diagram
- ⑤ Conceptual Diagram
- ⑥ Principle or relationship Diagram
- ⑦ Graphs
- ⑧ Figure



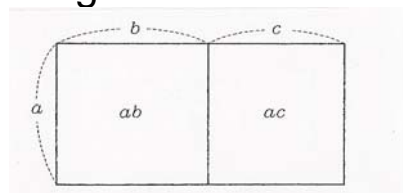
Scene Diagram



Procedural Diagram



Structure Diagram



Principle Diagram

○ Characteristics of Illustrative Rep.

- Analogical relationship
- **Visual and Intuitive rich**
- Imaging ▪ Totality

○ Utilization of Illustrative Rep.

- Situation and Scene diagram
- Procedural and Structure diagram
- Conceptual. Principle and relationship diagram
- Graph and Figure diagram

5. Utilization of Rep. System

(1) Utilization in Class

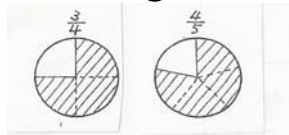
- **Introduction Scene**: Realistics rep.
 - **Solving Scene**: Manipurative rep.
Illustrative rep.
 - **Final Scene**: Linguistics rep.
Symbolic rep.
- ◎ **Mutual translations** between representational modes are important.

(2) Examples

① $3+2$

② Which is larger, $3/4$ or $4/5$?

A1. Figure:



A2. Decimal Fraction: $3/4=0.75$ $4/5=0.8$

A3. Integer: multiplied by 20, 15 and 16

B1. Numerator: $12/16$ and $12/15$

B2. Denominator: $15/20$ and $16/20$

C. Remainder: $1-3/4=1/4$

$$1-4/5=1/5$$