Assessment of Mathematics in Korea

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Abstract

Assessment is an important part of teaching and learning of mathematics. Assessment should reflect what mathematics education should value. However, present practices of assessment in Korea do not necessary reflect the above concerns. Recently, the Ministry of Education Science and Technology(MEST) in Korea has announced some changes about the 7th Korea National Curriculum in order to reinforce the policy to improve the quality of curriculum through the National Assessment of Educational Achievement and other assessment instruments. This report explains the National assessment of Educational Achievement (NAEA) and the College Scholastic Ability Test (CSAT**)** in Korea. At the end of this article, 30 mathematics problems from the 2009 College Scholastic Ability Test for college entrance examination were included to allow us to have a better understanding of the types of questions and levels of difficulty of the mathematics problems in Korea at the high school level.

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1. Introduction

The goal of Mathematics education is that students understand the basic mathematical concepts and principles, observe and interpret the phenomena from a mathematical perspective, think logically and solve problems in everyday situations systematically. The Goals of Mathematics education in the 7th National Curriculum are as follows:

- To understand the basic mathematical concepts, principles and their relationship through observing everyday phenomena from a mathematical perspective.
- To observe, analyze, organize, think and solve diverse problems in everyday situations by applying mathematical knowledge and functions.

In December, 2009, the Ministry of Education Science and Technology (MEST) in Korea announced some changes in the 7th National Curriculum. New assessment guidelines were included. These guidelines are designed to complement the 7th National Curriculum. We should avoid uniformity in mathematics assessment among in schools. The MEST in Korea reinforces the school management to improve the quality of curriculum through the National Assessment of Educational Achievement. Since 2008, the National Assessment of Educational Achievement (NAEA) has been expanded to include all students in the chosen grade in order to grasp individual scholastic achievement. By analyzing individual student's results, teacher can monitor the students' level of achievement in comparison with the curriculum goal of the corresponding grade. The MEST decided to open the results of NAEA to the public in order to improve the quality of mathematics education and to establish the direction of education, and to provide basic criterion for practical policies. In 2010, the MEST has a plan to hire part time teachers to help students who are "below-basic" and live rural area.

2. NAEA(National Assessment of Educational Achievement)

The National Assessment of Educational Achievement (NAEA) is a nationwide test that is implemented to evaluate elementary and secondary school students' achievements, and set clear the educational goals and standards defined in the National Curriculum. The test framework of Mathematics was decided based on the 7th National Curriculum. Its major goal is to produce reliable resources required for diagnosis and improvement in quality of teaching and learning and aims to measure students' academic achievement based on the goals of Mathematics education in the National Curriculum. It is to monitor the educational qualities in order to pursue a high quality school education. It includes not only diagnosing the trend of educational achievement but also applying the results to manage the qualities and effectiveness of school education. Then, as a monitoring system with reinforced feedback, the NAEA reports its results to educators and positively utilizes the data to improve school education. It also serves as a regular indicator of performance of the educational system.

The purpose of the NAEA

- ◆ To systematically diagnose students' achievements in different school levels and to monitor the trend of educational achievement. To establish the direction of education for the enhancement of national competitiveness based on the previous observations, and to provide basic criterion which creates detailed and practical policies.
- ◆ To analyze the degrees of students' achievements according to the educational goal of the National Curriculum and to provide useful information to improve the curriculum by examining its problems found in the classroom.
- ◆ To produce information to set up a policy of the teaching and learning methods by analyzing the items and the relationship between students' achievement and their background variables. To ascertain the key factors relating to student achievement by investigating such background factors as a student's personality, environment, school, and teachers.
- ◆ To guide schools toward better assessment methods by developing and utilizing original and appropriate assessment instruments.
- ◆ To investigate new research designs and methods to attain the fundamental goals of the study on the NAEA such as trend analysis, and the technique and the methods of setting goals and achievement criteria, and in depth analysis of the relationship among the background variables.

History of NAEA in Korea

In 1998, the Korea Institute of Curriculum & Evaluation (KICE) established a fundamental plan for the NAEA. In 1999, KICE had chosen the NAEA as its major task. It planned to develop test items for Social Studies and Mathematics, develop survey questions on students' background, and administer a pretest. Then in 2000, according to the request of the Ministry of Education and Human Resources Development (MOE & HRD), KICE has also developed achievement standards and test items - in Korean, Science and English and administered pre-tests on these subjects. The assessment on Social Studies and Mathematics was administrated planned as first testing subjects.

In 2001, the NAEA was administered to one percent of the population of the sixth, ninth, 10th and 11th grades in five subjects such as Korean, Social Studies, Mathematics, Science and English and KICE continued to perform a study on the results of the assessment.

In 2002, one percent of the target population of the sixth, ninth and 10th grades was sampled and assessed in five subjects: Korean, Social Studies, Mathematics, Science and English. These two

NAEAs have been carefully carried out considering the practical reasons such as staff and budget. The principles of the study on the 2003 National assessment of educational assessment reflected the nature of the NAEA, and with the request of the MOE & HRD; an annual NAEA will carry on woth the five subjects.

In 2003, NAEA 2003 was administered to the sixth, ninth and 10th grades on five subjects including Korean, Mathematics, Social Studies, Science and English. The whole procedure from deciding achievement standards to developing and analyzing the test items will be thoroughly investigated according to the purpose of the study. Raw scores have been moderated using the same scale to produce equivalent scale scores. Then the scale scores are classified into 4 performance levels: Advanced, Proficient, Basic, and Below-basic. Raters were trained and the test was double rated to aim at high inter-rater reliability.

Subjects and the scope of the NAEA 2003

School Year	Scope	Test Period	Subjects	Number of Subjects
6th Grade	the content covering the 4th to 6th grade	Oct.22~23, 2003	228 schools 228 classes	1% of the population (app 7,000)
9th Grade	the content covering 7th to 9th grade	Oct.22~23, 2003	175 schools 175 classes	1% of the population (app 7,000)
10th Grade	the content covering the 10th grade	Oct.22~23, 2003	172 schools 172 classes	1% of the population (app 7,000)

In 2004, the trend analysis include the main test items for the analysis in those major subjects. In order to secure the equating, the common test items will take at least 30% of the whole number of the main test items. The constructed response items will be enforced on the major subjects. The items that faithfully represent the nature of these subjects will be developed.

Until 2007, NAEA was not a comprehensive test but a sample test of 1% of the population. Therefore, it is impossible to provide the NAEA result of all students. In 2007, NAEA took a sample test of 3%~5% of the population, 3% from Grades 6 and 9, 5% from Grade 10.

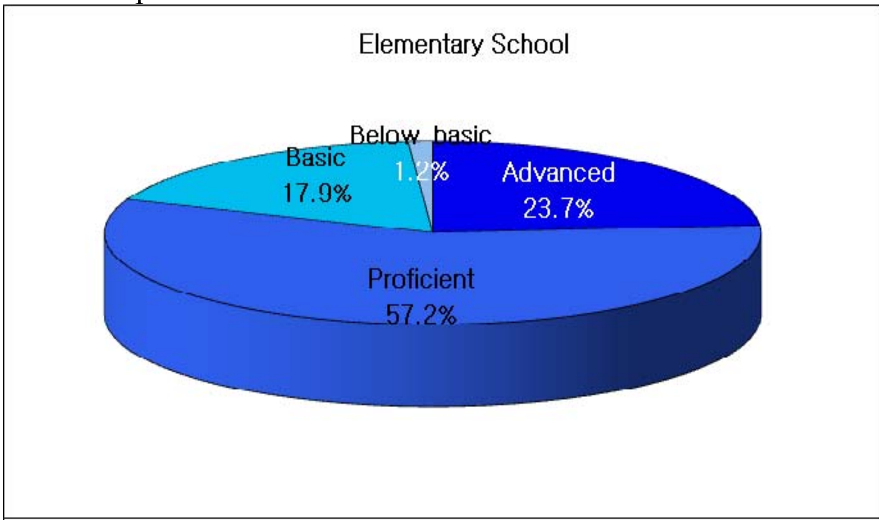
Subjects and the scope of the NAEA 2007

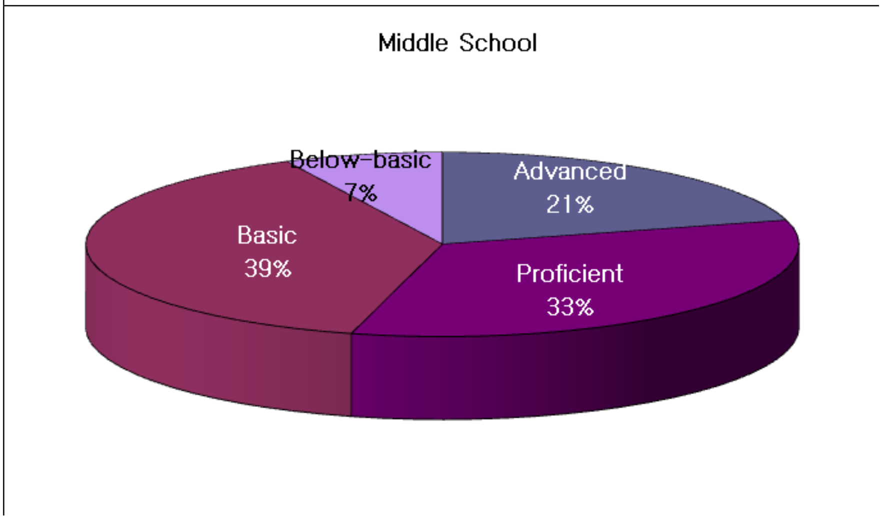
School Year	Scope	Test Period	Subjects	Number of Subjects
6th Grade	the content covering the 4th to 6th grade	Oct.16~17, 2007	345 schools 345 classes	3% of the population (app 20,000)
9th Grade	the content covering 7th to 9th grade	Oct.16~17, 2007	306 schools 306 classes	3% of the population (app 20,000)
10th Grade	the content covering the 10th grade	Oct.16~17, 2007	433 schools 433 classes	5% of the population (app 30,000)

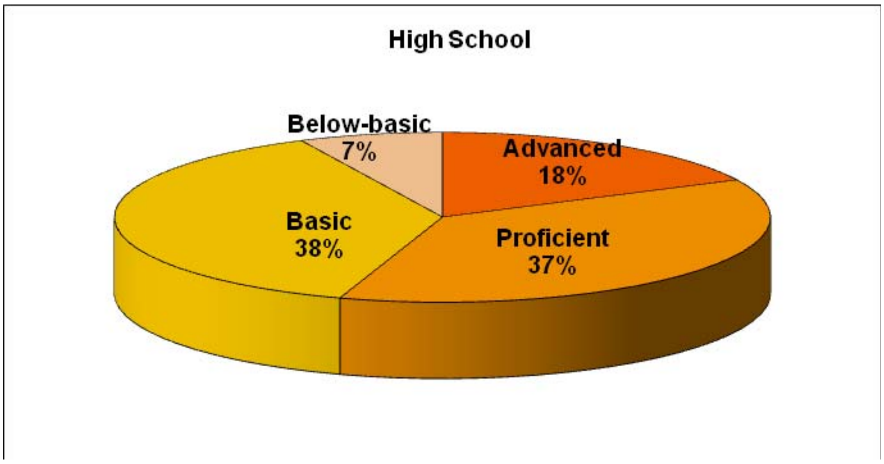
Since 2008, all students of the sixth, ninth and 10th grades were assessed. NAEA become a comprehensive test and it is possible to report accurate picture of achievement levels of all the students in Korea.

Analysis of NAEA 2008

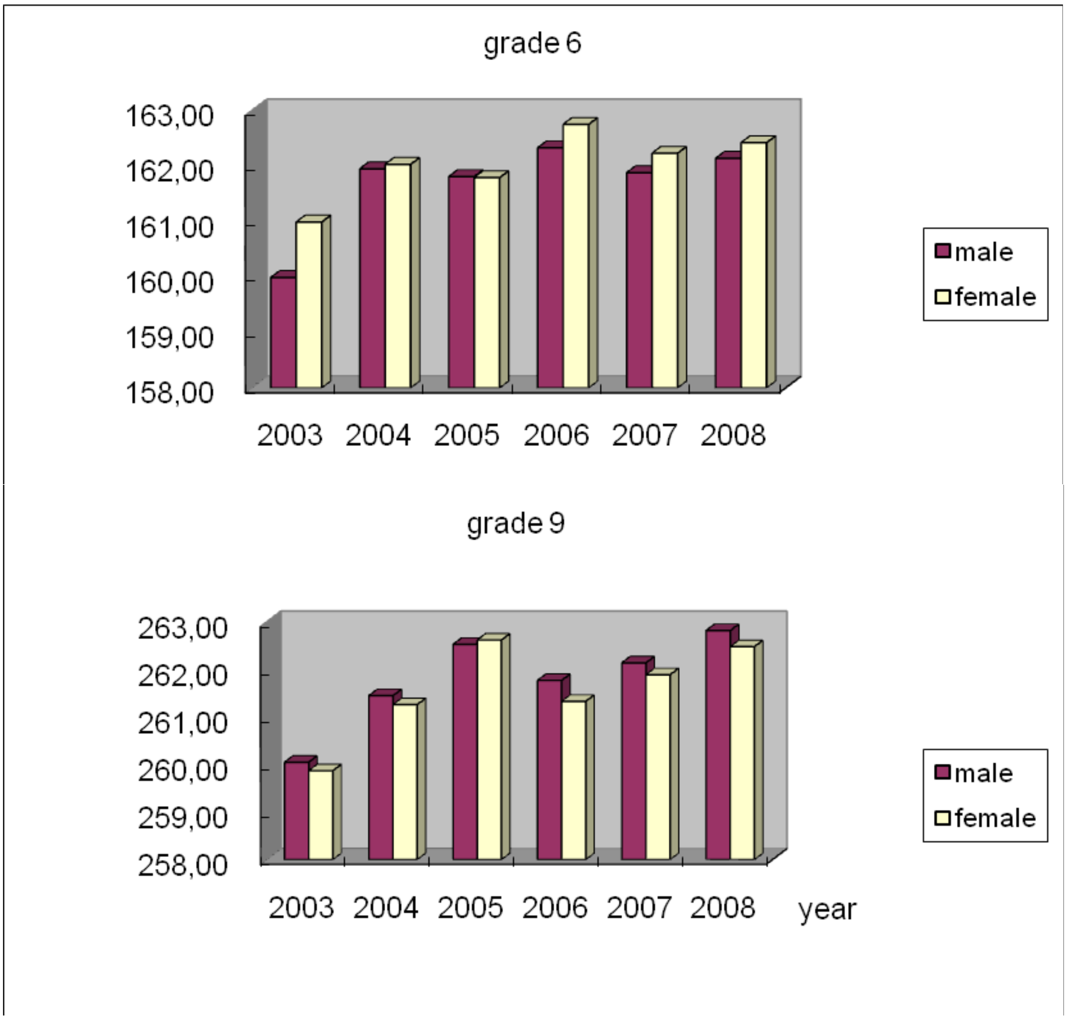
In 2008, Questionnaires were also administered to students, teachers, and principals in order to investigate the relationships between contextual variables and students' achievements in NAEA. After a session of pilot tests was administered to a randomly sampled group of the entire population in May 2008, the main test was implemented in mid October of 2008. Based on performance on the test, students' achievement was separated into in 4 levels, Advanced (understand curriculum content more than 80%), Proficient (understand curriculum content between 50% & 80%), Basic (understand curriculum content between 20% &50%), and Below-basic (understand curriculum content less than 20%). The results obtained from the Mathematics tests are as follows: Grade 6 students were labeled as proficient in general, while Grade 9 and Grade 10 students were generally labeled as proficient or basic.

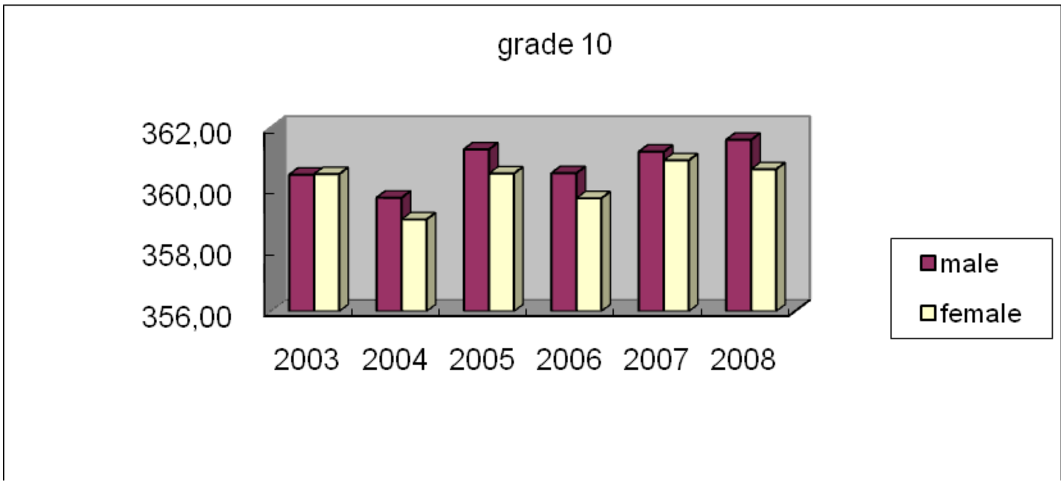




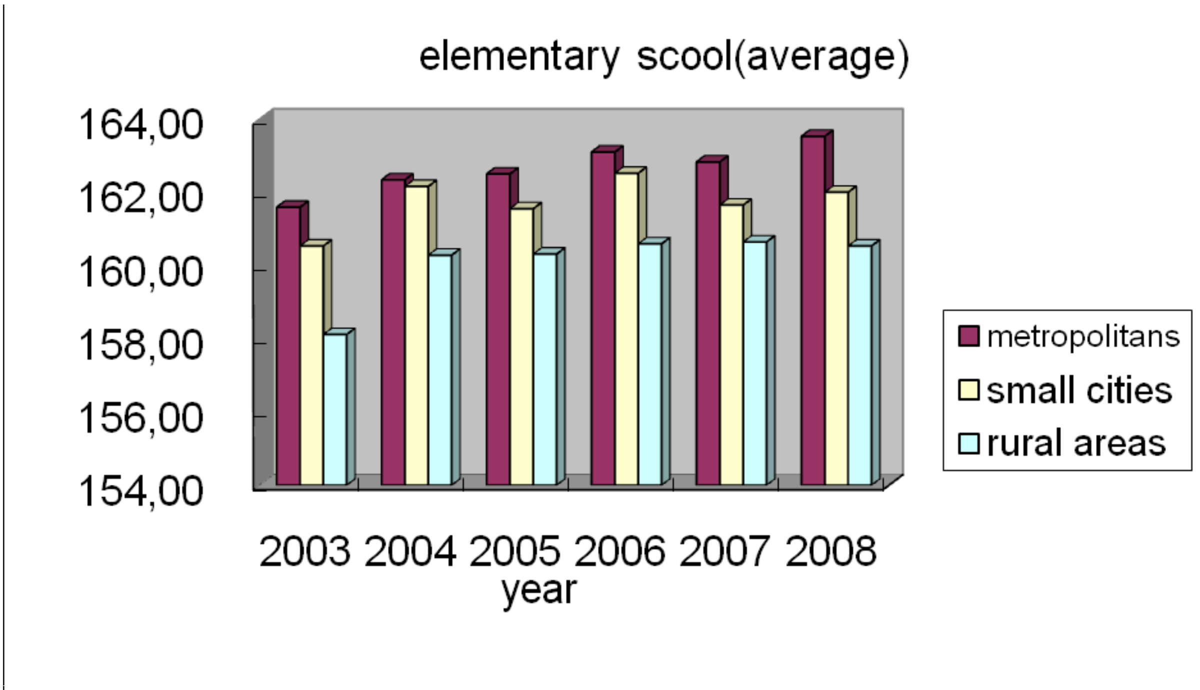


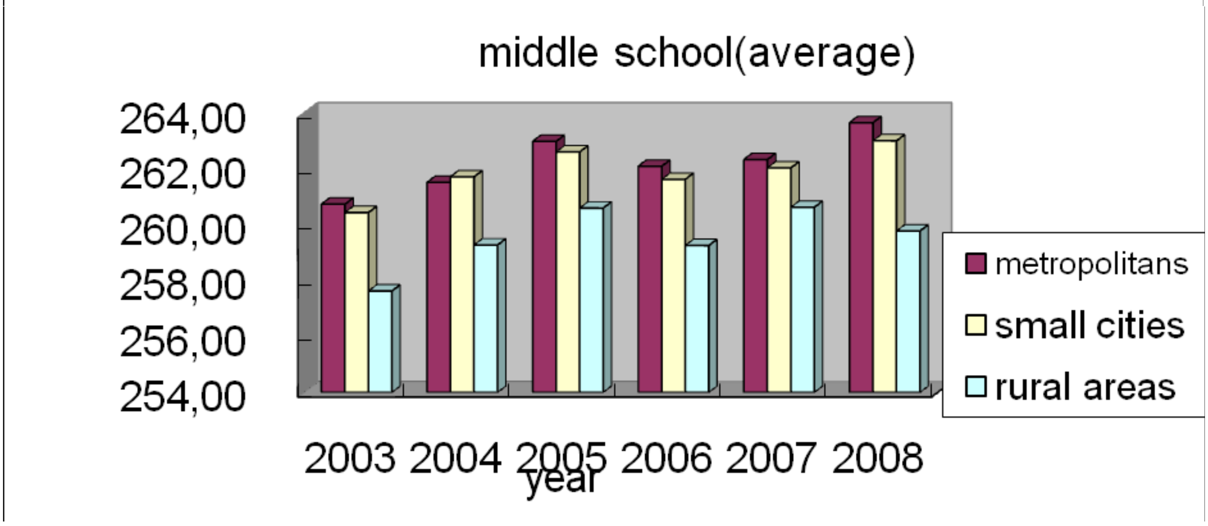
The results showed that there were statistically significant differences in test scores according to the two contextual variables, gender and region.

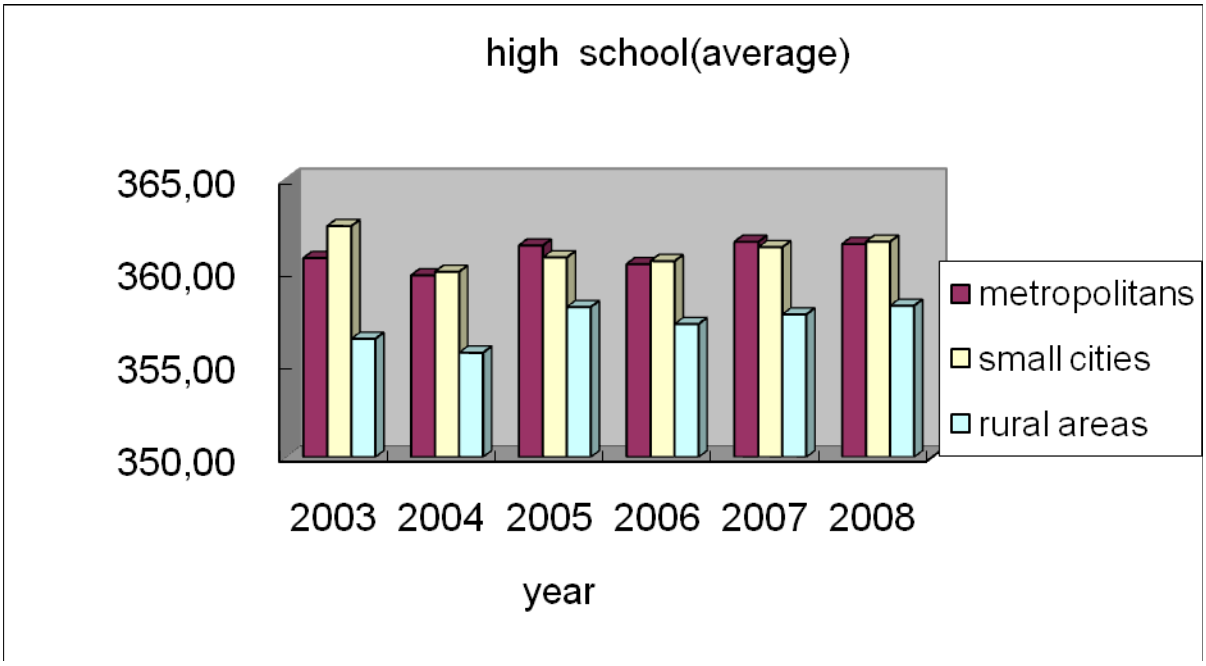




More specifically, male students' test scores were higher than female students' scores in Grades 9 and 10, while female students' test scores were higher than male students' scores in Grade 6.







In Grades 6 and 9, the students in small cities and rural areas obtained significantly lower scores than those in metropolitan areas. But In Grades 10, the students in small cities achieved similar scores level in compare to those in metropolitan areas.

The school, teacher, and student variables were obtained from three types of questionnaires: school-administrator, teacher, and student. Throughout the six years of comparison, the scores of private schools were higher in all the five subjects compared to public schools. High schools which emphasized on 'college entrance' and 'development of creativity' had the highest test performance. When school administrators perceived their teachers having the following aptitude: 'enthusiasm for teaching', 'a good understanding of students', 'high expectations for the students', and 'a good relationship with other teachers', their students performed well in all the five subjects. When teachers perceived low level of concerns regarding the performance gaps of students, students' low interest in learning, students' low attention to the lesson, low class participation, and low respect for teachers, students did not performed well. The more time students spend on communicating with their parents, and the longer hours students spent on self-study and supplementary lessons after school, the higher their achievement score. In Grade 6 and Grade 9 students, students who read more books achieve higher than those student who read less. Generally, the higher the students’ self-regulated learning, the higher their achievement. Generally, the more students have private lessons, the higher their achievement was. In Grade 10, the more time students spent on their hobby, the higher their achievement. Since students comprehend mathematics through various learning strategies, a variety of evaluation techniques have to be employed. Evaluation on students' knowledge base and problem-solving ability must not be limited to paper-and-pencil testing alone. Various evaluation methods should be encouraged.

3. Korea Scholastic Aptitude Test

The basic policy is to maintain the current framework of the test. It emphasizes its fairness and objectivity in measuring students' scholastic ability in college education. Students select one out of two types of the test ('A' type and 'B' type) in Mathematics. Test items will be focused on 11th and 12th grade level subjects. Since the contents of 11th and 12th grade subjects are based upon the previous subjects, it is feasible that test items could be related to those subjects indirectly.

Number of Problems and Time Limit

Classification Area		Number of Problems		Total Time (Time in each problem)	Problem Type
Math (Select one)	'A' type	30	High school Math I : 12 items (40%) High school Math I : 13 items (43%) Electives: 5 items(17%)	100 Minutes (3.3 minutes)	multiple choice (70%) short answer (30%)
	'B' type	30	Highs school Math I	100 minutes (3.3 minutes)	multiple choice (70%) short answer (30%)

Examination Scope in each Area

Area		Examination Scope
Mathematics (select 1)	'A' type	High school math I + High school math II +(Select 1 among Calculus, Statistics and probability, Discrete mathematics)
	'B' type	High school math I

Standardized scores

classification	Number of problems	Original Perfect Score	Standardized Scores		
			Mean	Standard Deviation	Range
Mathematics	**30**	**100**	**100**	**20**	**0~200**

4. College Scholastic Ability Test(2009)- Science Major Bound

1. (2 points) Evaluate $9^{\frac{5}{4}} \times 27^{-\frac{1}{2}}$.

① $\frac{1}{3}$ ② 1 ③ $\sqrt{3}$ ④ 3 ⑤ $3\sqrt{3}$

2. (2 points) Find the sum of all the elements in the matrix $(A+B)A$,

Where $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$, and $B = \begin{pmatrix} -1 & -2 \\ 1 & 0 \end{pmatrix}$.

① 9 ② 10 ③ 11 ④ 12 ⑤ 13

3. (2 points) Find the value of the constant a such that $f(x) = 6x^2 + 2ax$ and $\int_0^1 f(x)dx = f(1)$ is true.

① -4 ② -2 ③ 0 ④ 2 ⑤ 4

4. (3points) Find the sum of all real roots of the equation

$\sqrt{x^2 - 2x + 1} - \sqrt{x^2 - 2x} = \frac{1}{2}$.

① 1 ② 2 ③ 3 ④ 4 ⑤ 5

5 (3points) The right diagram shows a circle with radius 1, centered at the origin O, and a quadratic function $y = f(x)$ passing through point (0,-1). Find the number of real roots x in the equation

$\frac{1}{f(x)+1} - \frac{1}{f(x)-1} = \frac{2}{x^2}$.

① 6 ② 5 ③ 4 ④ 3 ⑤ 2

6. (3 points) Let $h(x) = f(x)g(x)$ where $f(x) = x^2 - 4x + a$ and $g(x) - \lim_{n\to\infty}\frac{2|x-b|^n+1}{|x-b|^n+1}$. Find the sum $a = b$, when the constants a and b are numbers which makes function $h(x)$ continuous in the real domain.

① 3 ② 4 ③ 5 ④ 6 ⑤ 7

7. (3 points) Find the sum of constants a and b for the two exponential functions $f(x) = a^{bx-1}$ and $g(x) = a^{1-bx}$ which satisfy the following conditions:

a) The graph of function $y = f(x)$ and the graph of function $y = g(x)$ are symmetric with respect to the line $x = 2$
b) $f(4) + g(4) = \frac{5}{2}$
c) $(0 < a < 1)$

① $\frac{3}{2}$ ② $\frac{11}{8}$ ③ $\frac{5}{4}$ ④ $\frac{9}{8}$ ⑤ 1

8. (3points) A company, recognized by the World Handball Confederation, produces handballs for women hand-ball games. The ball produced by this company has an average weight of 350g with a standard deviation of 16g. The company randomly weighs 64 balls produced in a certain time period. If the average weight of the balls is less than 346g, or greater than 355g, the company

reviews the manufacturing procedure. What is the probability for the review to occur based on the normal distribution on the bottom.

z	$P(0 \le Z \le z)$
2.00	0.4772
2.25	0.4878
2.50	0.4938
2.75	0.4970

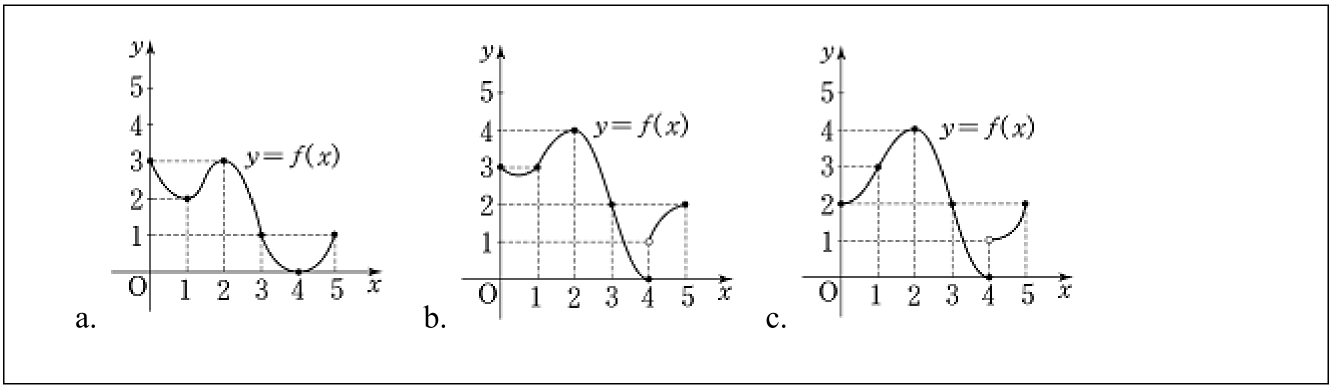
① 0.0092 ② 0.0152 ③ 0.0184

④ 0.0258 ⑤ 0.0290

9. (4 points) A function $y = f(x)$ is defined on a closed interval [0,5].

Let function $g(x)$ be defined as $g(x) = \begin{cases} \{f(x)\}^2, & 0 \le x \le 3 \\ (f \circ f)(x), & 3 < x \le 5 \end{cases}$

What is the graph of the function $y = f(x)$ which makes the function $g(x)$ continuous on the closed interval [0, 5] ?



① a ② b ③ c ④ a, b ⑤ b, c

10. (3 points) The sequence $\{a_n\}$ is defined as

$$\begin{cases} a_1 = \frac{1}{2} \\ (n+1)(n+2)a_{n+1} = n^2 a_n, n = 1,2,3 \cdots \end{cases}$$

The following is a proof by mathematical induction that $\sum_{k=1}^{n+1} a_k = \sum_{k=1}^{n} \frac{1}{k^2} - \frac{n}{n+1} \cdots$ (*) is true for all natural number n.

<Proof>

1) When $n = 1$, left side = $\frac{1}{2}$, right side $= 1 - \frac{1}{2} = \frac{1}{2}$. Therefore (*) is true.

2) When $n = m$, if (*) is assumed to be true

$\sum_{k=1}^{m+1} a_k = \sum_{k=1}^{m} \frac{1}{k^2} - \frac{m}{m+1}$ is true.

The following shows that (*) is also true for $n = m + 1$

$$\sum_{k=1}^{m+1} a_k = \sum_{k=1}^{m} \frac{1}{k^2} - \frac{m}{m+1} + a_{m+1}$$

$$= \sum_{k=1}^{m} \frac{1}{k^2} - \frac{m}{m+1} + \alpha \; a_m = \sum_{k=1}^{m} \frac{1}{k^2} - \frac{m}{m+1} + \frac{m^2}{(m+1)(m+2)} \cdot \frac{(m-1)^2}{m(m+1)} \cdot \cdots \frac{1^2}{2 \cdot 3} a_1$$

$$= \sum_{n=1}^{m} \frac{1}{k^2} - \frac{m}{m+1} + \beta$$

$$= \sum_{n=1}^{m} \frac{1}{k^2} - \frac{m}{m+1} + \frac{1}{(m+1)^2} - \gamma$$

$$= \sum_{n=1}^{m+1} \frac{1}{k^2} - \frac{m+1}{m+2}$$

Therefore (*) is true for $n = m + 1$, So (*) is true for all natural number n.

What are the correct equations for α, β, γ.

	α	β	γ
①	$\frac{m}{(m+1)(m+2)}$	$\frac{1}{(m+1)^2(m+2)}$	$\frac{1}{(m+1)(m+2)^2}$
②	$\frac{m}{(m+1)(m+2)}$	$\frac{m}{(m+1)^2(m+2)}$	$\frac{1}{(m+1)(m+2)}$
③	$\frac{m^2}{(m+1)(m+2)}$	$\frac{1}{(m+1)^2(m+2)}$	$\frac{1}{(m+1)(m+2)^2}$
④	$\frac{m^2}{(m+1)(m+2)}$	$\frac{1}{(m+1)^2(m+2)}$	$\frac{1}{(m+1)(m+2)}$
⑤	$\frac{m^2}{(m+1)(m+2)}$	$\frac{m}{(m+1)^2(m+2)}$	$\frac{1}{(m+1)(m+2)^2}$

11. (4 points) The following is true for a polynomial function $f(x)$ and two natural number m, n.

$$\lim_{x\to\infty}\frac{f(x)}{x^m}=1,\quad \lim_{x\to\infty}\frac{f'(x)}{x^{m-1}}=a$$

$$\lim_{x\to 0}\frac{f(x)}{x^n}=b,\quad \lim_{x\to\infty}\frac{f'(x)}{x^{n-1}}=9$$

(a and b are real numbers)

Which of the following is true?

a. $m \geq n$

b. $ab \geq 9$

c. If $f(x)$ is a cubic function, am = bn.

① a ② c ③ a, b ④ b, c ⑤ a, b, c

12. (4 points) Let a set U be defined as

$U = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle|\ a, b, c\ \&\ d \text{ are any positive numbers except for } 1\right\}$

If S is a subset of U, and is defined as $S = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle|\ \log_a d = \log_b c,\ a \neq b,\ bc \neq 1\right\}$

Which of the following is true?

a. If $A = \begin{pmatrix} 4 & 9 \\ 3 & 2 \end{pmatrix}$, then $A \in S$.

b. If $A \in U$ and an inverse matrix of A exists, then $A \in S$.

c. If $A \in S$, an inverse matrix of A exists.

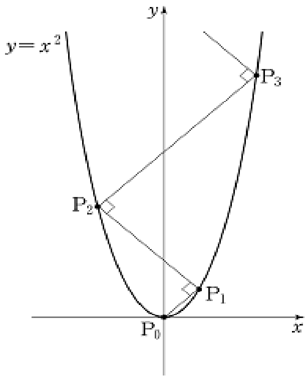
① a ② b ③ a, c ④ b, c ⑤ a, b, c

13. (3 points) For natural number n, there are two points P_{n-1} and P_n which are lying on a graph of a function $y = x^2$. Point P_{n+1} is determined by the following rules.

a. The coordinates for two points P_0 and P_1 are (0, 0) and (1, 1).

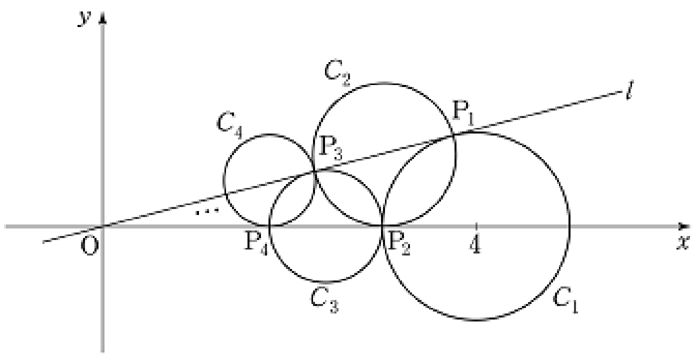
b. A point P_{n+1} is an intersection of a function $y = x^2$ and a straight line which is perpendicular to a line $\ell_n = P_{n-1}\,P_n$ and passes through point P_n. (P_n and P_{n+1} are different points)

What is the value of $\lim_{n\to\infty}\frac{\ell_n}{n}$?



□ $\sqrt{2}$ □ $\sqrt{3}$ □ 2 □ $2\sqrt{2}$ □ $2\sqrt{3}$

14. (4 points) There is a circle $C_1 : (x-4)^2 + y^2 = 1$ on the coordinate plane. Line ℓ has a positive slope, passes through the origin, and is tangent to the circle C_1. The tangent point is P_1. Circle C_2 is tangent to the x axis, passes through point P_1, and its center lies on the line ℓ. The point of intersection of circle C_2 and the x axis is called P_2. Circle C_3 is tangent to line ℓ, passes through point P_2, and its center lies on the x-axis. The point of intersection of circle C_3 and line ℓ is P_3. Circle C_4 is tangent to the x-axis, passes through point P_3, and its center lies on the line ℓ. If the above procedure is continued, What is the value of $\sum_{n=1}^{\infty} S_n$, where S_n is the area of the circle C_n. (The radius of the circle C_{n+1} is smaller than the radius of the circle C_n)



① $\frac{7}{2}\pi$ ② 3π ③ $\frac{5}{2}\pi$ ④ 2π ⑤ $\frac{3}{2}\pi$

15. (4 points) A community service center runs 4 daily programs.

Program	A	B	C	D
Service Hours	1	2	3	4

Sam wants to participate in 1 program per day, for 5 days. Sam is trying to make a service hour plan sheet, so that the total hour adds up to 8 hours. What is the total possible number of plan sheets that can be made?

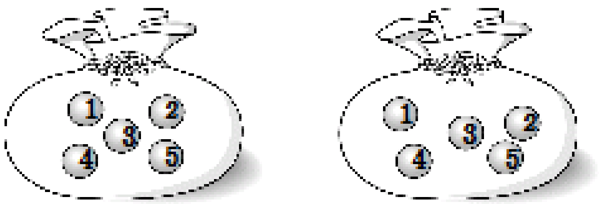
Service hour plan sheet

name :

Date	Program	Service hour
2009.1.5		
2009.1.6		
2009.1.7		
2009.1.8		
2009.1.9		
Total service hours		8 hours

① 35 ② 38 ③ 41 ④ 44 ⑤ 47

16. (4 points) Each bag A and B contains five marbles, inscribed 1, 2, 3, 4, 5. Sam takes 1 marble from bag A, and Ann takes one marble from bag B, they read the number inscribed on the marble, and do not put the marbles back in the bag. If this is continued, what is the probability of having first set of marbles with different inscribed numbers, and the second set of marbles with the same inscribed numbers?



① $\frac{3}{20}$ ② $\frac{1}{5}$ ③ $\frac{1}{4}$ ④ $\frac{3}{10}$ ⑤ $\frac{7}{20}$

17. (4 points) Information theory states that, when event E happens, the amount of information of event E, I(E) is defined as follows.

$$I(E) = -\log_2 P(E)$$

Which of the following are true? (P(E) is the probability of event E happening, is positive, and the unit of "the amount of information" is bit.)

a. If E is an event of getting an odd number on the dice when a dice is thrown, then $I(E) = 1$

b. If two events A and B are independent and if $P(A \cap B) > 0$,

then $I(A \cap B) = I(A) + I(B)$.

c. For two events A and B which are $p(A) > 0$ and $p(BA) > 0$,

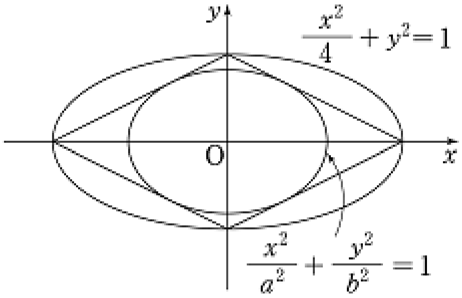
it is true that $2I(A \cup B) \leq I(A) + I(B)$.

① a ② a, b ③ a, c ④ b, c ⑤ a, b, c

18. (3 points) Let $\lim_{x \to 2} \frac{f(x+1)-8}{x^2-4} = 5$ for the polynomial function $f(x)$. Find the value of $f(3) + f'(3)$.

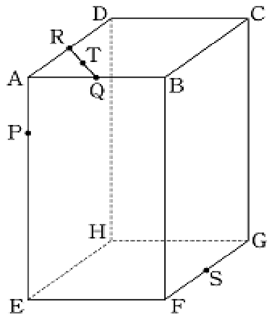
19. (3 points) A square is made by 4 vertices of an ellipse $\frac{x^2}{4} + y^2 = 1$ And the square contains an inscribed ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are $F(b, 0)$ and $F'(-b, 0)$, then $a^2b^2 = \frac{q}{p}$ is true.

Find the value of p+q. (p and q are natural numbers which are relatively prime)



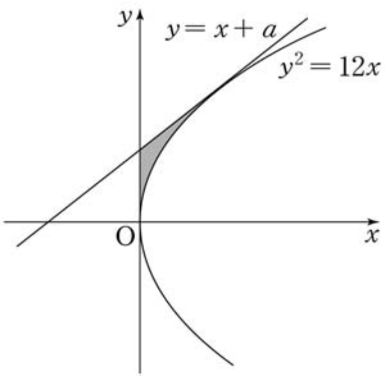
20. (3 points) A Right hexahedron ABCD-EFGH has length $\overline{AB} = \overline{AD} = 4$ and $\overline{AE} = 8$.

A point P divides line AE in the ratio of 1:3. The centers of lines, AB, AD & FG are Q, R & S respectively.



The center of the line QR is T. Find the dot product of the vectors $\overrightarrow{TP}$ and $\overrightarrow{QS}$.

21. (3 points) A straight line $y = x + a$ meets a parabola $y^2 = 12x$. A solid of revolution is created by rotating an area with respect to the x axis, where the area is enclosed by parabola $y^2 = 12x$ line $y = x + a$, and y axis. The volume of the solid of revolution is $b\pi$. Find ab, for two constants a and b.

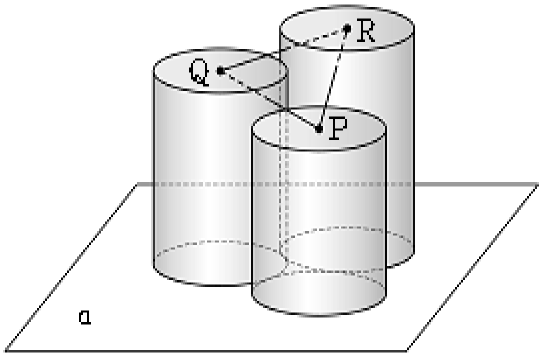


22. (4 points) Point P exists between points A(3, 3, 3) and a sphere $x^2 + y^2 + z^2 = 9$ centered at the origin. The value of $\left| \frac{2}{3}\overrightarrow{OA} + \frac{1}{3}\overrightarrow{OP} \right|$ has a maximum value of $a + b\sqrt{3}$. Find the value of $10(a + b)$. (a and b are rational numbers)

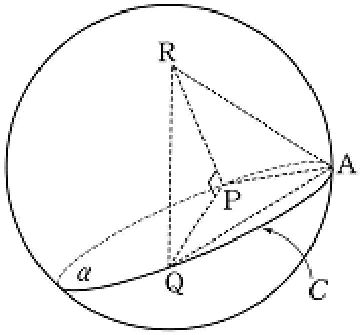
23. (4 points) a_n is the sum of all numbers, which have the same quotient and remainder when divided by natural number n $(n \geq 2)$. For example, the natural numbers with the same quotient and remainder when divided by 4 are 5, 10 & 15 so $a_4 = 5 + 10 + 15 = 30$

. Find the smallest possible natural number n which satisfies $a_n > 500$

24. (4 points) Three circumscribed cylinders, all with radius of $\sqrt{3}$ are lying on the plane α . The base of the cylinder, which doesn't lie on the plane has three centers, P, Q, and R. Triangle QPR is an isosceles triangle. The angle formed by plane QPR and plane α is 60^0. If the heights of the cylinders are, 8, a & b, find the value of a+b when 8 < a < b.



25. (4 points) On the coordinate space, circle C is the intersection of the sphere $S: x^2 + y^2 + z^2 = 4$ and plane $\propto : y - \sqrt{3}\,z = 2$. For point A(0, 2, 0) on circle C, two end points , P & Q of the diameter satisfy the condition $\overline{AP} = \overline{AQ}$. Point R is a point of intersection of sphere S and a line perpendicular to plane $\propto$. The area of a triangle ARQ is s. What is the value of s^2.



Calculus (problem 26 through 30 are for students who chose Calculus as their elective).

26. (3 points) Find the sum of all the x for $0 \le x < 2\pi$ which satisfies the following equation,

$\sin 2x = 2\cos x - 2\cos^2 x$.

① 2π ② $\frac{7}{4}\pi$ ③ $\frac{3}{2}\pi$ ④ $\frac{5}{4}\pi$ ⑤ π

27. (3 points) A continuous function $f(x)$ is defined on a closed interval $[0, 1]$. $f(0) = 0,\ f(1) = 1$. If the second derivative of $f(x)$ exists on an open interval $(0, 1)$, $f'(x) > 0$ & $f''(x) > 0$, which of the following has the same value as $\int_0^1 \{f^{-1}(x) - f(x)\}dx$?

① $\lim_{n\to\infty}\sum_{k=1}^{n}\{\frac{k}{n} - f(\frac{k}{n})\}\frac{1}{2n}$

② $\lim_{n\to\infty}\sum_{k=1}^{n}\{\frac{k}{n} - f(\frac{k}{n})\}\frac{2}{n}$

③ $\lim_{n\to\infty}\sum_{k=1}^{n}\{\frac{k}{n} - f(\frac{k}{n})\}\frac{1}{n}$

④ $\lim_{n\to\infty}\sum_{k=1}^{n}\{\frac{k}{2n} - f(\frac{k}{n})\}\frac{1}{n}$

⑤ $\lim_{n\to\infty}\sum_{k=1}^{n}\{2\frac{k}{n} - f(\frac{k}{n})\}\frac{1}{n}$

28. (3 points) Which of the following are true for the function $f(x) = 4\ln x + \ln(10 - x)$.

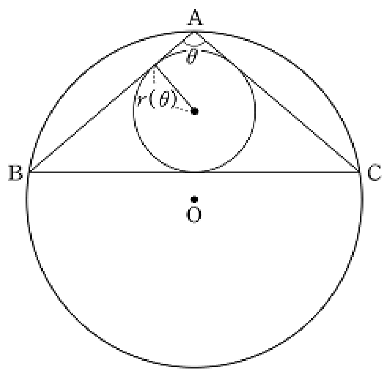
a. The maximum value of the function $f(x)$ is $13\ln 2$
b. Equation $f(x) = 0$ has two distinct real roots.
c. Function $y = e^{f(x)}$ is concave up in the interval (4, 8).

① a ② c ③ a, c , ④ b, c ⑤ a, b, c

29. (4 points) Function $f(x)$ is defined as $f(x) = \int_a^x \left(2 + \sin\left(t^2\right)\right) dt$. . If $f''(a) = \sqrt{3}\,a$, find the value of $\left(f^{-1}\right)''(0)$. (a is a constant with the range $0 < a < \sqrt{\frac{\pi}{2}}$)

① $\frac{1}{2}$ ② $\frac{2}{5}$ ③ $\frac{3}{10}$ ④ $\frac{1}{5}$ ⑤ $\frac{1}{10}$

30. (4 points) Point A lies on a circle of radius 1. Place two other points B & C on the circle so that $\angle BAC = \theta$ where θ is a positive number and $\overline{AB} = \overline{AC}$. If the radius of a circle inscribed in a triangle ABC is $r(\theta)$, $\lim_{\theta \to \pi - 0} \frac{r(\theta)}{(\pi - \theta)^2} = \frac{q}{p}$. Find $p^2 + q^2$, (natural numbers p and q are relatively prime.)



Probability and Statistics (problem 26 through 30 are for students who chose Probability and Statistics as their elective)

26(3 points). A basketball player practices 40 free throws a day. The following stem-and-leaf plot represents the number of successful throws each day, for the first 10 days. The tens are represented as the stem, and ones are represented as the leaf. On the eleventh day, n successful free throws were made, out of 40 trials,
If the mean number of successes over the past eleven days was equal to the mode of the stem-and-leaf plot, what is the value of *n*?

stem	leaf
0	9
1	7 9
2	1 4 4 6
3	0 1 3

① 32 ② 30 ③ 28 ④ 26 ⑤ 24

27(3 points). Toss a fair coin three times in a row. Let a random variable X represent the number of identical faces in continuous trials. That is,

$X = 0$ if identical faces did not occur in a row.
$X = 1$ if identical faces occur only twice in a row.
$X = 2$ if identical faces occur three times in a row.

What is the variance of X?

① 9/8 ② 19/16 ③ 5/4 ④ 21/16 ⑤ 11/8

28(3 points). Suppose an urn contains nine balls numbered 1 to 9. If four balls were drawn at random, what is the probability that the sum of the largest number and the smallest number is greater than 7 and less than
9?

① 1/3 ② 7/18 ③ 4/9 ④ 1/2 ⑤ 5/9

29(4 points). Let X be a random variable having a normal distribution with mean 0 and variance σ^2 and Y a normal distribution mean with 0 and variance $\frac{\sigma^2}{4}$. If two positive numbers a and b satisfy $P(\,|X| \le a) = P(\,|Y\,| \le b)$; which of the following are true? Select all that apply.

a. $a > b$
b. $P\left(Z > \frac{2b}{\sigma}\right) = P\left(Y > \frac{a}{2}\right)$
c. $P(|X| \le a) = 0.3, \quad \text{when } P(Y \le b) = 0.7$

① a □ b □ a, b □ b, c □ a, b, c

30(4 points). Given a sample size of 100, suppose that a 95% confidence interval of a population proportion was $\left[\frac{1}{10} - c,\ \frac{1}{10} + c\right]$. If a data containing n people was chosen from the same population again and yielded a
95% confidence interval of the population proportion being $\left[\frac{1}{9} - s(n),\ \frac{1}{9} + s(n)\right]$, where $s(n) = \frac{50}{81}c$.
What is the value of n?

Discrete mathematics(problem 26 through 30 are for students who chose Discrete mathematics as their elective)

26. (3 points) A sequence $\{a_n\}$ satisfies the following recursion relation:

$$\begin{cases} a_1 = 1,\ a_2 = 2 \\ a_{n+2} + a_{n+1} + a_n = 6,\ n = 1,2,3 \cdots \end{cases}$$

Find $\sum_{k=1}^{11} a_k$.

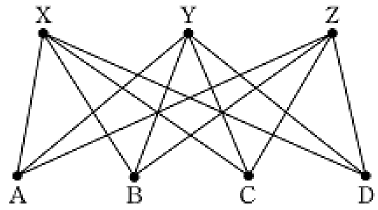
① 15 ② 18 ③ 21 ④ 24 ⑤ 27

27. (3 points) The following table shows the work, amount of days required for each work and the prerequisite works to be completed for each work. Find the least number of days required to finish the entire work..

Work	Required number of days	Prerequisite work(s)
A	2	None
B	3	A
C	5	A
D	3	A, C
E	2	B, C
F	4	D
G	3	E, F

① 17 ② 18 ③ 19 ④ 20 ⑤ 21

28. (3 points) A graph H can be formed by adding the minimum number of edges to the following graph with vertices A, B, C, D, X, Y, Z, so that the vertices of the graph H can be properly colored with 4 colors. How many different graphs are possible for H?



① 20 ② 18 ③ 16 ④ 14 ⑤ 12

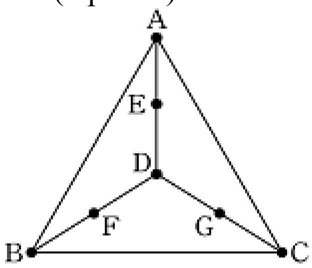
29. (4 points) How many combination of length of 6 is possible, with letters A, B, C, D, E, F if repetition of letters is not allowed and the following conditions are satisfied:

a. B can not directly precede A.
b. C can not directly precede B.
c. A can not directly precede C.

(for example, CDFBAE satisfies the above conditions, but CDFABE does not.)

① 380 □ 432 □ 484 □ 536 □ 598

30. (4 points) Find the total number of spanning tree from the following graph.



5. Concluding remarks

The main purpose of mathematics assessment should be to improve the teaching and learning of mathematics. Assessment should be in harmony with the national curriculum content and goals. However, there seems to be a gap between Korea national curriculum and present assessment systems. During the last decade, many Korean mathematics educators are attempting to overcome a gap between the assessment system and the goals of the current curriculum. So, there were dramatic changes in mathematics assessment in Korea. Portfolio, reports and attitudes were adopted in assessment. Specially, performance Assessment which includes student participation, degree of efforts, attitudes, individual and group reports was introduce. However, there could be problems to guaranteeing objectivity in evaluating students' mathematics performance. Because Korea is the third most densely populated country in the world and an extremely competitive society and also mathematics is the major screening subject in the College Scholastic Ability Test. It is very important to evaluate students' performance without causing any problem. In order to avoid such problems, some teachers adopted the assessment in classrooms which is focused on written paper and pencil tests and examinations. There is a need to encourage many teachers with the know-how to evaluate their students fairly in a variety of assessments other than written pen and paper tests and examinations.

Based on the aims and goals, the Korea Institute of Curriculum & Evaluation (KICE) has been conducting many research projects regarding mathematics assessment. The results of the studies, however, were not adopted in the classrooms. Without changing the way of the College Scholastic Ability Test, we can not reform present assessment systems in the classroom in Korea. And Without changing present assessment systems in the classroom, It is impossible to reform teaching and learning of mathematics because the assessment system greatly related the way of teaching

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