How materials were influenced in 20th century: Klein movement

Mariolina Bartolini Bussi Università di Modena e Reggio Emilia (Italia)

> <u>bartolini@unimore.it</u> <u>www.mmlab.unimore.it</u>

ITALIA



Università di Modena e Reggio Emilia

Laboratorio delle Macchine Matematiche



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Main References: Instruments



In Italian, with Dr. Michela Maschietto



In Japanese, thanks to prof. Masami Isoda and his team





Maria G. Bartolini Bussi · Daina Taimina · Masami Isoda

Structure of this presentation

- Klein's legacy in the 20° century
- Examples of Japanese and Italian Textbooks
- Italy: the Mathematical Laboratory
- The Laboratory of Mathematical Machines in Modena
 - ➤ A regional program
 - Example (secondary school)
 - Example (primary school)
- The need of a theoretical framework
- Tentative conclusions



History: Klein's Legacy

As the first president of the

International Commission on Mathematical Instruction (1908)

Klein promoted the approach to mathematics through resources and technologies towards the applications of mathematics

Japan and Italy: Textbooks

In the 40s, a similar development can be observed in some Japanese and Italian textbooks for students in grades 6-9.





Japan



From everyday mechanism to mathematical instruments (Monbusyo, 1943)



Italy



Intuitive geometry : the sum of angles of a triangle (Amaldi, 1941)

Source: Menghini in http://www.mat.uniroma1.it/ricerca/gruppi/education/ISLANDA2copia.htm

Italy

Intuitive geometry (E. Castelnuovo, 1948)



I realised that the construction of a figure with a ruler and a compass limits the thoughts's freedom, because you can consider only a finite number of cases: drawing is static and does not stimulate the observation nor leads to new discoveries. I understood that it is better to construct geometric figures with concrete material that can be handled and you can do and undo (Interview, 2008).

Source: Menghini in http://www.mat.uniroma1.it/ricerca/gruppi/education/ISLANDA2copia.htm

Italy



Emma Castelnuovo, http://www.icmihistory.unito.it/clips.php



Mathematical exhibition organized together with her students (grades 6-8)



Mathematical Laboratory

A mathematics laboratory activity involves people, structures, ideas, as well as a Renaissance workshop, in which the apprentices learn by doing, seeing, imitating and communicating with each other, namely practicing. In the activities, the construction of meanings is strictly bound, on one hand, to the use of tools, and on the other, to the interactions between people working together.

(*Matematica 2003*, prepared by the UMI-CIIM on behalf of the Italian Ministry of Education)



As a part of this trend



What is a mathematical machine?

A mathematical machine (concerning geometry) is a tool that forces a point to follow a trajectory or to be transformed according to a given law.

A mathematical machine (concerning arithmetics) is a tool that allows counting, ordering, reckoning.

What is a mathematical machine? Geometry

A mathematical machine (concerning geometry) is a tool that forces a point to follow a trajectory or to be transformed according to a given law.



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Examples









<image><image><image><image>

Bonaventura Cavalieri (1632)

The Laboratory of Mathematical Machines (Modena)

More than 200 mathematical instruments

- To draw curves
- To realize correspondences between plane regions
- To realize perspective drawings
- To solve problems

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The Laboratory of Mathematical Machines (Modena)

2008-2010: 4 new laboratories in the region Emilia – Romagna An impressive network of teachers (all grades), teacher educators, mentors

The Laboratory of Mathematical Machines (Modena)

Academic staff: M. Bartolini, M. Maschietto, F. Martignone, R. Garuti
Association "Macchine Matematiche"
5 Centers for educational documentation
Local authorities for schools

What happens in a laboratory session?

When? Where?	PRE - PRIMARY	PRIMARY	SECONDARY	TEACHER EDUCATION
LESSONS IN SCHOOL				PRE SERVICE
SPECIAL PLACE OUTSIDE SCHOOL				IN SERVICE

What happens in a laboratory session?

When? Where?	PRE - PRIMARY	PRIMARY	SECONDARY	TEACHER EDUCATION
LESSONS IN SCHOOL				PRE SERVICE
SPECIAL PLACE OUTSIDE SCHOOL				IN SERVICE

What happens in a laboratory session?

What happens in a laboratory session?

SECONDARY SCHOOL

A CLASSROOM VISIT TO THE MMLAB (LAB OPERATOR & STUDENTS)

A two-hours activity in the MMLab Lab. format

- Short introduction of the Lab operator.
- Small group work of students guided by an exploration sheet
- Production of a conjecture about the trajectory of the tracing point and of a proof in the analytic frame
- Public presentation of the results of each small group work made by a spokeperson
- Summary of the Lab operator

A two-hours activity in the MMLab Lab. format

• Short introduction of the Lab operator.

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A two-hours activity in the MMLab Lab. format

- Small group work of students guided by an exploration sheet
- Production of a conjecture about the trajectory of the tracing point and of a proof in the analytic frame

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Exploration sheet

Two parts

Build a mathematical model of the physical instrument (measuring, identifying important parts modelling)

Produce a conjecture about the functioning and prove it.

A two-hours activity in the MMLab Lab. format

• Summary of the Lab operator

In the classroom

- Short introduction of the Lab operator.
- Small group work of students guided by an exploration sheet
- Production of a conjecture about the trajectory of the tracing point and of a proof in the analytic frame
- Public presentation of the results of each small group work made by a spokeperson
- Summary of the Lab operator

In a similar way we have designed classroom lessons in cooperation with or completely run by the teacher

The Laboratory of Mathematical Machines (Modena)

Ongoing studies (cognitive analysis)

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(Martignone & Antonini)

What happens in a laboratory session? When? **TEACHER** PRE -PRIMARY **SECONDARY EDUCATION** PRIMARY Where? LESSONS **IN SCHOOL PRE SERVICE** SPECIAL PLACE OUTSIDE **IN SERVICE** SCHOOL 24

What happens in a laboratory session?

PRIMARY SCHOOL

INSIDE THE CLASSROOM (TEACHER and STUDENTS)

What is a mathematical machine? Arithmetics

A mathematical machine (concerning arithmetics) is a tool that allows counting, ordering, reckoning.

What happens in a laboratory session?

PRIMARY SCHOOL

A Japanese book for Elementary School

Publ. GAKKOH TOSHO 3° grade

Thanks to Prof. Isoda

A Chinese book for Elementary School

Historical perspective 2001 standards

Thanks to Beijing Normal University Press (4° grade)

Pascal's mechanical calculator (1642)

Schickard's mechanical calculator (1623)

1623

Others

Leibiniz, 1671

Florence: Burattini XVII cent.

A mechanical calculator in the mathematics classroom

www.quercetti.com

Teaching experiments in primary and secondary school and with prospective primary school teachers

Zero+1

Epistemological analysis.

What is the mathematics knowledge at stake?

The generation of the number sequence by means of the operator +1

Relationship between syntactical and semantic properties of natural numbers

Place value conventions

(composition/decomposition)

Addition (and subtraction) algorithms.

•••••

TASK 1

Write down the instruction for making an addition with the Pascaline. For instance

28 + 14

Thanks to Franca Ferri, primary school teacher-researcher

Two exemplary solutions

Christian:

I have written the first number (28) and then I have added the second one, rotating clockwise the unit wheel four steps and the ten wheel only one step. The result is 42.

Orlando:

I have written the number 28, then I have turned clockwise 14 times the wheel on the bottom right, the unit one. The number is 42.

TASK 2

Look carefully what Christian and Orlando have written to reckon on the Pascaline

28 + 14

Try and write the mathematical expressions that represents the two different solutions.

TASK 2

Christian:

I have written the first number (28) and then I have added the second one, rotating clockwise the unit wheel four steps and the ten wheels only one step. The result is 42.

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I have written the number 28, then I have turned clockwise 14 times the wheel on the bottom right, the unit one. The number is 42.

TASK 2

Look carefully what Christian and Orlando have written to reckon on the Pascaline

28 + 14

Try and write the mathematical expressions that represents the two different solutions

Only mathematical signs

Christian
= (20 + 10) + (4 + 8) =
= 30 + 12 =
= 42
Orlando
= (20 + 8) + (1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 +
= 20 + (8 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 +
= 20 + 22 =
= 42

Mathematical signs and drawings

ORLANDO:	CHRISTIAN:
28+15=52 28 	$\frac{\frac{1}{284}}{\frac{145}{52}}$ $28+19=52$ $\frac{1}{284}$ $\frac{145}{\frac{145}{52}}$ $\frac{145}{52}$ $\frac{145}{52}$ $\frac{1}{52}$

Comments, mathematical signs and drawings

Orlando Christian 28 +1+1 + 1+1+1+1+1+1+1+1+ 28+14 = ノナノナノナノナノ= 42 =20+10+8+4= 28+14= =-28+14= = 42 Secondo-me Christian= 62 28+14 = ha fatto l'operazione 4 yolt siamo altusti Invece Orlando ei sta facendo seoprire

A mechanical calculator in the mathematics classroom

In this way the teacher encourages students' involvement constructs the condition for transforming situated texts into mathematical texts

A mechanical calculator in the mathematics classroom

The evolution of situated texts into mathematical texts is not "spontaneous"

Task (Euclid, 1,1)

On a given finite straight line to construct an equilateral triangle.

To be solved by ruler and compass

To avoid break and failure

A lot of didactical "ingenuity" is required

Theoretical Framework

handbook of INTERNATIONAL RESEARCH in MATHEMATICS EDUCATION second edition

> LYN D. ENGLISH ASSOCIATE EDITORS MARIA BARTOLINI BUSSI GRAIIAM A. JONES RICHARD A. LESIR BUARTII SHRAMAN

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Artifacts and signs after a Vygotskian perspective

Maria G. Bartolini Bussi Università di Modena e Reggio Emilia

Maria Alessandra Mariotti Università di Siena

High manipulability artifacts (from the historical phenomenology of mathematics) and Information and Communication Technologies

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lini Bussi 1a e Reggio Emilia

Maria Alessa dra Mariotti

Università di Siena

MARIA BARTOLINI BUSSI GRAHAM A. JONES Richard A. Lesh Bharath Srikaman

То

My tentative conclusions (February 2010)

Maybe we have something to share with the international experts of Lesson Study But definitively in Italy we have to study more how Lesson Study is conceived and implemented in Japan and in other countries.

on the education system and on "standard" schools problematic, but possible

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How materials were influenced in 20th century: Klein movement

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