CLASSROOM TASKS FOR THE MATHEMATICAL PROBLEM SOLVING ON THE SITUATIONS OF THE TSUNAMI

(for high school level)

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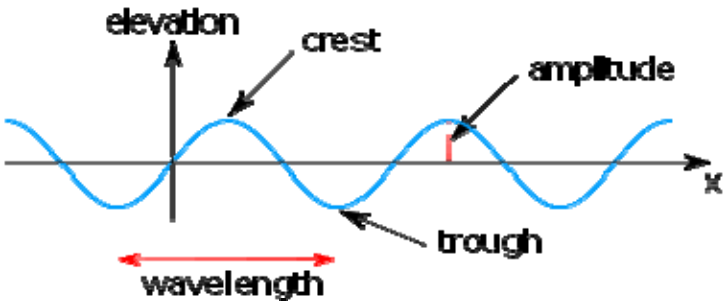
Due to geographical characteristics, the Vietnamese coastal areas are likely to be greatly affected by any tsunami in the East Sea area. The waters to the north of the Philippines can pose the most danger to Vietnam because it would take around two hours for a tsunami to hit the central coastal areas of Vietnam from this area. In such a case, the coastal area in Danang City will be the hardest-hit in Vietnam.

Wave propagation and dispersion



Usually a tsunami has a main crest whose height h was not much more than a metre. The wavelength L would have been roughly of the order 300 km. So the maximum slope of the surface would have been roughly $2\pi h/L$ - only about 1 in 50,000. Such a tiny slope would be quite invisible from any of our vantage points.

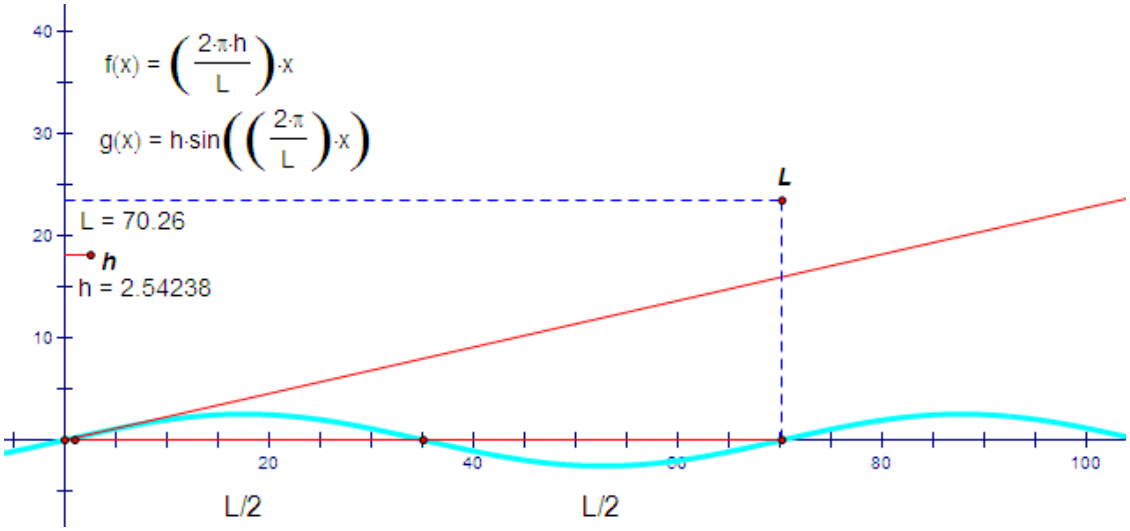
Task 1. **The maximum slope of a long tsunami wave**

It is explored that the surface of a tsunami has roughly the shape of a sine wave at a given instant. In a sine of length L the surface elevation is given by the function:

$$f(x)=h\sin(\frac{2\pi}{L}x)$$

where x is distance along the wave.

a) Investigate the maximum slope of $f(x)$.

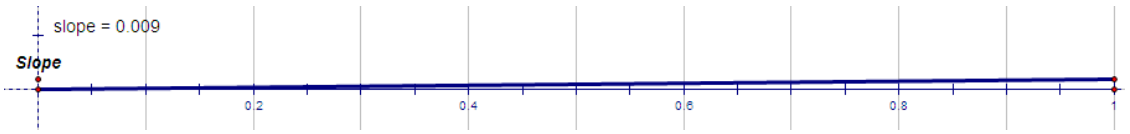


b) Estimate the maximum slope in the case of L = 300 km, and h = 1 m.

c) If we consider that π is roughly equal to 3, and h = 1, find the maximum slope?

d) We assume that the maximum slope is $\frac{6}{L}$. From the dynamic model in Cartesian Coordinates, drag slide L to see when the sine wave is flat?

e) Can we see the sine wave when L is big enough (bigger than 6 km)?

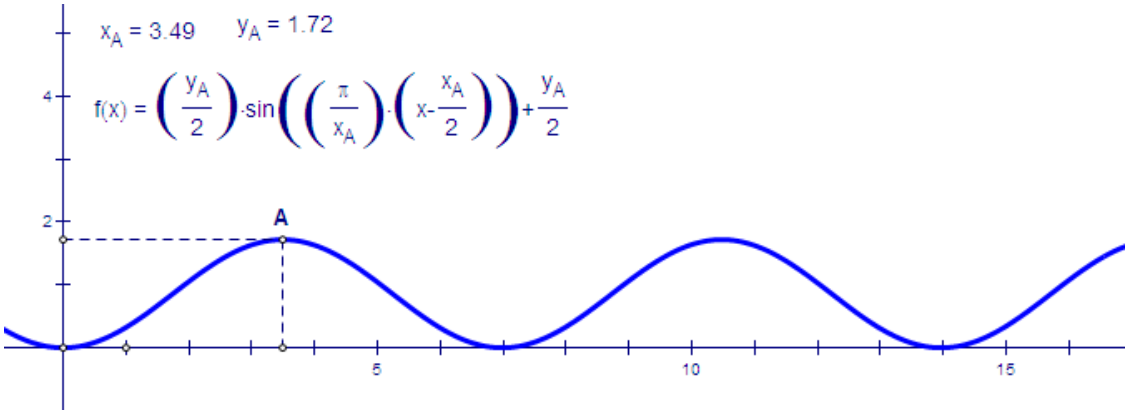


Maximum slope = 6/6000 = 1/1000 = 0.001. Imagine that we are putting one meter ruler on a table with the slope = 0.001, we see that the ruler is still horizontal.

Task 2. The bichromatic wave group of a tsunami

Given a point A (x_A, y_A) on the first quarter of the Cartesian coordinate plane.

a) Write a sine function $f(x)$ whose graph receiving A as one of its crest and origin O as one of its trough. Find the wave length and the amplitude of this sine wave.



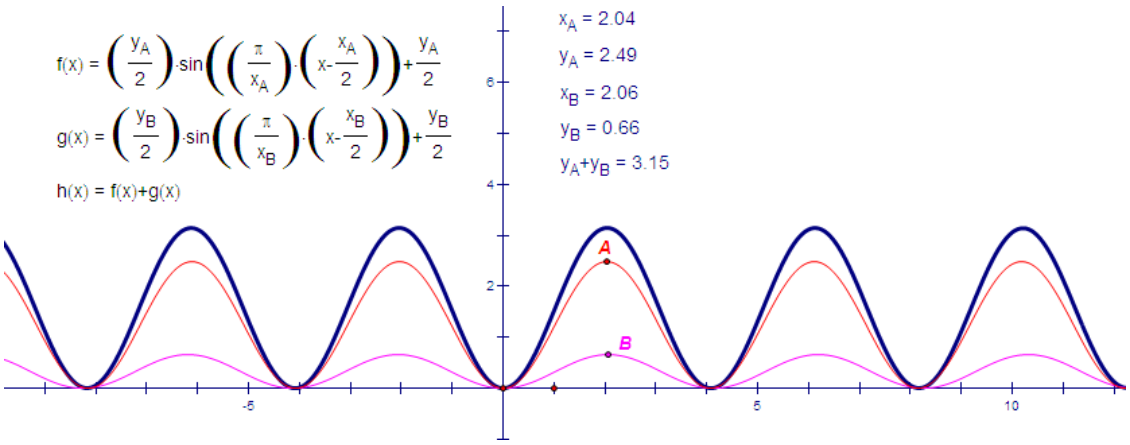
$$f(x)=\frac{y_A}{2}\sin\left(\frac{\pi}{x_A}\left(x-\frac{x_A}{2}\right)\right)+\frac{y_A}{2}$$

Wave length $L = 2\,x_A$. Amplitude $h = y_A$.

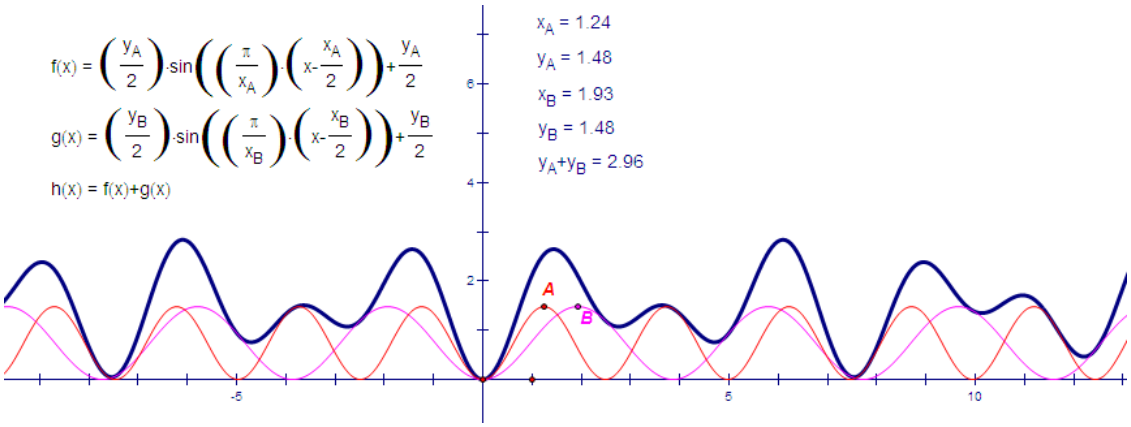
b) Similarly write a sine function $g(x)$ for another point B.

c) Investigate the addition $h(x)$ of two functions $f(x)$ and $g(x)$.

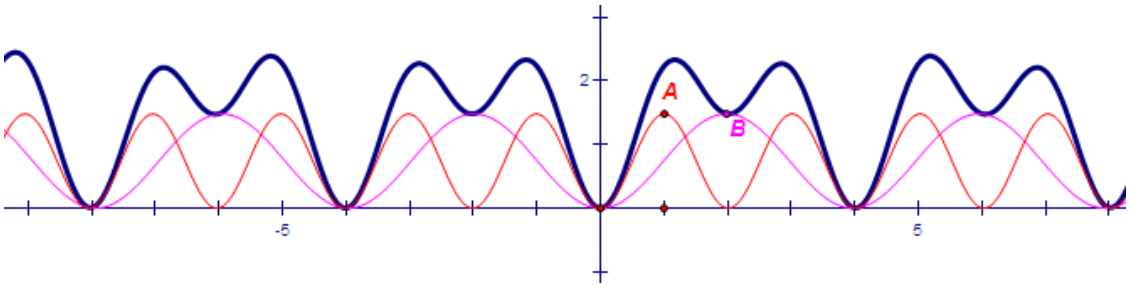
- What about the wave period and the amplitude of the addition function $h(x)$ when AB is vertical?



- What about the wave period and the height (amplitude) of the wave of the addition function $h(x)$ when AB horizontal?



d) Investigate the wave period of the addition function $h(x)$ when the length wave $2x_A = n$ and $2x_B = m$ are natural numbers.



The length wave = LCM (n, m).

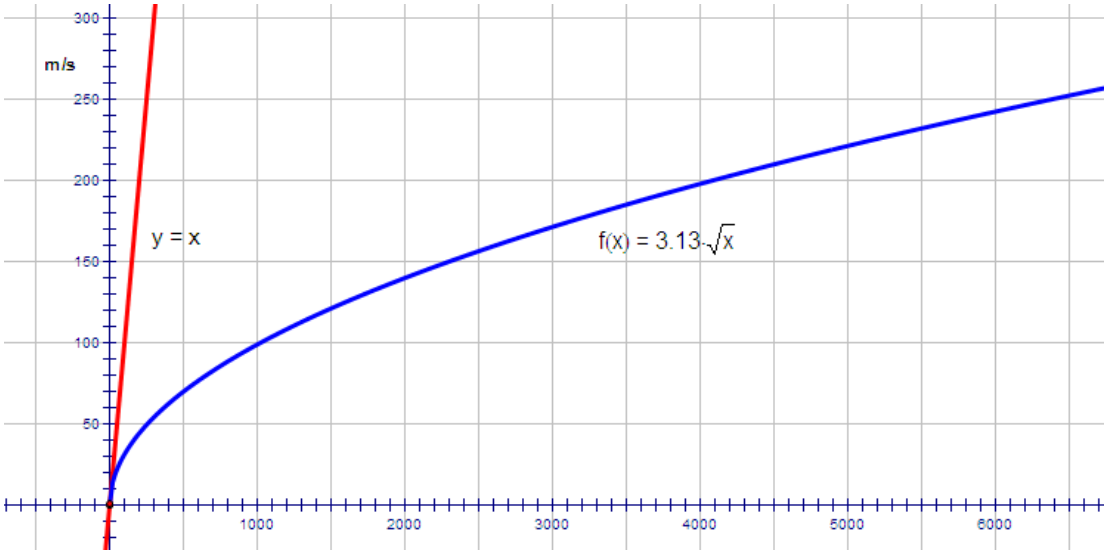
Task 3. Speed of a long tsunami wave

The speed of a long tsunami wave is given by the following formula:

$$U = \sqrt{gD}$$

where g is the acceleration of gravity, g = 9.8 m/s^2 and D is the ocean depth (m).

1. Graph the function $f(x)=3.13\sqrt{x}$ in a coordinate plane with rectangular grid so that we can see 6000 m on x-axis and 300 m/s on y-axis.



2. Graph the function $y = x$ on this coordinate plane. When the value of speed is equal to the value of the ocean depth?

$$x - 3.13\sqrt{x} = \sqrt{x}\,(x - 3.13) = 0 \Rightarrow x = 3.13.$$

3. The epicenter of an earthquake at the East Sea area with the ocean depth is 6500m. The distance from the epicenter to Da Nang city is 1350 km. How long the tsunami will reach to the coastal area in Da Nang?

The speed $U = \sqrt{gD} = \sqrt{9.8 \times 6500} = 252\,m/s$ = 908 km/h.

The time t = 1 hour and 23 minutes.