The Southeast Asian Ministers of Education Organisation
Regional Centre for Education in Science and Mathematics
(SEAMEO RECSAM, Malaysia)

筑波大学
University of Tsukuba
University of Tsukuba, Japan

Mathematics Challenges for Classroom Practices at the UPPER PRIMARY LEVEL

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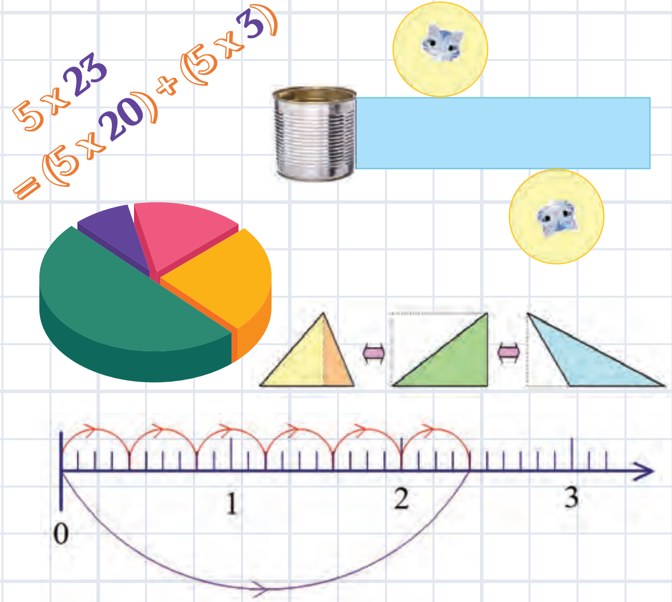
Based on SEAMEO Basic Education Standards: Common Core Regional Learning Standards in Mathematics



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A Collaboration Project:

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FOREWORD

Congratulations to CRICED, University of Tsukuba and SEAMEO RECSAM for another collaboration project in the publication of three series of guidebooks entitled Mathematics Challenges for Classroom Practices based on the SEAMEO Basic Education Standards: Common Core Regional Learning Standards (SEA-BES CCRLS) in Mathematics. Two series are for the lower and upper primary level while the third is for the lower secondary level. Generally, curriculum standards of subjects are not widely scrutinised by classroom practitioners and teacher educators compared to curriculum specialists. The publication of these three new guidebooks anchors well and consolidates the role and importance of the SEA-BES Common Core Regional Standards document which was already introduced to all the 11 SEAMEO member countries and beyond since published in 2017.

The content of this guidebook series covers across Grade 1 to Grade 9 and consists of tasks written to understand the learning standards in Mathematics. The transfer of information from the SEA-BES CCRLS to the newly published book series will create awareness among classroom teachers and teacher educators the importance and relevance of curriculum standards in formulating and designing learning specifications for students. The presentation of the book series emphasised three aspects, namely highlighting the misconceptions, developing new ideas from the previously learned knowledge and explanation of new mathematical concepts. The task-based approach will surely help readers to enhance their mathematical understanding and ultimately provide better support for classroom teaching and learning.

I sincerely hope that the Minister of Education of SEAMEO members would support and promote this guidebook series among educators and teachers in their respective countries. This effort and spirit of cooperation among SEAMEO members and associate members can be realised to bring benefits to classroom practices, which will eventually benefit children of our future.

My sincere appreciation and congratulations to CRICED, the University of Tsukuba as a project proponent and provided financial support, SEAMEO RECSAM as the main collaborator, other collaborating partner institutions and individual educators and specialists for their expertise, commitment and contributions in this endeavour.

Dr ETHEL AGNES PASCUA VALENZUELA
Director, SEAMEO Secretariat, Bangkok, Thailand

FOREWORD

On behalf of SEAMEO RECSAM, I would like to express my sincerest appreciation to the Centre for Research on International Cooperation in Educational Development (CRICED), the University of Tsukuba for inviting the Centre as the main collaborator in the publication of the guidebook series, titled “Mathematics Challenges for Classroom Practices” for the i) Lower Primary Level, ii) Upper Primary Level and iii) Lower Secondary Level. Besides the involvement of SEAMEO RECSAM, many educators and specialists of other collaborating partners such as Khon Kaen University, Thailand; the Institute for the Promotion of Teaching Science and Technology (IPST), Thailand, SEAMEO QITEP in Mathematics, Indonesia; mathematics specialists in APEC economies, educators, local teacher educators, and curriculum specialists involved in the SEAMEO Basic Education Standards: Common Core Regional Learning Standards (SEA-BES CCRLS) in Mathematics had contributed their writings in this guidebook series.

SEA-BES CCRLS in Mathematics and Science was first published in 2017 by SEAMEO RECSAM but had limited and restricted usage despite being shared with all SEAMEO member countries and beyond. Today, SEA-BES CCRLS has been given a new life where the learning standards of the Mathematics component have been adopted and used as the main reference for this guidebook series. Having said that, I am grateful to the outstanding writing team who made this possible. I shall start by acknowledging the contribution of Professor Dr Masami Isoda, who initiated the idea and the project and graciously invited SEAMEO RECSAM to produce this guidebook series; Ms Teh Kim Hong who coordinated the project with Mr Pedro Jr. Montecillo; Mr Gan Teck Hock who was later recruited to join the writing team and other mathematics specialists who were also invited to contribute their writings. Despite facing time constraints and changes in staff members, the writing team stayed intact with their contributions and commitment until this guidebook series is published. Besides the writing team, I also like to thank the panel reviewers of RECSAM who provided their constructive suggestions to improve the content of this guidebook series.

The guidebook series covers the mathematics content across Grades 1 to Grade 9 with the focus of utilising written tasks to understand the learning standards of SEA-BES CCRLS in Mathematics. The transfer of information from the latter to the newly published guidebook series will create awareness among classroom teachers and teacher educators regarding the importance and relevance of curriculum standards in the planning of teaching and learning. The presentation of this guidebook series emphasised highlighting misconceptions, and contradictions, and developing new ideas from previously learned knowledge to enhance learning and develop mathematical thinking. Such an approach to contradiction will foster deeper thinking among readers, thus enhancing mathematical understanding, and translating into better support for classroom teaching and learning.

Without a doubt, much commitment and hard work had been invested to produce these guidebooks. I hope that this mathematics guidebook series will be used widely by teachers and educators of SEAMEO member countries for classroom practices. I sincerely hope SEAMEO Secretariat will also provide their support by promoting this guidebook series in the classrooms of educators and teachers in all SEAMEO member countries.

I am therefore proud to present this guidebook series as the contribution of SEAMEO RECSAM and CRICED, University of Tsukuba, to the promotion and development of mathematics education in this region. This would not have been possible without CRICED, the University of Tsukuba’s content expertise and financial support. I hope this valuable collaboration and cooperation will continue in other future projects to benefit education development in this region.

Dr SHAH JAHAN BIN ASSANARKUTTY
Centre Director, SEAMEO RECSAM

FOREWORD

In addition to the Japanese Ministry of Education, Culture, Sports, Science Technology (MEXT), the University of Tsukuba has been playing the role of an affiliate member to collaborate with SEAMEO. As the Director of the Centre for Research on International Cooperation in Educational Development (CRICED), it is my pleasure to continue working with SEAMEO RECSAM on the SEAMEO Basic Education Standards for Mathematics (SEA-BES-M). This project was launched in 2014 as a reference book for curriculum reformers and teachers to develop 21st-century skills and OECD competency (2005) in education. For SEA-BES-M, I collaborated with Pedro Montecillo Jr., Kim Hong Teh, and the late Mohd Sazali bin Hj. Khalid of RECSAM, along with the contribution from curriculum developers in SEAMEO countries, specialists in APEC economies and several internationally leading researchers. Finally, SEA-BES-M was incorporated into the ‘SEAMEO Basic Education Standards (SEA-BES): Common Core Regional Learning Standards (CCRLS) in Mathematics and Science’ which was published by RECSAM in 2017.

Before SEA-BES-M was published, a comparative analysis of the mathematics curriculum documents of SEAMEO countries revealed that the higher-order thinking presented as process standards are compartmentalised between every content description. Therefore, a major contribution of SEA-BES-M to the world, particularly in the SEAMEO countries is its clear description of the meaning of higher-order thinking for the mathematics curriculum standards to develop mathematical thinking. After the publication, we recognised that there are difficult for readers to understand the intended meaning of every standard. This is mainly due to the fact that readers will interpret what is read based on their curriculum knowledge and experience teaching mathematics in their own countries. As such, it is crucial to develop a book series to be used as references for interpreting SEA-BES-M.

This book series is prepared, particularly for teacher educators, textbook authors, and curriculum developers as well as teachers to understand higher-order thinking for developing mathematical thinking in their classrooms. In this book series, the authors collaborated with leading researchers and educators in major teacher education institutions, CRME-IRDTP at KKU (Thailand), IPST (Thailand) and SEAMEO QITEP in Mathematics (Indonesia) with the support of the coordinators.

Furthermore, the authors made some minor revisions to the SEA-BES-M to align with the needs of the Era of the 4th Industrial Revolution to develop stakeholders and users of Artificial Intelligence and Big Data in business as well as establish a successful life under the reality of humanity with technology. The minor revision was made based on the curriculum reform recommendation (2020) by the APEC InMside Project with the purpose of promoting mathematical capitalism under mathematical-statistical-informational sciences on the demands of the Era. It is hoped that this book series is used in teacher education to develop new curriculum content knowledge for teaching in this Era. I would like to acknowledge the SEAMEO secretariat and centres for their collaborations, especially Shah Jahan Bin Assanarkutty, the Director of RECSAM, who made possible this publication. Last but not least, I would like to convey my sincere appreciation to all contributing writers stated in the contributor list and Gakko Tosho (the Japanese Textbook Publisher) who provided us with innovative ideas.

ISODA MASAMI
Director of CRICED, University of Tsukuba, Japan

PREFACE

Realising reform in school curricula beyond the 21st century and revitalising teacher education have been set as prioritised agendas in SEAMEO countries. On this demand, SEAMEO Basic Education Standards: Common Core Regional Learning Standards (SEA-BES: CCRLS) in Mathematic was published in 2017 for the main purpose of strengthening collaboration on curriculum standards and learning assessment across different educational systems in SEAMEO countries. In order for this document to be understood beyond the curriculum developers, supporting materials need to be developed for helping other users such as classroom teachers and teacher educators to acquire a deeper understanding of the standards. This book is an initiative to provide such support. With this support, it is anticipated that teachers and teacher educators will be able to innovate their classroom practices for developing competency and professional development aligning with the trends of the 4th Industrial Revolution.

Mathematics Challenges for Classroom Practices at the Upper Primary Level consists of mathematical tasks for the following strands:

- Extension of Numbers and Operations
- Measurement and Relations
- Plane Figures and Space Figures
- Data Handling and Graphs

These tasks are prepared to be used for pre-service and in-service mathematics teacher education. Its main purpose is to help readers develop mathematical knowledge for teaching (MKT) which consists of subject matter knowledge (SMK) and pedagogical content knowledge (PCK) (see, Ball, Thames, and Phelps, 2008). In developing the tasks, the English edition of Japanese mathematics textbooks published by Gakko Tosho had been used as the main reference. These textbooks provided major guides for learning mathematics through problem solving approach to develop mathematical thinking. As such, the tasks in this book are focused on mathematical ideas and ways of thinking. Basically, the tasks are developed in three ways: (a) analysing misconceptions of ideas, (b) developing ideas from previously learnt knowledge, and (c) using the inquiry-based investigation to learn new ideas. In designing the tasks, the importance of local contexts of the SEAMEO community had been considered. However, some essential elements of Japanese school mathematics were also incorporated into the tasks. It is hoped that these elements will set off a new breath of mathematics learning in the SEAMEO community, shaping our students to be critical and creative thinkers in the era of artificial intelligence and data science.

Each task is written based on a standard in SEA-BES: CCRLS in Mathematics and it serves to clarify (a) the curriculum knowledge of teaching in PCK, and (b) the mathematical ideas and ways of thinking on SMK related to the standard. Apart from that, SEA-BES CCRLS in Mathematics is used as the basic source for MKT. Since SEA-BES CCRLS in Mathematics was developed with curriculum specialists of SEAMEO countries, solving the tasks will also provide a bird’s eye view of their national curriculum to the readers. Furthermore, it will broaden their perspective of mathematical ideas, ways of thinking and curriculum sequence with respect to their use of local textbooks.

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1. Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What makes it special? Journal of Teacher Education, 49(5), 389-407.
2. Hitotsumatsu, S. et al (2005). Study with your friends: mathematics for elementary school (G. 1-6). Tokyo: Gakko Tosho./ Isoda, M., Murata, A., Yap, A. (2011, 2015, 2020) Study with your friends: mathematics for elementary school (G. 1-6). Tokyo: Gakko Tosho/ Isoda, M., Tall, D. (2019). Junior High School Mathematics (G. 1-3). Tokyo: Gakko Tosho. For developing tasks in this book, authors are inspired by the tasks and task sequence of these mathematics textbooks.

Although the targeted readers of this book are teachers and teacher educators, most of the tasks can also be solved by students in classrooms as they are also aligned with the school mathematics curriculum. In addition, teachers and teacher educators are expected to solve them without many difficulties. However, studying the tasks carefully will raise awareness of the depth of SMK and PCK required to complete the tasks. This will trigger off a need to upskill their understanding of mathematical ideas and function effectively to develop students' mathematical competency. Thus, solving tasks in this book will provide readers with opportunities to relearn the mathematics content for teaching. Furthermore, it will also help them to identify (a) the objectives of teaching the content, (b) the gap between students' prior knowledge and what is to be learnt, (c) what and how students reorganise the content knowledge of their learning, (d) students' difficulties in learning the content, and (e) what ideas will be developed through their new learning.

Readers may also choose to work with any task according to their interests. It is not necessary to work out all the tasks according to the sequence in the book. However, for a deeper understanding of the mathematical ideas embedded in a task, it is recommended that readers should solve the task in the following manner: (a) solve the task by themselves and read the related standards, (b) communicate their solutions with others to identify what is really new content for them, and (c) paraphrase and summarise the communication with others based on the perspective of mathematical ideas and ways of thinking that align with the framework of SEA-BES CCRLS as described in Chapter One.

This book is recommended for use in many ways and in various contexts. Firstly, as all the tasks were designed based on the school mathematics curriculum, so they can be used directly by students as learning tasks. In addition, teachers can also use the book as a quick guide to creating similar mathematical tasks that incorporate mathematical thinking. Secondly, when the book is used in the context of in-service teacher education such as in lesson study, teachers can solve the tasks in this book as a step to gain a deep understanding of the mathematical ideas in order to prepare a unit of a lesson plan based on the standard chosen. This may help to improve the effectiveness of lesson planning and anticipate the responses of students to the tasks. Thirdly, in the context of pre-service teacher education, the tasks in this book can serve as a means to acquire MKT which may be required for any teacher employment examination or entrance examination for an education graduate programme. Fourthly, in the context of mathematics education research, this book can be used as a reference for MKT. Last but not least, when the book is used in the context of curriculum reform and textbook revision, it could serve as a guide to formulate new objectives and tasks which do not exist in their current curriculum and textbooks.

Mathematics Challenges for Classroom Practices at the Upper Primary Level is the outcome of many contributions of educators and academia from different institutions. In order to ensure good quality of content and streamline the presentation of the writings, many rounds of editing and rewriting were unavoidable. It is our hope that the mathematical ideas and ways of thinking promoted through this book will enhance the teachers' capacities to develop their students' potential in facing the challenging and demanding era ahead.

ISODA Masami
GAN Teck Hock
TEH Kim Hong

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The authors/editors would like to thank and express our gratitude to the following contributors, institutions and organisations for making this project and publication possible:

Dr Ethel Agnes Pascua-Valenzuela, Director of SEAMEO Secretariat, Bangkok, Thailand for providing CRICED with the opportunities to collaborate with RECSAM and other SEAMEO centres;

Dr Shah Jahan Bin Assanarkutty, the Centre Director of SEAMEO RECSAM for his encouragement and support in realising the publication of the books;

Dr Johan Bin Zakaria, the ex-Centre Director of SEAMEO RECSAM for his confidence and generosity in granting permission to use the SEABES: CCRLS in mathematics to develop this guidebook series;

The University of Tsukuba generously supported the funding of this guidebook series project. The books will be disseminated to SEAMEO member countries and recommended for use by teachers, and teacher educators for the benefit of the SEAMEO community;

Assoc. Professor Dr Maitree Imprasitha, the Vice President of Khon Kaen University(KKU) and Director of the Institute for Research and Development in Teaching Profession (IRDTP) for ASEAN, Thailand, for providing support in organising a workshop to lead the KKU and other affiliated Universities lecturers in contributing writings to this guidebook series;

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Dr Wahyudi, the former director of SEAMEO QITEP in Mathematics (SEAQIM) and currently the Director of SEAMOLEC, together with Dr Sumardyono, the current Director of SEAQIM, provided the support to lead their mathematics lecturers in contributing the writings;

The specialists of APEC economies of the year September 2017 during the 13th APEC-Khon Kaen University Conference, and the year February 2018, in providing suggestions and contributions of resources in helping to conceptualise the content and outcome of the books;

All curriculum specialists and educators who attended the SEA-BES CCRLS in Mathematics workshop in March 2017, held in SEAMEO RECSAM, contributed ideas and suggestions on shaping the outcome of the guidebooks, as well as those who followed the sessions on how to write the tasks and submitted the writings for considerations;

Pedro Lucis Montecillo Jr, mathematics specialist of RECSAM (until May 2018) for assisting in the coordination at the early stage of the project;

All staff of SEAMEO RECSAM supported the production of these books by providing ideas and coordinating and managing the distribution of books to other institutions.

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TABLE OF CONTENTS

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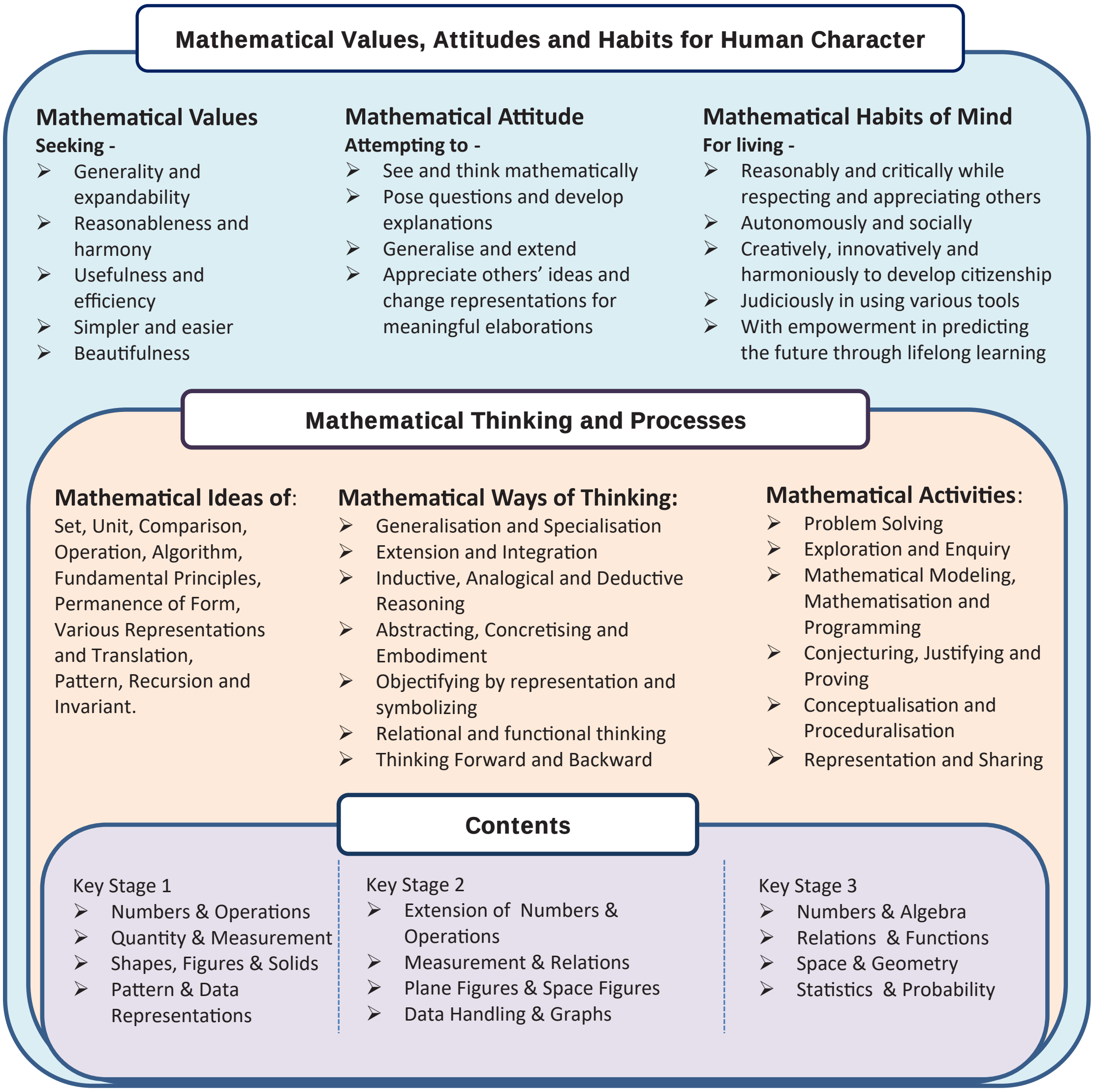
CHAPTER 1

Guide to the SEA-BES CCRLS Framework in Mathematics

The Southeast Asia Basic Education: Common Core Regional Learning Standards (SEA-BES CCRLS) was developed and directed to create a harmonious SEAMEO community through mutual understanding in the era of artificial intelligence and data science. In this respect, the CCRLS framework in Mathematics (2017) outlines three basic components towards developing creative, competent and productive global citizens essential for achieving this aim. A comprehensive illustration of the framework is attached in Appendix A. The revised framework in Figure 1 shows the interconnection of the three components.



Mathematical Values, Attitudes and Habits for Human Character

Mathematical Values
Seeking -
- Generality and expandability
- Reasonableness and harmony
- Usefulness and efficiency
- Simpler and easier
- Beautifulness

Mathematical Attitude
Attempting to -
- See and think mathematically
- Pose questions and develop explanations
- Generalise and extend
- Appreciate others' ideas and change representations for meaningful elaborations

Mathematical Habits of Mind
For living -
- Reasonably and critically while respecting and appreciating others
- Autonomously and socially
- Creatively, innovatively and harmoniously to develop citizenship
- Judiciously in using various tools
- With empowerment in predicting the future through lifelong learning

Mathematical Thinking and Processes

Mathematical Ideas of:
Set, Unit, Comparison, Operation, Algorithm, Fundamental Principles, Permanence of Form, Various Representations and Translation, Pattern, Recursion and Invariant.

Mathematical Ways of Thinking:
- Generalisation and Specialisation
- Extension and Integration
- Inductive, Analogical and Deductive Reasoning
- Abstracting, Concretising and Embodiment
- Objectifying by representation and symbolizing
- Relational and functional thinking
- Thinking Forward and Backward

Mathematical Activities:
- Problem Solving
- Exploration and Enquiry
- Mathematical Modeling, Mathematisation and Programming
- Conjecturing, Justifying and Proving
- Conceptualisation and Proceduralisation
- Representation and Sharing

Contents

Key Stage 1
- Numbers & Operations
- Quantity & Measurement
- Shapes, Figures & Solids
- Pattern & Data Representations

Key Stage 2
- Extension of Numbers & Operations
- Measurement & Relations
- Plane Figures & Space Figures
- Data Handling & Graphs

Key Stage 3
- Numbers & Algebra
- Relations & Functions
- Space & Geometry
- Statistics & Probability

Figure 1. Revised CCRLS Framework in Mathematics

This book is written to guide readers in acquiring a better understanding of this framework, particularly on mathematical thinking and processes which are embedded in all the tasks. A detailed explanation of mathematical ideas, mathematical ways of thinking and mathematical activities can be found in Appendix B. Mathematical ideas and mathematical values are elaborated in Appendix

C. The standards for the strand on Mathematical Processes-Humanity are attached in Appendix D to provide readers with challenging activities to promote metacognitive thinking at different levels of arguments to make sense of mathematics. In order to understand the development and progression of learning from the primary level to the secondary level (Key Stage 3), the learning standards of Key Stage 1 and Key Stage 3 can be referred to in Appendix E and Appendix F, respectively.

The interconnection of the three components is shown in Figure 2. The ultimate aim of the CCRLS framework is to develop mathematical values, attitudes and human characters which are the essence of a harmonious society. This component is closely related to the affective domain of human character traits which correspond to soft skills that can be developed through appreciation. In relation to this, the acquisition of mathematics contents as hard skills and reflection on the thinking processes are needed to inculcate the capability of appreciation. The reflection is necessary for learners to recognise their cognitive skills derived from the contents. Even though contents appeared to be learned independently through acquisition, the mathematical thinking and process, and the appreciation of mathematical values, attitudes and habits for the human character are possible to be developed through reflective experiences.

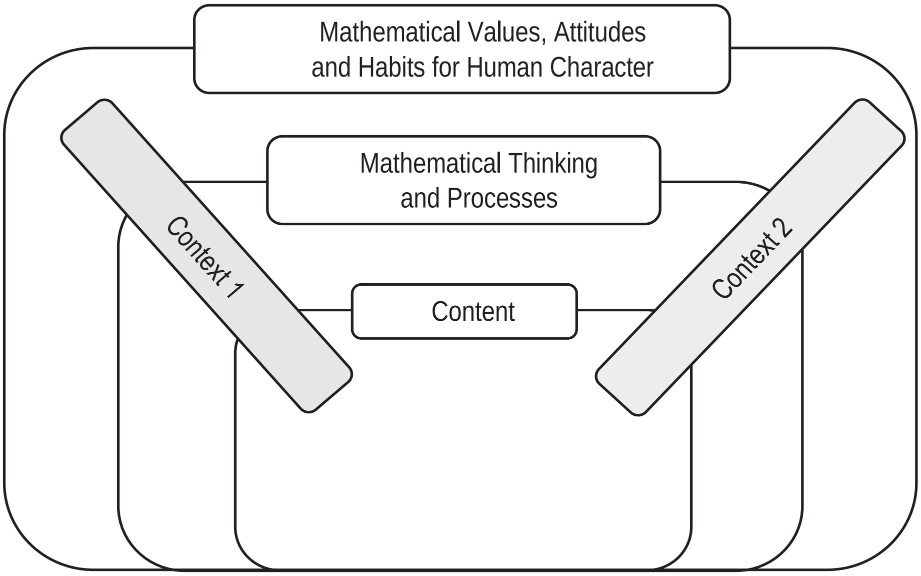


Figure 2. Interconnection of Components in CCRLS Framework in Mathematics

The three components will not be ideally operationalised without appropriate contexts. The tasks in this book provide the contexts for developing mathematical thinking and processes, which are the key learning objectives. Completing the tasks will correspond to gauging the readers' acquired mathematical knowledge for teaching. Thus, it is recommended that readers should constantly reflect on the appropriateness of their solutions to the tasks. Other than this, comparing solutions and discussing with others should always be done habitually in order to gain a deeper understanding of mathematical processes. This may enable the readers to discover any hidden mathematical ideas and structures in the tasks with appreciation. The tasks are specifically designed to cater for this purpose. Context 1 is real world problem solving and context 2 is task sequence in mathematics.

In a nutshell, an important target of solving the tasks in this book is to enable you to acquire a better insight into the learning standards. This insight will help you understand and appreciate your national mathematics curriculum from the perspective of SEABES CCRLS. Furthermore, since the learning standards are developed based on the framework which emphasised the components of contents, thinking and processes, and mathematics values, ultimately, you will be able to acquire mathematical teaching knowledge with appreciation.

CHAPTER 2

Extension of Numbers and Operations

Topic 1: Extending Numbers with Base Ten up to Billion and also to Thousandths with Three-Digit Numeral System Gradually

Standard 1.1:

Extending numbers using base-ten system up to billion with three-digit numeral system

i. Adopt the three-digit numeral system, extend numbers up to billion with the idea of relative size of numbers

ii. Compare numbers such as larger, smaller with base-ten system of place values through visualisation of relative size of numbers using cube, plane (flat), bar (long) and unit

Sample Tasks for Understanding the Standards

Task 1: How Large is 1 Billion?

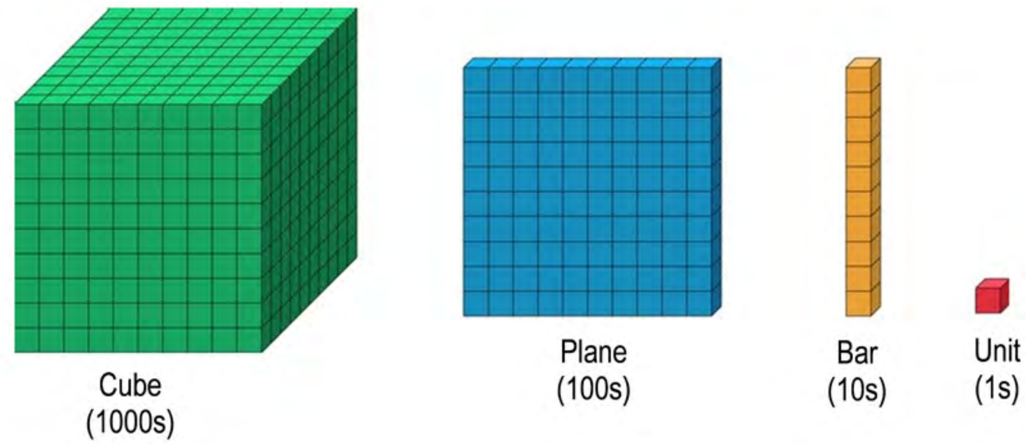


Diagram 1

i. Diagram 1 shows four base-ten blocks, a cube, a plane, a bar and a unit, normally used to represent 1000s, 100s, 10s and 1s, respectively.

- How many sets of 1s are there in 1 thousand?
- How many sets of 10s are there in 1 thousand?
- How many sets of 100s are there in 1 thousand?

			
?	?	10 000	1 000
1 000 000 000	100 000 000	?	?

Diagram 2

ii. These base-10 blocks can be used to represent different values as shown by the two examples in Diagram 2.

- In the first exampes, if the unit represents 1000, the bar will represent 10 000.
  - ❖ State the values represented by the plane and the cube.
  - ❖ How many thousands are there in a million?
- In the second example, if the cube represents 1 billion, then the plane will represent 100 million.
  - ❖ State the values represeted by the bar and the unit.
  - ❖ How many times is 1 billion compared to 1 million?
  - ❖ How many thousands are there in 1 billion?

iii. According to the latest *United Nations Population Division* estimates, China and India are the only two countries in the world with a population of more than 1 billion. Table 1 shows the population of these two countries as of 1 July 2020.

Table 1
Population of China and India as of 2020

Country	Population
China	1 439 323 776
India	1 380 004 385

Retrieved from WorldOmeter website
https://www.worldometers.info/world-population/population-by-country/

- Read and write the population of China and India in words, respectively.

iv. Diagram 3 shows the three-digit numeration system extended to billion.

Billions			Millions			Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

Diagram 3

- How can this chart be used to help your students read the population of China and India, respectively?

v. As of 1 July 2020, the world population is estimated to be 7 794 798 739.

- What is the value of the underlined digit 9 in each of the following cases?

  (a) 7 794 7$\underline{9}$8 739

  (b) 7 7$\underline{9}$4 798 739

- What is the value of the underlined digit 7 in each of the following cases?

  (a) 7 794 $\underline{7}$98 739

  (b) $\underline{7}$ 794 798 739

vi. Consider the number 8 820 406 200.

- Read and write the number in words.
- How many times is the 8 on the left compared to the 8 on the right?
- How many times is the 2 on the left compared to the 2 on the right?

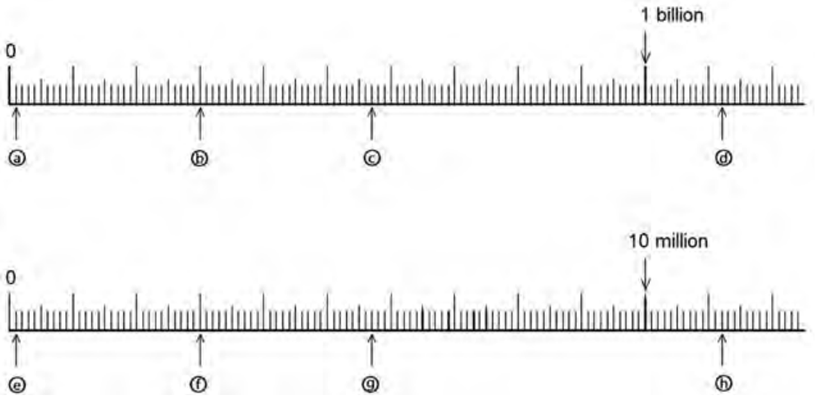


Diagram 4

vii. Diagram 4 shows two different number lines.

- Find the number indicated by each ↑ on the number lines.

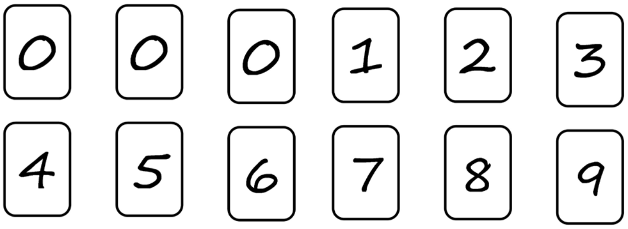


Diagram 5

viii. Diagram 5 shows 12 number cards. Use all the 12 cards to make various numbers.

- What number is the largest?
- What number is the smallest?

ix. About how many years is 1 billion seconds? (Take 1 year as 365 days.)

Standards 1.2:

Extending decimal numbers to hundredths, and to thousandths

i. Use the idea of quantity and fractions, extend decimal numbers from tenths to hundredths
ii. Compare decimal numbers such as larger, smaller with base ten system of place value
iii. Adopt the ways of extension up to thousandths and so on, and compare the relative sizes

Sample Tasks for Understanding the Standards

Task 1: Reading Decimal Numbers

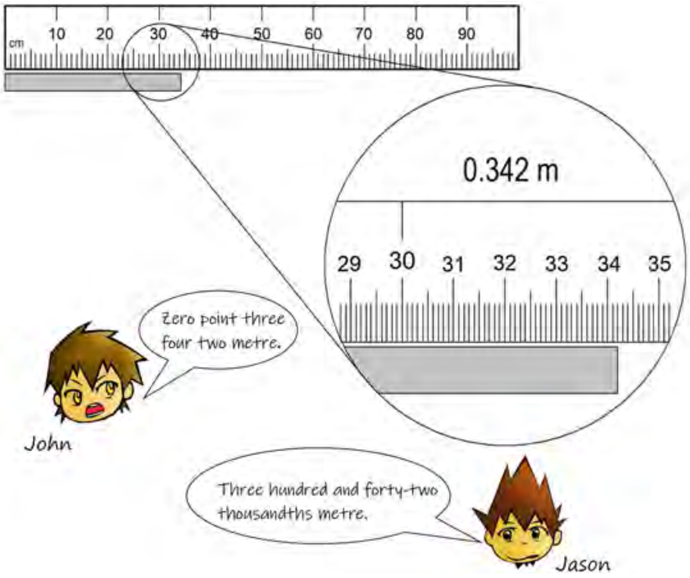


Diagram 1

i. A ribbon was measured using a metre rule and the length was recorded as 0.342 m. Two students, John and Jason, read the length in different ways as shown in Diagram 1.

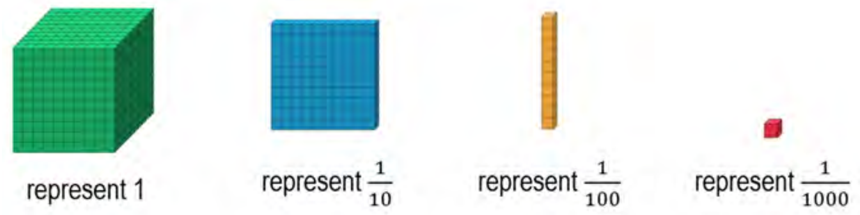
  - Explain the two different ways of reading decimal numbers.
  - Which way will you encourage your students to learn? Justify your choice.

ii. Write and read the following decimal numbers in words.

  - 2.6
  - 2.06
  - 2.46
  - 0.308
  - 0.038

Task 2: Visualising Decimal Numbers

① Decimal Number System

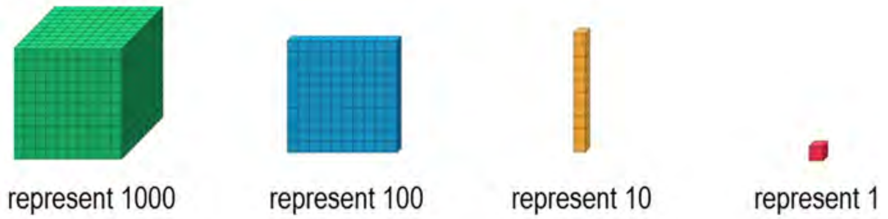


$$5.267 = 5 + 0.2 + 0.06 + 0.007$$
$$= (5x1) + (2x0.1) + (6x0.01) + (7x0.001)$$
$$= (5x1) + (2x\tfrac{1}{10}) + (6x\tfrac{1}{100}) + (7x\tfrac{1}{1000})$$

There are 5 sets of 1, 2 sets of $\frac{1}{10}$ and 6 sets of $\frac{1}{100}$ and 7 sets of $\frac{1}{1000}$ in 5.267.

5.267 is read as *"five and two hundred sixty-seven thousandths.*

② Whole Number System



$$5267 = 5000 + 200 + 60 + 7$$
$$= (5x1000) + (2x100) + (6x10) + (7x1)$$

There are 5 sets of 1000, 2 sets of 100, 6 sets of 10 and 7 sets of 1 in 5267.

5267 is read as *"five thousand two hundred and sixty-seven".*

Diagram 2

i. Diagram 2 shows the decimal number and the whole number systems. Compare the two systems.

   - Explain the similarities and differences between the two systems.

ii. Fill in the blanks in the following statements.

   - 734 is made up from ____ sets of hundreds, ____ sets of tens and ____ sets of ones.
   - 7.34 is made up from ____ sets of ones, ____ sets of tenths and ____ sets of hundredths.
   - 0.734 is made up from ____ sets of tenths, ____ sets of hundredths and ____ sets of thousandths.

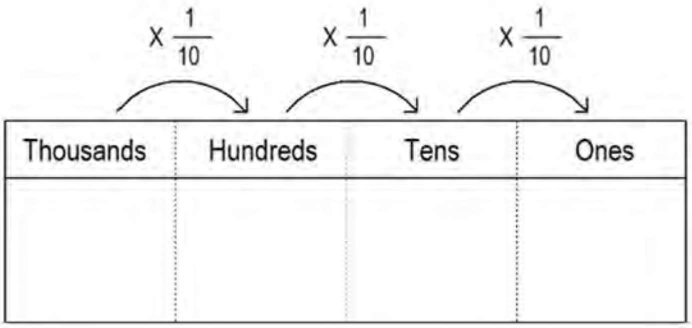


Diagram 3

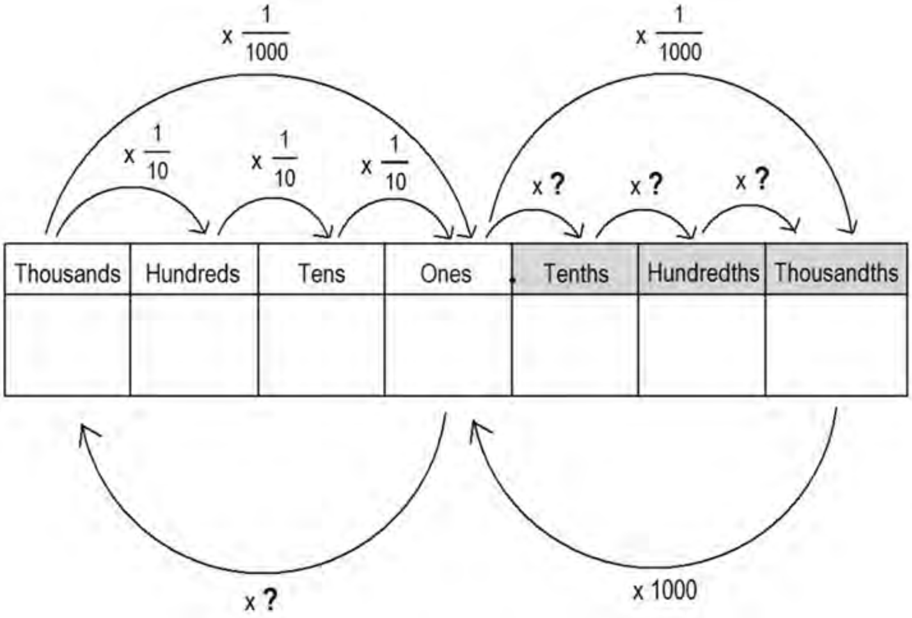


Diagram 4

iii. Diagram 3 shows a place-value chart for whole numbers whereas Diagram 4 shows the extension of the chart to include decimal numbers.

- What rules are there for the whole number place-value system?
- Do the same rules still apply for the extended decimal number place-value system?
- Delete the wrong word in the (   ).

  "For both the whole and decimal numbers, a digit is shifted to the next (higher/lower) place when there are 10 in a place, and a number is shifted to the next (higher/lower) place when it is divided into 10."

iv. How are decimal numbers related to fractions? Illustrate your answers with appropriate examples.

Task 3: Comparing Decimal Numbers

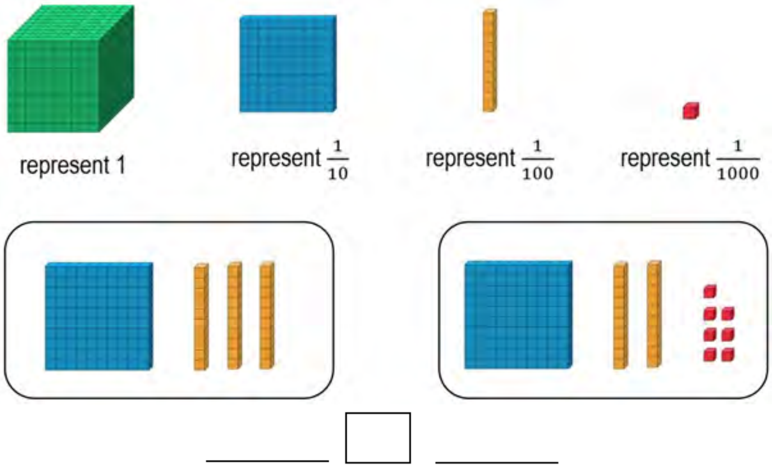
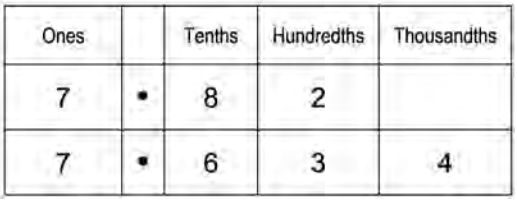


Diagram 5

i. Diagram 5 shows a pair of decimal numbers represented by base-ten blocks.

- Compare the pair of decimal numbers.
- Write the result of comparison using the symbols > or < .



Ones		Tenths	Hundredths	Thousandths
7	•	8	2	
7	•	6	3	4

Diagram 6

ii. Diagram 6 shows the comparison between a pair of decimal numbers, 7.82 and 7.634, using a place-value chart.

- Write the result of comparison using the symbols > or < .

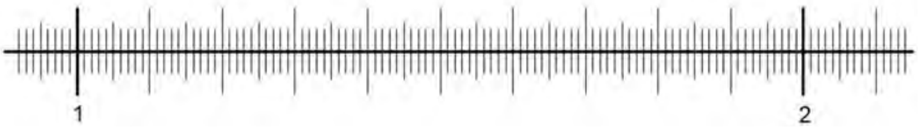


Diagram 7

iii. Diagram 7 shows part of a number line.

- Compare the numbers 1.4 and 1.27 using the number line.

iv. How would you sequence the above three comparison tasks in order to develop your students’ understanding on comparison of decimal numbers by themselves.

Topic 2: Making Decisions of Operations on Situations with Several Steps and Integrate Them in One Expression and Think about the Order of Calculations and Produce the Rule (PEMDAS)

Standard 2.1:

Finding easier ways of calculations using the idea of various rules of calculations such as the associative, commutative and distributive rules

i. Find the easier ways of addition and subtraction and use it, if necessary, such as answer is the same if add the same number to the subtrahend and minuend
ii. Find the easier ways of multiplication and division and use them in convenient ways such as 10 times of multiplicand produce the product 10 times
iii. Use associative, commutative and distributive rules of addition and multiplication for easier ways of calculation, however, commutative property does not work in subtraction and division
iv. Appreciate the use of simplifying rules of calculations

Sample Tasks for Understanding the Standards

Task 1: Addition and Subtraction Made Easier

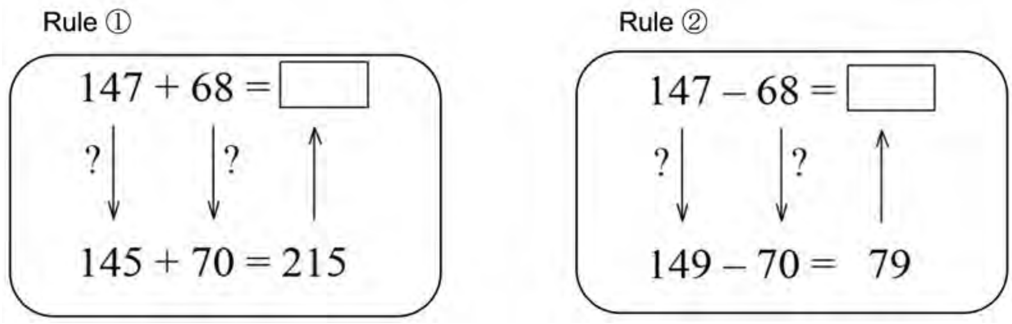


Diagram 1

i. Diagram 1 shows two rules used to make addition and subtraction easier.
  - Explain each rule.
  - Why does each rule work?

ii. Use the rules to make the following calculations easier.
  - 126 + 78
  - 214 – 59
  - $27.50 + $6.85
  - 125 m – 34 m

Task 2: Multiplication Made Easier

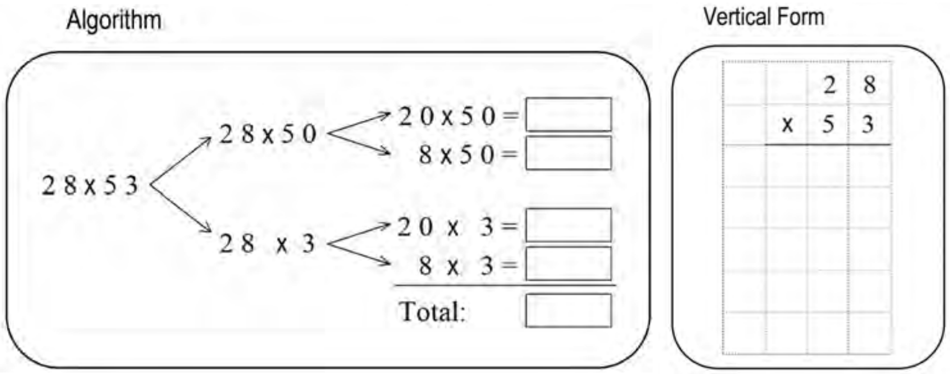
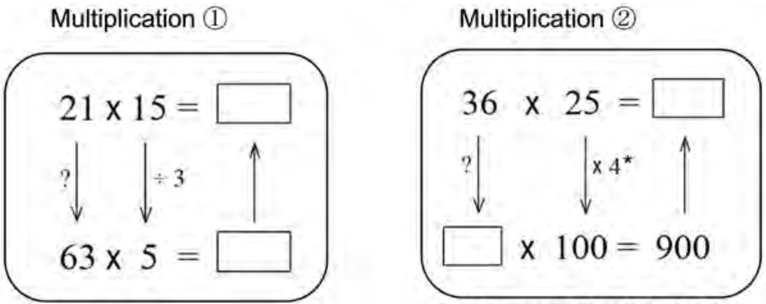


Diagram 2

i. Diagram 2 shows an algorithm and the vertical form multiplication for 28 x 53.

- What multiplication rule is used in the algorithm? Explain your answer.
- Explain how the algorithm can be used to explain the vertical form of multiplication.



* [x 4 means 25 x 4 or 4 x 25.]

Diagram 3

ii. Diagram 3 shows two multiplications made easier using a rule.

- Complete the sentences by filling in the missing number in each ☐.
- What rule is used to make the multiplications easier?
- Why does the rule work?

iii. Use the rule to make the following multiplications easier.

- 628 x 5
- 244 x 125

Task 3: Mixed Operations Made Easier

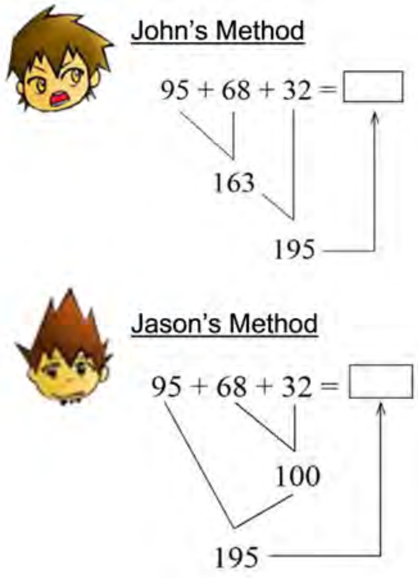


Diagram 4

i. John and Jason used different methods to calculate 95 + 68 + 32 as shown in Diagram 4.

Compare the two methods.

- Which method is easier for your students? Why?
- What rule of addition was used to make the calculation easier?

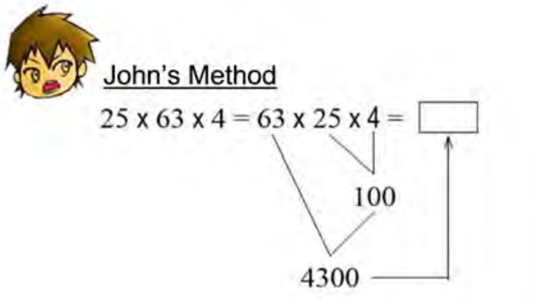


Diagram 5

ii. Diagram 5 shows John's method to calculate 25 x 63 x 4. He used two rules of multiplication to make the calculation easier.

- What are the two rules?
- Explain how the rules were used.

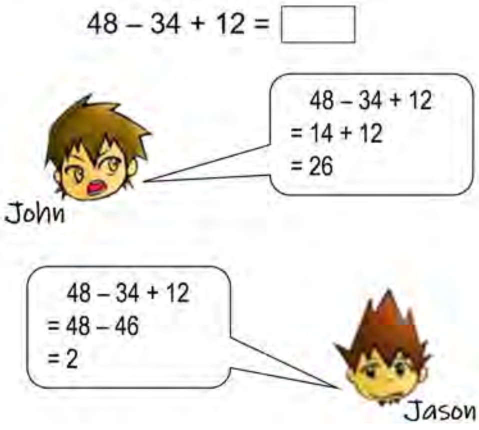


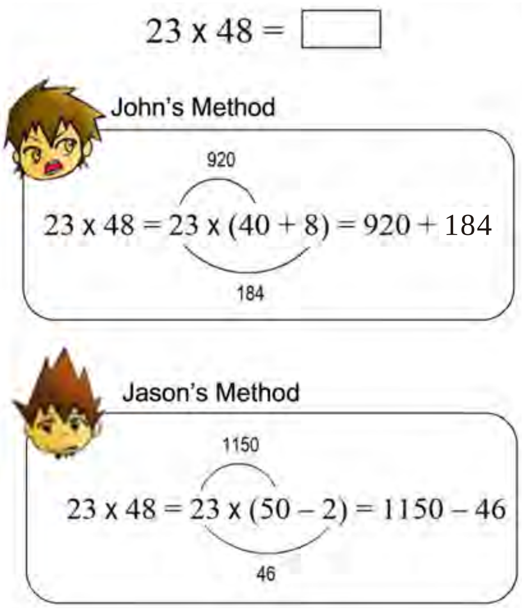
Diagram 6

iii. When trying to calculate 48 – 34 + 12, John and Jason interpreted the PEMDAS rule differently as shown in Diagram 6.

- Explain the two interpretations.
- Which of the interpretations is correct? Why?

iv. Explain why each of the following sentences is true.

- 60 – 20 – 15 = 60 – 15 – 20
- 60 – 20 – 15 = 60 – (20 + 15)



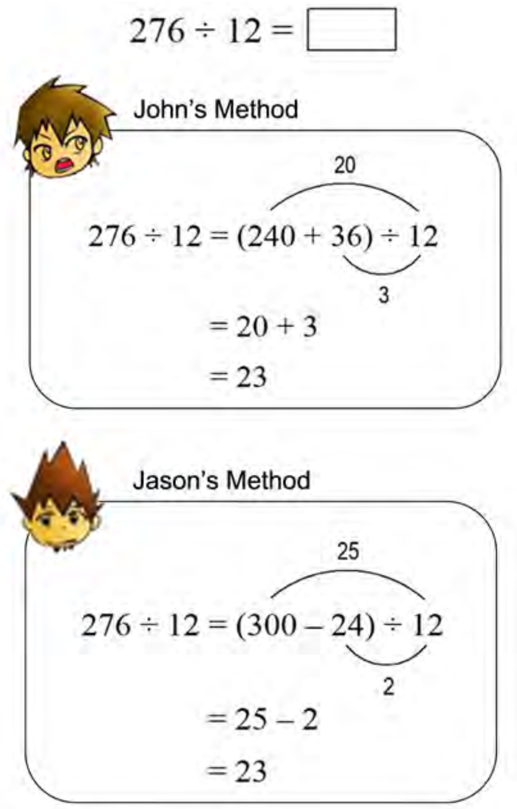
$$23 \times 48 = 23 \times (40 + 8) = 920 + 184$$

$$23 \times 48 = 23 \times (50 - 2) = 1150 - 46$$

Diagram 7

v. John and Jason used the distributive rule to simplify 23 x 48 as shown in Diagram 7.

- Compare the two methods.
  - ❖ Did John and Jason get a same answer?
  - ❖ Why or why not?
- Which method will you recommend to your students? Justify your choice.
- Do you think it is worth for students to see more than one method of doing computational problem? Why or why not?



$$276 \div 12 = (240 + 36) \div 12 = 20 + 3 = 23$$

$$276 \div 12 = (300 - 24) \div 12 = 25 - 2 = 23$$

Diagram 8

vi. John and Jason used different methods to simplify 276 ÷ 12 as shown in Diagram 8.

- Compare the two methods.
  - ❖ What rule was used to simplify the division in each method?
  - ❖ Which method will you recommend to your students? Justify your choice.

Standards 2.2 :

Thinking about the order of calculations in situations and produce rules and order of operations

i. Integrate several steps of calculation into one mathematical sentence
ii. Produce the rule of PEMDAS and apply it to the multi-step situation
iii. Think about the easier order of calculation and acquiring fluency of PEMDAS and rules with appreciation

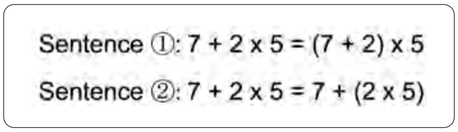
Sample Tasks for Understanding the Standards

Task 1: Mixed Operation in Daily Situations

A multi-level parking lot has 10 levels and there is a total of 1 799 parking lots. There are 152 parking lots on the first level. The other parking lots are distributed equally on the other 9 levels.

How many parking lots are there in the last two levels?

Diagram 1



Sentence ①: 7 + 2 x 5 = (7 + 2) x 5

Sentence ②: 7 + 2 x 5 = 7 + (2 x 5)

Diagram 2

i. Diagram 1 shows a daily-life problem.

- Write the calculation in **one** mathematical sentence.

ii. Build a daily-life story problem based on each of the following mixed-operation sentences.

- 12 x (6 – 3)
- 12 x 6 – 3
- 12 – 6 ÷ 3
- (12 + 6) ÷ 3

iii. Diagram 2 shows two mathematical sentences involving 7 + 2 x 5.

- Draw a diagram to show the difference between (7 + 2) x 5 and 7 + (2 x 5).
- Determine whether each of the sentences is true or false.

iv. Calculate 8 – 2 + 3. Is your answer 9 or 3? Why?

v. Calculate 12 ÷ 3 x 2. Is your answer 8 or 2? Why?

vi. Calculate each of the following mixed operations and explain each case using a real-life situation.

- 12 ÷ (3 x 2)
- 12 ÷ 2 x 3
- 12 ÷ 3 x 2

vii. The acronym PEMDAS is sometimes used by teachers to help students remember the rule involving order of calculation.

- Explain the rule.
- Why is the rule important?

viii. Calculate

- $7 + 3 \times 4^2$
- $7 + (3 \times 4)^2$

ix. Build a daily-life story problem for each of the following mixed operations.

- $12 \times (6 - 3)$
- $12 \times 6 - 3$
- $12 + 6 \div 3$
- $(12 + 6) \div 3$

Topic 3: Producing the Standard Algorithm Using Vertical Form Division with Whole Numbers

Standard 3.1:

Knowing the properties of division and use it for an easier way of calculation

i. Find the easier ways of division and use it, if necessary, such as answer is the same if multiplying the same number by the dividend and divisor

ii. For confirmation of the answer of division, use the relationship among divisor, quotient and remainder and appreciate the relationship

Sample Tasks for Understanding the Standards

Task 1: Division Made Easier

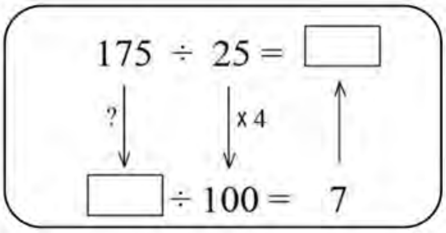


Diagram 1

i. Diagram 1 shows a division made easier using a rule.

- Complete the division sentence by filling in the missing number in each ☐ .
- What rule is used to make the divisions easier?
- Why does the rule work?

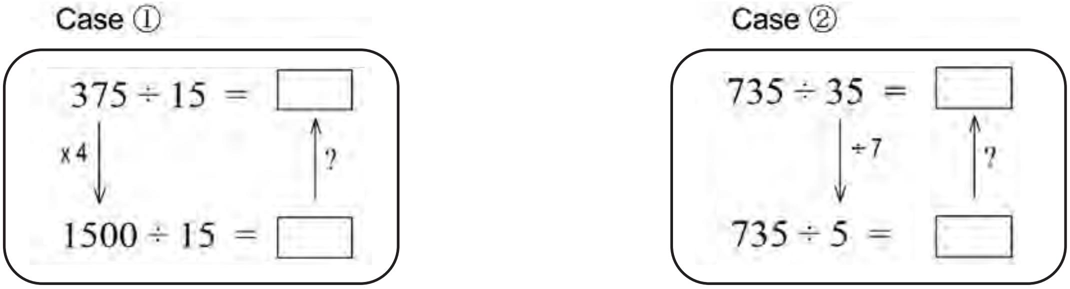
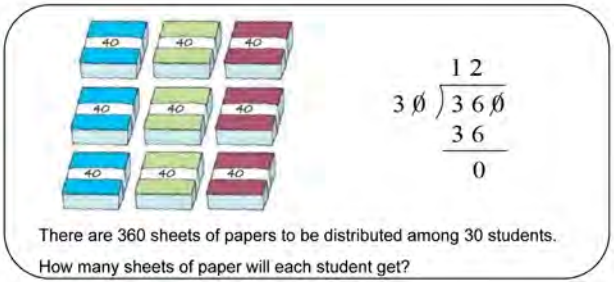


Diagram 2

ii. Diagram 2 shows another two cases of division made easier using some rules.

- Fill in the missing number in each ☐ .
- For each case, what rule is used to make the divisions easier?
- Why does the rule work?

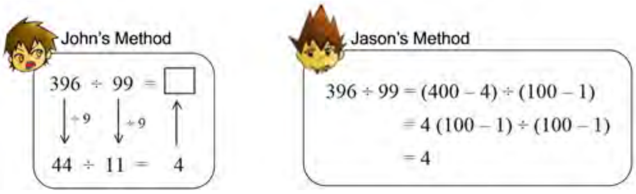


There are 360 sheets of papers to be distributed among 30 students.
How many sheets of paper will each student get?

Diagram 3

iii. A student used an easy method to solve a division problem as shown in Diagram 3. In this method, the student simply “cancelled off” the zero in both the dividend and divisor.

- What division rules can be used to justify this method? Explain your answer.
- Discuss the significance of writing down the remainder of zero in the vertical form division algorithm.
- How will this help your students learn other mathematical ideas such as the Remainder Theorem in higher grades?



$$396 \div 99 = (400 - 4) \div (100 - 1)$$
$$= 4\,(100 - 1) \div (100 - 1)$$
$$= 4$$

Diagram 4

iv. Two students, John and Jason simplify the division 396 ÷ 99 as shown in Diagram 4.

- Compare and justify the two methods.
- Which method will you recommend to your students? Justify your decision.

v. Use the division rules to make the following divisions easier.

- 220 ÷ 20
- 1300 ÷ 25
- 255 ÷ 15
- 348 ÷ 50
- 13800 ÷ 2300

vi. How can the division rules be used to make the following division involving fractions easier?

- $\frac{3}{4} \div 5$
- $8 \div \frac{2}{3}$

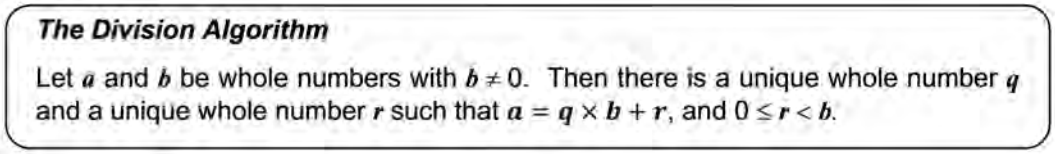
Standards 3.2 :

Knowing the Algorithm of Division in Vertical Form and Acquiring Fluency

1. Know the division algorithm with tentative quotient and confirm the algorithm by the relationship among divisor, quotient and remainder
2. Interpret meaning of quotient and remainder in situations
3. Acquire fluency for division algorithm in the case of up to 3-digit whole number divided by 2-digit
4. Think about the situations with or without remainder in relation to situations for quotative and partitive division

Sample Tasks for Understanding the Standards

Task 1: Division Algorithm in Vertical Form



The Division Algorithm

Let a and b be whole numbers with $b \neq 0$. Then there is a unique whole number q and a unique whole number r such that $a = q \times b + r$, and $0 \leq r < b$.

Diagram 1

i. Diagram 1 shows the definition of the division algorithm for whole numbers.

- Why is the phrase "and $0 \leq r < b$" included in the definition?
- In this definition, a, b, q and r are called the dividend, divisor, quotient, and remainder, respectively.
  - ❖ Explain the relationship among dividend, divisor, quotient, and remainder.
  - ❖ Explain how the relationship can be used to check the answer for 38 ÷ 4.

ii. Two conceptual models that are commonly used by teachers to develop division algorithm are the ***repeated-subtraction*** model, and the ***missing-factor*** model.

- Using appropriate examples, explain the meaning of division based on each of these models.

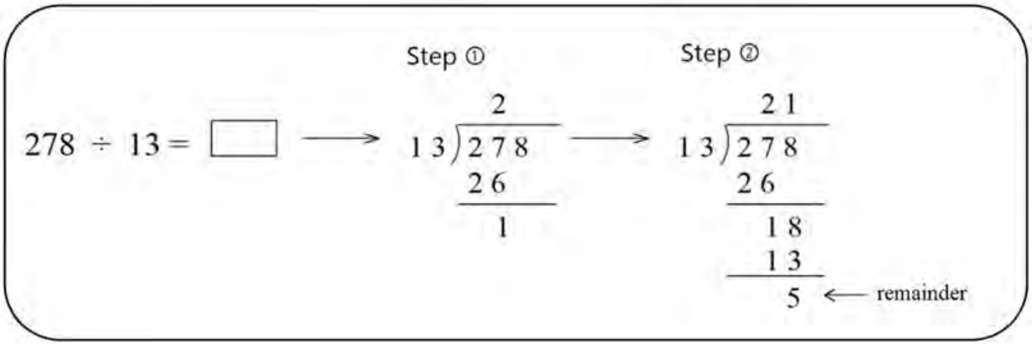


Diagram 2

iii. Diagram 2 shows the steps in calculating 278 ÷ 13 using a division algorithm in vertical form. In each of the steps, a teacher asked the following questions to guide students:

Step ① – "What number when multiplied by 13 will give you 27 or less?"

Step ② – "What number when multiplied by 13 will give you 18 or less?"

- In using this vertical form of division algorithm, what model of division did the teacher base on? Explain your answer.

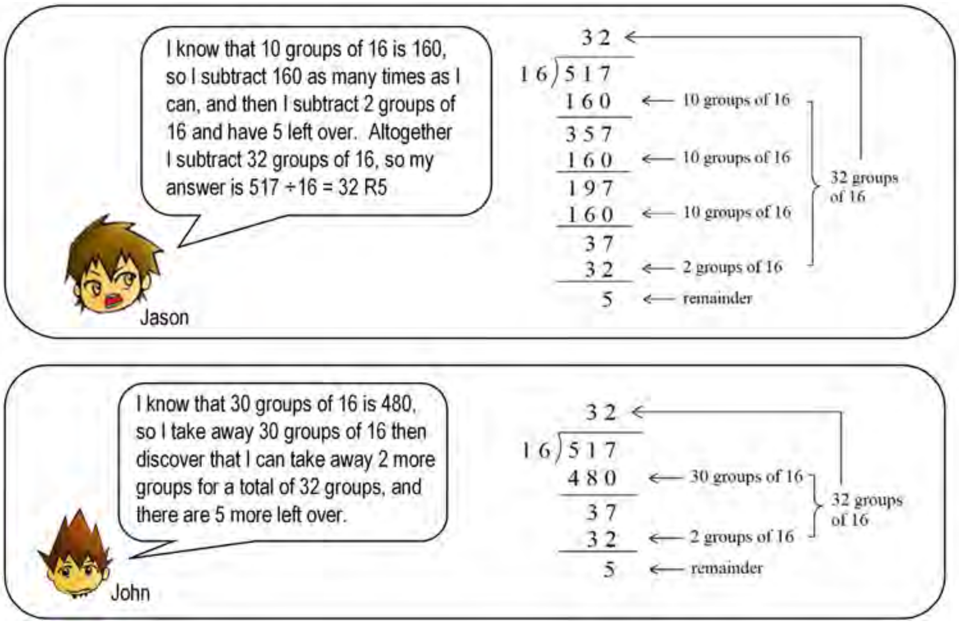


Diagram 3

iv. Two students, Jason and John used another algorithm to calculate 517 ÷ 16. Diagram 3 shows their calculation steps and explanations, respectively.

- In this algorithm, what model of division did Jason and John base on?

- Which student's thinking is more efficient? Why?

v. In using any division algorithm, how do you know when you are done with a long division calculation?

vi. What values in the SEA-BES CCRLS Framework in Mathematics (refer page 1) could be nurtured through this task? How would you do it?

Topic 4: Extending the Vertical Form Addition and Subtraction with Decimals to Hundredths

Standard 4.1:

Extending the vertical form addition and subtraction in decimals to hundredths

i. Extend the vertical form addition and subtraction to hundredths place and explain it with models

ii. Appreciate the use of addition and subtraction of decimals in their life

Sample Tasks for Understanding the Standards

Task 1: Relationships Among 1, 0.1 and 0.01

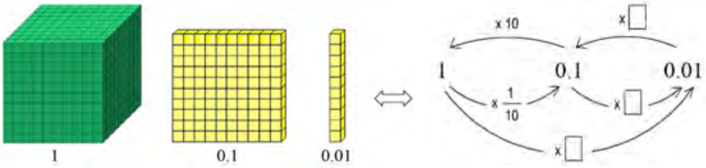


Diagram 1

i. A teacher used the base-10 blocks to represent numbers 1, 0.1 and 0.01 as shown in Diagram 1. The teacher then drew the picture in Diagram 1 to show the relationships among the three decimal numbers.

- Fill in the missing number in each □.
- What are the relationships among 1, 0.1 and 0.01?

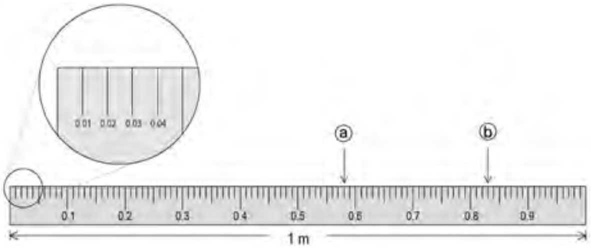


Diagram 2

ii. Another teacher, used a metre rule to represent decimal numbers up to hundredths as shown in Diagram 2.

- What number is represented by location ⓐ and ⓑ, respectively?
- Express the following measurements in metres.
  - ❖ 329 cm
  - ❖ 1 m 30 cm
  - ❖ 5 m 23 cm
  - ❖ 16 m 5 cm

iii. Compare the block model and the metre-rule model.

- Which model do you think will be easier for your students to see the relationships among 1, 0.1 and 0.01?
- Justify your answer.

iv. How will you use the two models to shows the procedure in doing the following calculations?

- 0.4 + 0.35
- 0.36 + 0.8
- 0.78 – 0.2
- 1.62 – 0.5

Task ①

Write the height of Mount Everest in the place-value chart.

Thousands	Hundreds	Tens	Ones	0.1	0.01

Task ②

Fill in the missing number in each □.

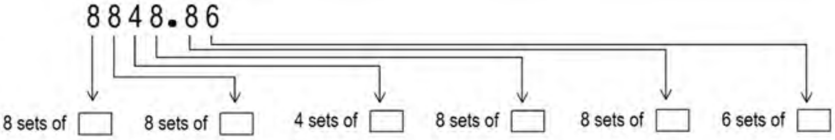


Diagram 3

v. Diagram 3 shows two tasks to help students understand the decimal place-value system.

- How will you sequence the order of the two tasks?
- Justify your choice.

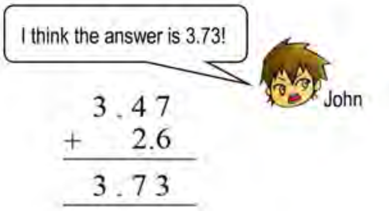


Diagram 4

vi. Diagram 4 shows the work of John involving addition of decimal numbers in vertical form.

- What is the error of John's calculation?
- Suggest a possible cause for the error.

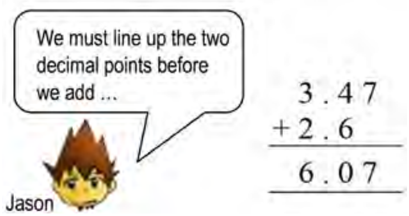
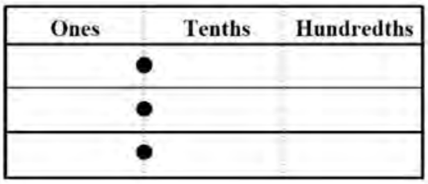


Diagram 5

vii. When asked to correct John's work, Jason explained his thinking as shown in Diagram 5.

- Why must the decimal points be lined up when adding two decimal numbers?



Ones	Tenths	Hundredths
●		
●		
●		

Diagram 6

viii. Diagram 6 shows an extended place value chart involving decimal numbers up to hundredths.

- What are the strengths and weakness of using this chart to help students add and subtract decimal numbers up to hundredths?
- How will you use this chart to help your students to solve the following problem?

Nadia did a long jump with a distance of 3.48 m and her friend Maria only cleared 2.5 m. How long did Nadia jump more than Maria in metres?

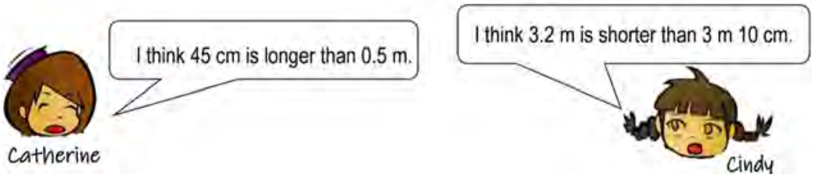


Diagram 7

ix. Many students have difficulties comparing measurements in different units. Diagram 7 shows two students' errors in comparing lengths.

- What are the possible reasons for each of them to think so?
- How will you help each of them to overcome the difficulty?

Topic 5: Extending the Vertical Form Multiplication and Division with Decimals and Find the Appropriate Place Value such as Product, Quotient and Remainder

Standards 5.1:

Extending the multiplication from the whole number to decimal numbers

i. Extend the meaning of multiplication with the idea of measurement by the number of unit length for multiplication of decimal numbers and use diagrams such as number lines to explain them with appreciation in situations

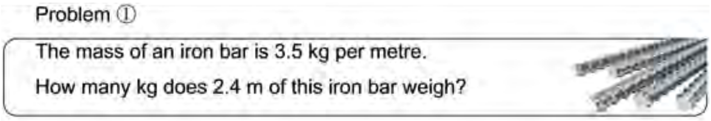
ii. Extend the vertical forms multiplication of decimals up to 3 digits by 2 digits with consideration of the decimal places step by step

iii. Obtain fluency using multiplication of decimals with sensible use of calculators in learners' life

iv. Develop number sense in multiplication of decimals such as comparing sizes of products before multiplying

Sample Tasks for Understanding the Standards

Task 1: Multiplication Involving Decimal Numbers



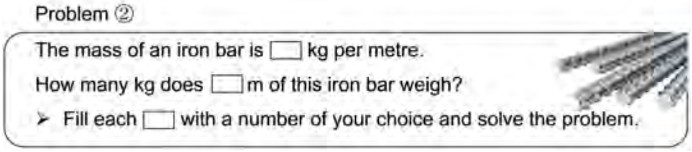
Problem ①

The mass of an iron bar is 3.5 kg per metre.

How many kg does 2.4 m of this iron bar weigh?

Diagram 1

i. Diagram 1 shows a problem used by a teacher to teach multiplication involving decimal numbers.
   - What are the possible difficulties a student may have in solving this problem?



Problem ②

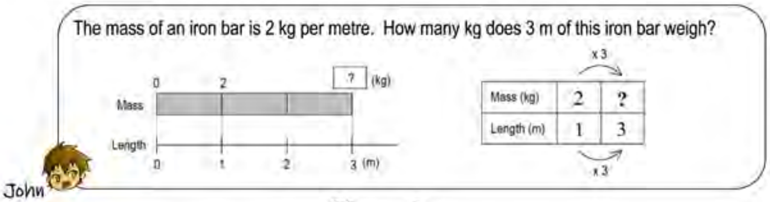
The mass of an iron bar is ☐ kg per metre.

How many kg does ☐ m of this iron bar weigh?

➤ Fill each ☐ with a number of your choice and solve the problem.

Diagram 2

ii. Another teacher modified Problem ① to become Problem ② as shown in Diagram 2.
   - What are the strengths of Problem ② as compared to Problem ① in helping students understand multiplication involving decimal numbers?



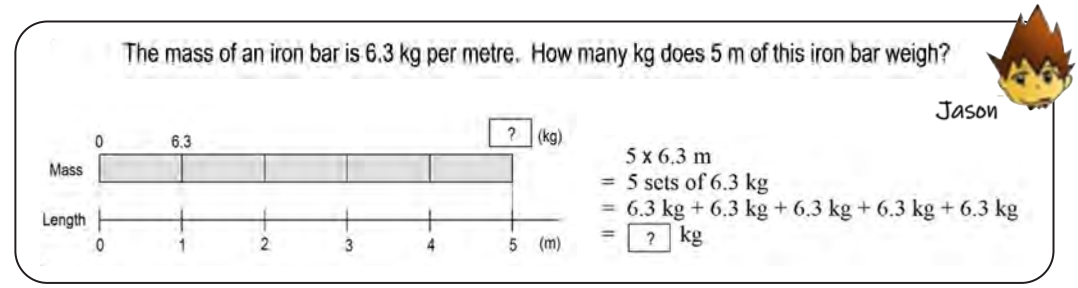
The mass of an iron bar is 2 kg per metre. How many kg does 3 m of this iron bar weigh?

Mass (kg)	2	?
Length (m)	1	3

Diagram 3

iii. John filled the ☐ with 2 kg and 3 m and drew a proportional number line and construct a table of Rule of Three as shown in Diagram 3.

   - Explain how the number line and the table can help John to understand the situation in this problem.
   - Find the missing number in $\boxed{?}$.



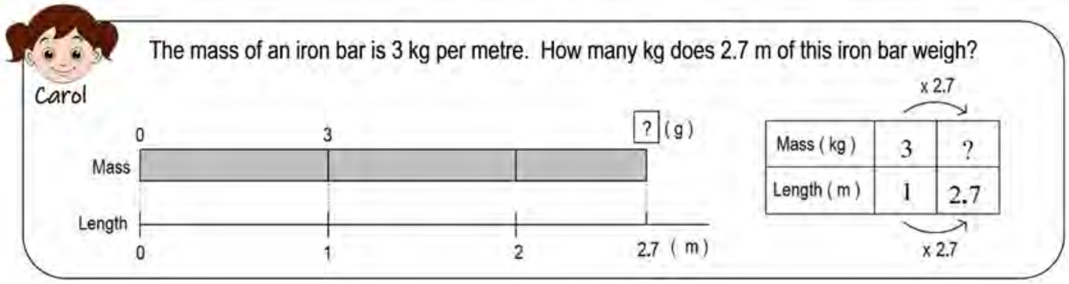
The mass of an iron bar is 6.3 kg per metre. How many kg does 5 m of this iron bar weigh?

$$\begin{aligned} & 5 \times 6.3 \text{ m} \\ = \; & 5 \text{ sets of } 6.3 \text{ kg} \\ = \; & 6.3 \text{ kg} + 6.3 \text{ kg} + 6.3 \text{ kg} + 6.3 \text{ kg} + 6.3 \text{ kg} \\ = \; & \boxed{?} \text{ kg} \end{aligned}$$

Diagram 4

iv. Jason filled the ☐ with 6.3 kg and 5 m and his solution is shown in Diagram 4.

- Explain the method used by Jason to solve the problem.
- Construct a table of Rule of Three to check Jason's solution.
- Find the missing number in $\boxed{?}$.



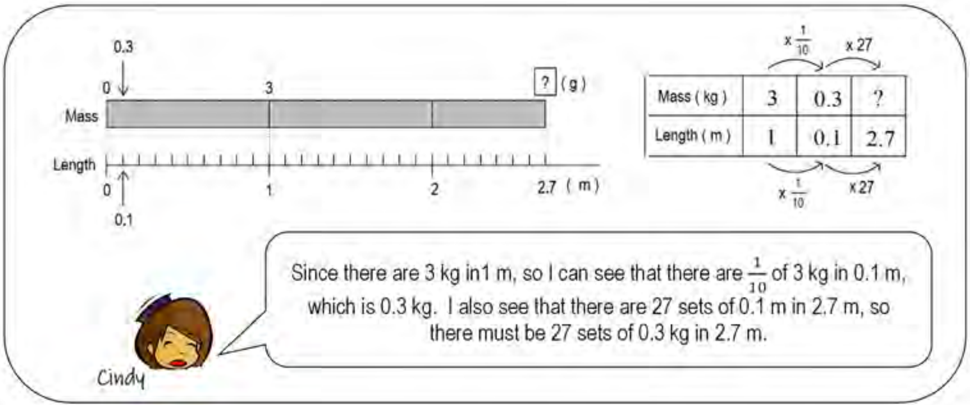
The mass of an iron bar is 3 kg per metre. How many kg does 2.7 m of this iron bar weigh?

Mass (kg)	3	?
Length (m)	1	2.7

Diagram 5

v. Carol filled the ☐ with 3 kg and 2.7 m and her solution is shown in Diagram 5.

- Write an expression for $\boxed{?}$.
- Explain Carol's solution.
- Find the missing number in $\boxed{?}$.



Mass (kg)	3	0.3	?
Length (m)	1	0.1	2.7

Diagram 6

vi. After studying Carol's solution, Cindy drew another proportional number line and a table of Rule of Three to explain her thinking as shown in Diagram 6.

- Write an expression for $\boxed{?}$.
- Compare Cindy's method with Carol's method.
  - ❖ What is the same and what is different about the two students' methods?
  - ❖ Which method will you recommend to your students? Why?

vii. Catherine filled the ☐ with 4.8 kg and 3.5 m.

- Draw a proportional number line and a Rule of Three table to help Catherine solve the problem.

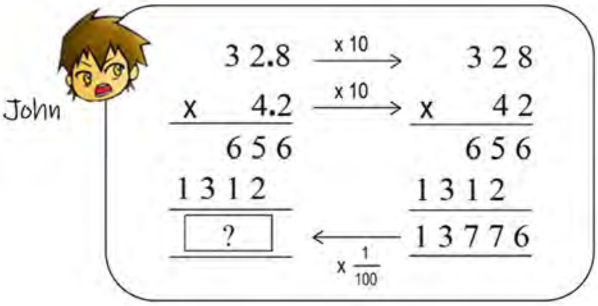


Diagram 7

viii. Diagram 7 shows John's idea to simplify the multiplication 32.8 x 4.2 in vertical form by changing both the multiplicands and the multipliers to whole numbers.

- Explain the rule used by John to simplify the multiplication.
- Find the missing number in $\boxed{?}$.

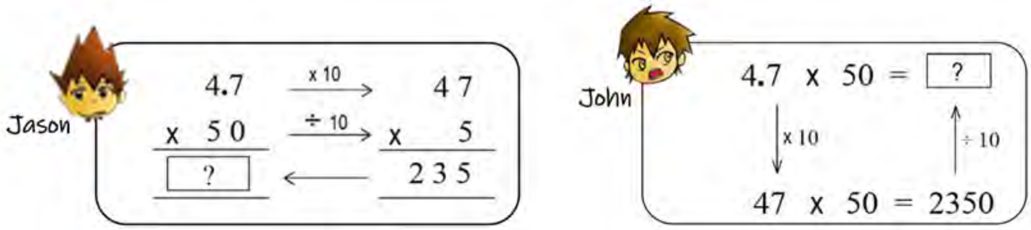


Diagram 8

ix. Diagram 8 shows ideas of Jason and John to simplify the multiplication 4.7 x 50.

- Compare the rules used by each of them.
- Find the missing number in each $\boxed{?}$.

x. Simplify and calculate the answer for each multiplication in vertical form.

- 8.4 x 320
- 12.64 x 70
- 7.35 x 26
- 72.4 x 3.65

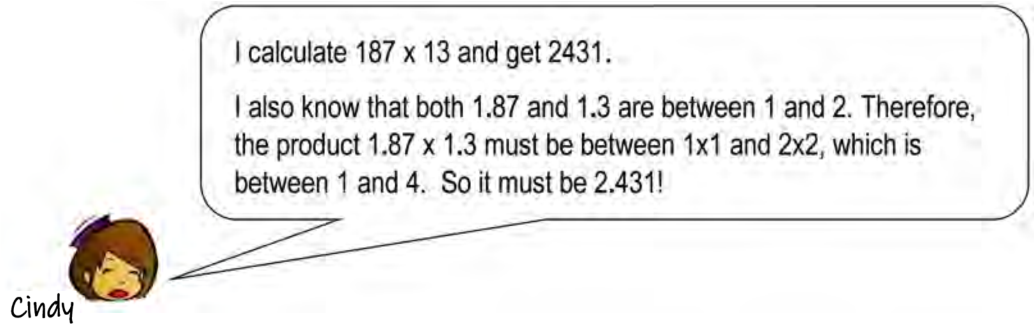


Diagram 9

xi. Diagram 9 shows Cindy's reasoning to get the answer for 1.87 x 1.3.

- Explain Cindy's reasoning.

xii. Without actual calculation, put the missing decimal point on the product for each of the calculations.

- 8.24 x 3.5 = 2 8 8 4
- 12.45 x 7.8 = 9 7 1 1
- 30.2 x 19.5 = 5 8 8 9

Check your answers with a calculator.

xiii. What is the area, in m², of a rectangular mat that is 1.3 m wide and 2.4 m long?

- Write an expression for the area of the rectangular mat.
- Approximately, what is the area in m^2 ?

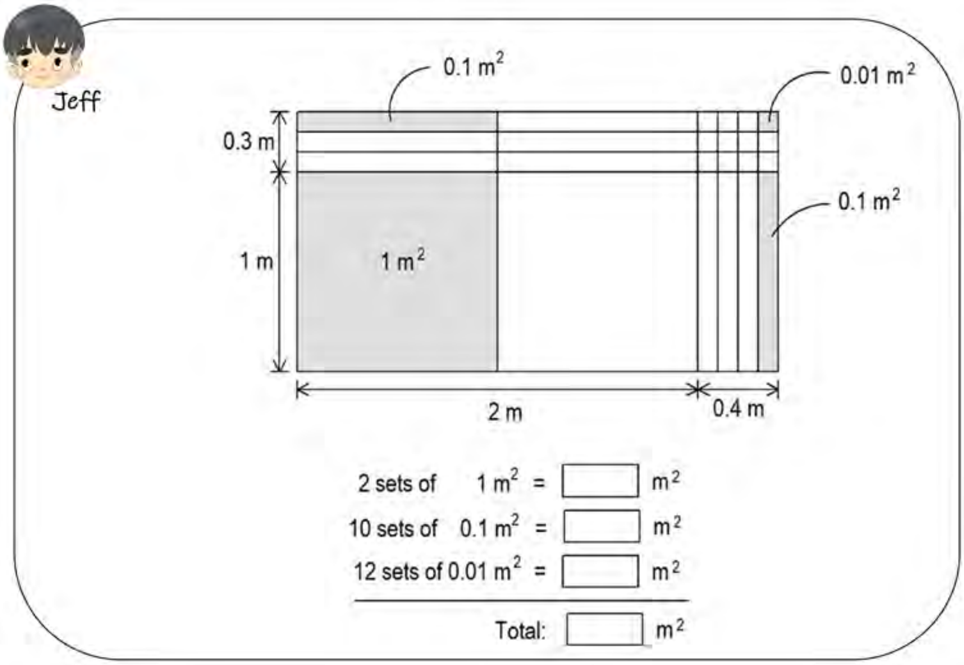


Diagram 10

- Diagram 10 shows how Jeff solved the problem by drawing an area diagram.
  - ❖ Fill in the missing number in each ☐.
  - ❖ Explain Jeff's method.
  - ❖ Calculate (1.3 x 2.4) m² to check the answer in Diagram 10.
  - ❖ What are the strengths of this problem in helping your students understand multiplication involving decimal numbers?

Standards 5.2:

Extending the division from the whole number to decimal numbers

i. Understand how to represent division situations using diagrams such as number lines, and extend the diagram of decimal numbers for explaining division by decimal numbers
ii. Extend the division algorithm in vertical form of decimal numbers and interpret the meaning of decimal places of quotient and remainder with situations
iii. Acquire fluency in division algorithm of decimals up to 3 digits by 2 digits with consideration of decimal places step by step
iv. Obtain fluency using division of decimals with sensible use of calculators in life
v. Develop number sense of division in decimals such as comparing sizes of quotient before dividing
vi. Distinguish the situations with decimal numbers of multiplication and division

Sample Tasks for Understanding the Standards

Task 1: Division Involving Decimal Numbers

Diagram 1

i. A supermarket in Manila sells orange juice in 2-litre and 1.8-litre cartons at 90 pesos and 72 pesos respectively as shown in Diagram 1.

- Compare the two packaging.
  - ❖ What calculations will you do to determine which packaging is the better buy?
  - ❖ Explain your answer.

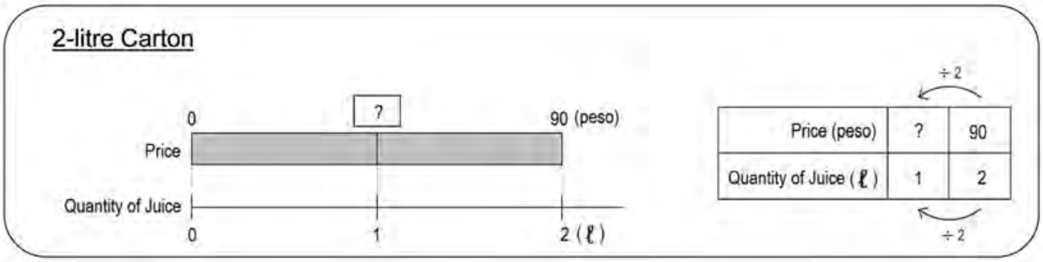


Diagram 2

- Diagram 2 shows a proportional number line and Rule of Three table to determine the calculation for the price of 1ℓ of orange juice for the 2- ℓ packaging.

  ❖ Write an expression for $\boxed{?}$.

  ❖ Calculate the price per litre for the packaging.

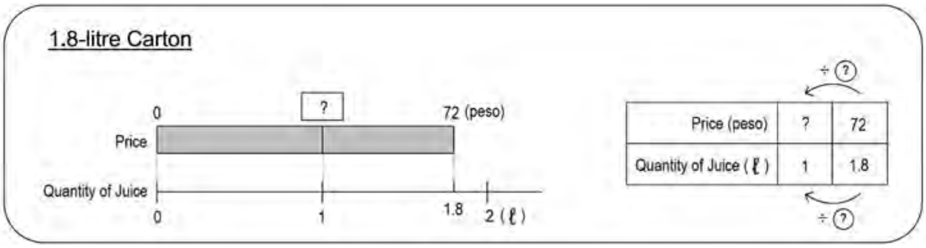


Diagram 3

- Diagram 3 shows a proportional number line and Rule of Three table to determine the calculation for the price of 1ℓ of orange juice for the 1.8 ℓ packaging.

  ❖ Fill in the missing number in each ?.

  ❖ Write an expression for $\boxed{?}$.

  ❖ Calculate the price per litre for the packaging.

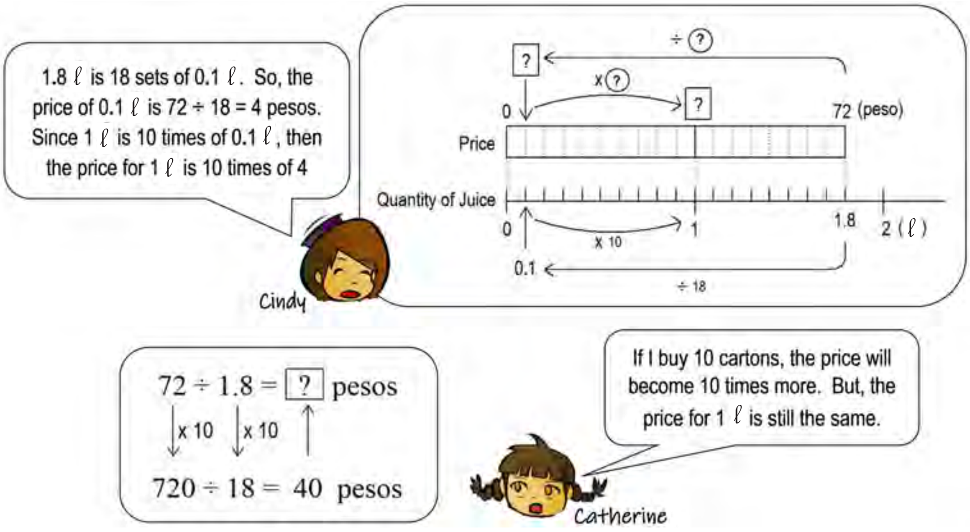


Diagram 4

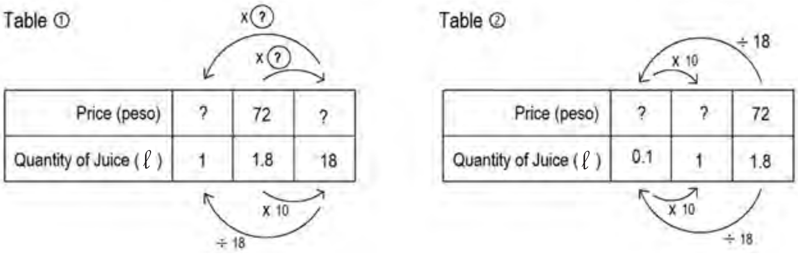


Diagram 5

ii. Diagram 4 shows two students' ideas to calculate 72 ÷ 1.8. The same ideas are also represented in tables shown in Diagram 5.

- Discuss what the two ideas have in common.
- Which idea corresponds to each of the two tables?

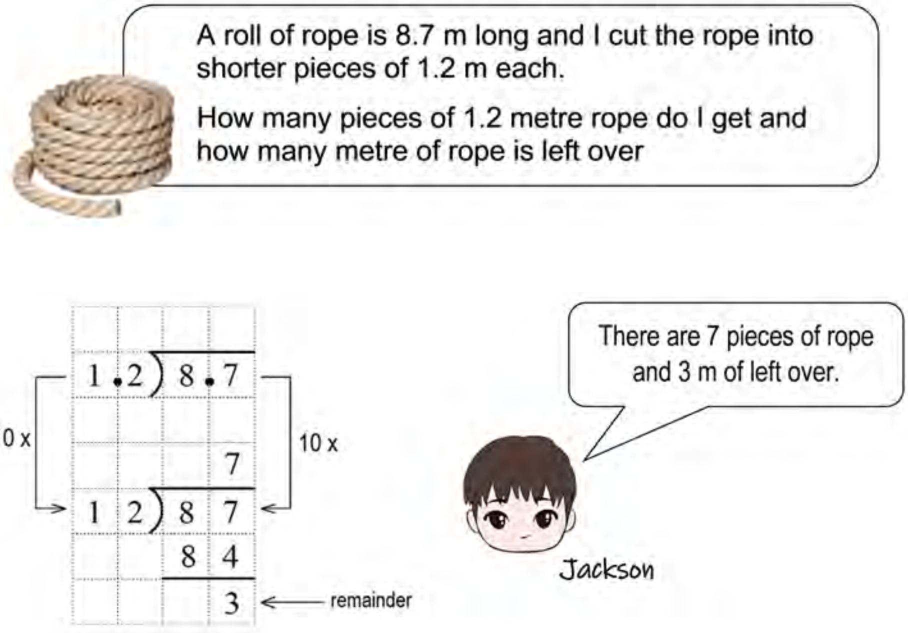


Diagram 6

iii. Diagram 6 shows a rope-cutting problem and Jackson's solution to the problem.

- Explain the rule used by Jackson to simplify the division?
- Is Jackson's solution correct?
- How many metre of rope was left over?

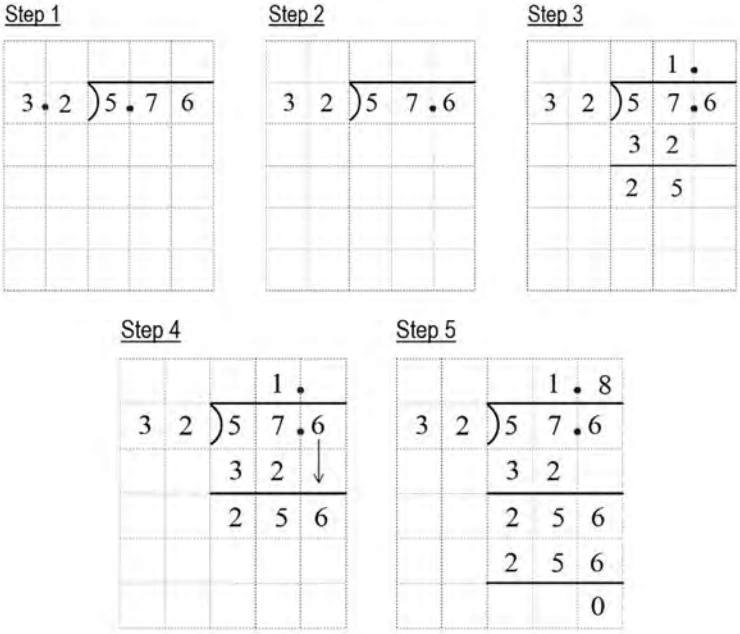


Diagram 7

iv. Diagram 7 shows the steps to do the division 5.76 ÷ 3.2 in vertical form.

- Explain each step.
- In Step 5, what do you think is the purpose of writing down the remainder 0?

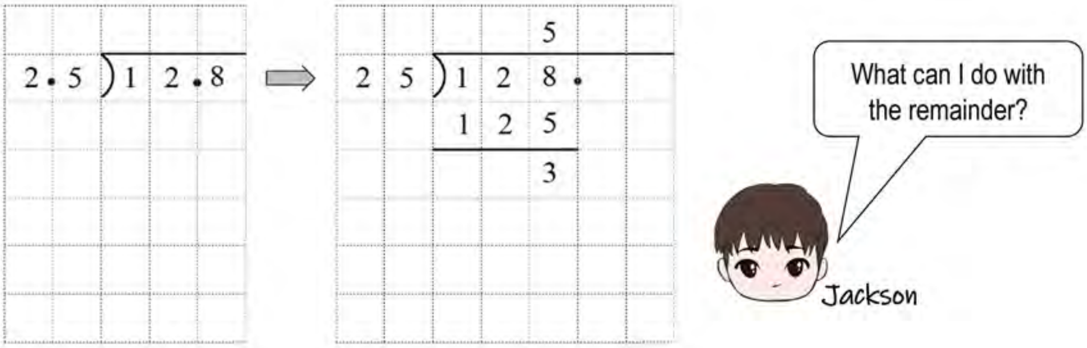
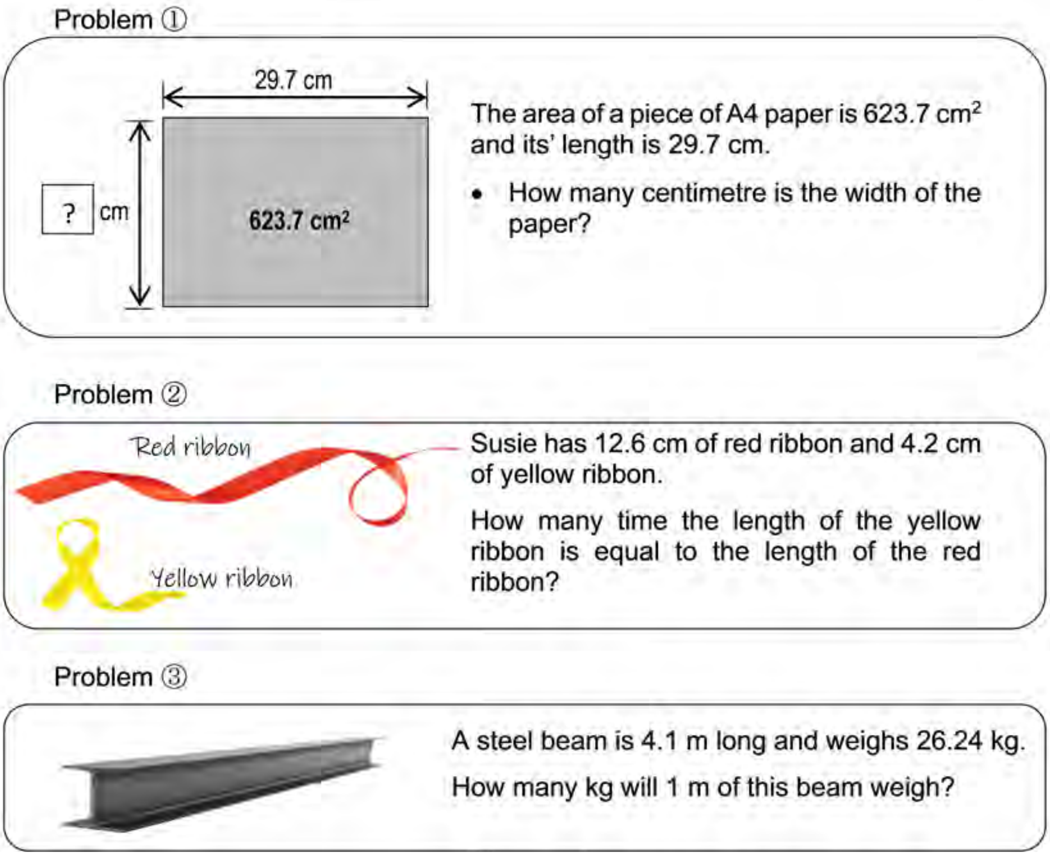


Diagram 8

v. Diagram 8 shows the procedure used by Jackson to do the division 12.8 ÷ 2.5 in vertical form.

- Discuss what Jackson could do with the remainder.
- Complete the division procedure for Jackson on Diagram 8.



Problem ①

The area of a piece of A4 paper is 623.7 cm² and its' length is 29.7 cm.

- How many centimetre is the width of the paper?

Problem ②

Susie has 12.6 cm of red ribbon and 4.2 cm of yellow ribbon.

How many time the length of the yellow ribbon is equal to the length of the red ribbon?

Problem ③

A steel beam is 4.1 m long and weighs 26.24 kg.

How many kg will 1 m of this beam weigh?

Diagram 9

vi. Diagram 9 shows three problems involving different contexts of division.

- Discuss the possibility of using a proportional number line and Rule of Three table to solve each of the problems.

vii. Diagram 10 shows a painter painting the interior wall of a house. He used 3.8 ℓ of paint to cover 45.6 m² of wall.

How many litres of paint will he need to cover 98.6 m² of wall?

Diagram 10

Topic 6: Using Multiples and Divisors for Convenience

Standards 6.1:

Using multiples and divisors for convenience with appreciation to enrich number sense

i. Understand set of numbers by using multiple and divisor
ii. Find common multiple and appreciate the use in situations, and enrich number sense with figural representations such as arrangement of rectangles to produce a square
iii. Find common divisor and appreciate its use in situations, and enrich number sense with figural representations such as dividing a rectangle into pieces of square
iv. Understand numbers as composite of multiplication of numbers as factors
v. Appreciate ideas of prime, even and odd numbers in situations using multiples and divisors
vi. Acquire number sense to see its multiple for convenience

Sample Tasks for Understanding the Standards

Task 1: Multiples and Divisors

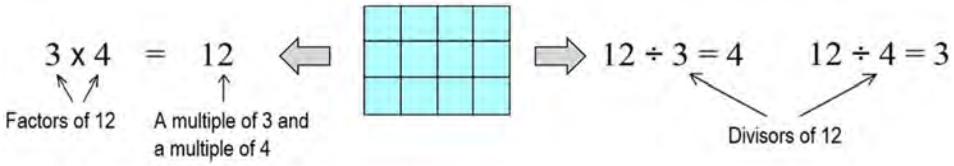


Diagram 1

i. Diagram 1 shows a figural representation of 3 x 4.

- Explain the relationships between multiples and divisors.
- Find all other divisors of 12.
- Is 1 a divisor of 12? Why or why not?
- Is 12 a divisor of 12? Why or why not?

[***Note***. In this topic, a divisor of a whole number is a whole number which evenly divides the number and leaves no remainder. A divisor is also known as a factor.]

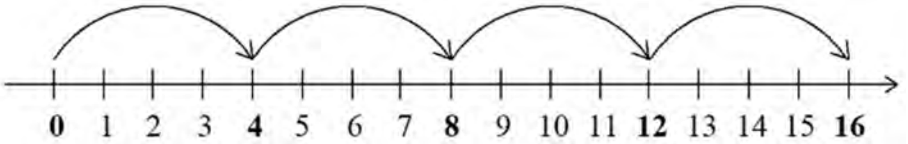


Diagram 2

ii. Diagram 2 is a number line used to represent skip counting by 4.

- How is skip counting related to multiples?
- Diagram 2 shows that 4, 8, 12 and 16 are multiples of 4.
  - ❖ Is zero a multiple of 4? Why or why not?

- Find the first five multiples of 13.
    - ❖ What is the least non-zero multiple of 13?
    - ❖ Can the greatest multiple of 13 be found? Why or why not?

iii. Diagram 3 shows a problem involving stacking boxes of chocolate.

- How does the task promote the understanding of multiples?

Stacking-Chocolate Problem

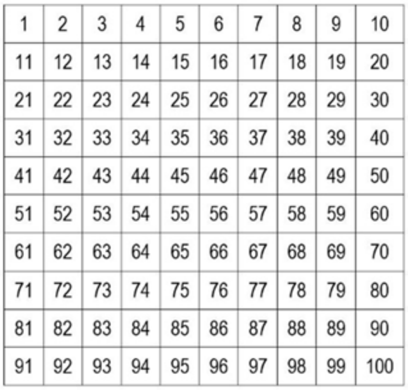
A box of chocolate is 3 cm high. Some chocolates are packed in a stack of 6 boxes.

- ➢ What is the total height of the stack?
- ➢ What is the height of a stack of 4 boxes? 7 boxes? 10 boxes?

Diagram 3

iv. Diagram 4 shows a hundred chart. Circle all multiples of 2 between 1 and 100.

- Describe the pattern of multiples of 2.

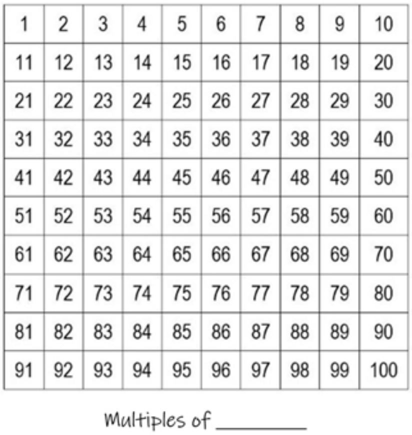


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Diagram 4

v. Explore the patterns of multiples of other numbers between 2 and 10 using the hundred chart in Diagram 5.

- Which of these patterns do you find them interesting? Why?
- How does this exploration of patterns promote your students' understanding of multiples?



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of ________

Diagram 5

January

Su	M	Tu	W	Th	F	Sa
1	2	3	④	5	6	7
⑧	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Diagram 6

vi. Krit's father intends to train him in managing his pocket money. So, instead of everyday, Krit's father decided to give him a bigger sum of pocket money every four days.

Diagram 6 shows the calendar for January of a year. The dates for the first two allowances are marked with ○.

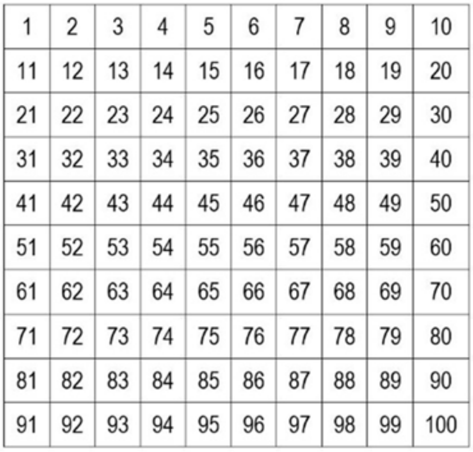
- What will be the date for Krit's third allowance?
- What are the remaining allowance dates in January for Krit?
- How can this problem inculcate your students' appreciation towards multiples?

Diagram 7

vii. Diagram 7 shows a big basket of 28 mangoes. José wants to pack all the mangoes into smaller baskets with equal number of mangoes, and with no left over.

- What are the different ways for José to pack the mangoes?
- How many smaller baskets of mangoes can he pack?
- How does this problem inculcate your students' appreciation towards divisors?

Task 2: Common Multiples



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Diagram 8

i. Mark with ○ on all multiples of 3, and ✕ on all multiples of 5 in Diagram 8.

- What numbers are marked with both the ○ and the ✕ ?
- These numbers the common multiples of 3 and 5.
  - ❖ What is common multiple?
  - ❖ What is least common multiple (LCM)?
  - ❖ Find the LCM of 3 and 5.

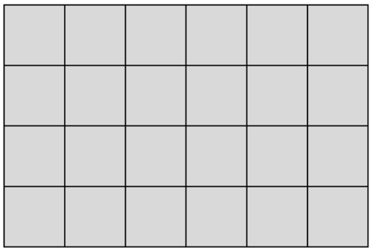


Diagram 9

ii. Diagram 9 shows a 4 x 6 rectangular card.

- Cut out the rectangular cards in Material Sheet 1 at the end of this topic.
- Arrange some of these cards to construct a square.
- Construct another square of different sizes.
- Explain how the squares are related to the common multiples of 4 and 6.
- Find the LCM of 4 and 6 using these squares.

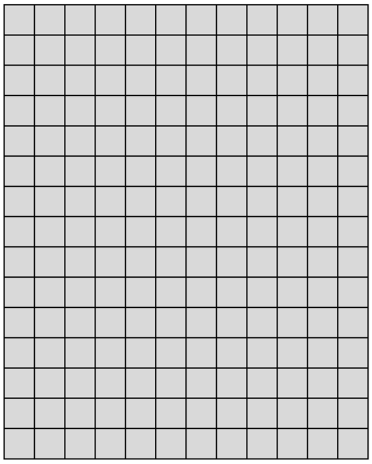


Diagram 10

iii. Diagram 10 shows a 15 x 12 rectangular card.

- Cut out the rectangular cards in Material Sheet 2 at the end of this topic.
- Explain how these cards can be used to find the LCM of 15 and 12.
- Find the LCM of 15 and 12 using these cards.

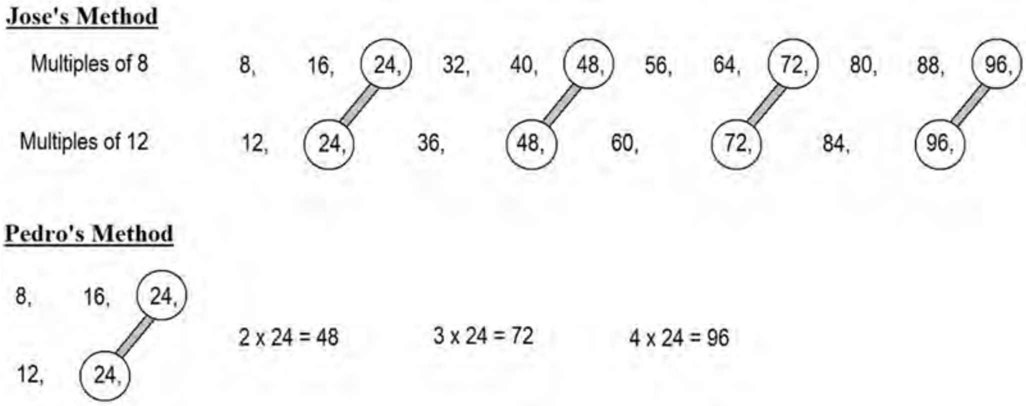


Diagram 11

iv. Diagram 11 shows two student's methods to find the common multiples of 8 and 12.

- Describe each method and explain the student's ideas.
- What are the strengths and weaknesses of each method?
- What is the LCM of 8 and 12?
- Find the LCM of 18 and 27.

Diagram 12

v. A supermarket sells buns that come in a pack of 8 and sausages in a pack of 6 as shown in Diagram 12. Tata plans to prepare hot dogs with these buns and sausages for a party. She tries to match the number of sausages with the buns in order not to leave with surplus of either one.

- How many packs of each will you recommend for Tata to buy? Show more than one way of solving the problem.
- What is the least number of packs for buns and sausages that Tata can buy?
- What concept of multiples is enhanced in solving this problem?

Task 3: Common Divisors

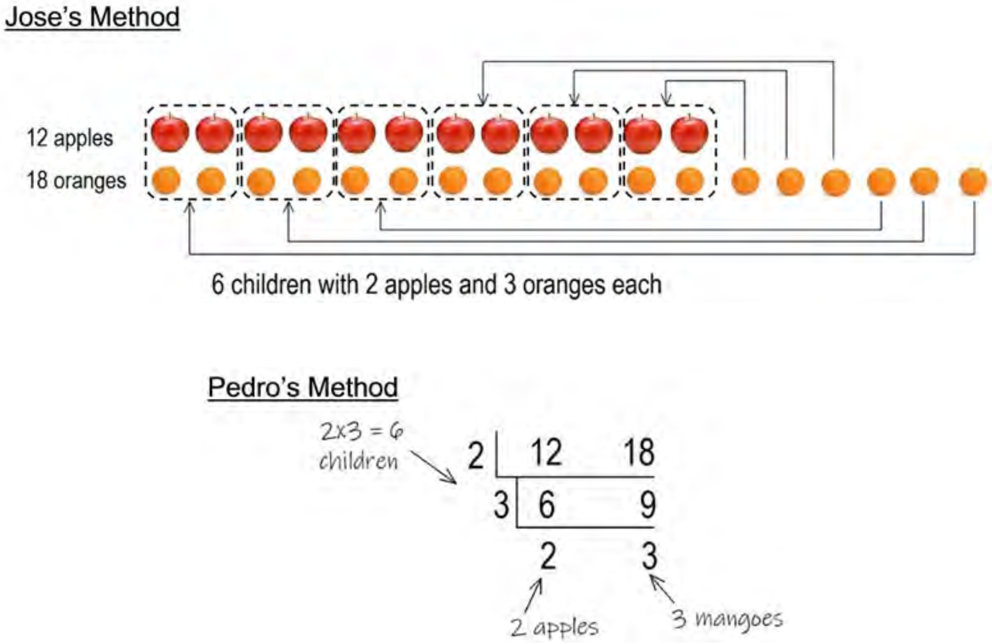


Diagram 13

i. Diagram 13 shows the methods used by two students to solve the following problem:

> 12 apples and 18 oranges are to be distributed equally amongst some children. What is the greatest number of children who can get the fruits ?

- Explain each method.
- What are the strengths and weaknesses of each method?
- What is the greatest common divisor (GCD) of 12 and 18?
- Find the GCD of 12 and 28.
- Find the greatest common divisor of 18 and 27.

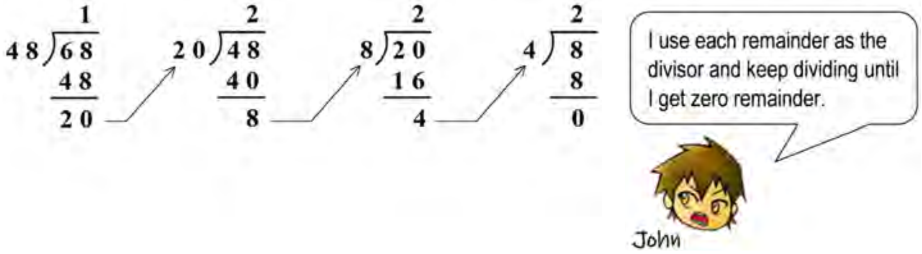


Diagram 14

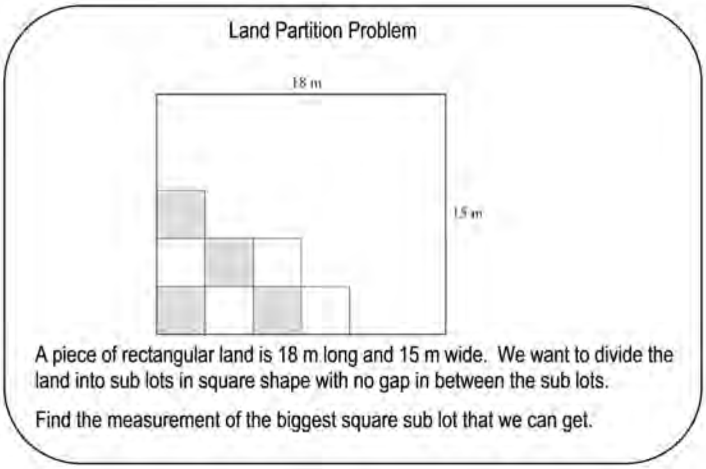
ii. Diagram 14 shows John's method of finding the GCD of 68 and 48 using the Euclidean algorithm.

- What is the GCD of 68 and 48?
- Explain John's method.
- Find the GCD of 210 and 45 using John's method.

iii. Both the numbers 12 and 8 have 6 divisors each.

Diagram 15

- What pattern do you see when the divisors are grouped as shown in Diagram 15?
- Make a conjecture about the number of divisors of a number.
    - ❖ Is your conjecture sometimes or always true? Justify your decision.



Land Partition Problem

A piece of rectangular land is 18 m long and 15 m wide. We want to divide the land into sub lots in square shape with no gap in between the sub lots.

Find the measurement of the biggest square sub lot that we can get.

Diagram 16

iv. Diagram 16 shows a problem to divide a piece of land.

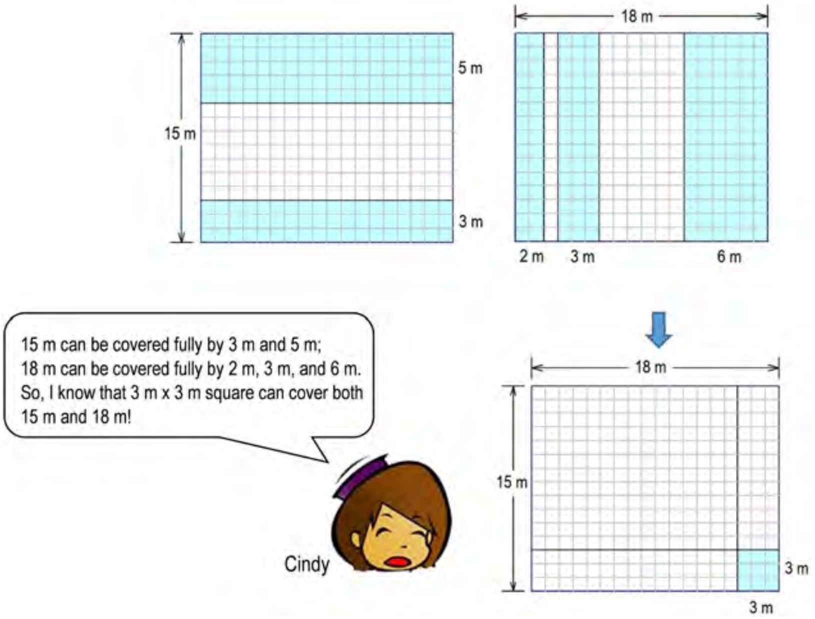


Diagram 17

Cindy figured out a solution to the Land Problem as shown in Diagram 17.

- Explain Cindy's ideas and solution.
- Solve the Land Problem using any other method.
- Compare your method with Cindy's method.
  - ❖ In what ways is your method different from Cindy's method?
  - ❖ Is there any similarity between your method and Cindy's method?

Task 4: Prime Numbers and Composite Numbers

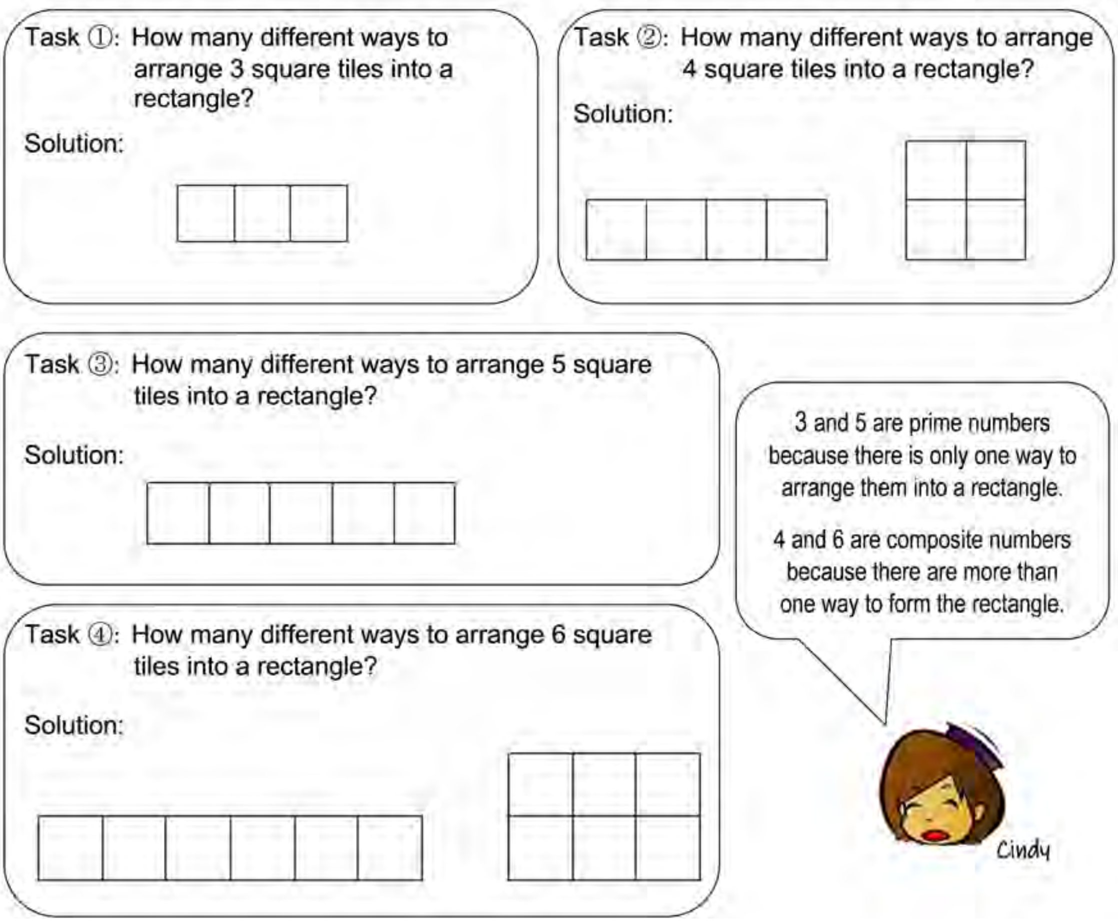
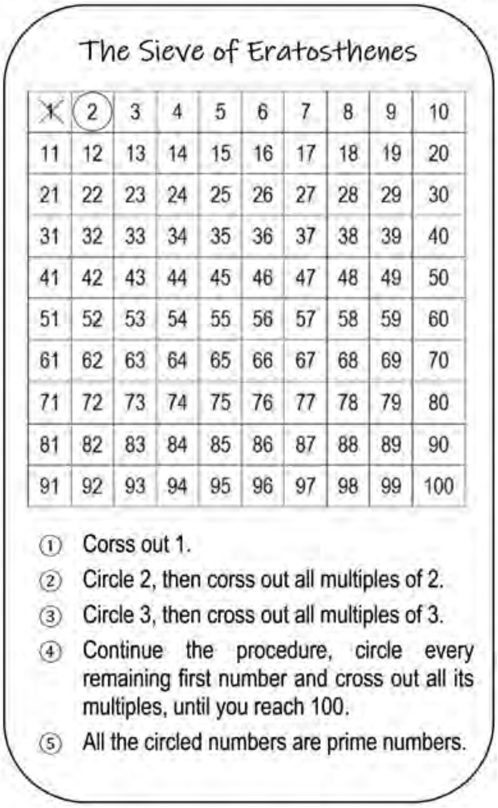


Diagram 18

i. Diagram 18 shows four tasks involving arrangement of square tiles and the solutions given by Cindy as well as her understanding on the meaning of prime and composite numbers.

- Cut out the square tiles in Material Sheet 3 at the end of this topic.
- How many different ways to arrange 15 square tiles into a rectangle?
  - ❖ How is this subtask related to the factors of 15?
  - ❖ Is 15 a prime or composite number? Justify your decision.
- How many different ways to arrange 19 square tiles into a rectangle?
  - ❖ How is this subtask related to the factors of 19?
  - ❖ Is 19 a prime or composite number? Justify your decision.



The Sieve of Eratosthenes

~~1~~	②	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

① Corss out 1.

② Circle 2, then corss out all multiples of 2.

③ Circle 3, then cross out all multiples of 3.

④ Continue the procedure, circle every remaining first number and cross out all its multiples, until you reach 100.

⑤ All the circled numbers are prime numbers.

Diagram 19

ii. Diagram 19 shows an ancient method of finding all prime numbers within 100 which is known as the Sieve of Eratosthenes.

- How many prime numbers less than 100 do you get from the Sieve of Eratosthenes?
- What is a prime number?
- Why is 1 not considered as a prime number?

Task 5: Even Numbers and Odd Numbers

Diagram 20

i. Diagram 20 shows a representation of the smallest even number 2. Using this as a unit, a student represented five other even numbers as shown in Diagram 21.

Diagram 21

Based on these representation of even numbers, a student makes a conjecture that “even numbers are multiples of 2”.

- Is the conjecture sometimes or always true? Justify your decision.
- How are odd numbers related to multiples of 2?

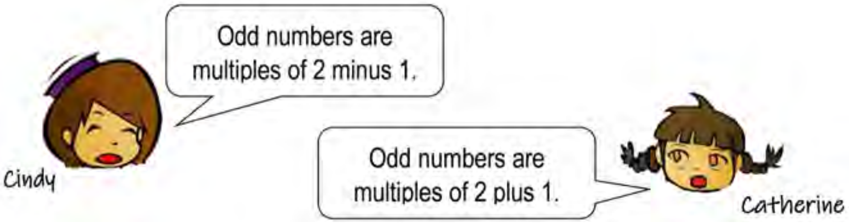


Diagram 22

ii. Two students make a conjecture respectively on the relationship between odd numbers and multiples of 2 as shown in Diagram 22.

- Which conjecture is true? Justify your decision.

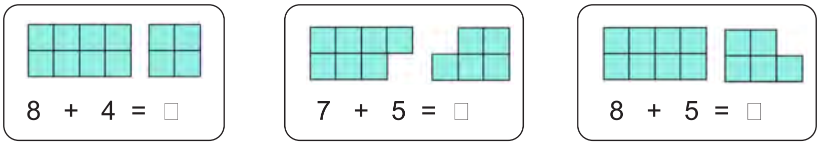


Diagram 23

iii. Diagram 23 shows three sums involving even and odd numbers.

- Fill in the missing sum in each ☐.
- Based on these sums, make a conjecture for each of the following case:
  - ❖ Sum of two even numbers
  - ❖ Sum of two odd numbers
  - ❖ Sum of an even number and an odd number
- Is each of your conjectures sometimes or always true? Justify your decision.

8 – 4 = □ 7 – 5 = □ 8 – 5 = □

Diagram 24

iv. Diagram 24 shows three cases of subtraction involving odd and even numbers.

- Fill in the missing number in each □.
- Make a conjecture involving subtraction of odd and even numbers for each case.
- Is each of your conjectures *sometimes or always* true? Justify your decision.

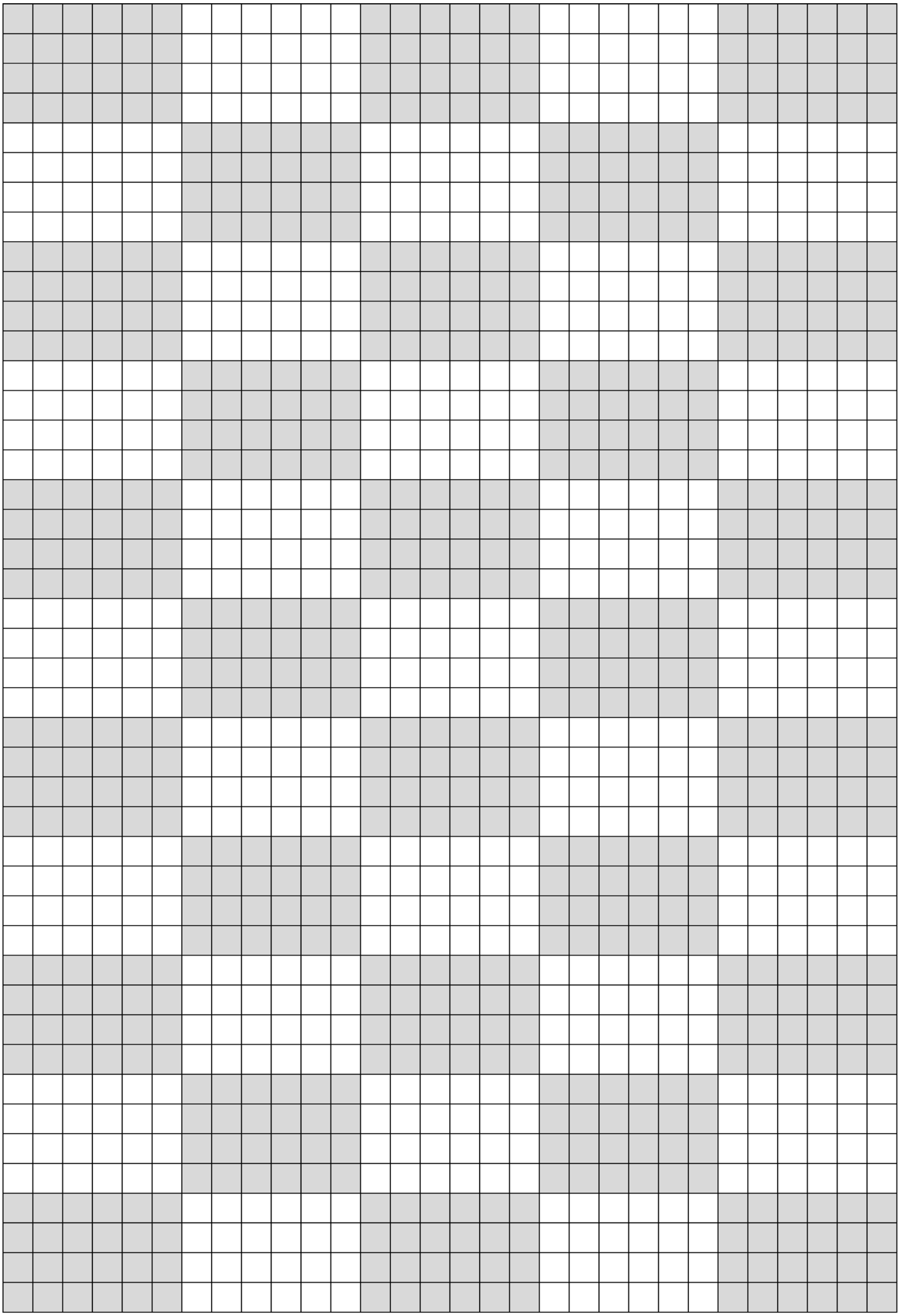
v. Decide whether each of the following statements is **true** or **false**. Justify your decision.

- All prime numbers are odd.
- An even number cannot be a prime number.
- Zero is an even number.

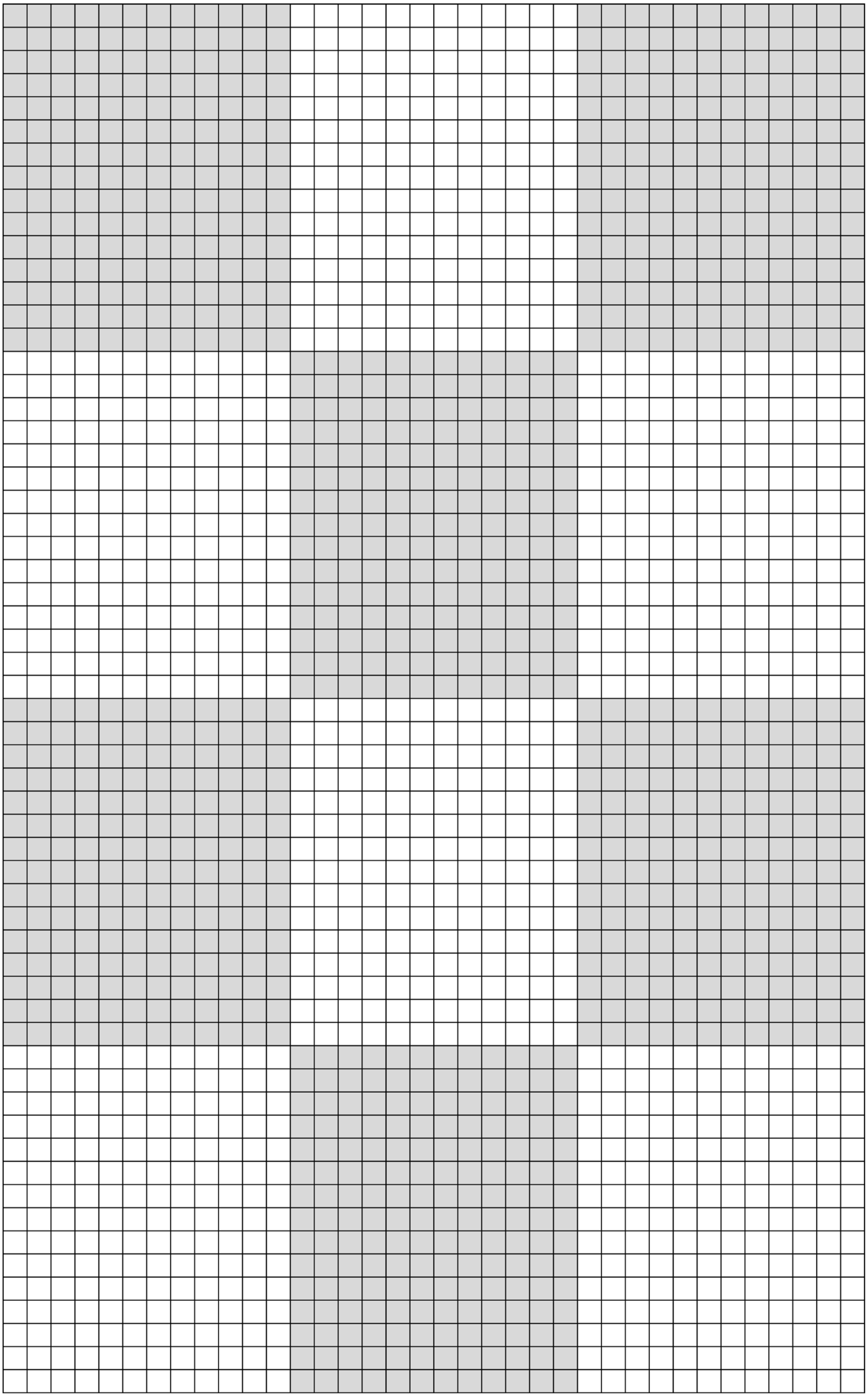
vi. In Malaysia, every citizen is issued an identification card with a 12-digit number such as xxxxxx-xx-5207 and xxxxxx-xx-2318. The last digit indicates the gender of the card holder, even digit for female and odd digit for male. So, the holder of xxxxxx-xx-5208 is a female and the holder of xxxxxx-xx-2317 is a male.

- Why do odd and even numbers work for Malaysian identification system?
- What other application of odd and even numbers can you find in your own country? Explain the application clearly.

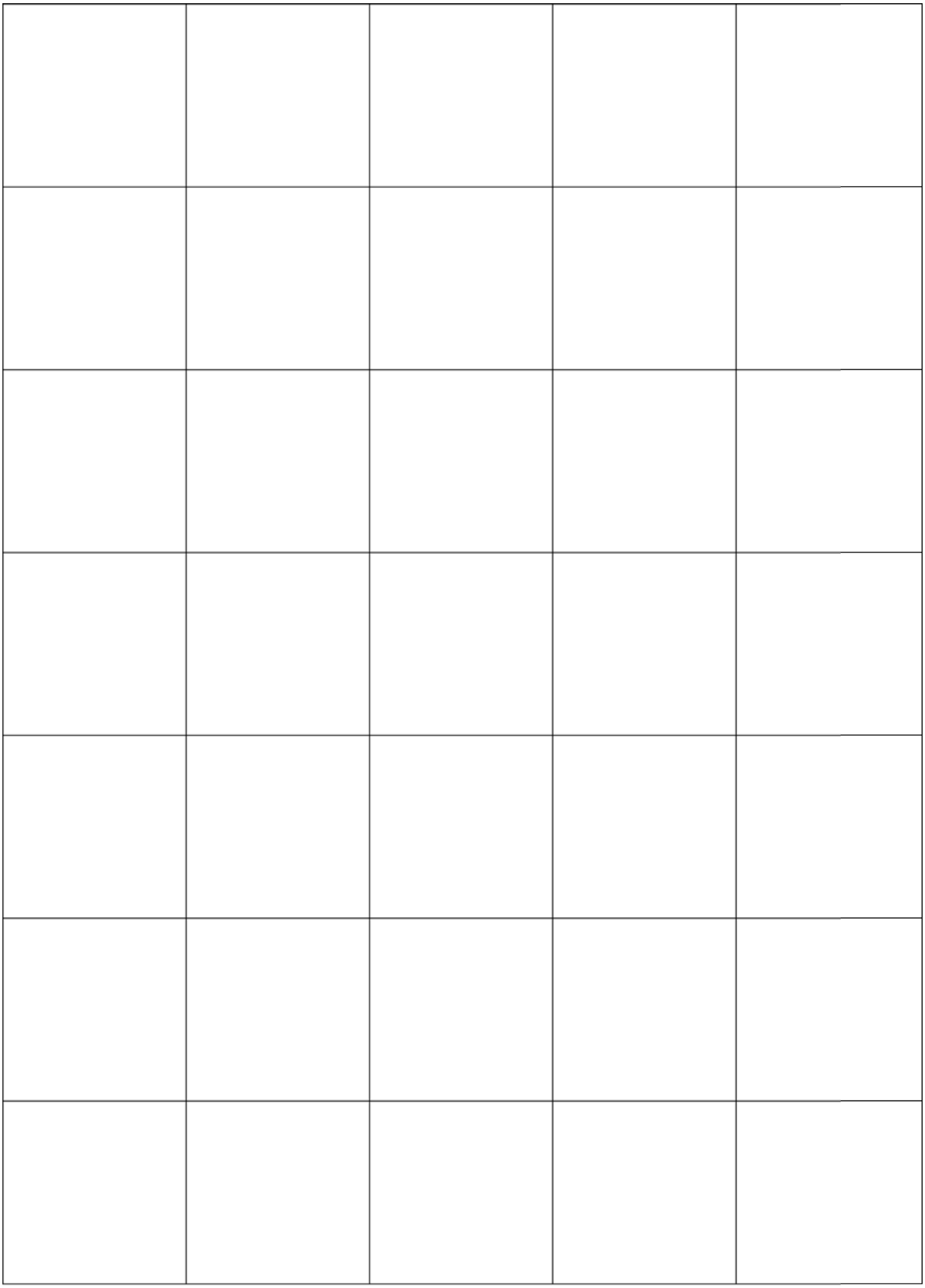
Material Sheet 1: Rectangular Card (4 x 6)



Material Sheet 2: Rectangular Card (15 x 12)



Material Sheet 3: Square Tile



Topic 7: Introducing Improper and Mixed Fractions and Extending to Addition and Subtraction of Fractions to Dissimilar Fractions

Standards 7.1: Extending fractions to improper, mixed and equivalent fractions

i. Extend fractions to improper and mixed fractions using number line of more than one by measuring with unit fraction

ii. Find ways of determine equivalent fractions with number lines and with the idea of multiple of numerator and denominator

iii. Compare fractions using number line and the idea of multiple

Sample Tasks for Understanding the Standards

Task 1: Fraction More than 1

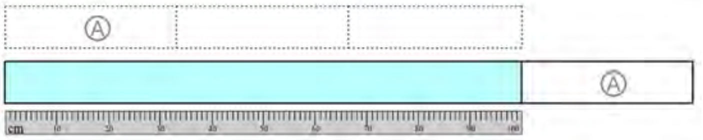


Diagram 1

i. A student measured the length of a tape using a metre rule as shown in Diagram 1. The tape was found to be 1 m and a remaining part Ⓐ. The student also found that 3 parts Ⓐ were equal to 1 m.

- How many metre is one remaining part Ⓐ?
- How many metre is the length of the entire tape?
  - ❖ Express your answer as an improper fraction and also a mixed fraction.

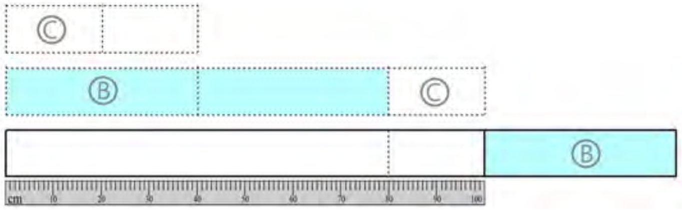


Diagram 2

ii. The student measured another tape and found that it was again 1 m and a remaining part, Ⓑ. He cut out part Ⓑ and found that two parts of Ⓑ is a little less than 1 m. He then continued the process and found that 2 parts of Ⓒ is the same as 1 part Ⓑ as shown in Diagram 2.

- How many part Ⓒ is the same as 1 m?
- How many metre is one part Ⓒ?
- How many metre is one part Ⓑ?
- How many metre is the entire tape?
  - ❖ Express your answer as an improper fraction and also a mixed fraction.

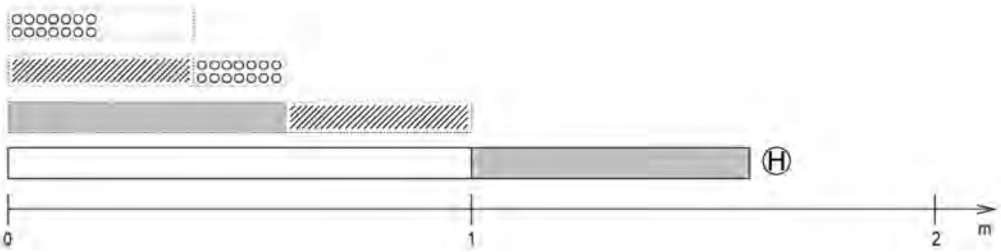


Diagram 3

iii. Both the procedures used in Diagram 1 and 2 involve the Euclidean algorithm to find the remaining part as a fraction are similar to the procedure of finding the GCD of 68 and 48 as illustrated in Diagram 14, Task 3 of Topic 6.

Diagram 3 shows the same procedure for finding the entire length of tape Ⓗ.

- Explain the procedure.
- Find the entire length of tape Ⓗ, in m.

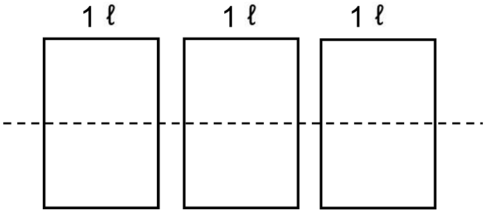


Diagram 4

iv. Diagram 4 shows 3 litres of water divided equally into 2 jugs.

- Shade the portion for 1 jug on Diagram 4.
- Express in litre, the amount of water in 1 jug.
  - ❖ Give your answer as an improper fraction and also a mixed fraction.

v. Fill in each ☐ with an appropriate number for the following statement to be true.

There are ☐ sets of $\frac{1}{5}$ m in $\frac{☐}{5}$ m.

- In what way can the unit fraction $\frac{1}{5}$ help your students understand the idea of improper fraction?
- Discuss how the idea of unit fraction can be used to compare $\frac{8}{5}$ m, $\frac{12}{5}$m and $1\frac{2}{5}$ m.

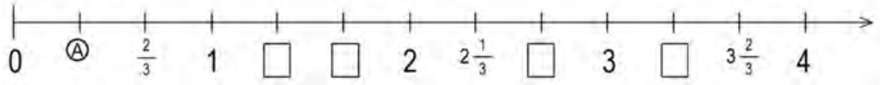


Diagram 5

vi. Diagram 5 shows a fraction number line.

- What is the unit fraction Ⓐ?
- Fill in the missing mixed fraction in each □ to complete the number lines.

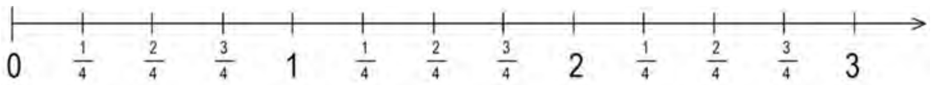


Diagram 6

vii. Diagram 6 shows a fraction number line drawn by a student.

- What is wrong with the number line?

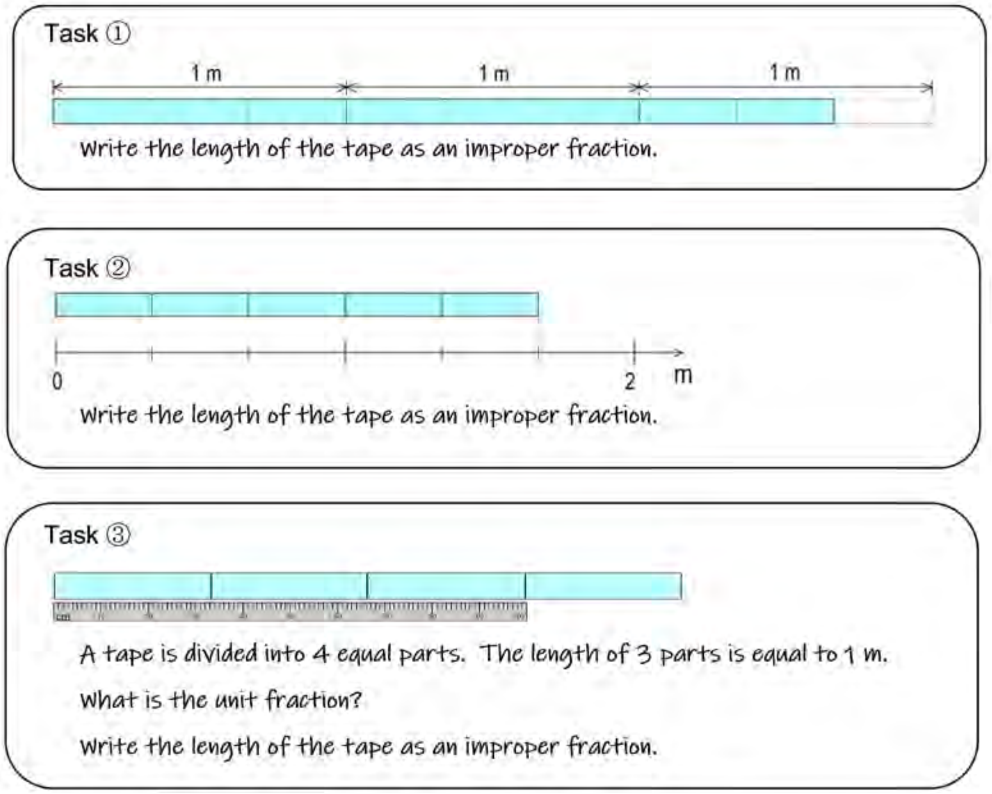


Diagram 7

viii. Diagram 7 shows three tasks involving mixed fractions and improper fractions.

- How will you order the sequence of the three tasks? Justify your decision.

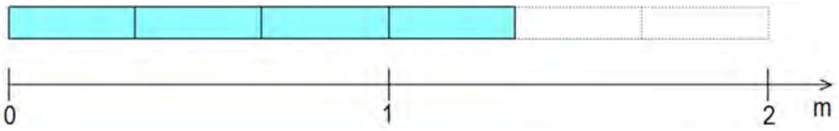


Diagram 8

ix. Diagram 8 shows a tape divided into 4 equal parts. A student said that the length of the tape is $\frac{2}{3}$ of 2 m.

- Is the student's claim correct?
- What is the length of the tape in m?

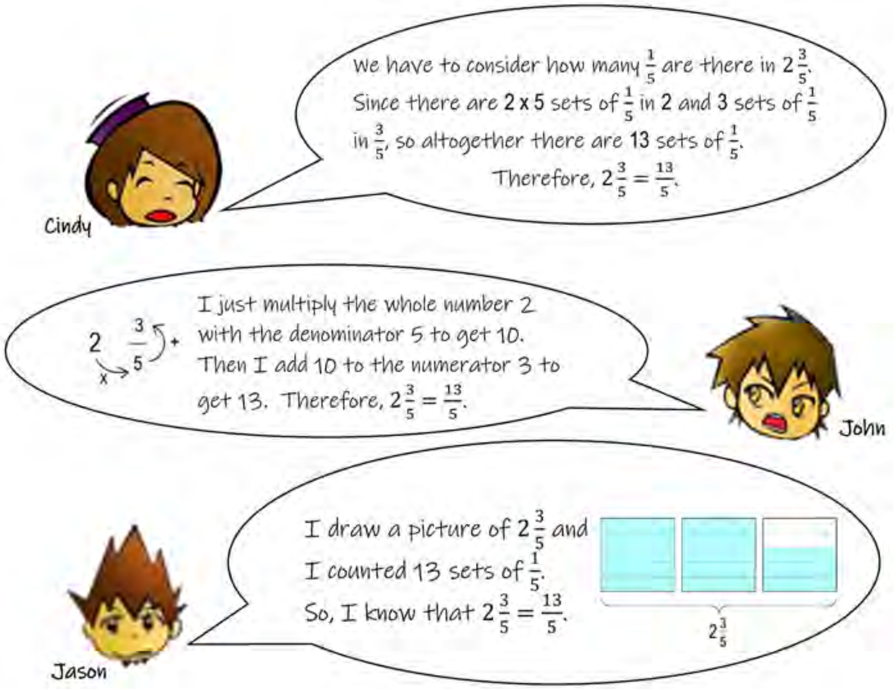
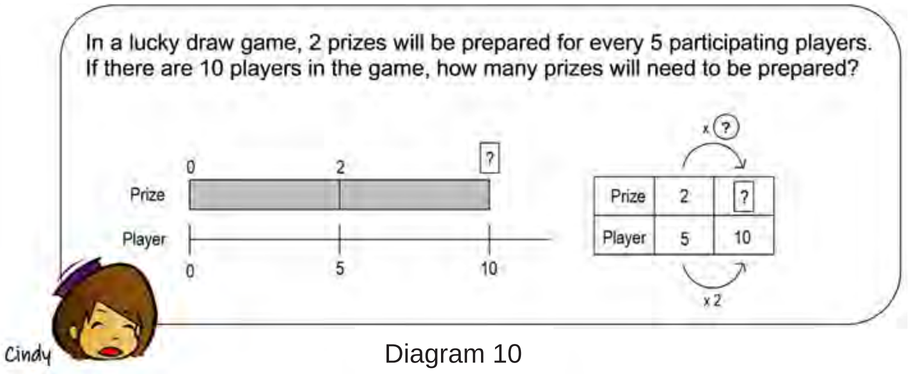


Diagram 9

x. Diagram 9 shows three students' reasoning process when changing $2\frac{3}{5}$ to an improper fraction.

- Compare the three students' reasoning.
  - ❖ Why is John's method possible?
  - ❖ How is John's method related to Cindy's method?
  - ❖ How is Cindy's method related to Jason's method?
- Which methods will you promote to your students, Cindy's, John's, Jason's or all the three?
  - ❖ Justify your decision.

Task 2: Equivalent Fractions



In a lucky draw game, 2 prizes will be prepared for every 5 participating players. If there are 10 players in the game, how many prizes will need to be prepared?

Prize	2	$\boxed{?}$
Player	5	10

Diagram 10

i. Diagram 10 shows a problem involving a ratio of lucky draw prizes to a number of players. Cindy drew a proportional number line and constructed a table of Rule of Three to find the solution as shown in the diagram.

- Find the missing number in (?) and $\boxed{?}$.
- The ratio of prize to 5 players is $\frac{2}{5}$. What is the ratio of prize to 10 players?
- If there are 45 players participating in the game, draw a proportional number line and construct a table of Rule of Three to find the number of prizes.
- What is the ratio of prize to player for the 45-player case?
- What can you conclude on the ratios of these three cases?
- How are these ratios related to equivalent fractions?

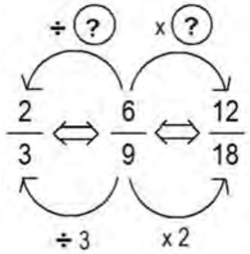


Diagram 11

ii. Diagram 11 shows three equivalent fractions $\frac{2}{3}$, $\frac{6}{9}$ and $\frac{12}{18}$.

- Find the missing number in each (?) .
- Draw a proportional number line to show the relationship between $\frac{2}{3}$ and $\frac{12}{18}$.
- Make a conjecture on methods of finding equivalent fractions.
- Test your conjecture with other fractions.
- Make a conclusion about your conjecture.

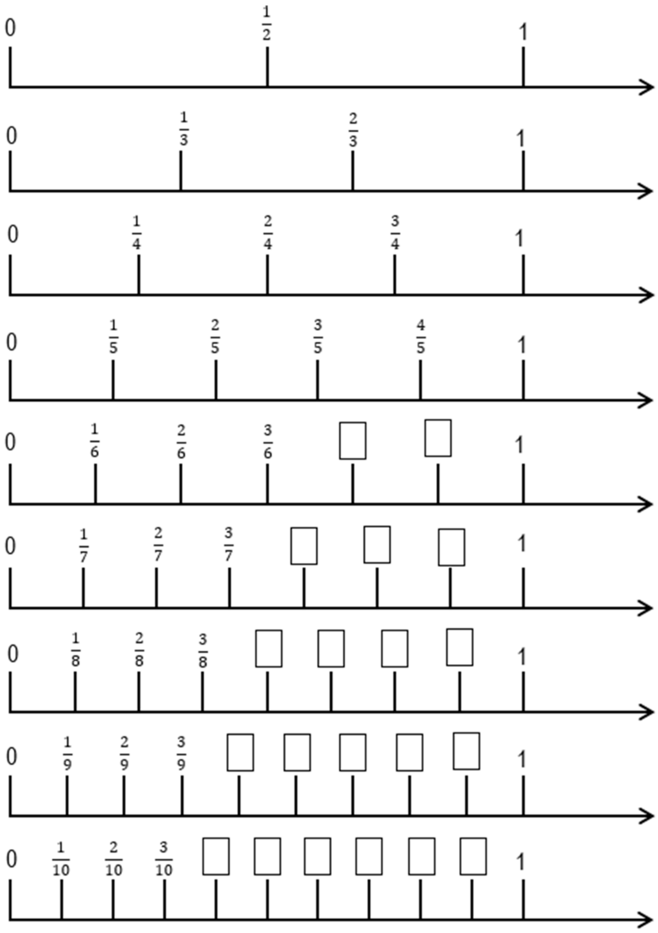


Diagram 12

iii. Diagram 12 shows nine fraction number lines for unit fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$ and $\frac{1}{10}$ respectively.

- Fill in the missing fraction in each ☐.

- Use the number lines to compare the values of the following pairs of fractions.
    - ❖ $\frac{1}{2}$ and $\frac{2}{5}$
    - ❖ $\frac{2}{3}$ and $\frac{5}{8}$
    - ❖ $\frac{3}{4}$ and $\frac{6}{8}$

- Explain how the number lines can be used to find equivalent fractions of $\frac{2}{6}$.

- Use the number lines to find equivalent fractions of
    - ❖ $\frac{1}{2}$
    - ❖ $\frac{6}{8}$

Task 3: Comparing Fractions

i. Pick any two fractions with the same numerator, such as $\frac{3}{5}$ and $\frac{3}{10}$ from the number lines in Diagram 12.

- Compare the values of the two fractions.
- Make a conjecture about the values of fractions with the same numerator.
- Test your conjecture.
- Make a conclusion about your conjecture.
- Justify your conclusion.

ii. Pick any two fractions with the same denominator, such as $\frac{3}{7}$ and $\frac{6}{7}$.

- Compare the values of the two fractions.
- After investigating the values of some pairs of fractions with the same denominator, a student concluded that “when the denominators are the same, a fraction becomes larger as the numerator increases”.
  - ❖ Justify the student’s conclusion.

Diagram 13

iii. Diagram 13 shows some fractions with similar numerator 3.

- Arrange the fractions in order, from least to greatest.

Diagram 14

iv. Diagram 14 shows some fraction with similar denominator 9.

- Arrange the fractions in order, from greatest to least.

Diagram 15

v. Diagram 15 shows three fractions with different numerators and denominators.

- Using common multiples, compare the values of the three fractions in two different ways.

Standards 7.2:

Extending addition and subtraction of similar fractions to improper and mixed fractions, and dissimilar fractions

i. Extend addition and subtraction of similar fractions to proper and mixed fractions with explanations using models and diagrams
ii. Extend addition and subtraction into dissimilar fractions with explanations using diagrams and common divisors
iii. Acquire fluency of addition and subtraction of fractions with appreciation of idea to produce the same denominators

Sample Tasks for Understanding the Standards

Task 1: Addition and Subtraction Involving Mixed Fractions

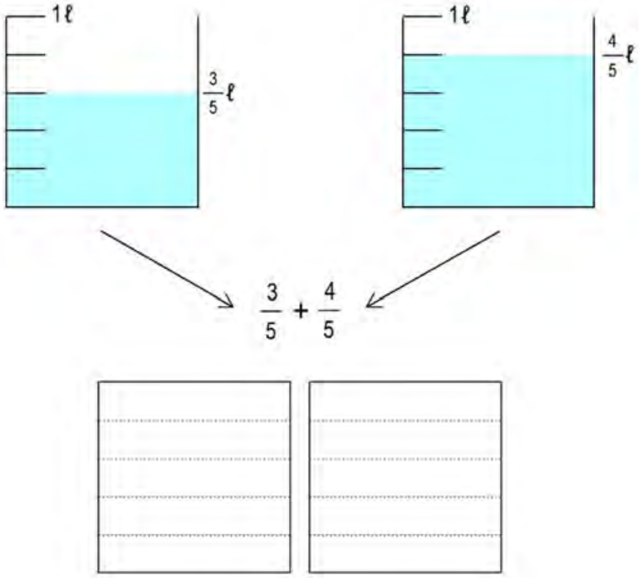


Diagram 1

i. Diagram 1 shows two containers with $\frac{3}{5}\ell$ and $\frac{4}{5}\ell$ of water, respectively.

- Shade on Diagram 1 to show the sum of $\frac{3}{5}\ell+\frac{4}{5}\ell$.
  - ❖ Write the sum as an improper fraction.
  - ❖ Convert the sum to a mixed fraction.

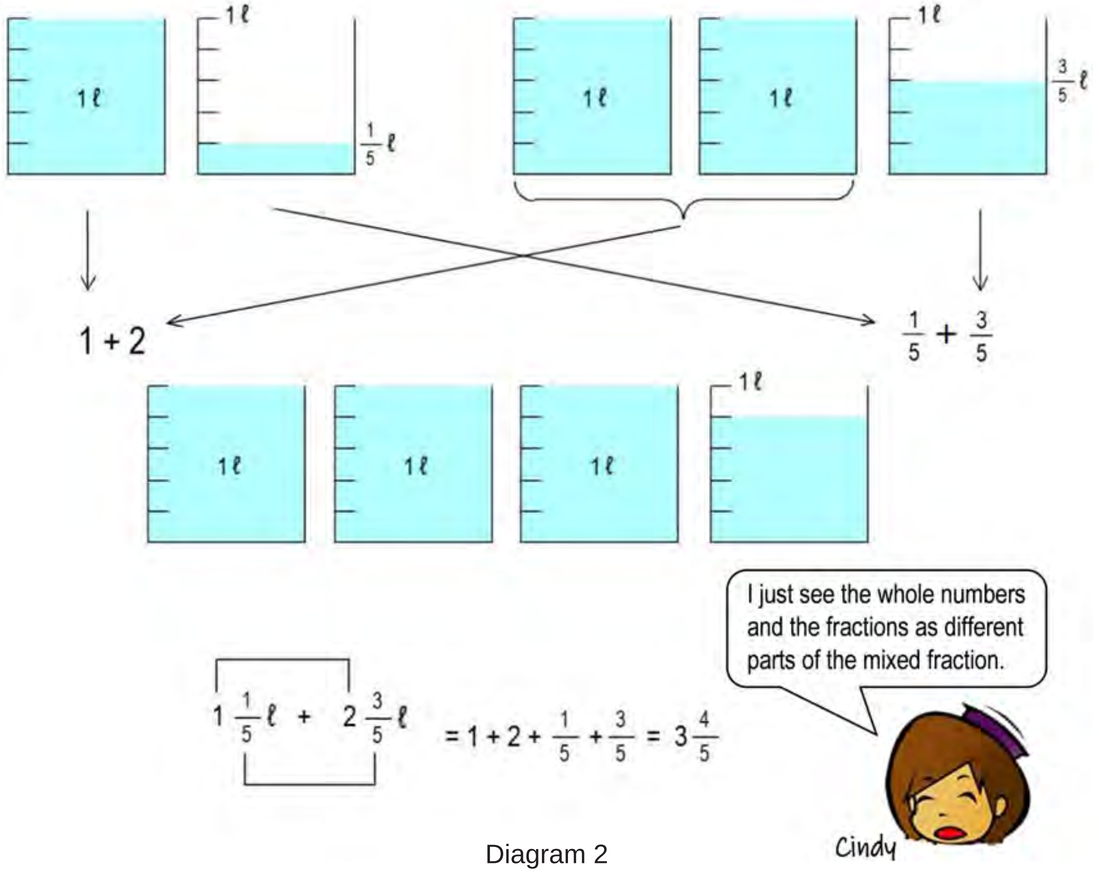


Diagram 2

ii. Diagram 2 shows Cindy's method to add two mixed fractions.

- Explain Cindy's method.
- Use Cindy's method to add the following mixed fractions.
  - ❖ $1\frac{2}{3} + 2\frac{1}{3}$
  - ❖ $7 + 3\frac{5}{6}$
  - ❖ $2\frac{3}{7} + 4\frac{6}{7}$

iii. With the help of a diagram, explain how you will extend Cindy's thinking to do subtraction involving mixed fractions such as $3\frac{4}{5} - 1\frac{3}{5}$.

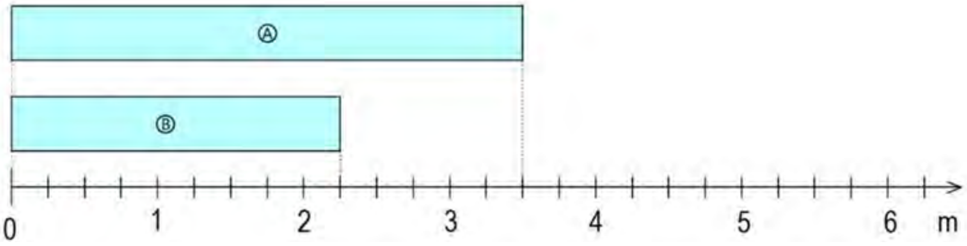


Diagram 3

iv. Diagram 3 shows two tapes, Ⓐ and Ⓑ. Use the number line to answer the following questions:

- What is the total length of the two tapes in metres?
- What is the difference in metres between tape Ⓐ and tape Ⓑ?

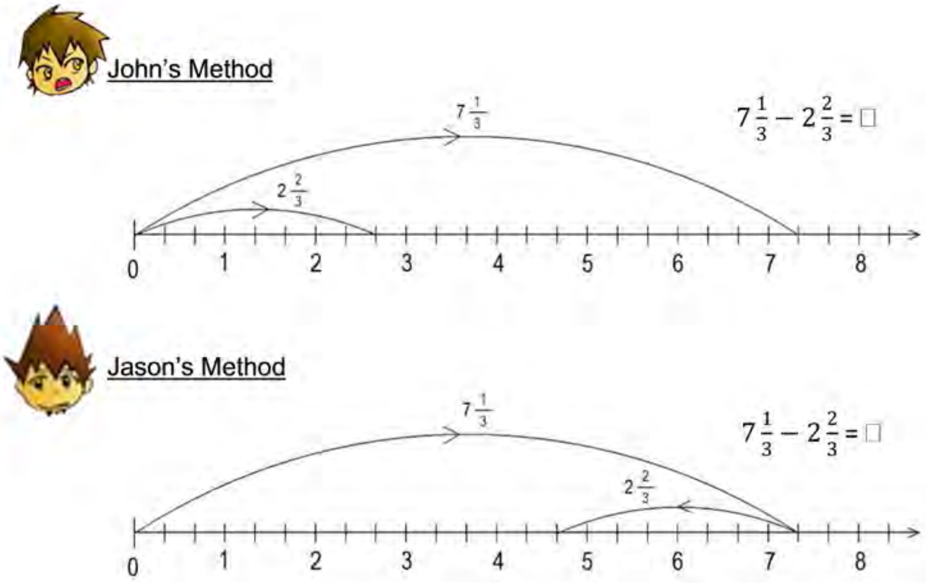


Diagram 4

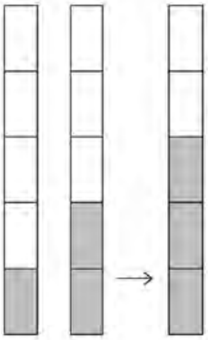
v. Diagram 4 shows John's and Jason's methods to subtract mixed fractions using a number line.

- What is the difference between the two methods?
- Which method will you promote to your students? John's, Jason's or both?
- Justify your decision.

vi. Explain how you use the strategy of "carrying" and "borrowing" to do the following calculations.

- $2\frac{3}{5} + 1\frac{4}{5}$
- $3\frac{2}{7} - \frac{5}{7}$

Task 2: Addition and Subtraction of Dissimilar Fractions



$$\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5}$$

$$= \frac{3}{5}$$

Diagram 5

i. Diagram 5 shows how a student calculated the sum of $\frac{1}{5}$ and $\frac{2}{5}$. The student also drew a diagram as shown to explain his method.

- Referring to the diagram, explain why the denominator of the sum remain unchanged.

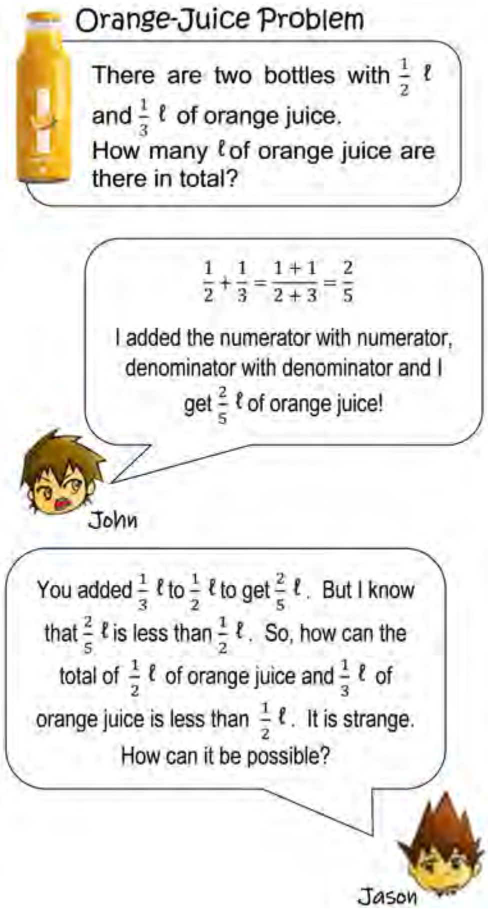


Diagram 6

ii. Diagram 6 shows contradicting ideas of John and Jason regarding the solution of the Orange-Juice problem.

- How will you use the contradicting ideas to promote a dialectic discussion among your students?

iii. John drew a diagram as shown in Diagram 7 to argue for his case.

- Conceptually, what is wrong with John diagram?

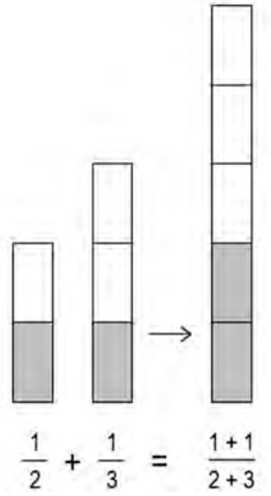


Diagram 7

iv. Jason tried to draw a diagram to support his argument that $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$. However, he got stuck with his drawing as shown in Diagram 8.

- Help Jason to complete the drawing and use it to explain that $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$.

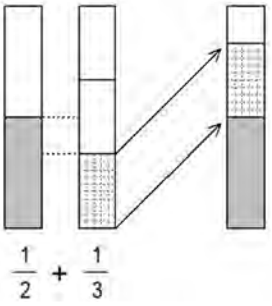


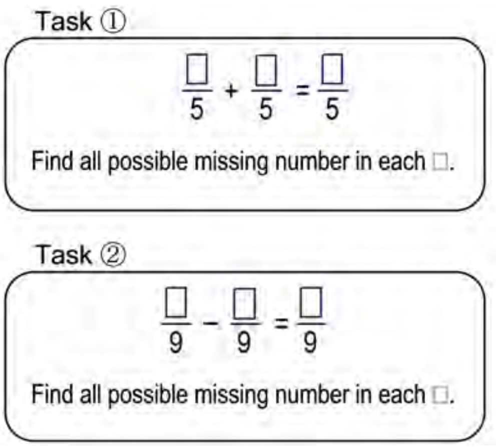
Diagram 8

v. Diagram 9 shows a common error of a student when subtracting fractions.

- What is the misconception of the student?
- With help of a diagram, explain how you will guide the student to correct the error.

$$\boxed{\frac{3}{5}-\frac{1}{2}=\frac{2}{3}}$$

Diagram 9



Task ①

$$\frac{\square}{5}+\frac{\square}{5}=\frac{\square}{5}$$

Find all possible missing number in each □.

Task ②

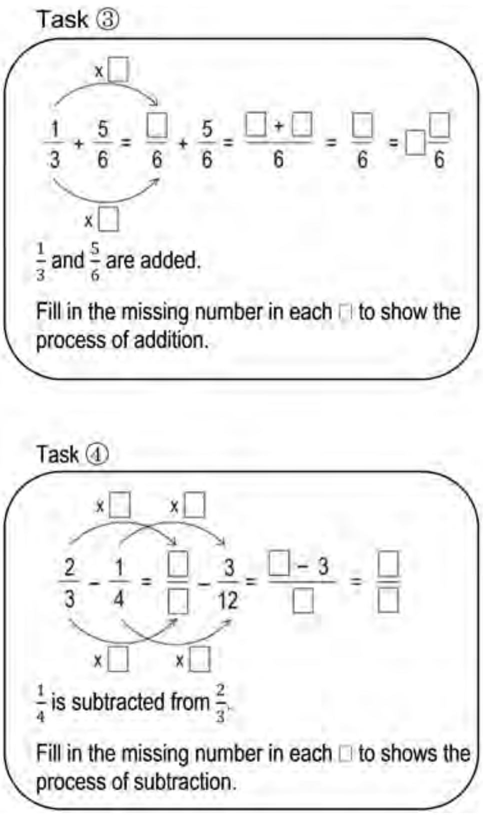
$$\frac{\square}{9}-\frac{\square}{9}=\frac{\square}{9}$$

Find all possible missing number in each □.

Diagram 10

vi. Diagram 10 shows two tasks, ① and ②, to develop fluency of addition and subtraction with similar fractions, respectively.

- Explain the respective rule for adding and subtracting similar fractions.
- Explain how the rules can be extended to addition and subtraction of dissimilar fractions such as $\frac{2}{3}+\frac{1}{4}$ and $\frac{2}{3}-\frac{1}{4}$.



Task ③

$$\frac{1}{3}+\frac{5}{6}=\frac{\square}{6}+\frac{5}{6}=\frac{\square+\square}{6}=\frac{\square}{6}=\square\frac{\square}{6}$$

$\frac{1}{3}$ and $\frac{5}{6}$ are added.

Fill in the missing number in each □ to show the process of addition.

Task ④

$$\frac{2}{3}-\frac{1}{4}=\frac{\square}{\square}-\frac{3}{12}=\frac{\square-3}{\square}=\frac{\square}{\square}$$

$\frac{1}{4}$ is subtracted from $\frac{2}{3}$.

Fill in the missing number in each □ to shows the process of subtraction.

Diagram 11

vii. Diagram 11 shows another two tasks, ③ and ④, to help students understand the respective process of adding and subtracting dissimilar fractions.

- Explain each process.
- How can these two tasks inculcate appreciation towards the idea of common multiples?
- In Task ④, another student used 24 instead of 12 as the common denominator.
  - ❖ Will this student get the same answer? Why or why not?

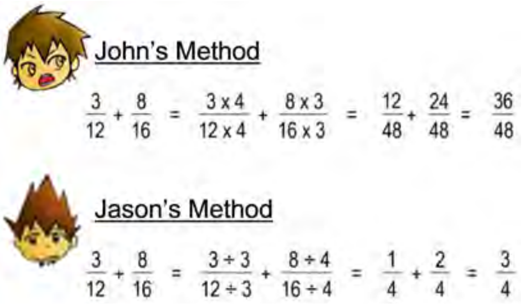


Diagram 12

viii. Diagram 12 shows two students' methods to add $\frac{3}{12}$ and $\frac{8}{16}$.

- Compare the two methods.
- Which method is correct? John's, Jason's or both?
- Justify your decision.

Topic 8: Extending Fractions as Numbers and Integrate

Standards 8.1: Seeing fractions as decimals and seeing decimals as fractions

i. See fractions as decimals using division and define quotient with divisible which includes repetition of the remainder
ii. See decimals as fractions such as hundredths are per hundred
iii. Compare decimals and fractions and order them on the number line

Sample Tasks for Understanding the Standards

Task 1: Fraction as a Quotient

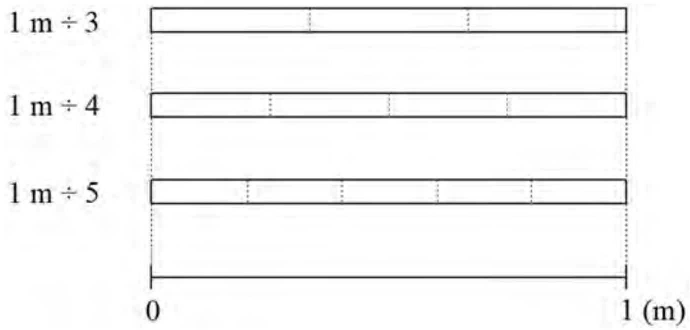


Diagram 1

i. Diagram 1 shows 3 pieces of 1-metre paper strips, folded into 3, 4 and 5 parts respectively.

- Shade on the diagram and find the quotient of (1 m ÷ 3), (1 m ÷ 4) and (1 m ÷ 5) as a fraction, respectively.

- What is the relationship between fraction and division?

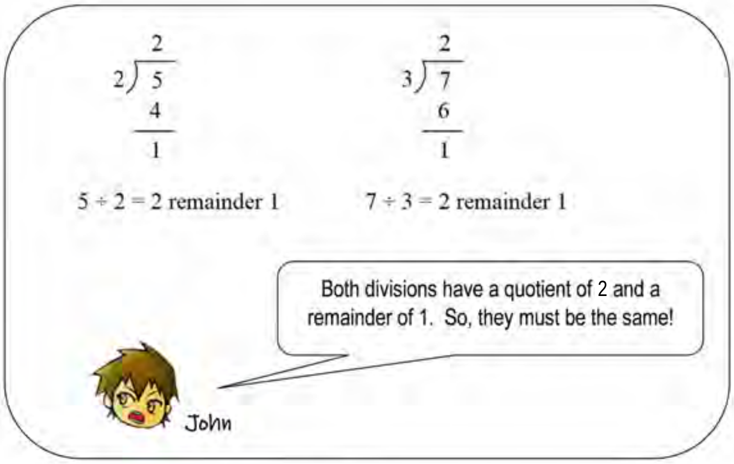


Diagram 2

ii. Diagram 2 shows John's reasoning when comparing the answers of 5 ÷ 2 and 7 ÷ 3. He concluded that 5 ÷ 2 = 7 ÷ 3 since both answers seem to be the same.

- Why is John's reasoning not correct?

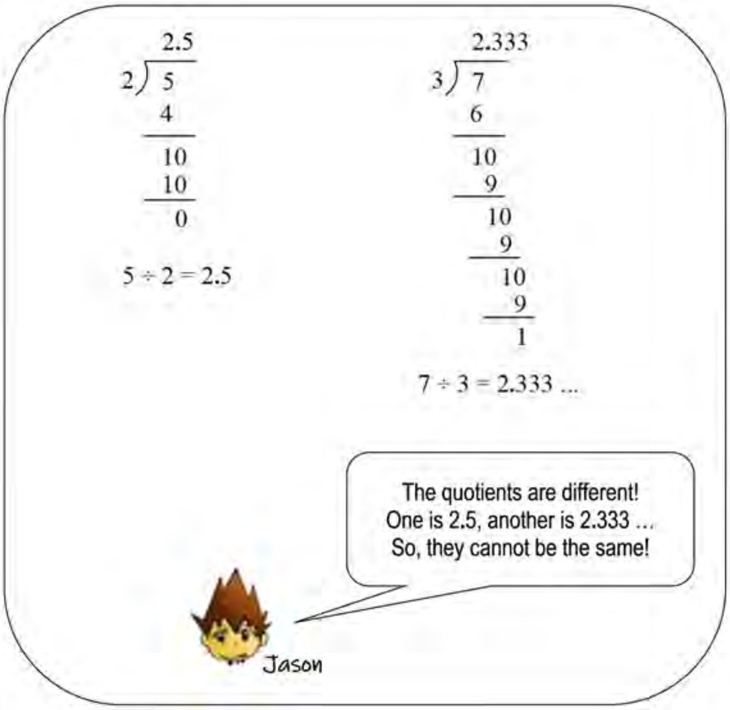
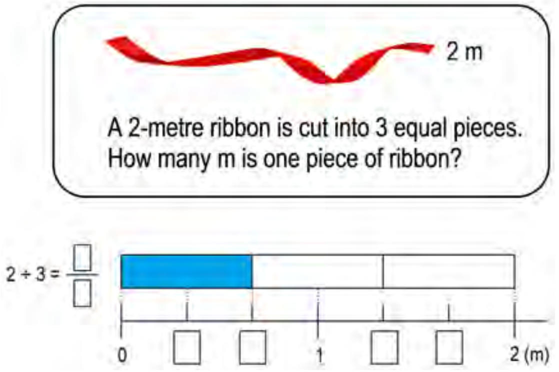


Diagram 3

iii. Another student, Jason argued differently based on his own calculations shown in Diagram 3.

- Explain how Jason’s argument can be used to help John correct his misconception.
- Why is decimal important in division of whole numbers?

Task 2: Expressing Fractions as Decimals



A 2-metre ribbon is cut into 3 equal pieces.
How many m is one piece of ribbon?

Diagram 4

i. Diagram 4 shows a ribbon problem and a student's solution.

- Fill in the missing fraction in each □.
- Convert each of the fractions to a decimal number.

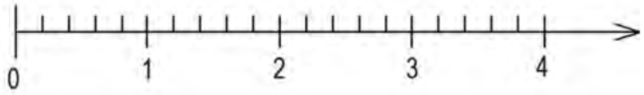


Diagram 5

ii. Diagram 5 shows a number line with unit fraction $\frac{1}{5}$.

- Using the number line, draw a diagram to show that $\frac{4}{5}$ is the quotient of 4 ÷ 5.
- Convert the quotient to a decimal number.

iii. Draw and use a number line to show that $\frac{4}{3}$ is the quotient of 4 ÷ 3.

- Convert the quotient to a decimal number.

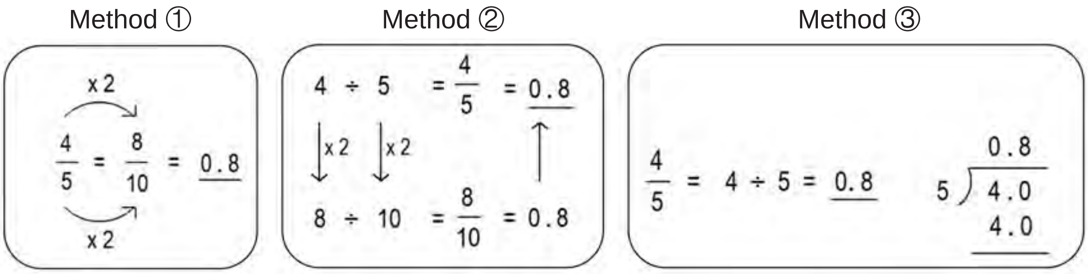


Diagram 6

iv. Diagram 6 shows three students' methods to convert $\frac{4}{5}$ to a decimal number.

- Compare the three methods.
  - ❖ Which method will you promote to your students?
  - ❖ Justify your decision.

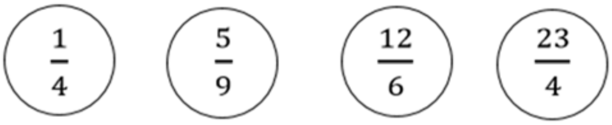


Diagram 7

v. Diagram 7 shows four fractions.

- Express each fraction as a decimal or whole number.
- Sort the fractions into three groups.

Ⓐ Whole number

Ⓑ Accurate decimal number

Ⓒ Repeating decimal number

Task 3: Expressing Decimals as Fractions

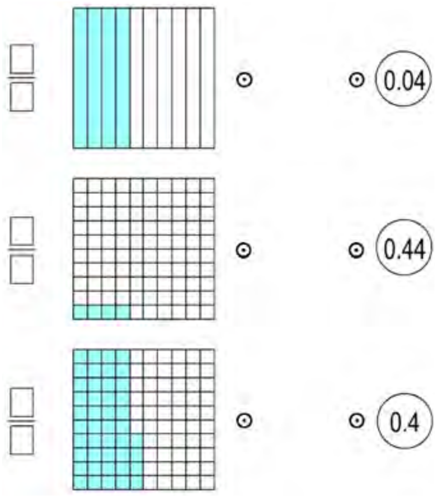


Diagram 8

i. How is a decimal related to a fraction?

ii. Diagram 8 shows three squares each representing 1. The shaded region of each square represents a fraction of the whole square.

- How many sets of $\frac{1}{10}$ are there in 1?
- How many sets of $\frac{1}{100}$ are there in 1?
- Fill in the missing number in each □ to show the fractions.
- Match each fraction with its corresponding decimal number.

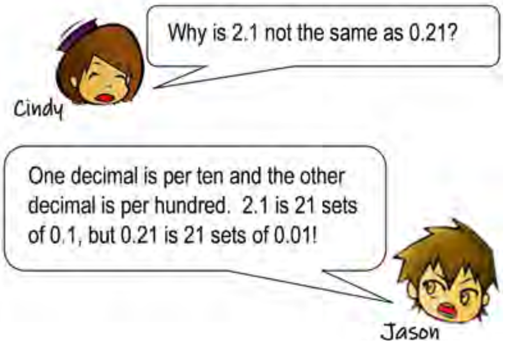


Diagram 9

iii. Jason tried to explain why 2.1 is not the same as 0.21 to Cindy as shown in Diagram 9.

- Explain Jason's way of thinking.
- How can Jason's way of thinking be used to compare the values of 2.1 and 0.21?

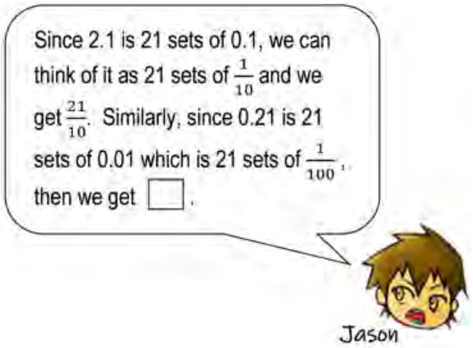


Diagram 10

iv. Diagram 10 shows Jason's way of converting 2.1 and 0.21 to fractions.

- Fill in the missing fraction in □.
- Will you recommend Jason's way of thinking to your students? Why or why not?

v. Express the decimal numbers as fractions.

- 2.8
- 2.08
- 2.80

Task 3: Ordering Fractions and Decimals

Which is longer, $\frac{3}{5}$ m or 0.5m?

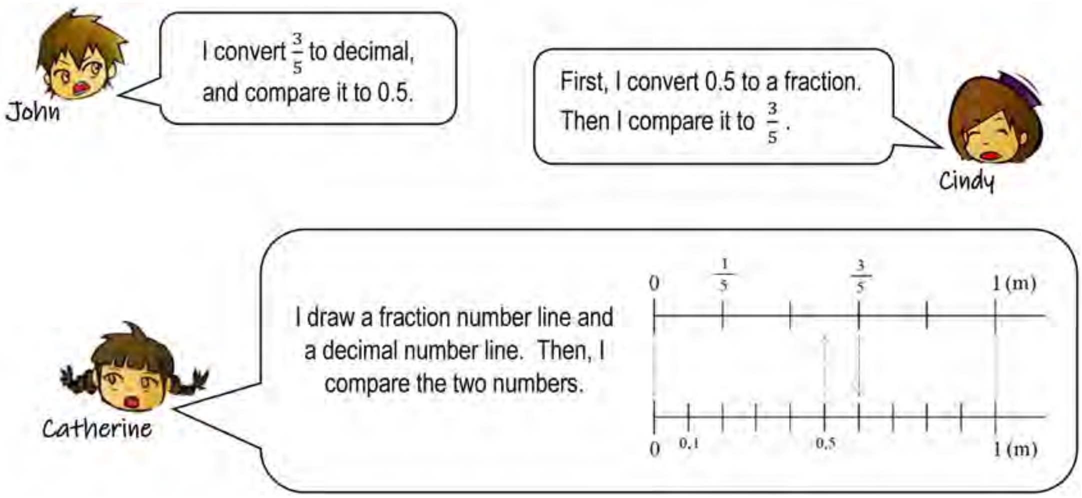


Diagram 11

i. Diagram 11 shows a problem about comparing a fraction with a decimal number and three students' ideas to find the answer.

- Which idea is easier to do the comparison?
- Justify your choice.

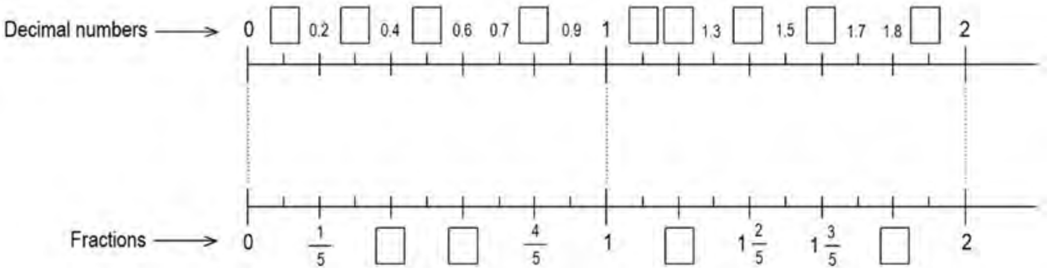


Diagram 12

ii. Diagram 12 shows a fraction number line and a decimal number line.

- Fill in the missing number in each □.
- Use the number lines to compare the values of
  - ❖ 0.4 and $\frac{1}{5}$
  - ❖ 0.8 and $\frac{4}{5}$
  - ❖ 1.3 and $1\frac{2}{5}$

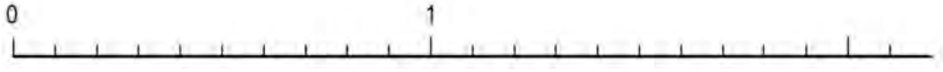


Diagram 13

iii. Mark with a ↓ on the number line in Diagram 13 to show each of the following numbers.

$\frac{1}{5}$ 0.7 $1\frac{9}{20}$ 2 1.75 $\frac{3}{4}$ $\frac{2}{3}$

iv. Arrange the following numbers in order from least to largest.

1.2 0.25 $\frac{6}{3}$ $1\frac{1}{2}$ $\frac{7}{10}$ $\frac{5}{6}$

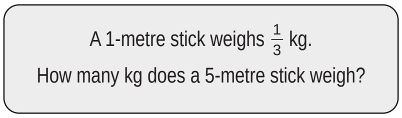
Topic 9: Extending Multiplication and Division to Fractions

Standards 9.1: Extending multiplication to fractions

i. Extend the multiplication to fractions with situations using diagrams such as number lines, step by step, and find the simple algorithm for the multiplication of fractions
ii. Acquire fluency with the multiplication of fractions
iii. Develop number sense of multiplication of fractions such as comparing sizes of products before multiplying

Sample Tasks for Understanding the Standards

Task 1: Multiplication of Fractions



A 1-metre stick weighs $\frac{1}{3}$ kg.
How many kg does a 5-metre stick weigh?

Diagram 1

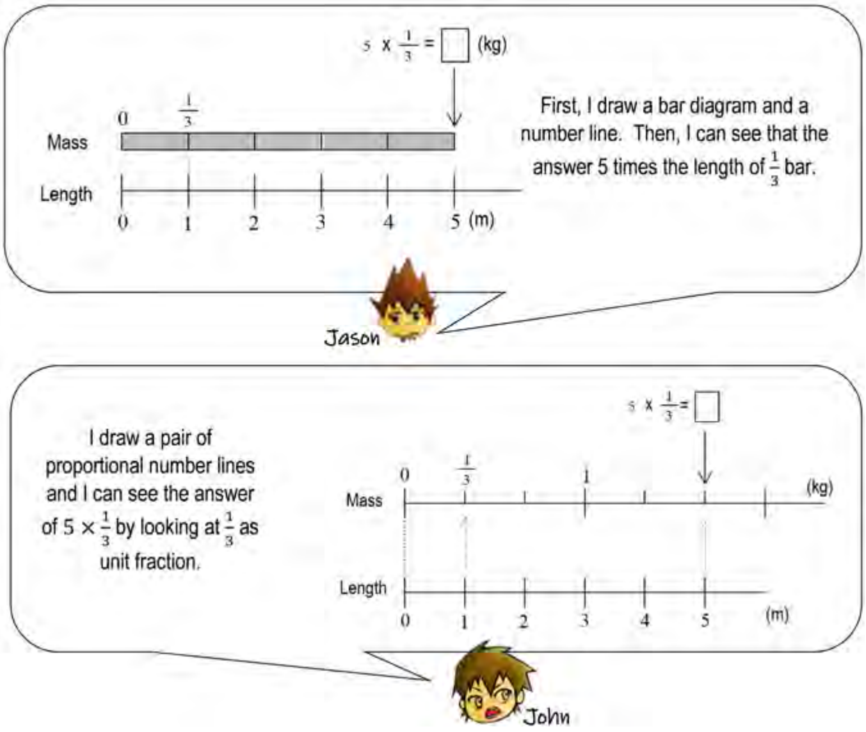


Diagram 2

i. Diagram 1 shows a problem involving the mass and length of a stick. Diagram 2 shows John's and Jason's methods to solve the problem.

- Compare the two methods.
  - ❖ What is the crucial learned knowledge required to use each of the methods?
  - ❖ Find the missing number in ☐.

Diagram 3

ii. Another student, Catherine, found the answer using a table of Rule of Three as shown in Diagram 3.

- Explain Catherine's method.

iii. Use each of the three methods to calculate $3\times\frac{2}{5}$.

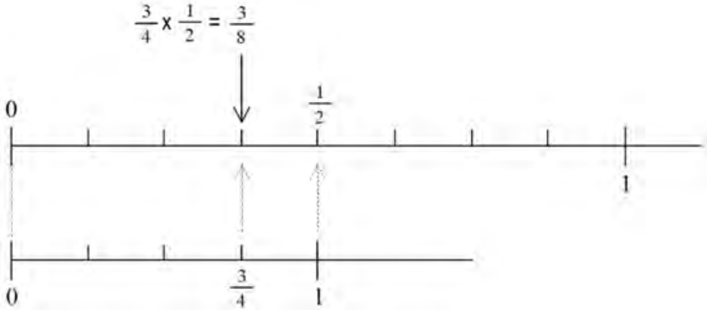


Diagram 4

iv. Diagram 4 shows John's method using two proportional number lines to calculate $\frac{3}{4}\times\frac{1}{2}$.

- Using the diagram, explain why $\frac{3}{4}\times\frac{1}{2}=\frac{3}{8}$.

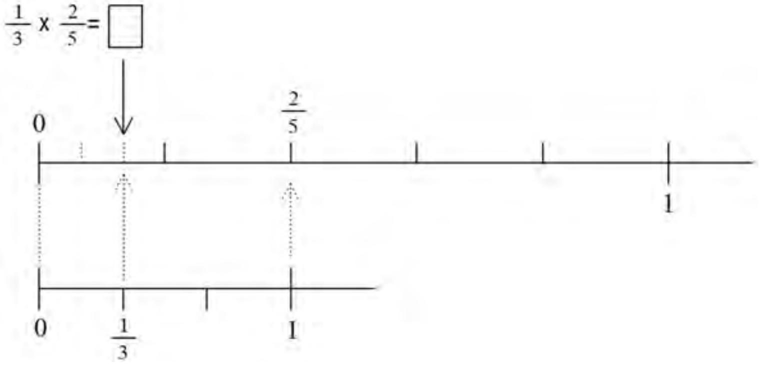


Diagram 5

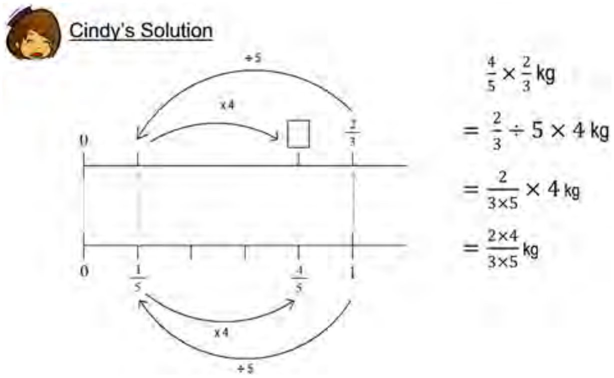
v. Diagram 5 shows John's method to calculate $\frac{1}{3}\times\frac{2}{5}$.

- Explain how the diagram can be used to find the missing number in □.

Iron Rod Problem

The mass of an iron rod is $\frac{2}{3}$ kg per metre.

How many kg will a $\frac{4}{5}$ m long iron rod weigh?

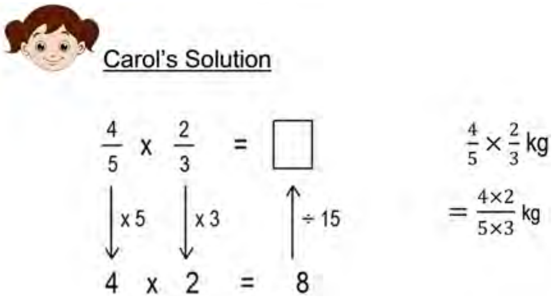


$$\frac{4}{5} \times \frac{2}{3} \text{ kg}$$
$$= \frac{2}{3} \div 5 \times 4 \text{ kg}$$
$$= \frac{2}{3\times5} \times 4 \text{ kg}$$
$$= \frac{2\times4}{3\times5} \text{ kg}$$

Diagram 6

vi. Diagram 6 shows a problem involving the mass of an iron rod and Cindy's solution to the problem.

- Find the missing number in □.
- Compare the proportional number lines drawn by Cindy to that drawn by John in Diagram 2.
    - ❖ What are the similarities and differences between the two ways of using the proportional number lines?
    - ❖ What are the strengths and weaknesses of each way?



$$\frac{4}{5} \text{ x } \frac{2}{3} = \square$$
$$4 \text{ x } 2 = 8$$

$$\frac{4}{5} \times \frac{2}{3} \text{ kg}$$
$$= \frac{4\times2}{5\times3} \text{ kg}$$

Diagram 7

vii. Diagram 7 shows Carol's solution to the Iron Rod Problem.

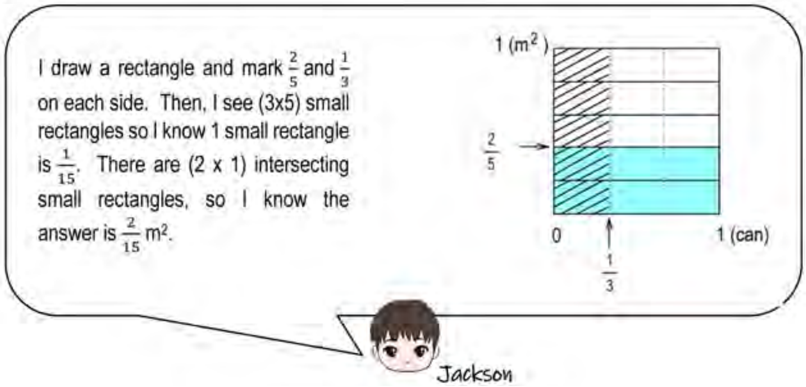
- Fill in the missing number in □.
- Explain Carol's method of finding the solution.
- Between Cindy's and Carol's methods, which will you promote to your students? Justify your decision.

Printable Area Problem

A can of paint will cover $\frac{2}{5}$ m^2 of a wall.

What area in m^2 will $\frac{1}{3}$ can of this paint cover?

Jason's Solution



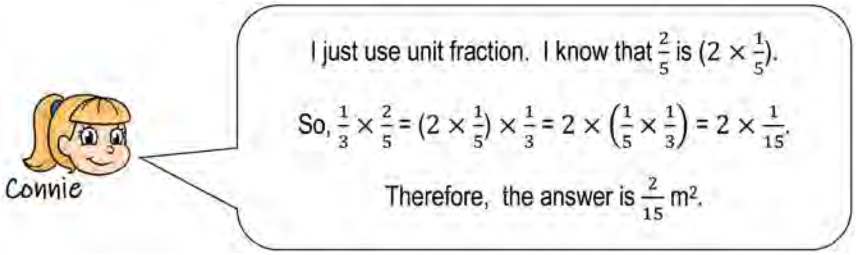
I draw a rectangle and mark $\frac{2}{5}$ and $\frac{1}{3}$ on each side. Then, I see (3x5) small rectangles so I know 1 small rectangle is $\frac{1}{15}$. There are (2 x 1) intersecting small rectangles, so I know the answer is $\frac{2}{15}$ m^2.

Diagram 8

viii. Diagram 8 shows a problem involving a paintable area on a wall and Jackson's solution for the problem.

- Explain Jackson's method of solving the problem.
- Write a mathematical expression for the problem.
- Compare Jackson's method of multiplying fractions with John's and Cindy's methods.
  - ❖ What is the main mathematical idea supporting each method?
  - ❖ Which method will you promote to your students? Why?

[Note. It is important to ensure the two fractions are represented by two sides of a square instead of a rectangle. Otherwise, the shaded regions of the area diagram may be a major source of misconceptions if students recognise fractions only by the part-whole relationship.]



I just use unit fraction. I know that $\frac{2}{5}$ is $(2 \times \frac{1}{5})$.

So, $\frac{1}{3} \times \frac{2}{5} = (2 \times \frac{1}{5}) \times \frac{1}{3} = 2 \times \left(\frac{1}{5} \times \frac{1}{3}\right) = 2 \times \frac{1}{15}$.

Therefore, the answer is $\frac{2}{15}$ m^2.

Diagram 9

ix. Diagram 9 shows Connie's method of calculating $\frac{1}{3} \times \frac{2}{5}$.

- Compare Connie's method with Jackson's method.
  - ❖ What are the similarities and differences between the two methods?
  - ❖ Which method will you recommended to your students? Why?

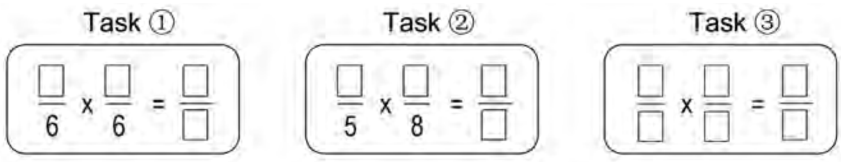


Diagram 10

x. Diagram 10 shows three open-ended tasks for multiplication of fractions.

- Find as many solutions as possible for the three tasks.
- How do these tasks help your students acquire fluency with multiplication of fractions?

xi. Calculate.

- $5\times \frac{2}{3}$
- $\frac{2}{5}\times\frac{3}{4}$
- $2\frac{3}{5}\times\frac{1}{7}$

Task 2: Comparing Products of Fractions

i. Justify each statement without calculate the products.

- $\frac{2}{3}\times\frac{5}{7} > \frac{1}{3}\times\frac{5}{7}$
- $\frac{1}{4}\times\frac{7}{2} < \frac{1}{2}\times\frac{35}{7}$

ii. Which multiplication has a product that is less than 8?

- $8\times\frac{3}{7}$ $\quad$ $8\times(1\frac{2}{5})$ $\quad$ $\frac{9}{5}\times 8$
- Justify each answer without calculate the product.
- Calculate each product to verify your answer.

Standards 9.2: Extending division to fraction

i. Extend the division to fraction with situations using diagrams such as number lines, step by step
ii. Acquire fluency with the division of fractions
iii. Develop number sense of division of fractions such as comparing sizes of quotients before dividing

Sample Tasks for Understanding the Standards

Task 1: Modeling Division of Fraction

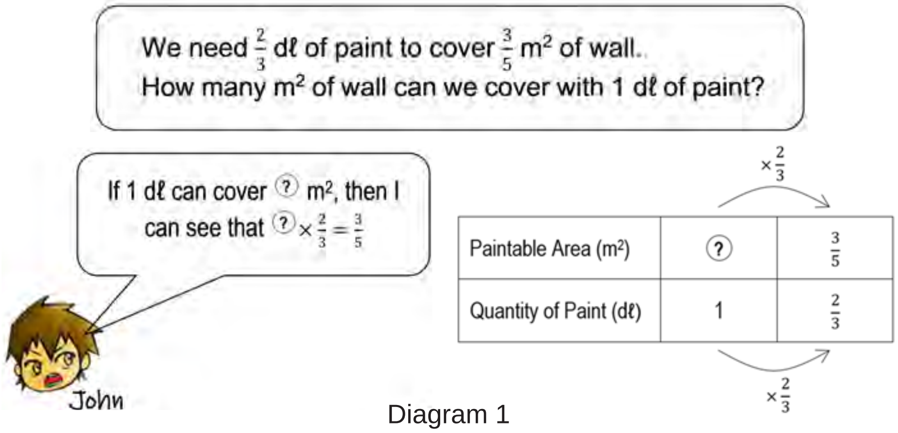


Diagram 1

i. Diagram 1 shows a problem and a Rule-of-Three table constructed by John to solve the problem.

- Explain why the solution to the problem is $\frac{3}{5} \div \frac{2}{3}$ (m²).
- Find the missing number in ?.

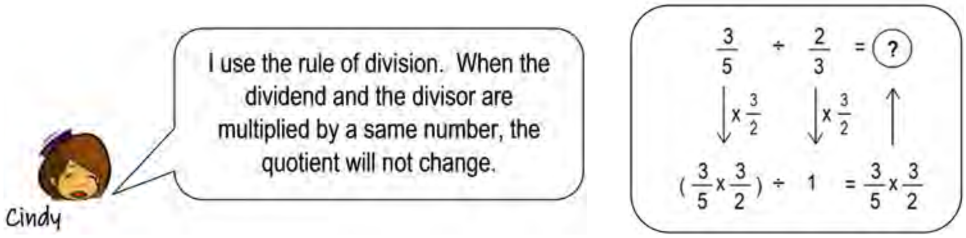


Diagram 2

ii. Diagram 2 shows Cindy's method to calculate $\frac{3}{5} \div \frac{2}{3}$.

- Find the missing number in ? using Cindy's method.
- What are the strengths of Cindy' method?
- Use Cindy's method to calculate $\frac{2}{7} \div \frac{3}{5}$.

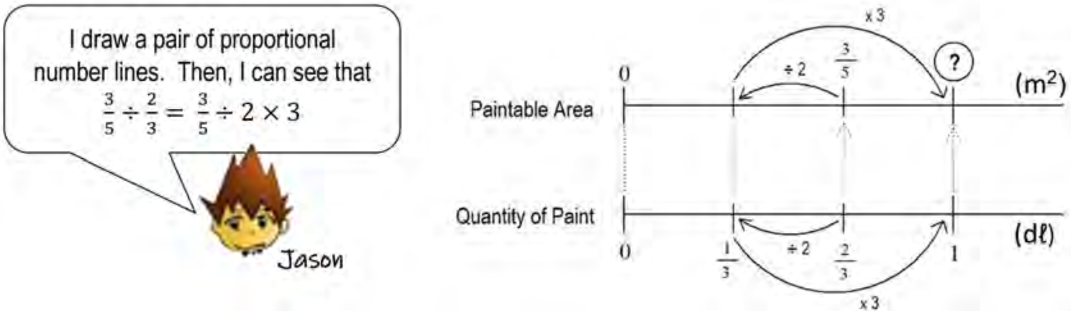


Diagram 3

iii. Diagram 3 shows Jason's method to calculate $\frac{3}{5} \div \frac{2}{3}$.

- Explain Jason's method.
- Find the missing number in ? using Jason's method.
- Use Jason's method to calculate $\frac{3}{4} \div \frac{7}{3}$.

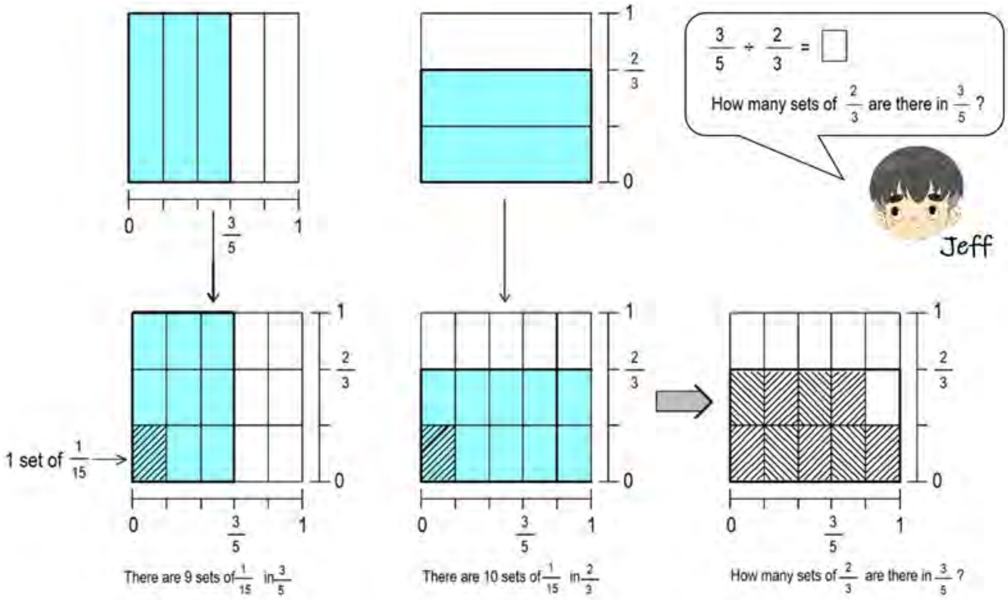


Diagram 4

iv. Diagram 4 shows Jeff's method to calculate $\frac{3}{5} \div \frac{2}{3}$. The procedure is based on the repeated-subtraction model of division.

- Explain the division procedure.
- Find the missing number in □.
- Use the same method to find $\frac{3}{4} \div \frac{1}{2}$.

Task 2: Fluency with Division of Fractions

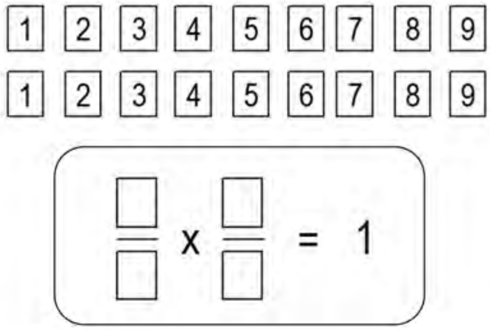


Diagram 5

i. Diagram 5 shows 18 digit-cards and a problem involving the multiplication of two fractions to get a product of 1.

- Pick and arrange any four cards to find as many solutions as possible for the problem.
- What rule is there between multiplicand and multiplier to make the product 1?

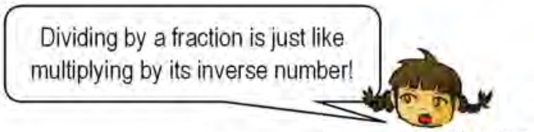


Diagram 6

ii. Catherine made a claim about the division of fractions as shown in Diagram 6.

- What is the inverse number of a fraction?
- Use Catherine's method to calculate
  - ❖ $\frac{3}{4} \div \frac{2}{5}$
  - ❖ $\frac{1}{5} \div \frac{2}{3}$

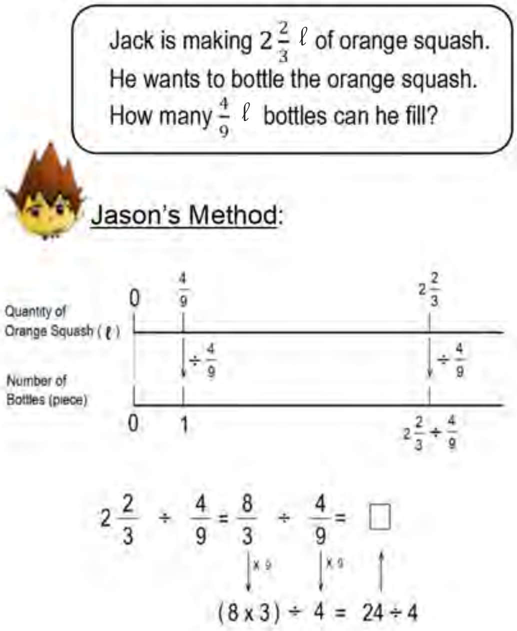


Diagram 7

iii. Diagram 7 shows a problem on bottling of orange squash and Jason's method in solving the problem.

- Explain Jason's method.
- Find the missing number in □.
- What rule of division did Jason use to calculate $\frac{8}{3} \div \frac{4}{9}$?
- Find $\frac{1}{5} \div \frac{2}{3}$ using Jason's method.
- What are the strengths and weaknesses of Jason's method?

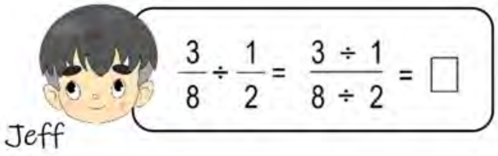


Diagram 8

iv. Diagram 8 shows a student's idea in dividing two fractions.

- Find the missing number in □.
- Make a conjecture base on Jeff's idea.
- Is your conjecture always true, sometime true, or never true?
- Make a conclusion on your conjecture.
- Find $\frac{1}{5} \div \frac{2}{3}$ using Jeff's method.

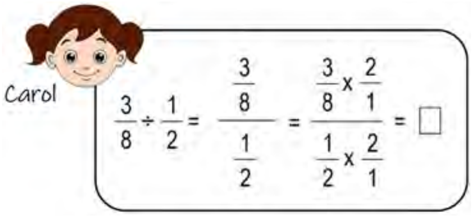


Diagram 9

v. Diagram 9 shows another student's idea in calculating the same division of fractions.

- Find the missing number in □.
- Explain Carol's method.
- Find $\frac{1}{5} \div \frac{2}{3}$ using Carol's method.

vi. How will you use Catherine's, Jason's, Jeff's or Carol's ideas to help your students acquire fluency with division of fractions?

vii. Calculate

- $\frac{3}{7} \div \frac{4}{5}$
- $8 \div \frac{3}{4}$
- $\frac{2}{3} \div 5$

viii. Which value is larger?

- $\frac{3}{4} \div \frac{2}{3}$ or $\frac{3}{4} \div 1\frac{1}{2}$
- $\frac{4}{5} \div \frac{3}{2}$ or $\frac{4}{5} \div \frac{5}{6}$
- Complete the following statements:
  - ❖ If the divisor is smaller than 1, then the quotient is __________ than the dividend.
  - ❖ If the divisor is larger than 1, then the quotient is __________ than the dividend."

ix. A bottle contains $1\frac{3}{5}\ell$ of mango juice. If you pour $\frac{2}{3}\ell$ of juice into a cup, how many cup of juice can you get?

x. The area of a rectangular board is $2\frac{4}{5}$ m² and its length is $1\frac{1}{5}$ m. Find its width in metres.

CHAPTER 3

Measurement and Relations

Topic 1: Introducing Angle and Measuring it

Standards 1.1:

Introducing angle by rotation, enabling measure and acquire fluency using the protractor

i. Compare the extent of rotation and introduce degree as a unit for measuring an angle
ii. Recognise right angle is 90 degrees, and adjacent angle of two right angles is 180 degrees, and 4 right angles are 360 degrees
iii. Acquire fluency in measuring angles using the protractor
iv. Draw equivalent angles with addition and subtraction using multiples of 90 degrees
v. Appreciate measurement of angles in geometrical shapes and situations in life

Sample Tasks for Understanding the Standards

Task 1: Introducing Angles and Unit for Measuring Angle

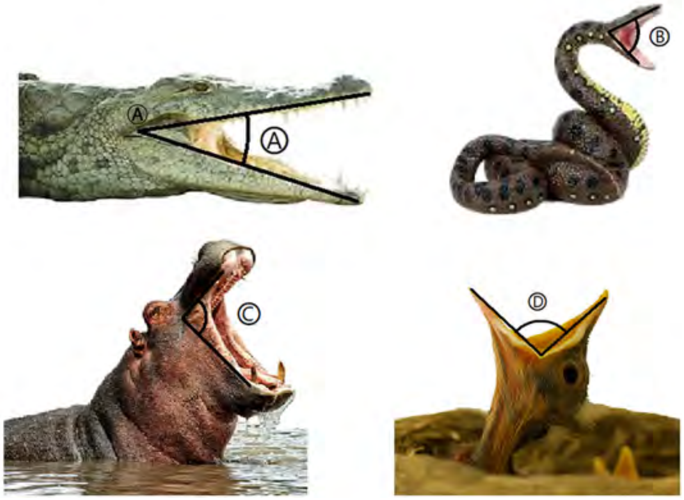


Diagram 1

i. Diagram 1 shows some animals that can open their mouths very wide.

- Which animal can open its mouth the widest?
- How do you compare how wide the mouths are opened?
  - ❖ Is the wideness same as the angle?
- Arrange the size of angles Ⓐ,Ⓑ,Ⓒ,and Ⓓ in ascending order.
- How do you make the comparison without any measuring tool?
- Explain the meaning of angle by rotation.

Diagram 2

ii. Diagram 2(a) shows a 2-arm plastic stick joined at a vertex. One arm rotates in anti-clockwise direction to form angle ⓐ as shown by the circular arrow in Diagram 2(b).

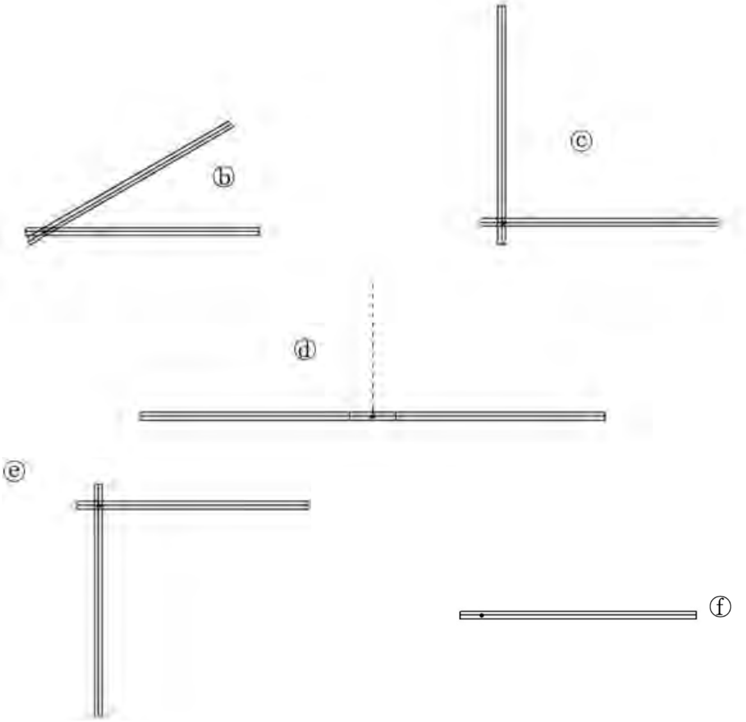


Diagram 3

Diagram 3 shows five other angles formed by rotating the arm in anti-clockwise direction.

- Draw a circular arrow to show each of the angles on Diagram 3.
- If ⓒ forms a right angle, how many right angles are there in, ⓓ, ⓔ, and ⓕ?
- For angle ⓕ, the arm rotates and making one complete revolution back to the original position.
  - ❖ What is the difference in the size of the angles formed in ⓕ and the original position?
- Which of the angles ⓐ to ⓔ show the angle of a half revolution?

iii. Angle of one revolution is divided into 360 equal parts. The size of one part is one degree and is a unit to measure the size of angles.

- State the following angles in degree.
  - 1 right angle
  - 2 right angles
  - 3 right angles
  - 4 right angles
- Historically, why is one revolution divided into 360 instead of 100 or 10?

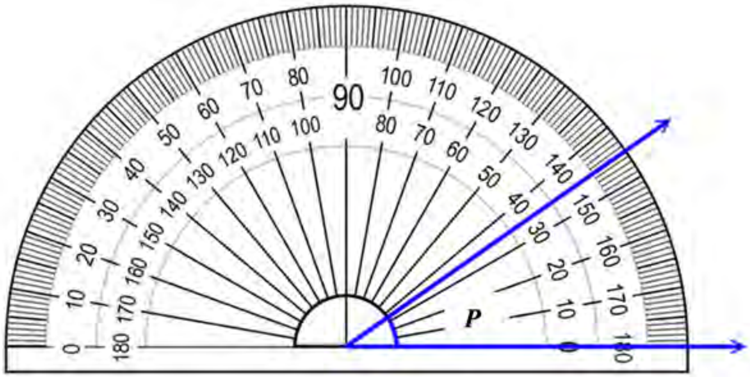


Diagram 4

iv. Diagram 4 shows a protractor for measuring angles. Students A, B and C read angle *P* as 35°, 145° and 155°, respectively.

- Discuss the three readings of the students.
- Use a circular arrow to mark 145° on Diagram 4.
- What is the relationship between 145° and angle *P* ?

Task 2: Measuring Angle

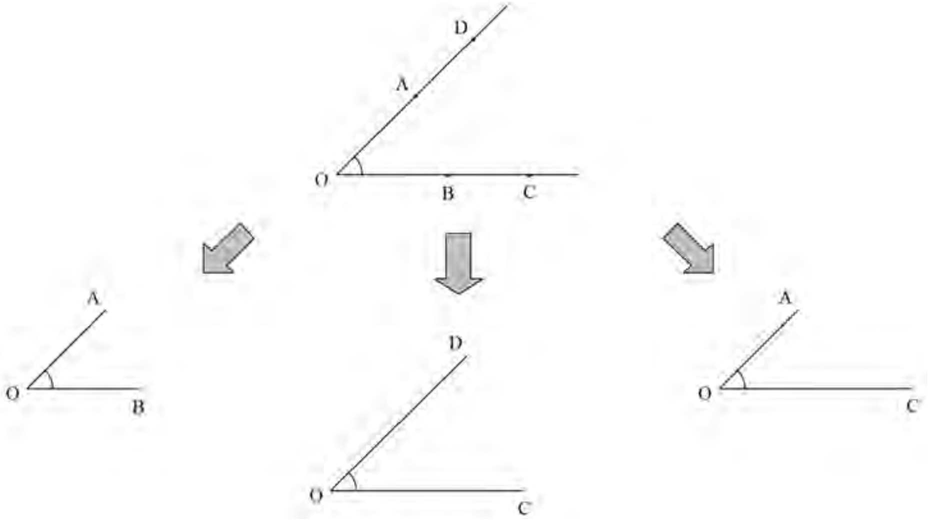


Diagram 5

i. Diagram 5 shows two sides of an angle extended from point O. A, B, C and D are 4 points on the two sides.

   Angle ∠AOB is formed by line segments OA and OB, angle ∠COD is formed by line segments OC and OD and ∠AOC is formed by line segment OA and OC.

   - A student claims that both ∠AOB and ∠AOC are smaller than ∠COD because they look smaller.

     ❖ Suggest and explain how you will help the student to overcome the misconception.

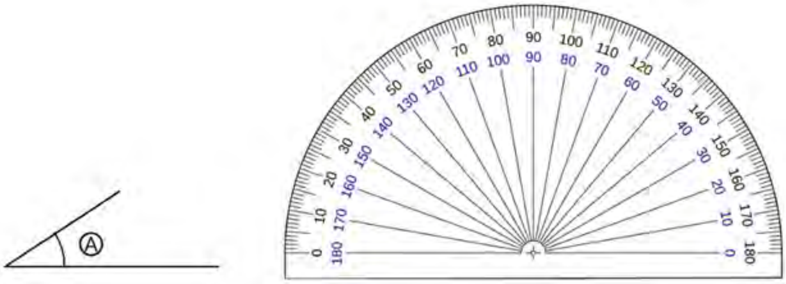


Diagram 6

ii. Diagram 6 shows an angle Ⓐ and a protractor. When a student, Wijaya was asked to measure the angle using the protractor, he was confused and claimed that the angle was too small to be measured.

   - Explain how your suggestion in question (i) is applied to help Wijaya overcome his confusion.

Diagram 7

iii. Diagram 7 shows two angles, Ⓑ and Ⓒ which are larger than 180°.

- What difficulties do you anticipate from your students if they are asked to measure these angles using the protractor in Diagram 6.
- Explain how the protractor is used to measure these two angles.

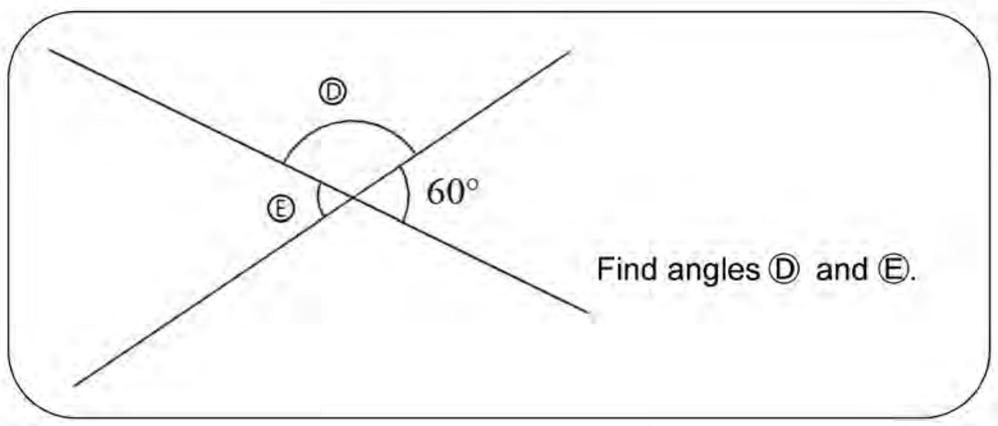


Diagram 8

iv. Diagram 8 shows a task involving angles formed by two intersecting straight lines.

- What is the learned knowledge required to do this task?
- How can this task be used to enhance your students' mathematical reasoning?
- What is the next task related to angles that you will give to your students?

❖ What mathematical contents do you expect the students to acquire from the task?

❖ Justify your answer.

Task 3: Angles in Daily-life Situations

Diagram 9

i. Diagram 9 shows a camping hut which has a unique geometrical shape with two slanting roofs and a floor base.

   Angles of large structure such as the angle between a slanting roof and the floor base of this camping hut are usually difficult to measure directly.

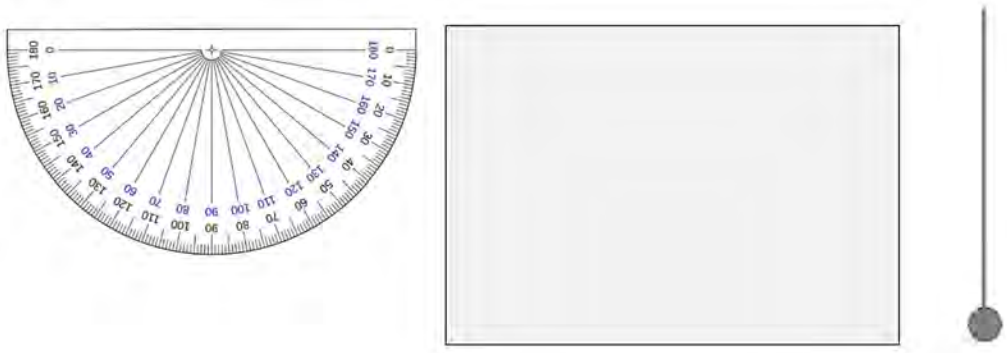


Diagram 10

Diagram 10 shows a protractor, a cardboard and a pendulum.

- Discuss how you will use these materials to design a tool that can measure the angle of large structure in an easier way.

- Discuss how this task can incalcate your students' appreciation of the usefulness of mathematics.

Topic 2: Exploring and Utilising Constant Relation

Standards 2.1:

Exploring equal constant relation with utilisation of letters to represent placeholders

i. Explore two possible unknown numbers such that their sum (or difference/ product/quotient) is constant, for example □+Δ=12 (□ and Δ are placeholders)
ii. Use letters instead of placeholder (empty box) to derive equivalent relation
iii. Understand the laws for operations (e.g. associative, commutative and distributive, etc.) to explain the simpler way of calculation
iv. Appreciate the use of diagrams such as number lines and area to represent relation when finding solutions

Sample Tasks for Understanding the Standards

Task1: Sorting Out the Possible Numbers

Diagram 1

i. Diagram 1 shows cupcakes with two different flavours. Erika plans to buy some cupcakes.

Let □ represents the number of chocolate cupcakes and ○ represents the number of vanilla cupcakes bought by Erika.

- Explain the meaning of the expression □ + ○ = 8.

- List all the possible values of □ and ○ using a suitable strategy.

Diagram 2

ii. Let ◊ represents the number of cupcakes and each piece of the cupcake costs 9 baht.

- Write an expression to represent the cost of all the cupcakes.

- Diagram 2 shows another box of cupcakes bought by Erika. Find the cost of this box of cupcakes.

iii. Instead of using symbols such as □, ○, and ◊, letters can also be used to represent the situation.

- Suggest a letter to replace □, ○, ◊ each.

- Use the letters to show the relationship between □, ○ and ◊.

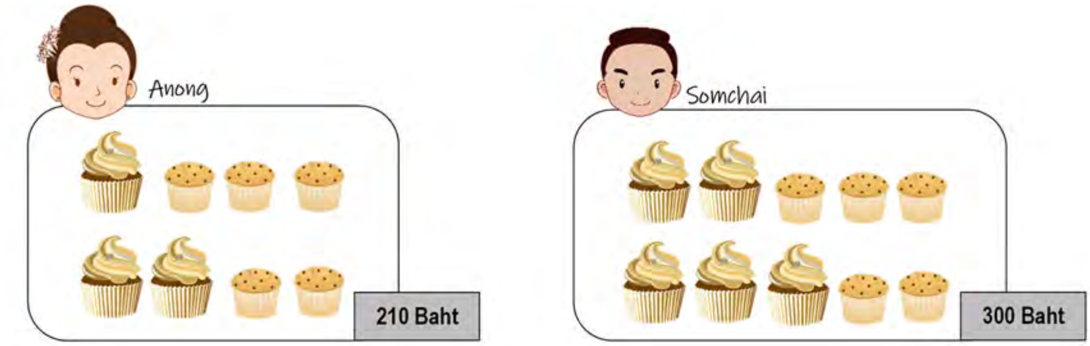


Diagram 3

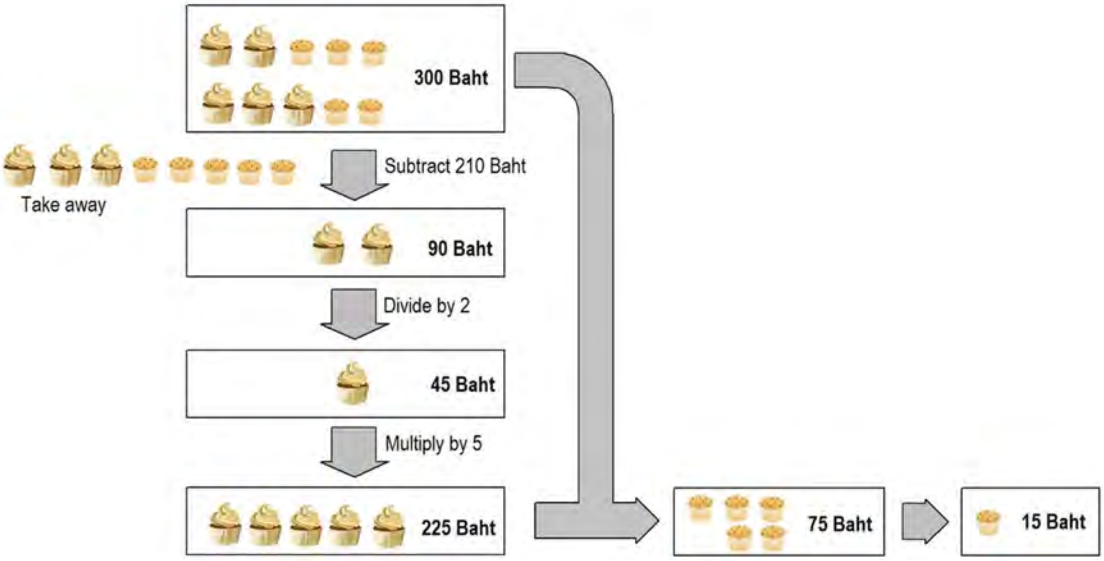


Diagram 4

iv. On New Year day, a bakery put on a sale for the cupcakes. Customers can pick any number of cupcakes. Diagram 3 shows the boxes of cupcake bought by Anong and Somchai and the price of each box.

Diagram 4 shows a student's reasoning process in finding the price of 1 chocolate and 1 vanilla cupcakes.

- Explain the student's reasoning.
- To what extend will this reasoning process help your students learn solving equations?

Diagram 5

v. A bakery in Hatyai sells cupcakes in two boxes as shown in Diagram 5.

- If □ and ○ represents the price of 1 chocolate cupcake and 1 vanilla cupcake, respectively, write an equation base on each of the boxes.
- Explain how the idea of substitute values for the symbols □ and ○ can be used to find the price of
  - ❖ 1 chocolate cupcake.
  - ❖ 1 vanilla cupcake.

Task 2: Laws of Operations

Diagram 6

Diagram 7

i. Diagram 6 shows a box of cupcakes, P, with vanilla and chocolate flavour. Let letter c represents the number of chocolate cupcakes and letter v the number of vanilla cupcakes.

- Explain how box P can be used to explain commutative law for $c + v$.
- Diagram 7 shows another box of cupcakes, Q. Based on this box, a student claims that $c - v = v - c$.
  - ❖ What is the possible cause of the student's misconception?
  - ❖ What can you do to help the student correct his misconception?

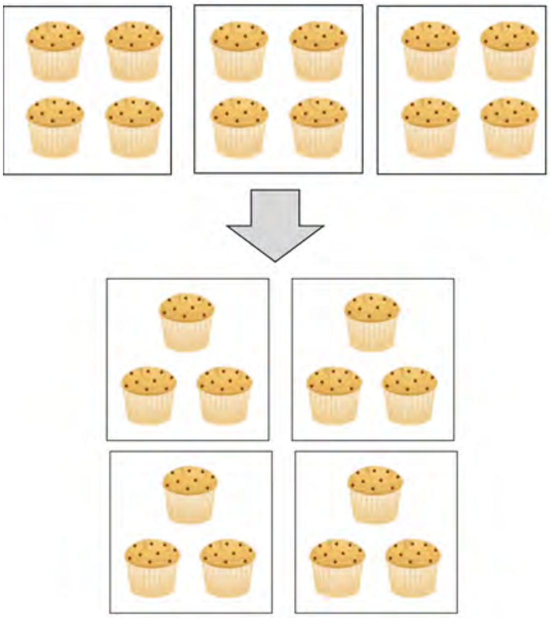


Diagram 8

ii. Diagram 8 shows 3 boxes of vanilla cupcakes repackage into 4 boxes. Let letter b represents the number of boxes and letter v the number of vanilla cupcakes.

- Explain how the situation can be used to explain commutative law for $b \times v$.

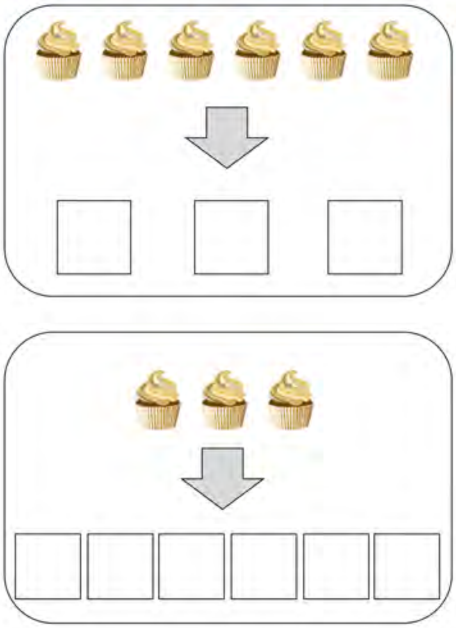


Diagram 9

iii. Diagram 9 shows two situations involving packaging chocolate cupcakes into boxes. Let letter b represents the number boxes and letter c the number of chocolate cupcakes.

- Explain how you will use the situations to help your students learn that the commutative law does not apply for $c \div b$.

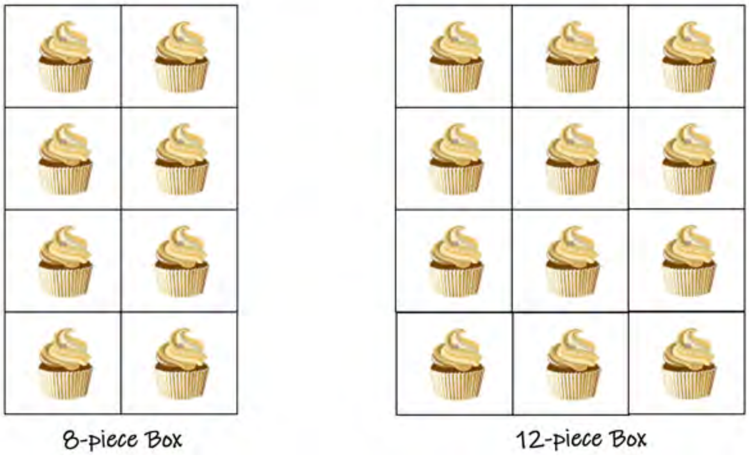


Diagram 10

iv. Diagram 10 shows an 8-piece box and a 12-piece box that contain only chocolate cupcakes. A bakery is offering a “Buy-one-free-one” promotion with buying a 12-piece box and get an 8-piece box for free.

- Mr Krit buys p 12-piece boxes during the promotion. How many cupcakes will he get in total?

- Ms Nisakorn buy some boxes of cupcakes. If she get a total of 60 cupcakes, what is the value of p?

- Explain how the distributive law can be used to simplify the calculation.

Task 3: Problem Solving

Diagram 11

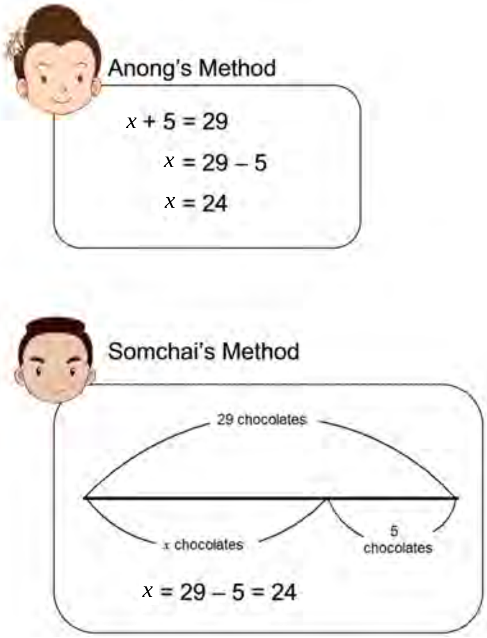


Diagram 12

i. Diagram 11 shows a problem involving a gift box filled with chocolates and another 5 pieces outside the box.

- Solve the Chocolate Problem.
- Diagram 12 shows Anong's and Somchai's methods to solve the problem.
  - ❖ Explain each of the method.
  - ❖ Is your method of solving the same as or different from the two students' methods?
  - ❖ Which of the methods will you promote to your students? Justify your choice.

Topic 3: Extending Measurement of Area in Relation to Perimeter

Standards 3.1:

Introducing area and produce formula for the area of rectangle

i. Compare extent of area and introduce its unit, and distinguish it from perimeter
ii. Introduce one square centimetre (cm^2) as unit for area and its operation using addition and subtraction
iii. Investigate area of rectangles and squares and produce the formula of area
iv. Extend square centimetre to square metre and to square kilometre for measure of large areas
v. Convert units and use appropriate units of area with fluency
vi. Draw the equivalent size of a rectangular area based on a given area with the factors of a whole number
vii. Appreciate the use of areas in daily life such as comparing land sizes

Sample Tasks for Understanding the Standards

Task1: Area and Perimeter

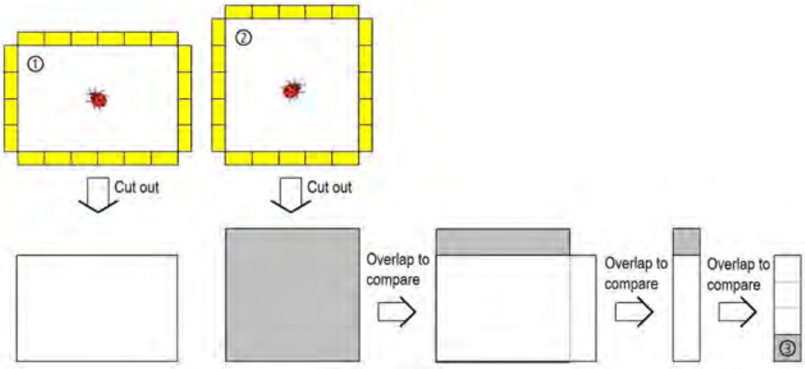


Diagram 1

i. Ms Apinya was making rectangular enclosures for ladybugs to move around using 1 cm-length plastic blocks. Diagram 1 shows two enclosures, ① and ②, each made using 20 pieces of the plastic blocks. Ms Apinya then cut out the two enclosed spaces. When she tried to compare the two enclosed spaces, she found two extra regions, one longer and another shorter. She then repeated the comparison process with the two extra regions until she finally got a square, ③, that fitted nicely onto the shorter extra region. The space of square ③ is 1 square centimetre.

- How many square centimetres is
  - ❖ the shorter extra region?
  - ❖ the longer extra region?
  - ❖ enclosure ①?
  - ❖ enclosure ②?
- Which enclosure is larger, ① or ②?

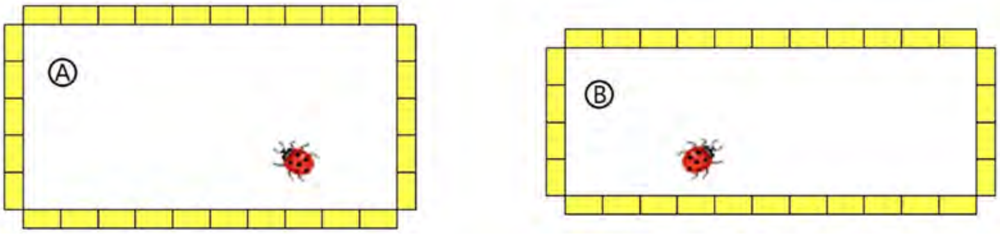


Diagram 2

ii. Ms Apinya gave 30 pieces of 1 cm-length plastic blocks for each of her students to make a rectangular enclosure. Two students made the shapes, Ⓐ and Ⓑ, as shown in Diagram 2.

- The two students agreed that the enclosed space in enclosures Ⓐ and Ⓑ are the same because both rectangles were made from the same number of plastic blocks.
  - ❖ Explain how direct comparison can be done to help the students rectify their misconception.

iii. The space 1 square cm, commonly written as 1 cm² is a unit used to measure area.

- Explain the meaning of 1 cm².
- Explain how you will guide your students to find the area in cm², of rectangles Ⓐ and Ⓑ, respectively
- Find the area, in cm², of rectangles Ⓐ and Ⓑ, respectively.

iv. What is the difference between area and perimeter?

v. Investigate other possible rectangles formed by the 30 pieces of 1 cm-length plastic blocks.

- What dimensions of rectangle will give the minimum area?
- What dimensions of rectangle will give the maximum area?
- Make a conjecture on the relationship between the dimensions of a rectangle to its area.
- Verify your conjecture with other perimeters such as 38 cm.
  - ❖ What if you use a 38-cm string instead of the plastic blocks?
- Make a conclusion on your conjecture.

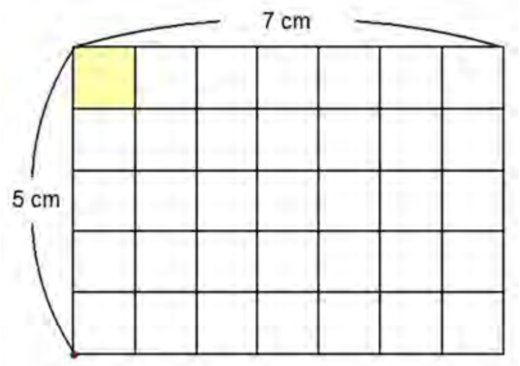


Diagram 3

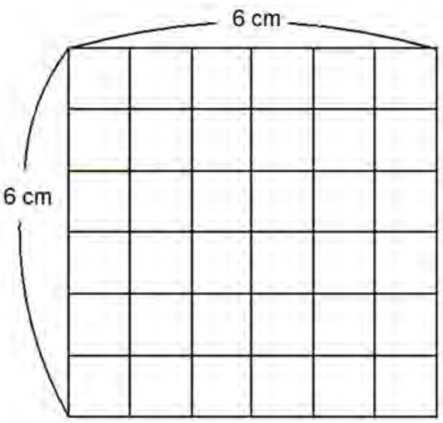


Diagram 4

vi. Diagram 3 shows a rectangle with square grids.

- One square grid is shaded.
  - ❖ What is the area of this square grid in cm²?
- How many square grids are there altogether?
  - ❖ How is this number of square grids related to the area of the rectangle?
  - ❖ What mathematical operation will enable you to find the number of square grids directly? Explain your answer.
- Make a conjecture on a formula to find the area of a rectangle.
- Verify your conjecture and make a conclusion.
- Does your conclusion also work for a square such as Diagram 4. Why or why not?

Task 2: Adding and Subtracting Areas

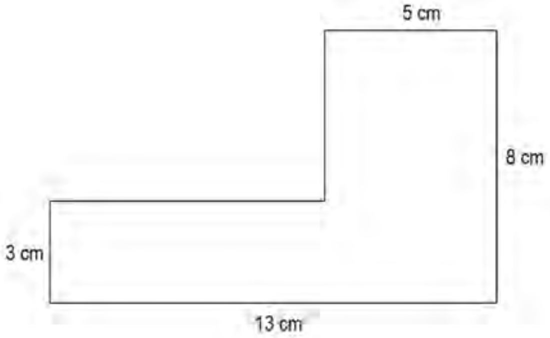


Diagram 5

i. Diagram 5 shows a composite shape. Four students, John, Jason, Cindy and Catherine find the area of the shape with different methods as shown in Diagram 6.

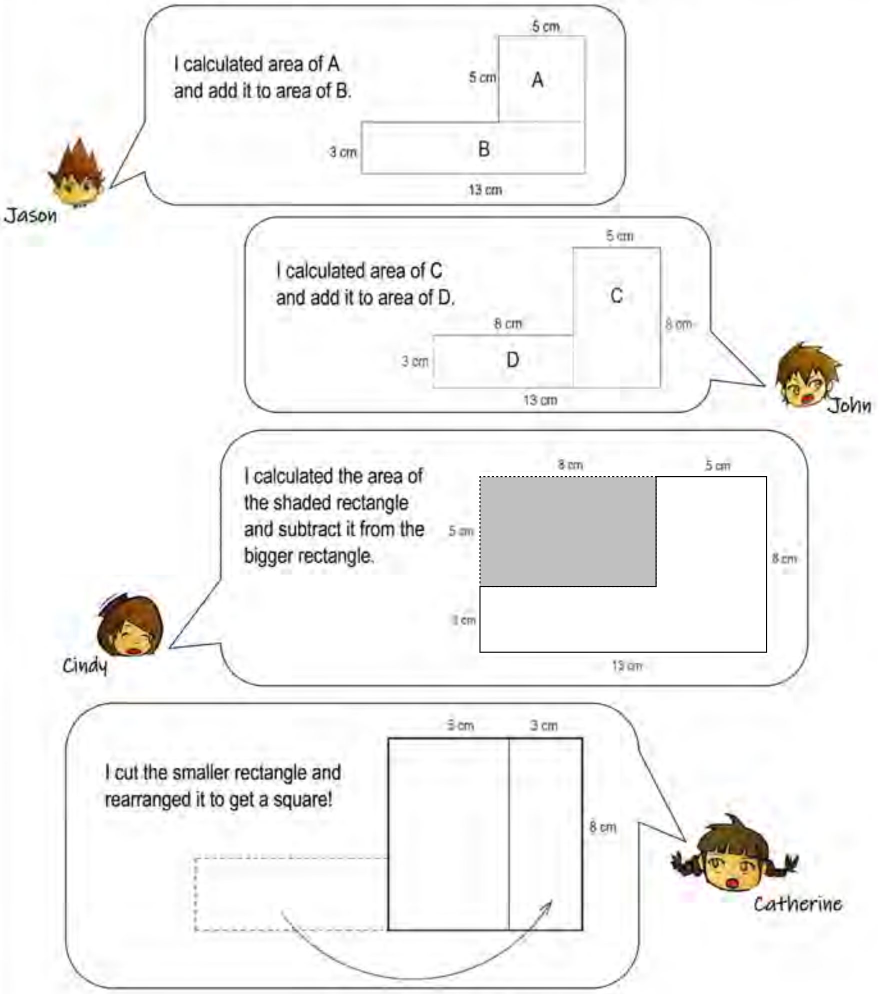


Diagram 6

- The main idea used in Jason's and John's methods is the same. What is the main idea?
- What is the main idea used in Cindy's method?
- Catherine explained that "when I rearrange the shapes, the total area is still the same."
  - ❖ Explain what does Catherine mean.

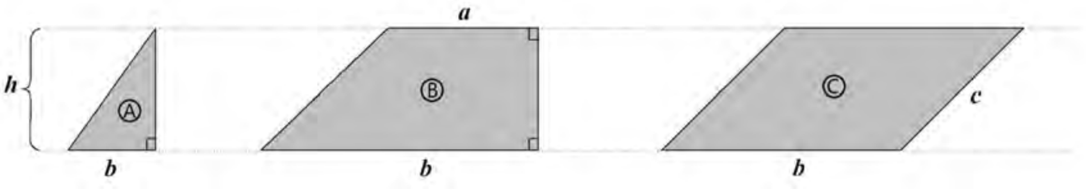


Diagram 7

ii. Explain how the main ideas in John's, Jason's, Cindy's and Catherine's methods can be used to develop formula of finding the area of triangle Ⓐ, trapezium Ⓑ, and parallelogram Ⓒ in Diagram 7.

- Find the formula for finding the area of triangle Ⓐ, trapezium Ⓑ, and parallelogram Ⓒ.

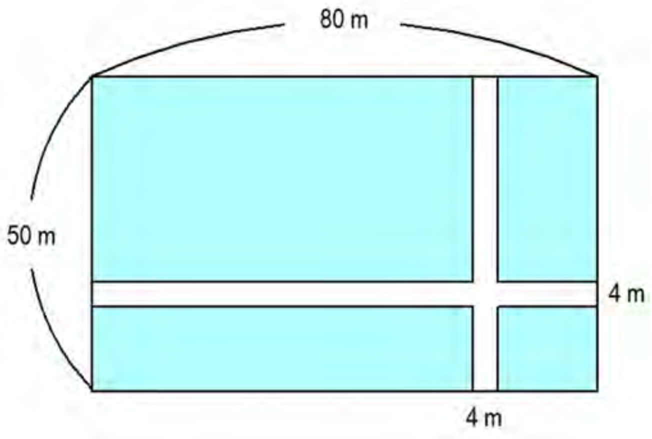


Diagram 8

iii. Diagram 8 shows a piece of rectangular land with two crossed paths. Each path is 4 m wide.

- Calculate the area of the shaded region.
- One student claimed that the area can be calculated using (46 m x 76 m). Explain how this calculation is possible.

Task 3: Larger Units of Area

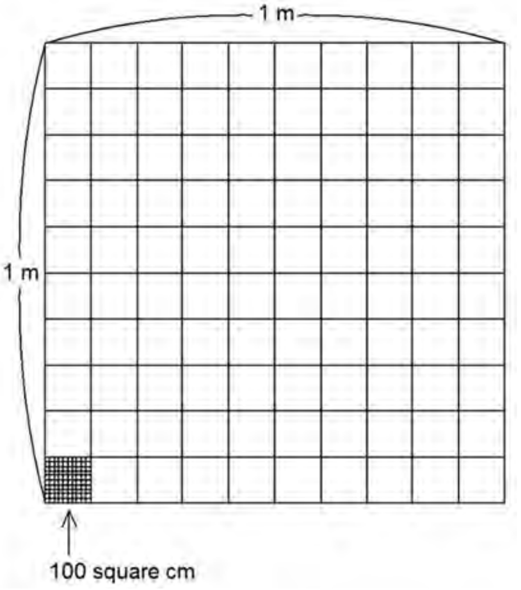
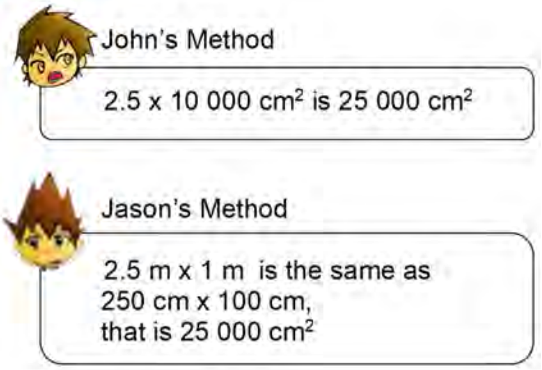


Diagram 9

i. Diagram 9 shows a 1 m x 1 m large square divided into 100 medium squares. One medium square at the bottom left corner is further divided into another 100 small squares. The area of the large square is 1 square metre.

- A medium square is a 10 cm x 10 cm square. What is the area of a medium square, in cm²?
- What is the area of one small square, in cm²?
- What is the relationship between 1 m² and 1 cm²?



John's Method

2.5 x 10 000 cm^2 is 25 000 cm^2

Jason's Method

2.5 m x 1 m is the same as
250 cm x 100 cm,
that is 25 000 cm^2

Diagram 10

ii. When asked to convert 2.5 m² to cm², two students, John and Jason, showed their methods of conversion as in Diagram 10.

- Compare the two methods.
  - ❖ Which method will you promote to your students?
  - ❖ Why?

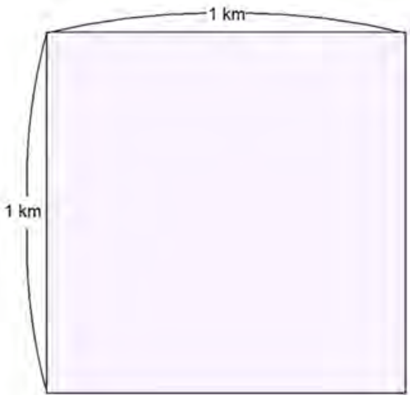


Diagram 11

iii. Diagram 11 shows a square with an area of 1 km².

- Explain how you will use the diagram to help your students derive the relationship between km² and m².

Task 4: Equivalent Size of Rectangular Area

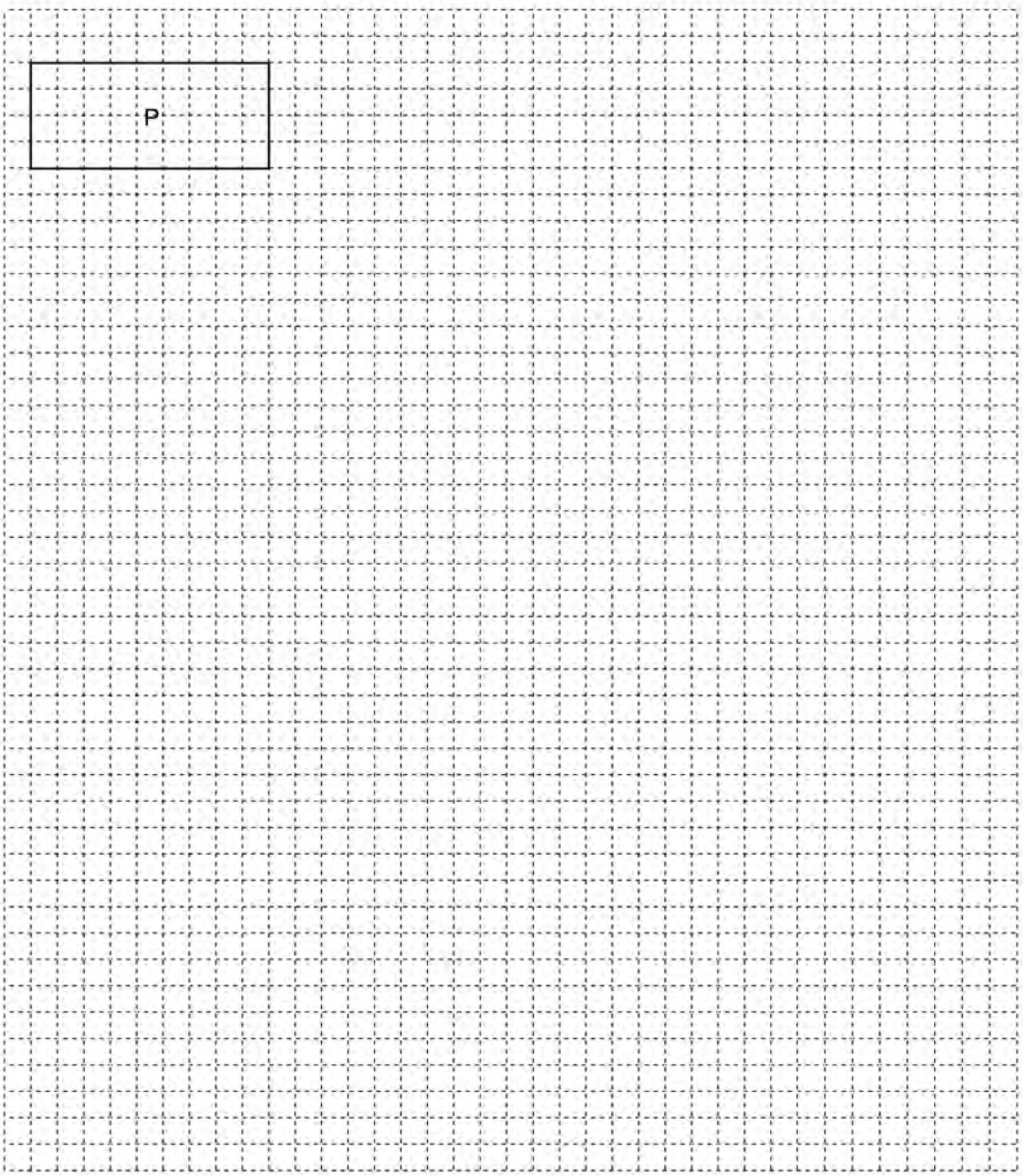


Diagram 12

i. Diagram 12 shows a rectangle P with an area of 36 m² drawn on a square grid.

- State the dimensions of rectangle P.
- Draw other rectangles with the same area as P on Diagram 12.
    - ❖ State the dimensions of each rectangle.
- Explain how you can find the dimensions of each rectangle without drawing.

Standards 3.2:

Extending area of a rectangle to other figures to derive formulae

i. Explore and derived a formula for the area of a parallelogram by changing its shape to a rectangle without changing its area
ii. Explore and derived a formula for the area of a triangle by bisecting a rectangle into two triangles without changing its area
iii. Appreciate the idea of changing or dividing shapes of a rectangle, parallelogram, or/and triangle for deriving the area of other figures
iv. Use formulae to calculate areas in daily life

Sample Tasks for Understanding the Standards

Task1: Finding Area of Parallelogram and Triangle

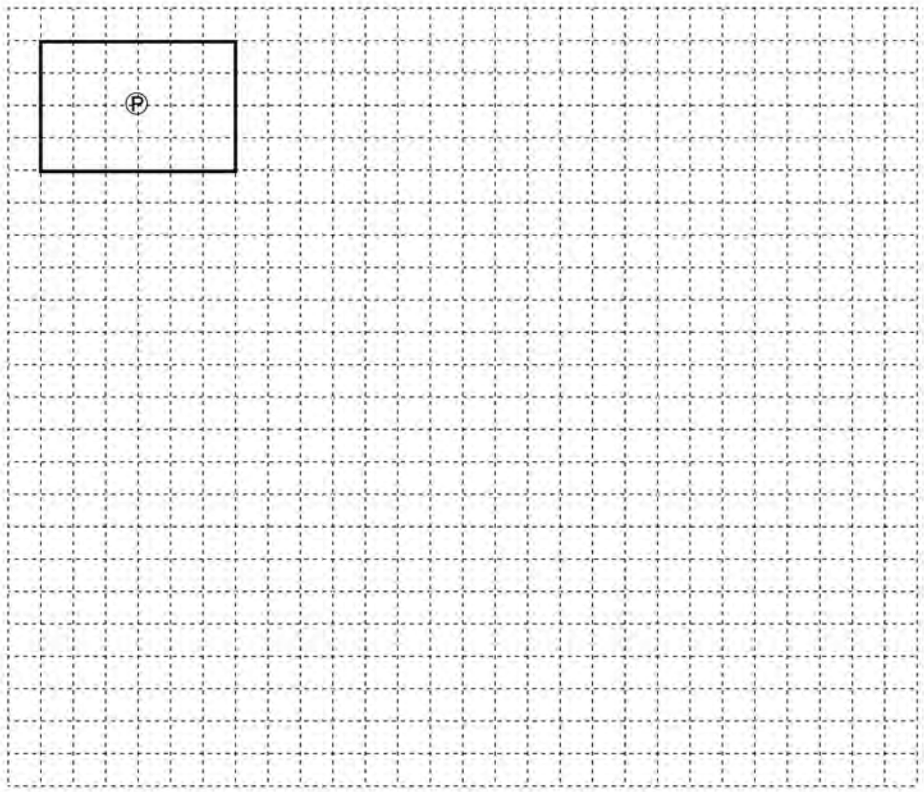


Diagram 1

i. Diagram 1 shows a rectangle Ⓟ drawn on a 1 cm² grid.

- What is the perimeter of rectangle Ⓟ?
- Draw others rectangles on Diagram 1, each with the same perimeter with rectangle Ⓟ.
  - ❖ Which rectangle has the maximum area?
  - ❖ What can you conclude on the rectangle with maximum area?

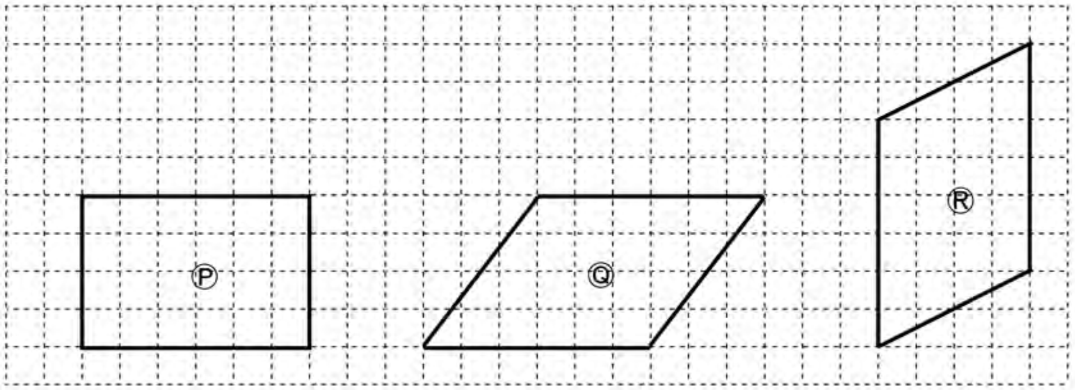


Diagram 2

ii. Diagram 2 shows a rectangle Ⓟ and two parallelograms, Ⓠ and Ⓡ.

- How could you show that Ⓟ, Ⓠ and Ⓡ have the same area, without any calculation,
  - ❖ Based on your answer, explain how you could derive a formula to find the area of a parallelogram.
  - ❖ Given that each square grid is 1 cm². Use your formula to find the area of parallelogram Ⓡ, in cm².

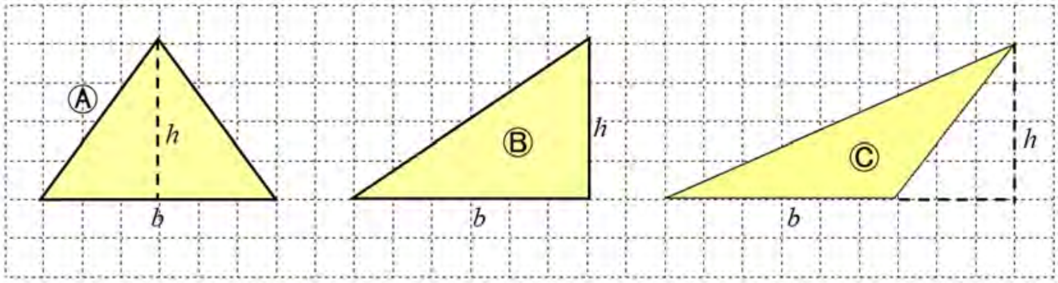


Diagram 3

iii. Diagram 3 shows three triangles, Ⓐ, Ⓑ,and Ⓒ, each with *a* base b, and *a* height h.

- Prove that the area of each triangle is $\frac{1}{2} \times b \times h$.
- How will you sequence the order of the three tasks in order to facilitate your students' reasoning process? Justify your decision.

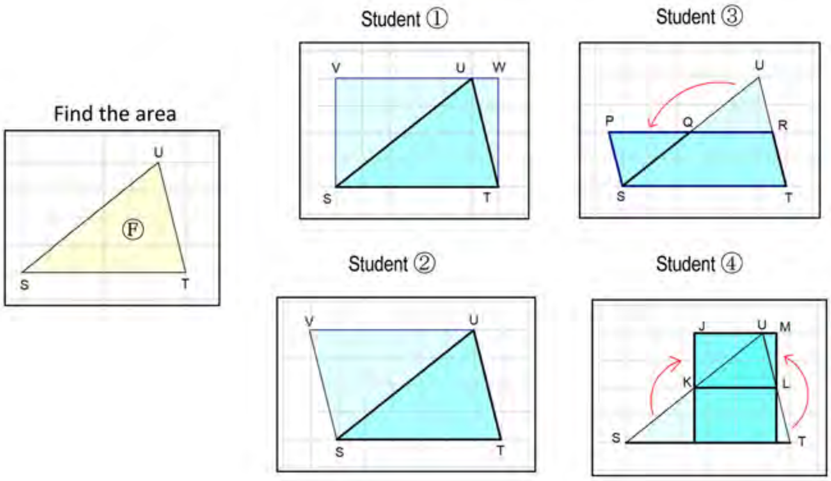


Diagram 4

iv. Mr. Ram gave triangle Ⓕ in Diagram 4 as a task for his students to think of different methods to find the area. He picked four samples among the students to share their ideas as shown in the diagram.

- Which student’s idea is the same as yours?
- What are the similarities and differences among the ideas of the 4 students?
- Student ① changed the triangle into a rectangle.
  - ❖ Write a mathematical expression to find the area of triangle Ⓕ.
- Student ② changed the triangle into a parallelogram.
  - ❖ Write a mathematical expression to find the area of triangle Ⓕ.
- Compare students ② and ③ methods.
  - ❖ What is the same and what is different about the two methods?
  - ❖ Explain how the formula to find the area of triangle Ⓕ can be derived from student ③ method.
- Compare students ① and ④ methods.
  - ❖ What is the same and what is different about the two methods?
  - ❖ Explain how the formula to find the area of triangle Ⓕ can be derived from student ④ method.

v. You intend to use the four students’ ideas in a classroom discussion.

- Which students’ ideas will you discuss first? Justify your decision.
- How will you sequence the order of discussion for the other three ideas? Justify your decision.

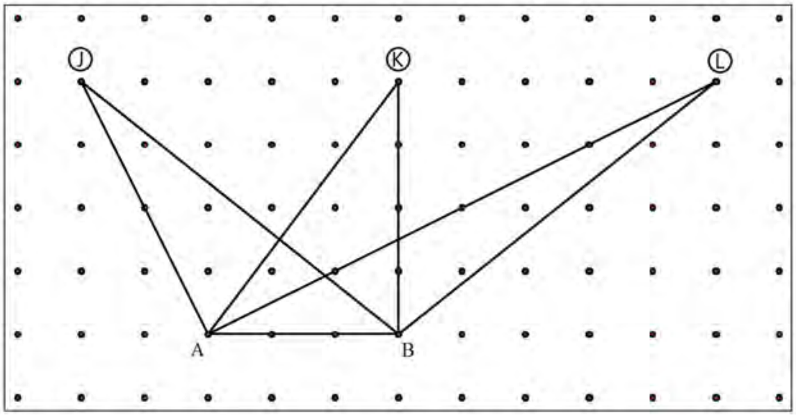


Diagram 5

vi. Diagram 5 shows a geoboard. Three pieces of rubber bands were used to construct triangles Ⓙ, Ⓚ, and Ⓛ with a common base AB.

- Without any calculation, explain why the areas of the three trianges are the same.
- If the distance between any two dots on the geoboard is 2 cm, find the area of the triangle in cm².

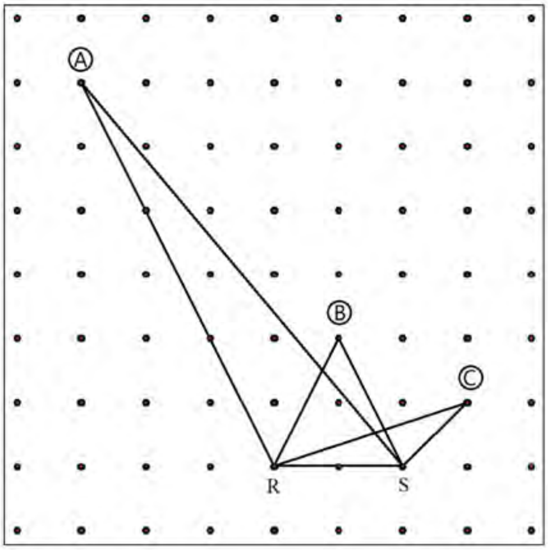


Diagram 6

vii. Diagram 6 shows three triangles Ⓐ, Ⓑ and Ⓒ, with a common base RS constructed on a geoboard.

- Without any calculation, explain why the area of triangle Ⓒ is half the area of triangle Ⓑ.
- Without any calculation, find the ratio of the area of triangle Ⓐ to the area of triangle Ⓑ.

Task 2: Area of Other Figures

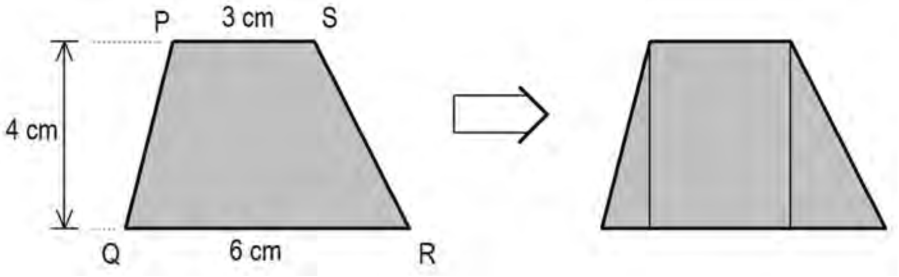
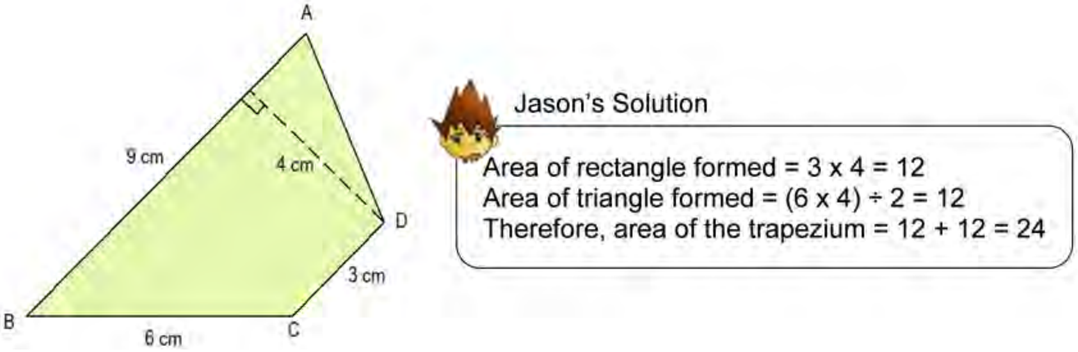


Diagram 7

i. Diagram 7 shows a trapezium PQRS. John found the area of the trapezium by cutting the trapezium as shown in the diagram.

- Find the area of the trapezium, in cm^2.
- Explore at least 3 other ways of drawing to find the area of the trapezium.
- What are the similarities and differences found in all the ways?
- In what way will this task inculcate the appreciation of knowing the formulae of figures such as triangle and rectangle?



Jason's Solution

Area of rectangle formed = 3 x 4 = 12
Area of triangle formed = (6 x 4) ÷ 2 = 12
Therefore, area of the trapezium = 12 + 12 = 24

Diagram 8

ii. Diagram 8 shows the trapezium ABCD and Jason's calculation to find the area of the trapezium.

- Explain how Jason did the calculation.
- Is Jason's solution correct?
- Justify Jason's solution.

Task 3: Problem Solving Involving Land Areas

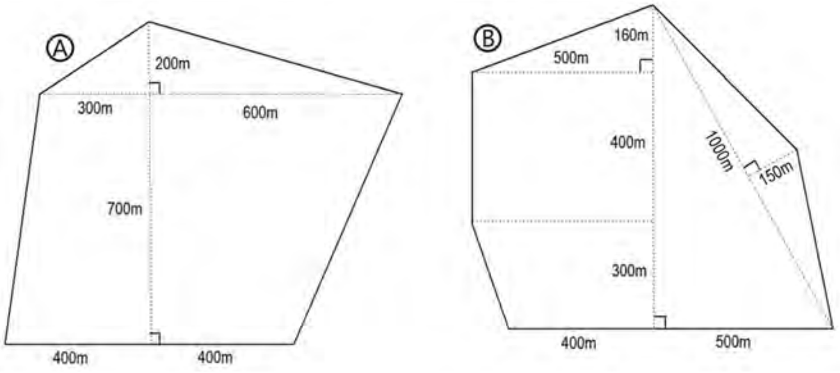


Diagram 9

i. Diagram 9 shows two plots of land, Ⓐ and Ⓑ owned by Uncle Nathan. He called his two sons to choose one plot each. Both the sons wanted plot Ⓐ because it appears larger to both.

   - What would you do to find out if the choice was correct?
   - Which piece of the land is actually larger? Justify your answer with calculations.

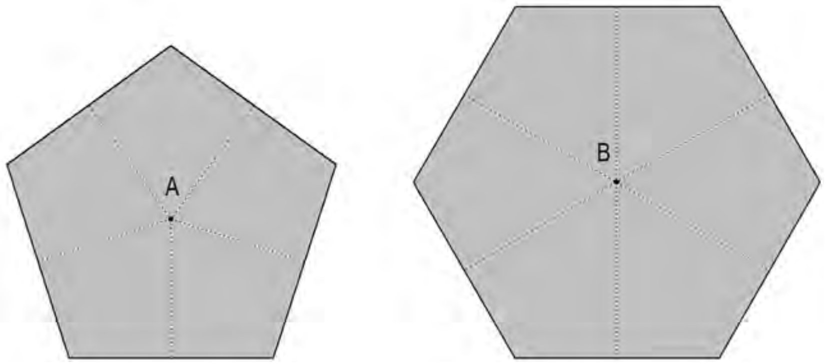


Diagram 10

ii. Diagram 10 shows a regular pentagon and a regular hexagon, with centres A and B respectively.

   - Use a ruler to make some measurements needed for the calculation of the area of the hexagon and pentagon, respectively.

     ❖ Use the measurements to calculate the area of each shape.

   - What is the minimum number of measurements needed for calculating the area of each shape?

     ❖ Calculate the area of each shape using the minimum number of measurements.

Topic 4: Extending Measurement of Volume in Relation to Surface

Standards 4.1:

Introducing volume from area and derive formula for cuboid

i. Compare the extent of volume and introduce its unit, and distinguish it from surface
ii. Introduce one cubic centimetre as the unit for volume and its addition and subtraction
iii. Investigate the volume of a cuboid and cube and produce the formulae
iv. Extend cubic centimetre to cubic metre to measure large volume
v. Convert units and use appropriate units of volume with fluency
vi. Appreciate the use of volume in life such as comparison of capacity of containers

Sample Tasks for Understanding the Standards

Task1: Relating Area to Volume

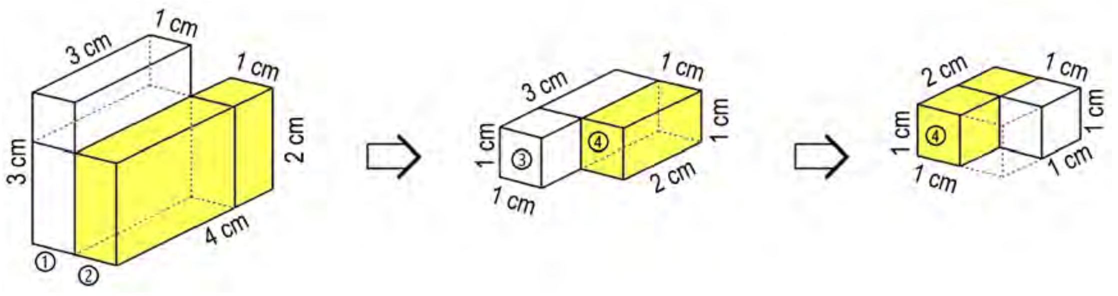


Diagram 1

i. Diagram 1 shows two boxes, ① and ②. Box ① is 3 cm x 3cm x 1cm and box ② is 4cm x 2cm x 1cm. A student compared the sizes of the two boxes and found that there was an "extra part" from each box, ③ and ④. He continued the process of comparison and finally got a 1 cm cubic box as shown in the diagram. The volume of the 1 cm cubic box is 1 cubic cm, written as 1 cm³.

- What is the volume of the following, in cm³?
  - ❖ Extra part ④
  - ❖ Extra part ③
  - ❖ Box ②
  - ❖ Box ①
- What previously learned knowledge is required for your student to learn the process of deriving 1 cm³ by Euclidean algorithm?

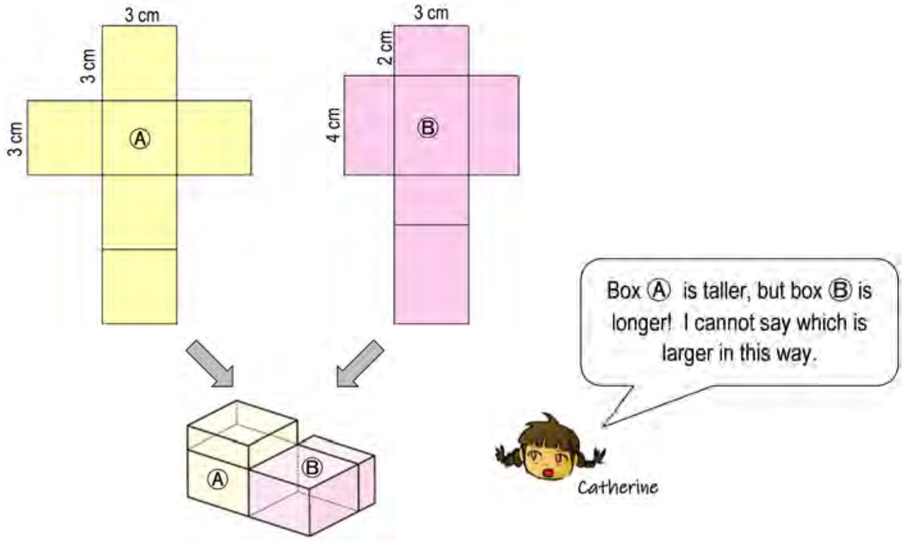


Diagram 2

ii. Diagram 2 shows two nets used to make two boxes, Ⓐ and Ⓑ, Catherine tried to compare the volume of the two boxes by puting them side by side. However, she seem to have some difficulties doing it.

- Explain how 1 cm cubic blocks can be used to help Catherine determine which box is larger.
- Which box is larger and how much larger?

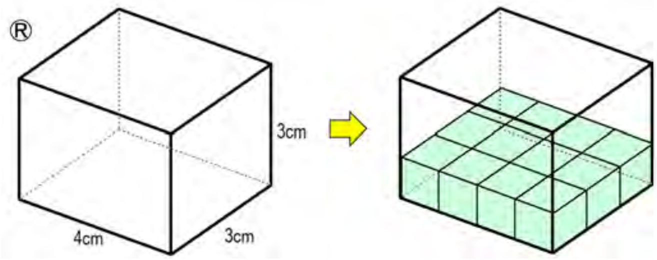


Diagram 3

iii. Diagram 3 shows a box, Ⓡ, filled with a layer of 1 cm cubic blocks.

- How many 1 cm cubic blocks are there in this layer?
- How many layers of 1 cm cubic blocks can fill the whole box?
- How many 1 cm cubic blocks altogether can fill the whole box?
- What is the volume of box Ⓡ, in cm^3?
- Explain the relationship between the base area of a rectangular box and its volume.
- Base on the relationship, state the formula of the volume of a rectangular box.

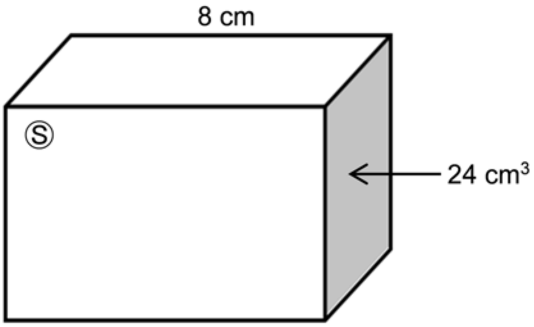


Diagram 4

iv. Diagram 4 show a rectangular box Ⓢ. The area of one of its vertical faces is 24 cm².

- Find the volume of box Ⓢ, in cm³.
- How can this task help your students learn other mathematical ideas such as volume of a prism in future grade?

Task 2: Larger Units for Volume

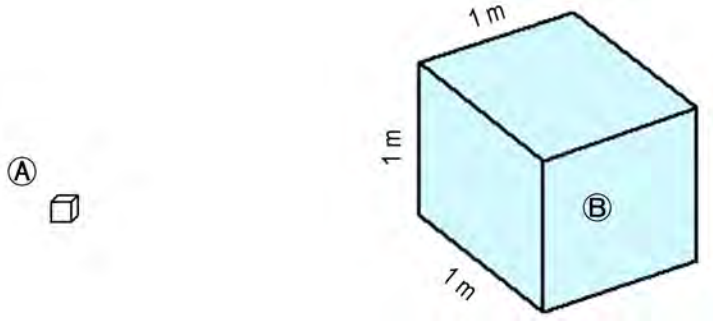


Diagram 5

i. Diagram 5 shows a 1 cm cubic block, Ⓐ, and another 1 m cubic block, Ⓑ.

- How many blocks Ⓐ are there in one block Ⓑ?
- What is the relationship between cm³ and m³?
- What is the volume of block Ⓑ, in m³?
- How can this task help your students understand the size of 1 m³?

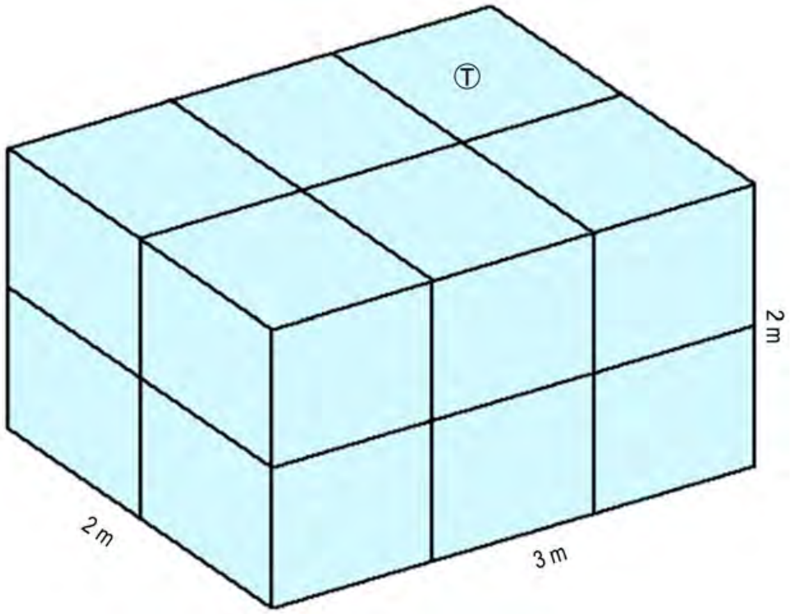


Diagram 6

- Diagram 6 shows a rectangular block Ⓣ which is made up of several blocks Ⓑ.
    - ❖ How many blocks Ⓑ are there in block Ⓣ?
    - ❖ What is the volume of block Ⓣ in m³?

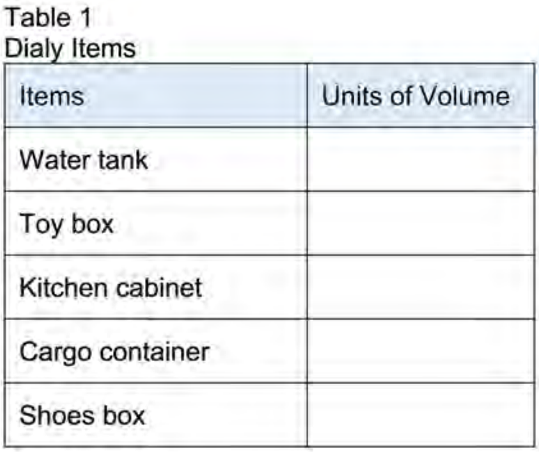


Table 1
Dialy Items

Items	Units of Volume
Water tank	
Toy box	
Kitchen cabinet	
Cargo container	
Shoes box	

ii. Table 1 shows a list of dialy items.

- Choose from cm^3 or m^3, as appropriate unit to measure the volume of each item.
- Justify each of your choices.

Standards 4.2:

Extending volume of a cuboid to other solid figures to derive formula

i. Extend the formula for the volume of a cuboid as base area x height for exploring solid figures such as prism and cylinder

ii. Extend the formula for the volume of a prism and a cylinder to explore and derive the volume formula of a pyramid and cone

iii. Use formulae to calculate volume in daily life

Sample Tasks for Understanding the Standards

Task1: Volume of Prisms and Cylinder

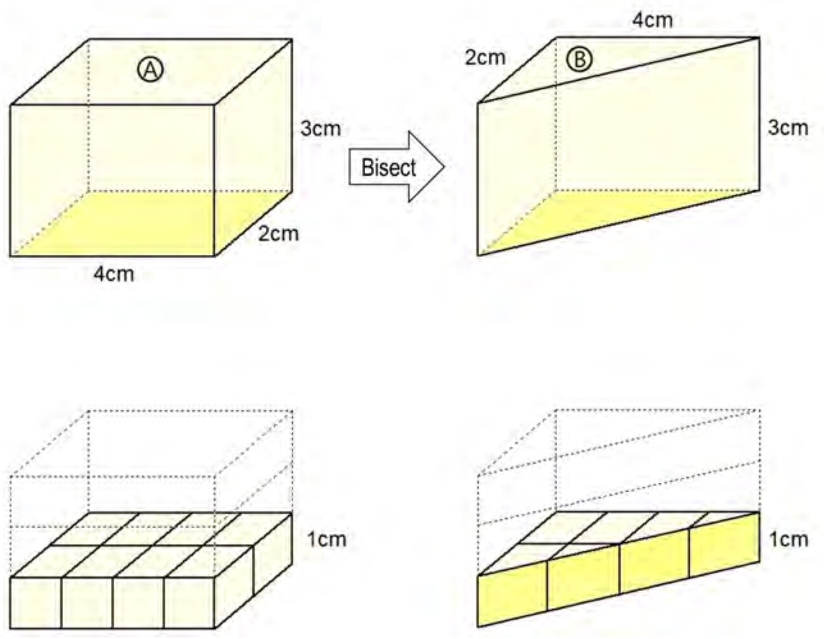


Diagram 1

i. Diagram 1 shows two prisms, Ⓐ and Ⓑ, where the base area of Ⓑ is ½ of Ⓐ.

- Name the two prisms.
- The two prisms are to be filled with 1 cm cubic blocks.
  - ❖ How many blocks will be needed to fill one layer of Ⓐ and Ⓑ, respectively?
  - ❖ How many layers is needed to fill each prism?
  - ❖ What is the volumn of each prism, in cubic cm?
- Explain the relationship between the base area of a prism and its volume.
- Base on the relationship, state the formula of the volume of a prism.

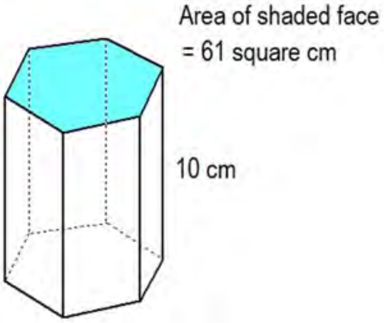


Diagram 2

ii. Diagram 2 shows a prism with two parallel hexagonal faces. The area of one face is 61 square cm and the perpendicular distance between the two faces is 10 cm.

- Find the volume of the prism, in cubic cm.

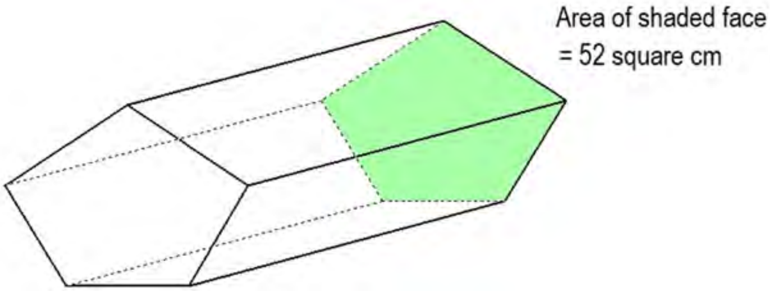


Diagram 3

iii. Diagram 3 shows a prism with two parallel pentagonal faces. The area of one face is 52 square cm.

- What missing information is required for you to calculate the volume of the prism?
- Provide a value for the missing information and find the volume of the prism.

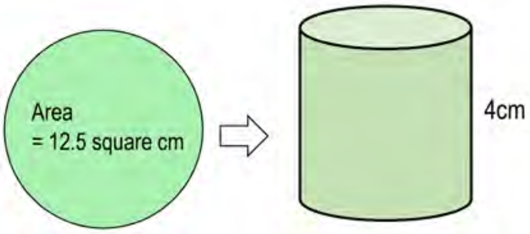


Diagram 4

iv. Diagram 4 shows a circular card with an area of 12.5 sqrare cm. The cards are stacked up to 4 cm high to form a cylinder.

- In what way can the knowledge of finding volume of a prism helps your students find the formula for the volume of a cylinder?
- Find the volume of the cylinder, in cubic cm.

Task 2: Volume of Various Space Solids

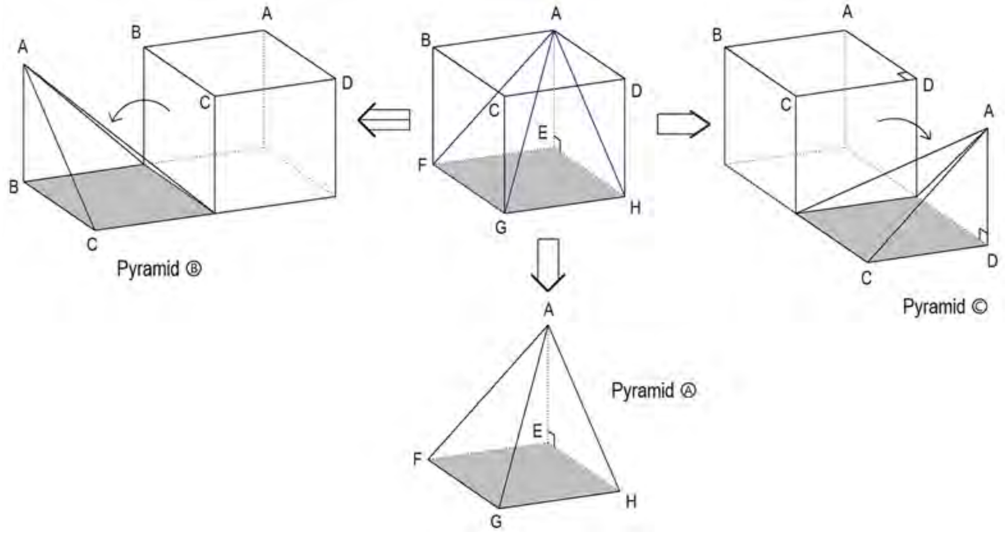


Diagram 5

i. Diagram 1 shows a cube ABCDEFGH. The cube is trisected into three identical pyramids, Ⓐ, Ⓑ and Ⓒ.

- Given the side AB = a cm.
    - ❖ Explain why the volume of the pyramid is V= $\frac{1}{3} \times a \times a \times a$.
- If a = 6 cm, find the volume of the pyramid, in cm³.

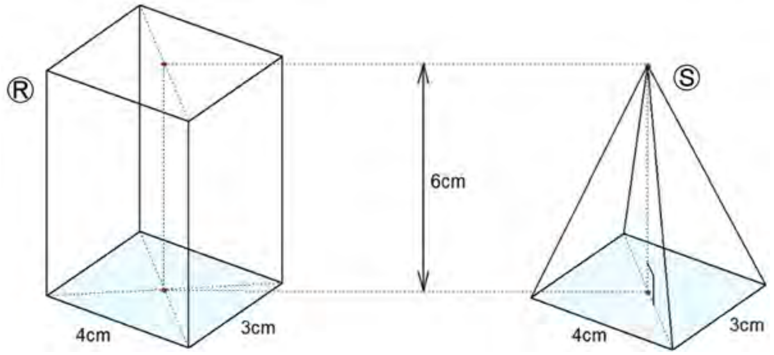


Diagram 6

ii. Diagram 6 shows two containers, Ⓡ and Ⓢ. Container Ⓡ is a rectangular prism with a base of 4 cm x 3 cm, and a height of 6 cm. Container Ⓢ is a pyramid with the same rectangular base and height.

- John filled container Ⓢ with sand fully. Then, he poured the sand into container Ⓡ.
    - ❖ Mark the level of sand in container Ⓡ on Diagram 7.

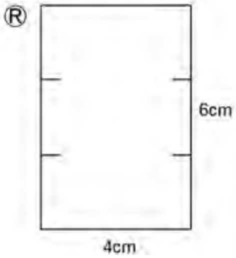


Diagram 7

- What is the relationship between the volume of Ⓡ and Ⓢ.
- Verify the relationship.
- Base on the relationship, state the formula for the volume of a pyramid.
  - ❖ What if the base of the pyramid is a square?

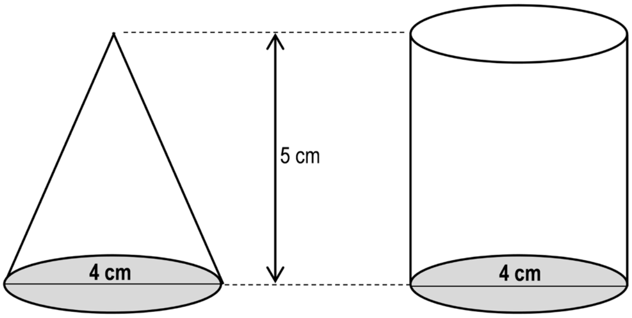


Diagram 8

iii. Diagram 8 shows a cone and a cylinder with the same base diameter and height.

- What is the relationship between the volume of the cone, V_1, and the Cylinder, V_2.
- Verify the relationship.
- Base on the relationship, state the formula for the volume of a cone.

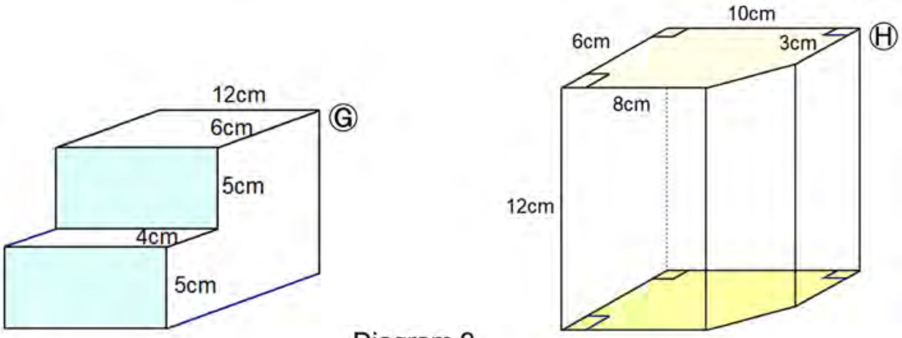


Diagram 9

iv. Digram 9 shows a L-shaped solid Ⓖ and a pentagonal prism Ⓗ.

- Calculate in different ways, the volume of each solid in cm³.

Topic 5: Approximating Quantities

Standards 5.1:

Approximating numbers with quantities depending on the necessity of contexts
i. Understand the ways of rounding such as round up and round down
ii. Use rounding as an approximation for making a decision on the quantity with related context
iii. Critique approximation beyond the context with a sense of quantity such as based on the relative size of units

Sample Tasks for Understanding the Standards

Task1: Different Ways of Rounding

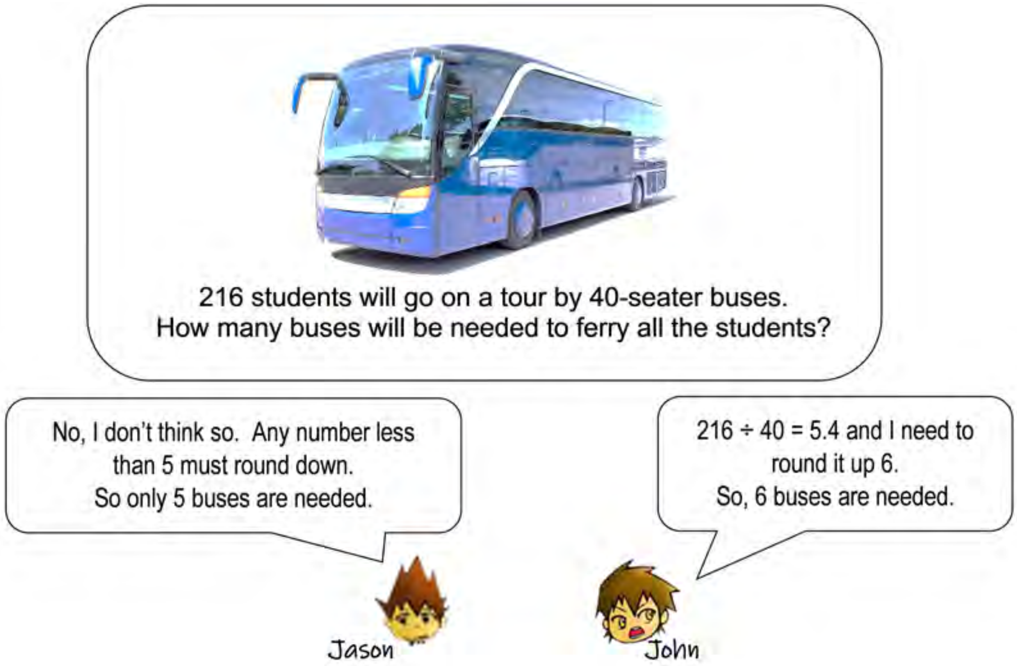


Diagram 1

i. Diagram 1 shows a problem and two students' ideas on the solution.
   - Explain the different ways of rounding used by each of the students.
   - Which way of rounding is inappropriate for this situation? Justify your decision.

ii. There are 1 685 sheets of greeting cards to be packed into boxes for sale. Each box must have 100 sheets of cards.
   - How many boxes of greeting cards can be packed for sale?
   - Which way of rounding is appropriate in this situation? Justify your decision.

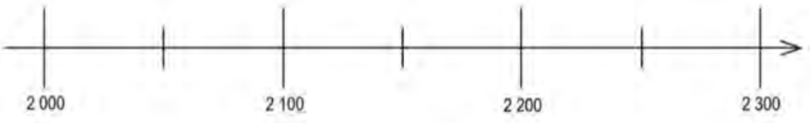


Diagram 2

iii. Diagram 2 shows part of a number line.

- Explain how the number line can be used to help your students round 2 134 to the nearest hundred.
- What way of rounding is used in this case? Explain your answer.
- Explain briefly, the rule used in this way of rounding.

iv. A number ⓐ is rounded to the nearest ten, and the answer is 60.

- What are the possible numbers for ⓐ?
- Show your answers on a number line.

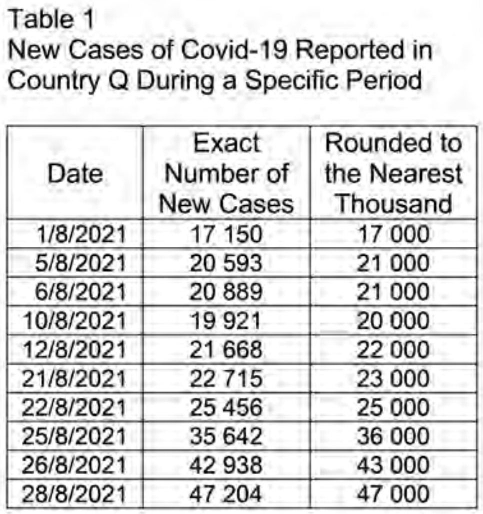


Table 1
New Cases of Covid-19 Reported in Country Q During a Specific Period

Date	Exact Number of New Cases	Rounded to the Nearest Thousand
1/8/2021	17 150	17 000
5/8/2021	20 593	21 000
6/8/2021	20 889	21 000
10/8/2021	19 921	20 000
12/8/2021	21 668	22 000
21/8/2021	22 715	23 000
22/8/2021	25 456	25 000
25/8/2021	35 642	36 000
26/8/2021	42 938	43 000
28/8/2021	47 204	47 000

v. Table 1 shows the exact number of new cases of COVID-19 reported on specific dates in a country. A student rounded these numbers to the nearest thousand.

- Which set of numbers should a health officer refer to for more efficient monitoring of the situation? Justify your decision.
- Which set of numbers is sufficient to provide a real situation to a commoner? Justify your decision.

vi. The following are some numbers.

56 476, 57 501, 58 573, 58 231, 57 499, 58 500, 57 634, 57 256

- Which numbers become 58 000 when rounded to the nearest thousand?
- Which numbers become 57 000 when rounded down to the nearest thousand?
- Which numbers become 57000 when rounded up to the nearest thousand?

Task 2: Making Decisions in Context

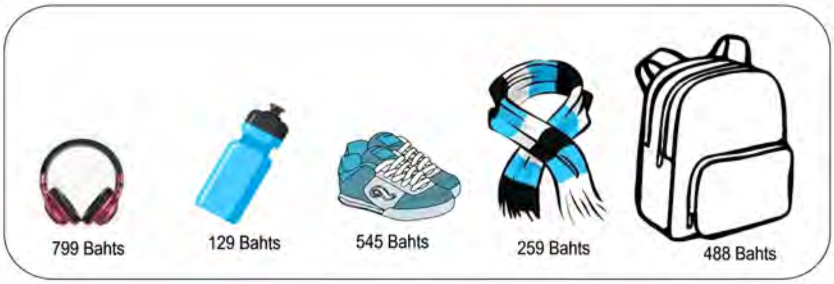
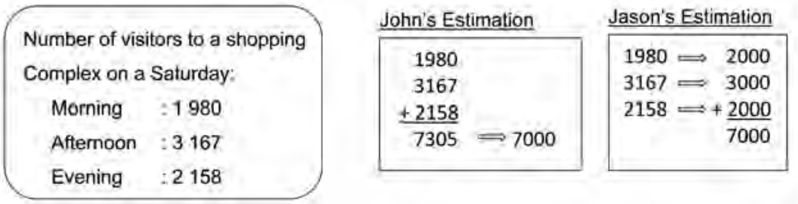


Diagram 3

i. Diagram 3 shows a list of items on Anada's shopping list. In one month, Anada had saved 1 000 Bahts.

- Using appropriate estimation, what is the most number of items he can buy?
- Estimate the amount of money required for him to buy all the items.
    - ❖ Explain how you do the estimation.
    - ❖ How do you ensure that your estimation can guarantee you enough money to buy all the items?



Number of visitors to a shopping Complex on a Saturday:

Morning : 1 980

Afternoon : 3 167

Evening : 2 158

John's Estimation

$$\begin{array}{r} 1980 \\ 3167 \\ \underline{+\,2158} \\ 7305 \Longrightarrow 7000 \end{array}$$

Jason's Estimation

$$\begin{array}{lr} 1980 \Longrightarrow & 2000 \\ 3167 \Longrightarrow & 3000 \\ 2158 \Longrightarrow & +\,\underline{2000} \\ & 7000 \end{array}$$

Diagram 4

ii. Diagram 4 shows how John and Jason did their estimation of visitors to a shopping complex in their trip report.

- Whose way of rounding is more sensible? Verify your decision.

iii. Why is it necessary to use estimation in our daily life?

iv. A primary school has 2 856 students. The headmaster informs the board of directors that there are about 3 000 students, whereas the assistant headmaster informs the parent-teacher association that the school has about 2 900 students.

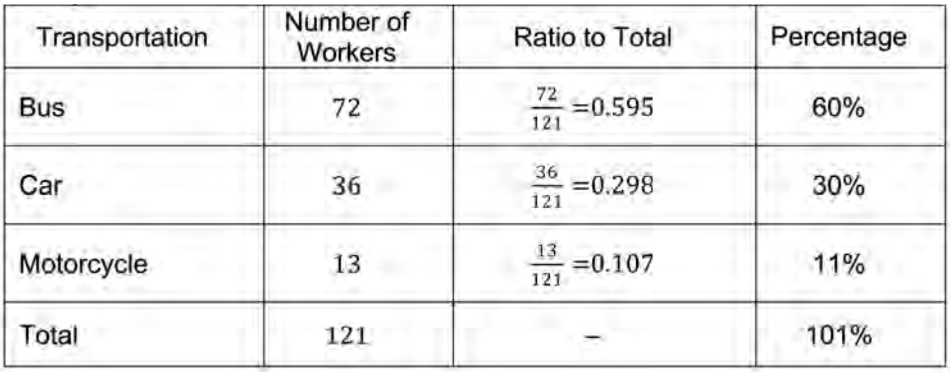
- Which approximation is more appropriate? Justify your decision.

v. In the recent mathematics test, Lina's teacher announced that 92% of the students passed the test. When reaching home, Lina told her mother that about 100% of her class students passed the test.

- Is it appropriate for Lina to tell the mother an estimated passing percentage? Justify your decision.

vi. Table 2 shows the information collected from 121 factory workers on the transportation they used to go to work. A student calculated the ratio of each group of workers to the total number of workers for each type of transportation. The ratios were then rounded to the nearest percentage as shown in the table.

Table 2
Transportation to work



Transportation	Number of Workers	Ratio to Total	Percentage
Bus	72	$\frac{72}{121}=0.595$	60%
Car	36	$\frac{36}{121}=0.298$	30%
Motorcycle	13	$\frac{13}{121}=0.107$	11%
Total	121	–	101%

- Explain how each of the ratios was rounded to the nearest percentage?
- Why the total percentage is more than 100%?

Topic 6: Extending Proportional Reasoning to Ratio and Proportion

Standards 6.1:

Extending proportional reasoning to ratio and percent for comparison

i. Understand ratio as the relationship between two same quantities or between two different quantities (the latter idea is rate)
ii. Express the value of a ratio by quotient such as the rate of two different quantities
iii. Understand percent as the value of a ratio with the same quantity and the necessity of rounding
iv. Understand proportional reasoning for ratio as part-whole and part-part relationships
v. Apply the rule of three in using ratio

Sample Tasks for Understanding the Standards

Task 1: Proportional Reasoning in a Context

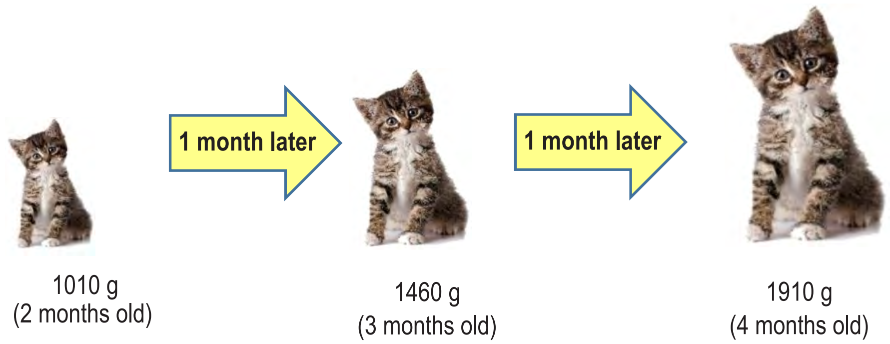


Diagram 1

Diagram 1 shows a kitten's body weight for three consecutive months.

i. What is your prediction on the weight of the kitten when it is 5 months old?
  - Explain the reason of your prediction.
  - Discuss the accuracy of your prediction.
    - ❖ What assumption do you make for your prediction?
    - ❖ How can you justify it?
  - How do you determine if the situation is proportional to help you make a decision?

ii. What is your guess on the weight of the kitten at birth?

iii. How many minimum sets of data do you need to make a reasonably reliable prediction?

Note. For this case, we are able to find any possible function that fit three known ordered pairs, mathematically.

Task 2: Comparing Quantities by Ratios

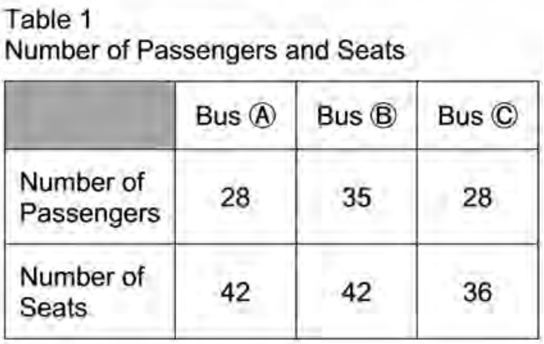


Table 1
Number of Passengers and Seats

	Bus Ⓐ	Bus Ⓑ	Bus Ⓒ
Number of Passengers	28	35	28
Number of Seats	42	42	36

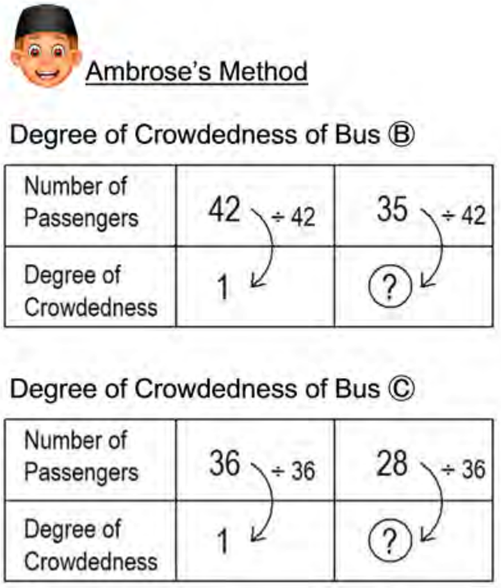
i. Table 1 shows the number of passengers and the number of seats available in three buses. A student compared the crowdedness between the buses as follows:

Comparison ①: Bus Ⓐ and Bus Ⓑ

Comparison ②: Bus Ⓐ and Bus Ⓒ

Comparison ③: Bus Ⓑ and Bus Ⓒ

- Why is comparison ③ more difficult to do than comparisons ① and ②?
- Explain how the idea of ratio can be used to make comparison ③.



Ambrose's Method

Degree of Crowdedness of Bus Ⓑ

Number of Passengers	42	35
Degree of Crowdedness	1	?

Degree of Crowdedness of Bus Ⓒ

Number of Passengers	36	28
Degree of Crowdedness	1	?

Diagram 2

ii. Ambrose constructed two tables as shown in Diagram 2 to compare the degree of the crowdedness of Bus Ⓑ and Bus Ⓒ.

- Explain Ambrose's method.
- Find the missing number in each ?.
- Which bus is more crowded ?

iii. There are two ways to look at the crowdedness of the bus: (a) the number of passengers per seat, and (b) the number of seats per passenger.

- Explain how the two ideas are used to determine the degree of the crowdedness of the bus.
- Which idea is used in Ambrose's method?
- Use the other idea to determine the degree of the crowdedness of the bus.

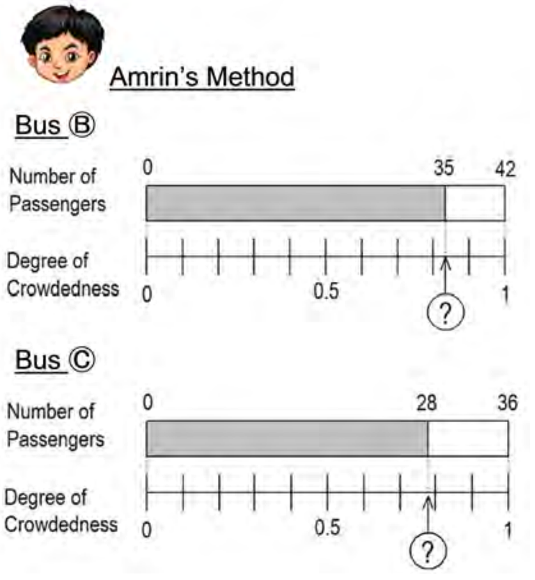


Diagram 3

iv. Amrin drew two graphs as shown in Diagram 3 to compare the degree of the crowdedness of Bus Ⓑ and Bus Ⓒ.

- Explain Amrin's method.
- Find the missing number in each (?).

v. Compare Ambrose's method with Amrin's method.

- What is the same and what is different between the two methods?

vi. Compare the degree of the crowdedness of Bus Ⓐ, Bus Ⓑ and Bus Ⓒ.

- Which bus is the most crowded?
- Which bus is the least crowded?

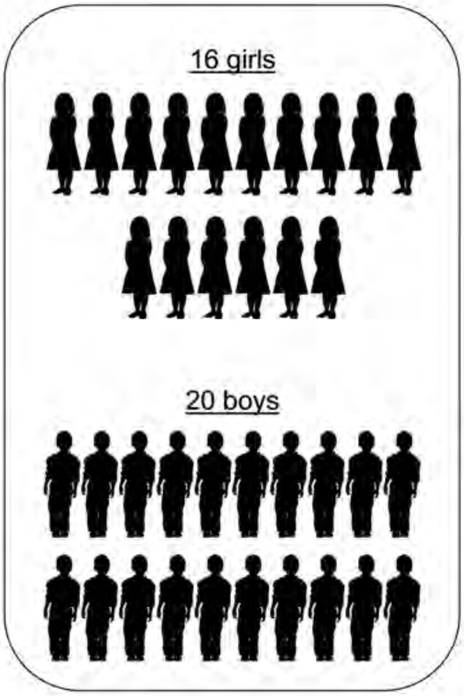


Diagram 4

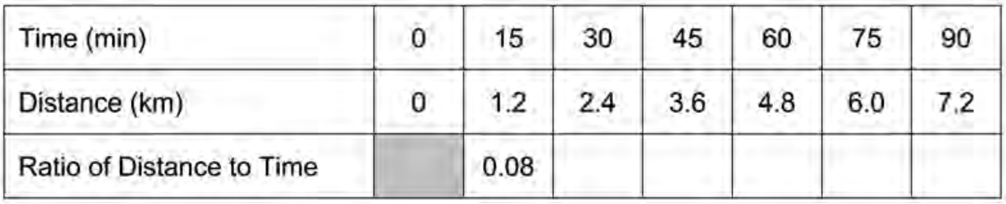
vii. Diagram 4 shows a class of 16 girls and 20 boys.

- Find the ratio of
  - ❖ the number of girls to the number of boys.

    (Note. The base value for comparison is the number of boys.)
  - ❖ the number of boys to the number of girls.
  - ❖ the number of girls to the total number of students.
  - ❖ the number of boys to the total number of students.
- Why are the values of the four ratios not the same?
- What is the purpose of using a different base value for comparison?
  - ❖ Explain your answer with a real-life example.

Task 3: Ratio and Rate

Table 2
Distance Travelled with Time



Time (min)	0	15	30	45	60	75	90
Distance (km)	0	1.2	2.4	3.6	4.8	6.0	7.2
Ratio of Distance to Time	▨	0.08					

A man went for a brisk walk. Table 2 shows the record of time taken and distance travelled by the man.

i. Distance and time are two different quantities with different units of measure. The ratio of distance to time is also known as speed, which is an example of rate.

   - What is the difference between ratio and rate?
   - What is the unit of measure for the ratio of distance to time?

ii. The ratio of distance to time for the interval 0 – 15 mins is shown in Table 2.

   - Find the ratio of distance to time for every other interval.
   - What can you say about the speed of walking of the man?

iii. The man walked for 240 minutes.

   - Calculate the total distance travelled, in km.
   - What assumption do you make in the calculation?

iv. The man walked 10 km.

   - Calculate the time taken, in minutes.
   - What assumption do you make in the calculation?

Task 4: Percent as the Value of Ratio

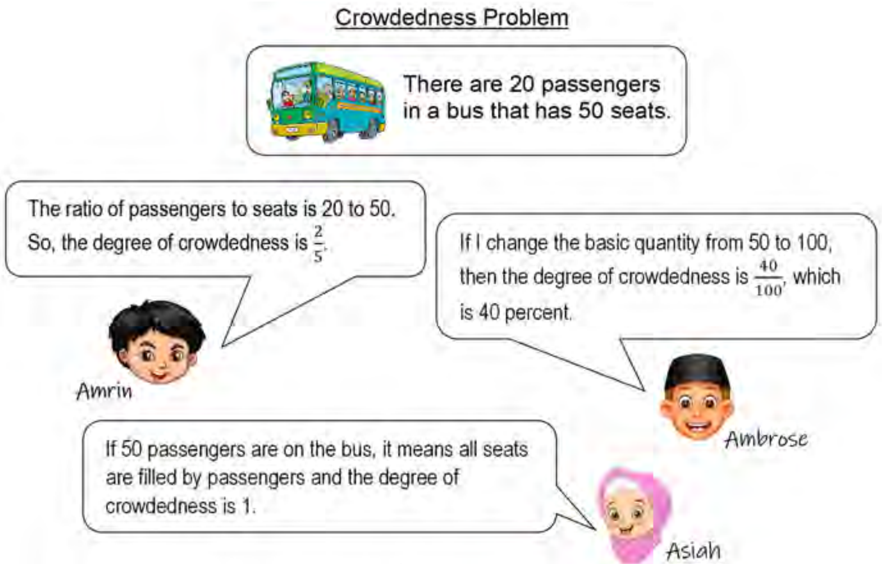


Diagram 5

i. Diagram 5 shows the conversation between three students, Amrin, Ambrose and Asiah, discussing the solution for the bus crowdedness problem.

- Compare the ideas of Amrin and Ambrose.
    - ❖ What is the same and what is different between the two ideas?
- How does Asiah obtain the degree of crowdedness as 1?
    - ❖ What does it mean by crowdedness of 1?

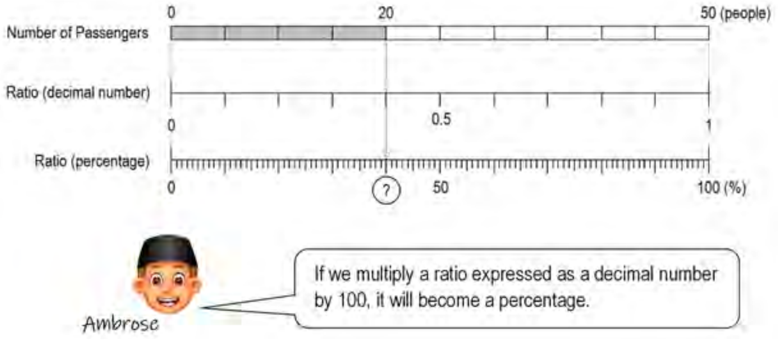


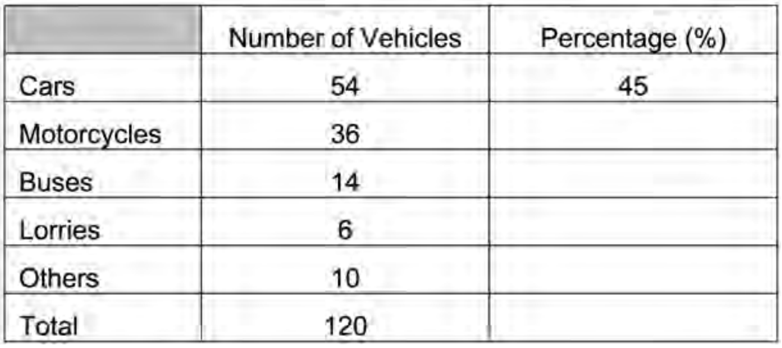
Diagram 6

ii. Ambrose explained his ideas further by drawing a graph as shown in Diagram 6.

- Explain Ambrose's ideas.
- Find the missing number in (?).
- Express the ratio 0.03 and 0.3 as a percentage, respectively.

- Express the ratio $\frac{9}{100}$ and $\frac{3}{4}$ as a percentage, respectively.
- On another day, taking the 50 seats in a bus as the base value, the degree of the crowdedness of the bus is found to be 90%.
  - ❖ How many passengers are there on the bus?

Table 3
Types of Vehicles Passed by a School Gate



	Number of Vehicles	Percentage (%)
Cars	54	45
Motorcycles	36	
Buses	14	
Lorries	6	
Others	10	
Total	120	

iii. Table 3 shows the number of various vehicles on the road in front of a school gate for 1 hour.

- Use Amborose’s method to find the ratio of each type of vehicle to the total number of vehicles.
- Fill in the blank in Table 3.
- What is the total of all the percentages?

iv. A bus with 50 seats is filled with 55 passengers.

- Calculate the degree of crowdedness as a percentage.
- What does it mean when the percentage is larger than 100%?

Task 4: Equivalent Ratios

Diagram 7

Diagram 7 shows an orange concentrate with the value of ratio 1 : 9 to make the drink with a dilution of water.

i. What does the ratio 1 : 9 mean?

ii. If you need to make a glass of 300 mℓ orange drink, how much of concentrate and water is required?

iii. There are 25 people in a meeting and each is served a glass of orange drink.

- How would you prepare the drink?

iv. Christ prepares his own drink by adding 20 mℓ concentrate to 180 mℓ water. Hardy prepares the drink for a few friends. He adds 120 mℓ concentrate into 1 ℓ 80 mℓ of water.

- Will both drinks taste the same? Explain why or why not.
- What is equivalent ratio?
- If Hardy wants to use only 80 mℓ of concentrate, how much water should he use?

Task 5: Applying Ratio and Rate to Solve Problems

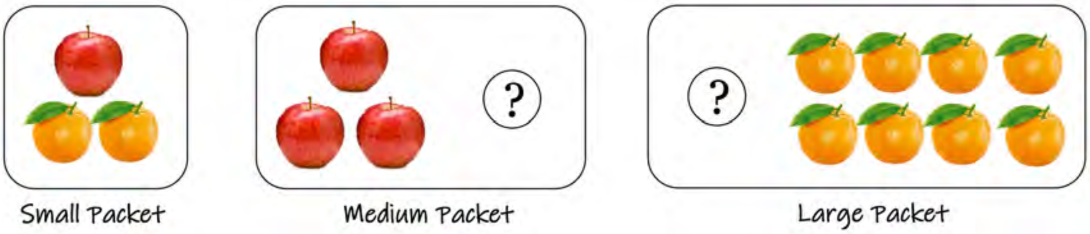


Diagram 8

i. A supermarket sells apples and oranges in three different packages, small, medium and large. As shown in Diagram 8, a small packet has 1 apple and 2 oranges, a medium packet has 3 apples, and a large packet has 8 oranges. Given that the number of apples and the number of oranges in all the packets are in the same proportion.

- What is the ratio of the number of apples to the number of oranges in the small packet?
- What does it mean by "the number of apples and the number of oranges is in the same proportion"?
- Find the number of oranges in the medium packet.
- Find the number of apples in the large packet.

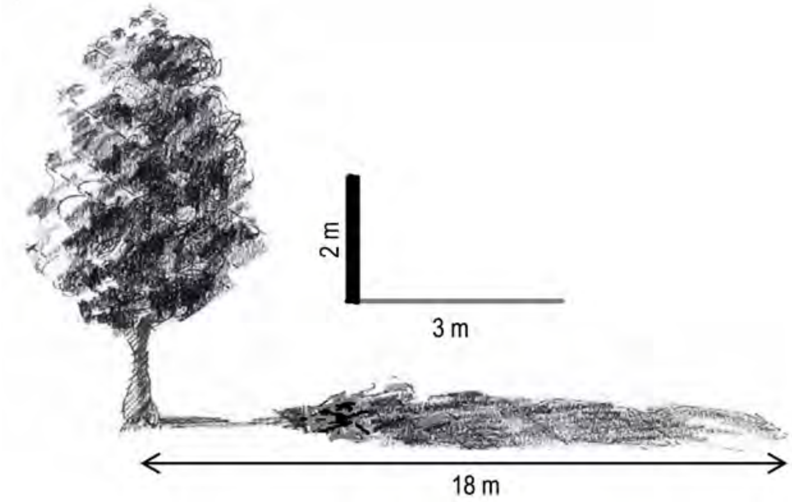


Diagram 9

ii. Diagram 9 shows a tree and a 2-metre wooden pole on a sunny day. The shadow of the wooden pole and the tree is 3 m and 18 m, respectively.

- Explain how the idea of proportion can be used to find the height of the tree.
- Using the rule of three, calculate the height of the tree.

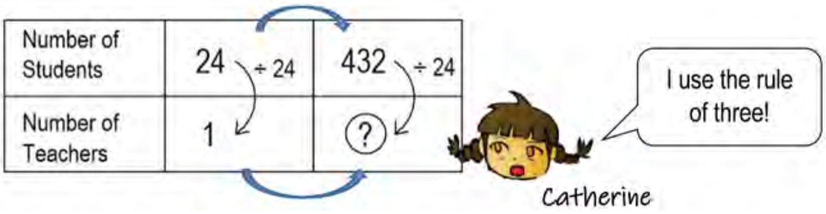


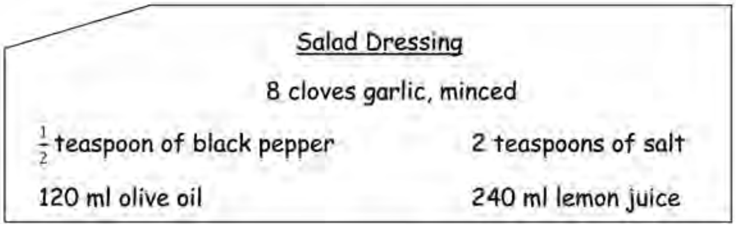
Diagram 10

iii. The student-teacher ratio for a school in a country is a maximum of 24 students to 1 teacher. Diagram 10 shows Catherine's method to find the number of teachers for a school with 432 students.

- Explain Cindy's method.
- Find the missing number ?.
- Find another way to get the answer. [Hint: Read the table horizontally.]
- Another school has 25 teachers. Use the rule of three to guess the number of students.

iv. Mr Bảo used 8 litres of emulsion paint to paint 2 rooms.

- How many litres of emulsion paint is needed to paint 6 similar rooms?
- How many similar rooms can he paint using 28 litres of emulsion paint?



Salad Dressing

8 cloves garlic, minced

$\frac{1}{2}$ teaspoon of black pepper	2 teaspoons of salt
120 ml olive oil	240 ml lemon juice

Diagram 11

v. A ratio of two quantities, a and b, can be written either in the form of $a : b$ or $\frac{a}{b}$. Diagram 11 shows a recipe to prepare a salad dressing using olive oil and lemon juice.

- Let *a* represent the volume of olive oil and *b* represents the volume of lemon juice.
    - ❖ Write the ratio $a : b$ in its simplest form.
    - ❖ Rewrite the ratio in the form of $\frac{a}{b}$.
    - ❖ How much lemon juice should be used with 40 mℓ of olive oil to prepare the salad dressing?
    - ❖ What is the unit of measure for the ratio of olive oil to lemon juice?

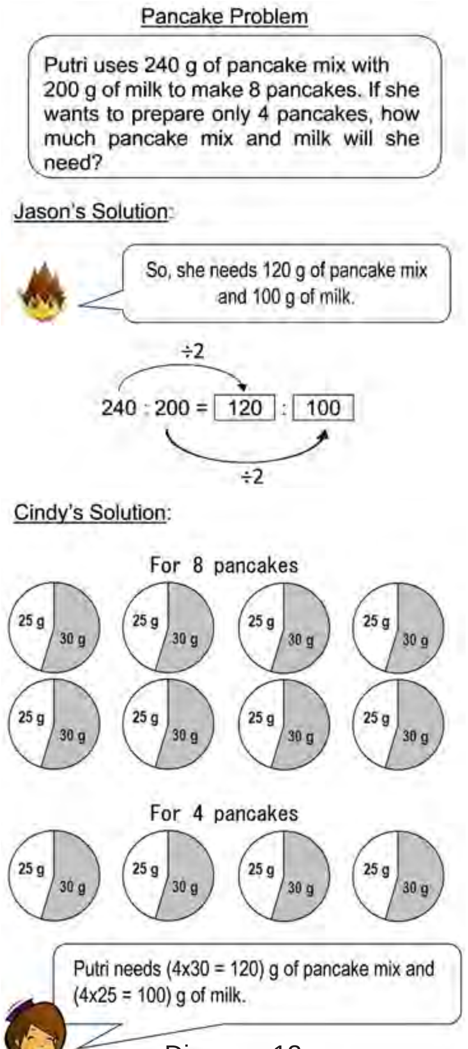
- Let *a* represent the volume of olive oil and *c* represents the number of teaspoons of salt.
  - ❖ Write the ratio $\frac{a}{c}$ in its simplest form.
  - ❖ Rewrite the ratio in the form of $a : c$.
  - ❖ How much salt should be used with 40 mℓ of olive oil to prepare the salad dressing?
  - ❖ What is the unit of measure for the ratio $\frac{a}{c}$?

vi. What are the major difference between the ratios $\frac{a}{b}$ and $\frac{a}{c}$?

vii. The recipe for salad dressing is the quantity for 6 people.

- What should be the quantity used in the recipe for 18 people?

viii. Diagram 12 shows a problem involving pancakes and two students' solutions to the problem.



Pancake Problem

Putri uses 240 g of pancake mix with 200 g of milk to make 8 pancakes. If she wants to prepare only 4 pancakes, how much pancake mix and milk will she need?

Jason's Solution:

So, she needs 120 g of pancake mix and 100 g of milk.

$$240 : 200 = \boxed{120} : \boxed{100}$$

Cindy's Solution:

Putri needs (4x30 = 120) g of pancake mix and (4x25 = 100) g of milk.

Diagram 12

- Why did Jason divide the ratio by 2?
- How did Cindy get 30 g of pancake mix and 25 g of milk for each cake?
- Compare Jason's and Cindy's solutions.
  - ❖ Which is easier for your students to understand the meaning of proportion? Justify your decision.

ix. Find the missing numbers in the (   ) for the following proportions.

- 4 : 6 = (   ) :150
- 3 : 7 = 63 : (   )
- 120 : (   ) = 10 : 18
- (   ) : 240 = 3 : 8

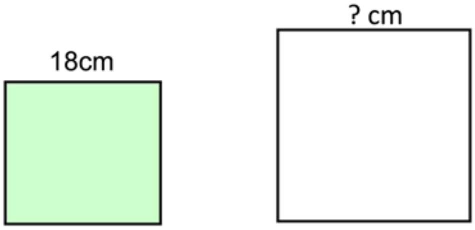


Diagram 13

x. Diagram 13 shows two squares. The proportion of the length of the sides is 3 : 4.

- When the length of a side of the smaller square is 18 cm, what is the length of the corresponding side of the larger square?
- If the length of a side of the larger square is 64 cm, how many cm is the length of the side of the smaller square?
- Is it possible to make the square becomes a rectangle with the same proportion?
    - ❖ Explain your answer with an example.

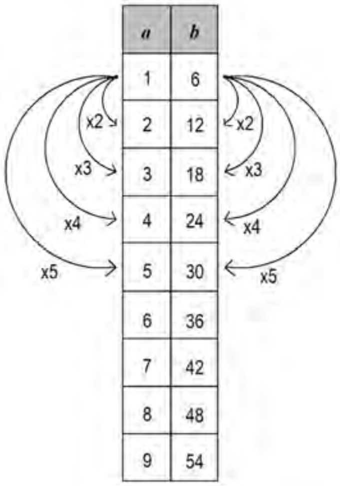
Standards 6.2:

Extending proportional reasoning to proportion

i. Extend proportional reasoning on multiplication tables as equal ratios and understand proportions
ii. Understand proportion by multiple and a constant quotient, not changing the value of the ratio
iii. Demonstrate simple inverse proportion by constant product
iv. Express proportion in a mathematical sentence by letters and graph
v. Use properties of proportionality to predict and explain phenomena in daily life

Sample Tasks for Understanding the Standards

Task1: Exploring Ratios in Multiplication Tables

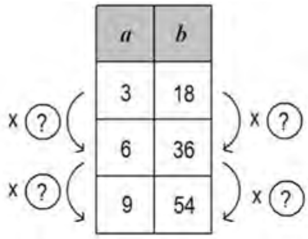


a	b
1	6
2	12
3	18
4	24
5	30
6	36
7	42
8	48
9	54

Diagram 1

i. Diagram 1 shows the multiplication table for the number 6.

- Study the first five pairs of numbers a and b.
  - ❖ What pattern do you observe in each successive pair of a and b?
  - ❖ Does the pattern also hold for the next four successive pairs of a and b?
  - ❖ Why do you think such a pattern exists?

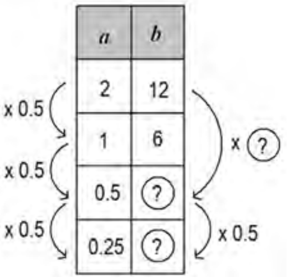


a	b
3	18
6	36
9	54

Diagram 2

ii. Pick any pairs of a and b such as the example shown in Diagram 2.

- Find the missing number in each (?).
- What pattern do you observe?
- Does the pattern also hold for any other pairs of a and b?
- Why do you think such a pattern exists?



a	b
2	12
1	6
0.5	(?)
0.25	(?)

Diagram 3

iii. The pairs of a and b are extended beyond the multiplication table as shown in Diagram 3.

- Find the missing number in each (?).
- Does the pattern in Diagram 2 still exist? Why or why not?

iv. The quotient $\frac{b}{a}$ is the ratio of b to a for each pair of a and b.

- What can you conclude about these ratios?
- What about the ratio of a to b?

7	84
13	?

Diagram 4

v. Diagram 4 shows two pairs of numbers from another different multiplication table.

- Find the missing number ?.
- Explain your reasoning for finding the missing number.

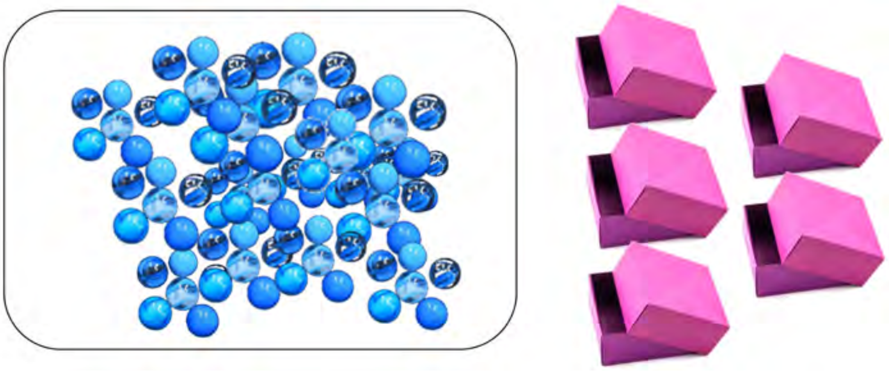


Diagram 5

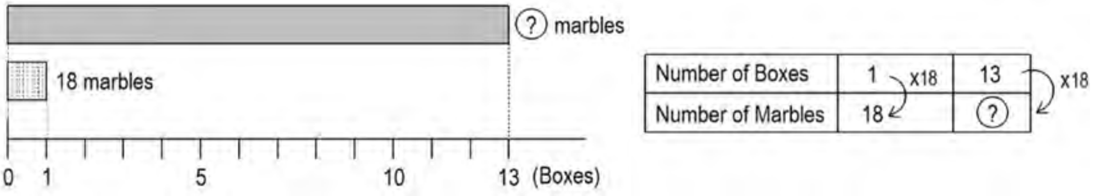
vi. Diagram 5 shows some marbles that will be kept in fancy boxes. Each box can keep 18 marbles. Table 1 shows the number of marbles for 1 to 5 boxes.

Table 1
Number of Marbles in Boxes

Number of Boxes	[0]	1	2	3	4	5
Number of Marbles	[0]	18	36	54	72	90

- The numbers 18, 36, 54, 72 and 90 are multiples of 18.
  - ❖ Explain the additive property in these multiples.
    Why is zero significant in the additive property?
  - ❖ Explain the multiplicative property in these multiples.
    Why is zero problematic in the multiplicative property?

- The number of marbles is said to be proportional to the number of boxes.
    - ❖ Find the ratio of the number of marbles to the number of boxes for each pair of numbers in Table 1 except the ‘zero’ pair.
    - ❖ What can you conclude about the ratios?
    - ❖ If there are 15 boxes, how many marbles are there?
    - ❖ How many boxes are needed to keep 240 marbles?



Number of Boxes	1	13
Number of Marbles	18	?

Diagram 6

- Diagram 6 shows a proportional number line drawn to determine the number of marbles kept in 13 boxes.
    - ❖ Using the Rule of Three, find the missing number in ?.
- Draw a proportional number line and use the Rule of Three to determine the number of boxes needed to keep 414 marbles?

Task 2: Proportions

i. A group of students collected some data to study the relationship between the number of marbles and mass. Table 2 shows the data collected.

- What pattern do you observe between x and y in Table 2?

Table 2
Number and Mass of Marbles

Number of marbles, x (pieces)	1	2	3	4	5	6
Mass, y (g)	8	16	24	32	40	48
Quotient, $\frac{y}{x}$	8					

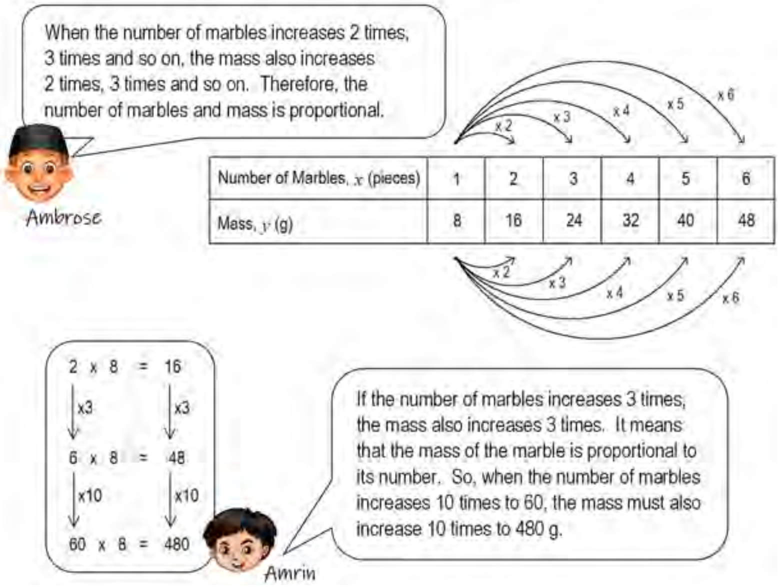


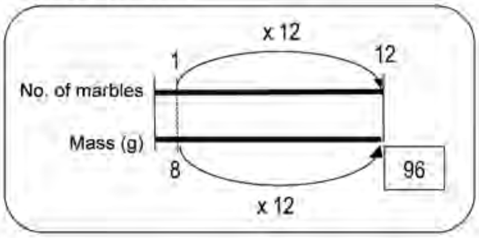
Diagram 7

- Ambrose and Amrin reported an interesting observation as shown in Diagram 7.
  - ❖ What does it mean by the mass of marbles is proportional to the number of marbles?
- For each pair of x and y, calculate the quotient $\frac{y}{x}$ and fill in the blank in Table 2.
  - ❖ Write a mathematical sentence involving x and y to represent the proportional relationship.
  - ❖ Use the mathematical sentence to find

    (a) the mass of 15 marbles.

    (b) the number of marbles when the mass is 360 g.

Rudy's Solution:



Prakash's Solution:

4 marbles → 32 g
8 marbles → (2 x 32) g = 64 g
12 marbles → (32 + 64) g = 96 g

Yaris' Solution:

6 marbles → 48 g
↓ x 2 ↓ x 2
12 marbles → (2 x 48) g = 96 g

Diagram 8

- Miss Sarsi asked her students to find the mass of 12 marbles. Diagram 8 shows three solutions from her students.
- Whose solution is the easiest to show the proportion? Why?

ii. Tables 3 and 4 show two sets of data.

Table 3
Distance Travelled with Volume of Petrol

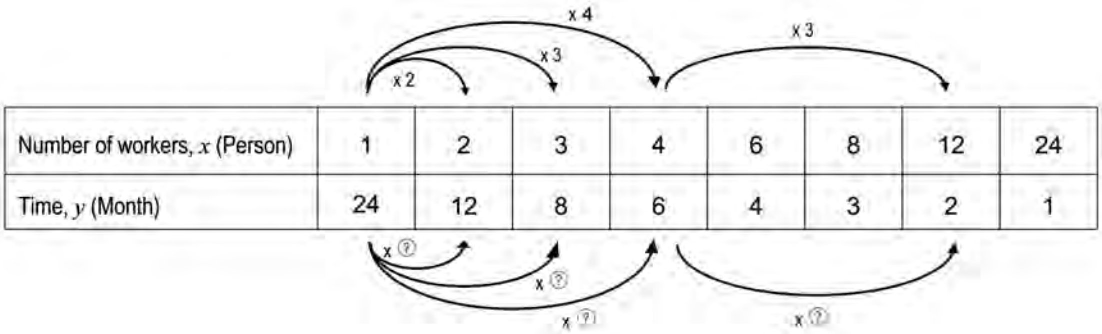
Volume of Petrol, x (ℓ)	1	2	4	6	9
Distance travelled, y (km)	9	18	36	54	81
Quotient, $\frac{y}{x}$	9				

Table 4
Temperature Change with Time

Time, t (min)	0	2	5	8	10
Temperature, T (°C)	32	52	85	100	100
Quotient, $\frac{T}{t}$					10

- Calculate the quotient, $\frac{y}{x}$, and fill in the blank in Table 3.
- Calculate the quotient, $\frac{T}{t}$, and fill in the blank in Table 4.
- Which table shows two quantities that are in direct proportion? Justify your decision.

Task 4: Inverse Proportion



Number of workers, x (Person)	1	2	3	4	6	8	12	24
Time, y (Month)	24	12	8	6	4	3	2	1

Diagram 9

i. Diagram 9 shows the duration of time in months, required by different numbers of workers to build a house.

- Find the missing number in each ?.
- If the value of x changes 2 times, 3 times and so on, how does the value of y change?

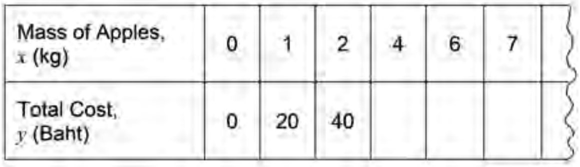
Table 5
Number of Workers and Duration of Time to Build a House

Number of workers, x (Person)	1	2	3	4	6	8	12	24
Time, y (Month)	24	12	8	6	4	3	2	1
Product, $x \times y$	24							

- For each pair of x and y, calculate the product $x \times y$ and fill in the blank in Table 5.
  - ❖ Fill in the blank in Table 5.
  - ❖ What pattern do you observe between the products of x and y?
- In this case, y is said to be inversely proportional to x.
  - ❖ Write a mathematical sentence involving x and y to represent the relationship.
  - ❖ Use the mathematical sentence to solve the following problems.
    - (a) If 10 workers are available to build the house, how many months will it take to complete the house?
    - (b) If we want to complete the house in 5 months time, how many workers do we need?

Task 5: Graphs of Proportion

Table 6
Mass of Apples and Total Cost



Mass of Apples, x (kg)	0	1	2	4	6	7	
Total Cost, y (Baht)	0	20	40				

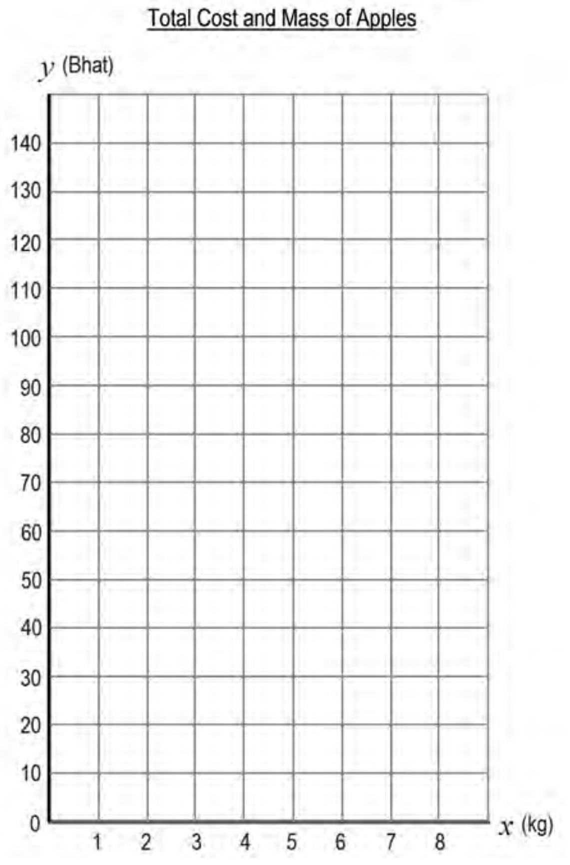


Diagram 10

i. A supermarket in Chengmai is having a “Crazy Sale” to sell apples by kilogrammes. 1 kg of apple is sold for 20 Thai Baht. Table 5 shows the mass of apple x, and the total cost y.

- Find each missing value of y and fill in each blank in Table 6.
- What is the relationship between the x and y?
- Write a mathematical sentence to represent the relationship between x and y.
- Plot points that represents all pairs of values, the value of x and its corresponding value of y, on the graph in Diagram 10.
- Connect the points with a line.
  - ❖ What kind of line do you get?
  - ❖ Use the graph to find the cost of 3 kg and 5 kg of apples, respectively.
- Explain how the graph can be used to find the cost of 8 kg of apples.

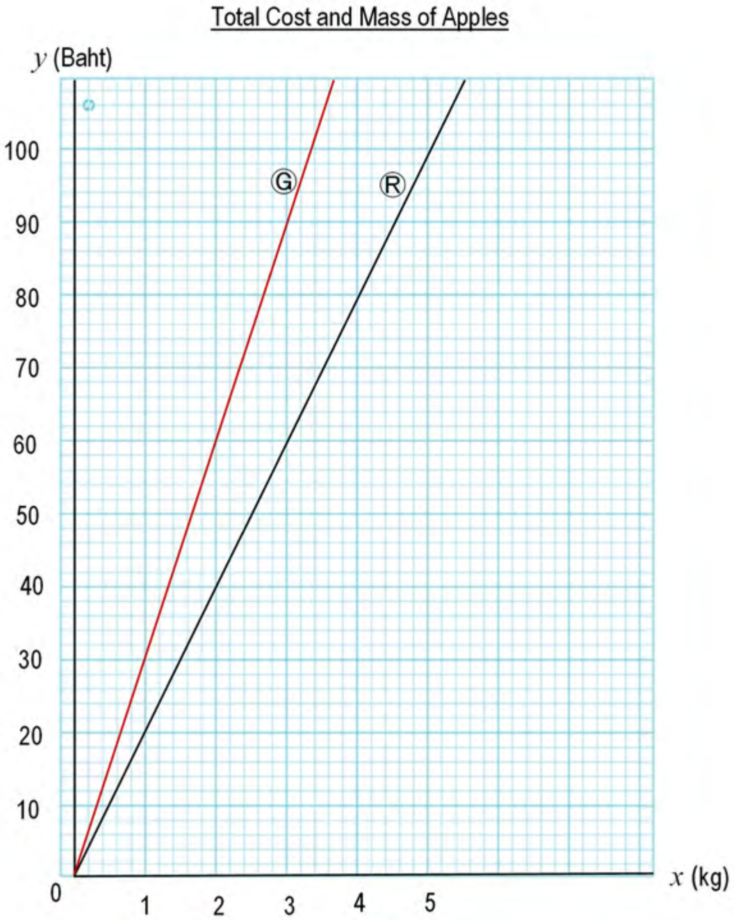


Diagram 11

ii. Diagram 11 shows graph Ⓖ and graph Ⓡ, representing the relationship between the mass of apples, x in kg and the cost, y in Thai Baht for green apples and red apples, respectively.

- Which type of apple is more expensive? Explain how you find it from the graph.
- Aki has 100 Baht only.
  - ❖ How many kg of red apples can he buy?
  - ❖ How many kg of green apples can he buy?
- Sui has 140 Baht only. She wants to spend all the money to buy apples.
  - ❖ Explain how the graphs can be used to find different combinations of green apples and red apples that she can buy.

Task 6: Graph of Inverse Proportion

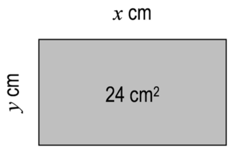


Diagram 12

i. Diagram 12 shows a rectangle with length, x cm and width, y cm. The rectangle has a fixed area of 24 cm². Table 7 shows some combinations of different values of x and y.

Table 7
Length and Width of a Rectangle with a Fixed Area of 24 cm²

Length, x (cm)	1	2	3	4	6	8	12	24
Width, y (cm)	24	12	8					

- Find and fill in each blank with the missing value of y in Table 7.

- What relationship do you observe between x and y?

- Plot a point that represents each pair of values, the value of x and its corresponding value of y, on the graph in Diagram 13. The first three points have been plotted for you.

  ❖ Connect the points with straight line.

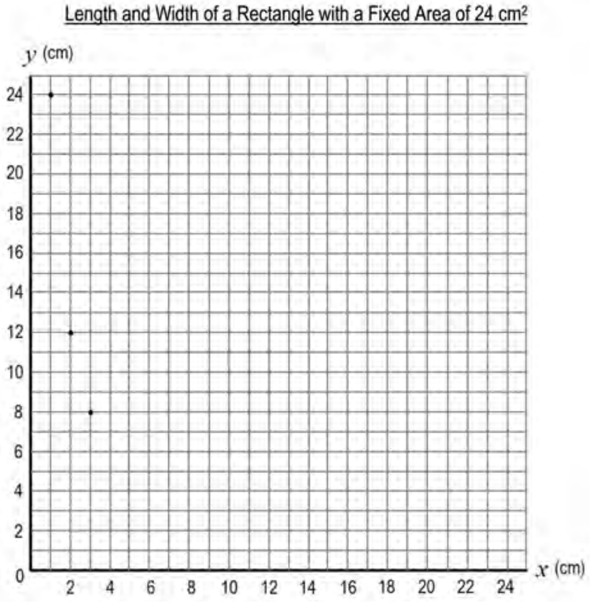


Diagram 13

- Compare the graph with the graph of direct proportion in Diagram 10.

  ❖ Describe the differences between the two graphs.

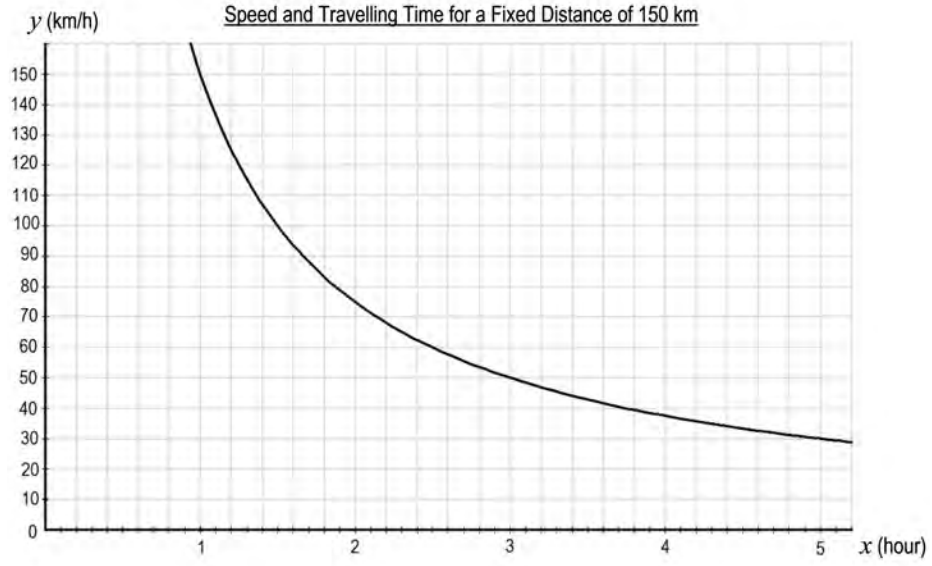


Diagram 14

ii You leave Kuala Lumpur and use the PLUS highway to drive to Melaka, the historical city of Malaysia. The distance between Kuala Lumpur and Melaka is 150 km. Diagram 14 shows the graph of speed, y (km/h) and travelling time, x (hour).

- What is the relationship between the speed, y and the travelling time, x?
  - ❖ Write a mathematical sentence to represent the relationship.
- Use the graph to solve the following problems.
  - ❖ If you want to arrive in Melaka in 1 hour 30 minutes, how fast in km per hour do you need to drive?
  - ❖ If you drive at 75 km/h, how long will the journey take?
  - ❖ Check your solutions using the mathematical sentence that represent the relationship between y and x.

Task 7: Making Prediction with Properties of Proportionality

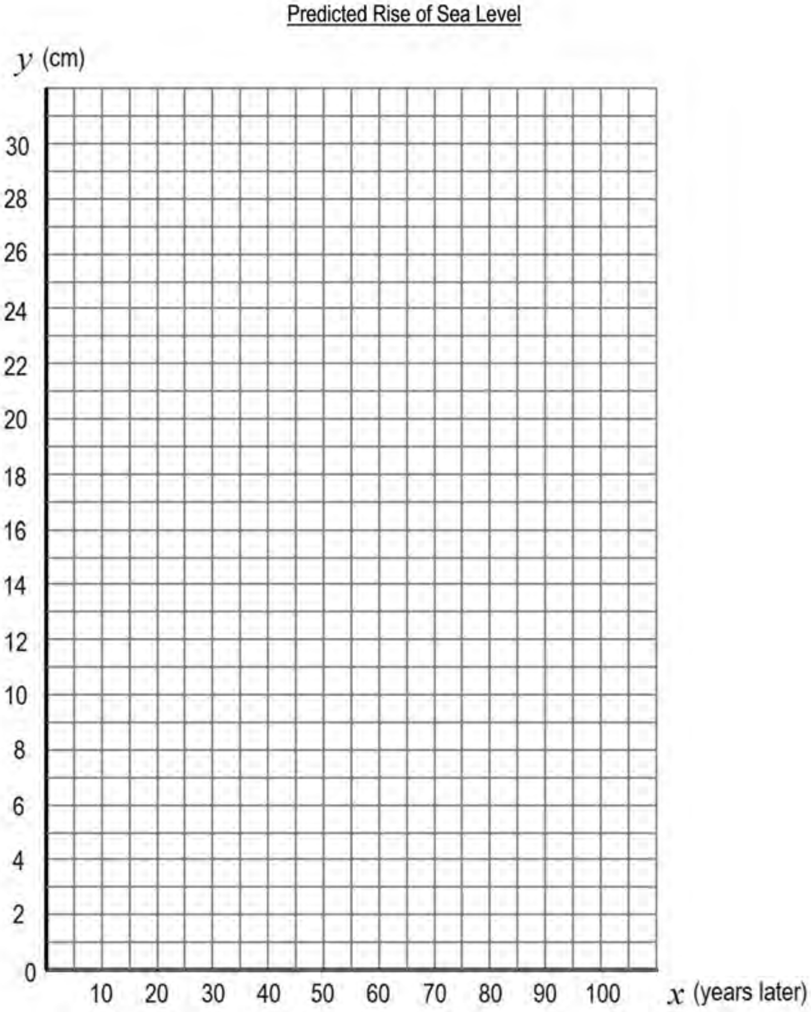


Diagram 15

Global warming is causing global sea level to rise mainly due to the melting of mountain glaciers and ice in the North and South poles. According to Climate Change Knowledge Portal, the current rise is approximately 3 mm per year.

[Source: https://climateknowledgeportal.worldbank.org/country/malaysia/impacts-sea-level-rise]

i. It is predicted that the sea level will rise at least 3 cm in next 10 years. Based on this prediction, draw a graph of rise of sea level, y cm and time, x years later on Diagram 15.

ii. Use the graph to make the following prediction.

- After how many years, will the land that is 24 cm above the sea level now be covered by the sea completely?
- After 100 years, how many cm will the sea level rise?

Topic 7: Producing New Quantities Using Measurement Per Unit

Standards 7.1:

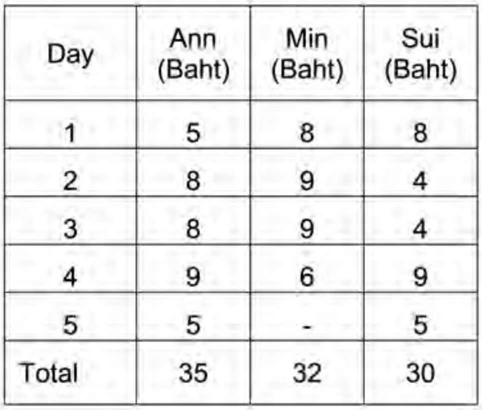
Producing new quantities using measurement per unit

i. Introduce average as units for distribution and comparison of different sets of values
ii. Introduce population density with the idea of average and appreciate it for comparison
iii. Introduce speed with the idea of average and appreciate it for comparison
iv. Appreciate using diagrams such as number lines and tables to decide the operations on the situations of measurement per unit quantity
v. Compare the context of different quantities with the idea of average as a rate
vi. Apply the idea of measurement per unit quantity in a different context

Sample Tasks for Understanding the Standards

Task1: Understanding Average

Table 1
Students Saving from Pocket Money



Day	Ann (Baht)	Min (Baht)	Sui (Baht)
1	5	8	8
2	8	9	4
3	8	9	4
4	9	6	9
5	5	-	5
Total	35	32	30

Diagram 1

i. Table 1 shows the pocket money saved by three students over five days. Min was absent for one of the days.

- Compare the saving of the three students.
  - ❖ Who had saved the most pocket money?
- John and Jason had different opinions on who had saved the most pocket money. Their argument is shown in Diagram 1.
  - ❖ What is the difference between John's and Jason's opinions?
  - ❖ Whose argument do you support? Why?

Cindy's Method

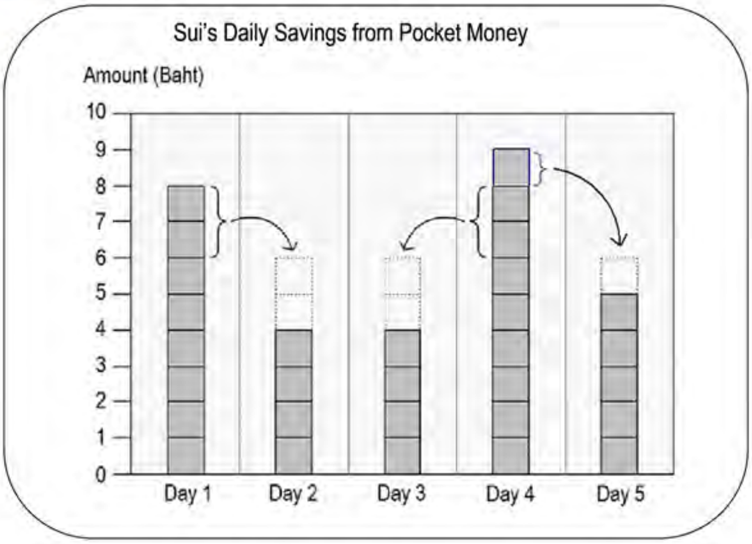


Diagram 2

ii. Diagram 2 shows Cindy's method to find the average daily saving of Sui by drawing a bar graph.

- Explain Cindy's method.
- Find the average saving of Sui from the diagram.

iii. Use Cindy's method to draw a separate bar graph on Diagram 3 to find the average daily saving of Ann and Min, respectively.

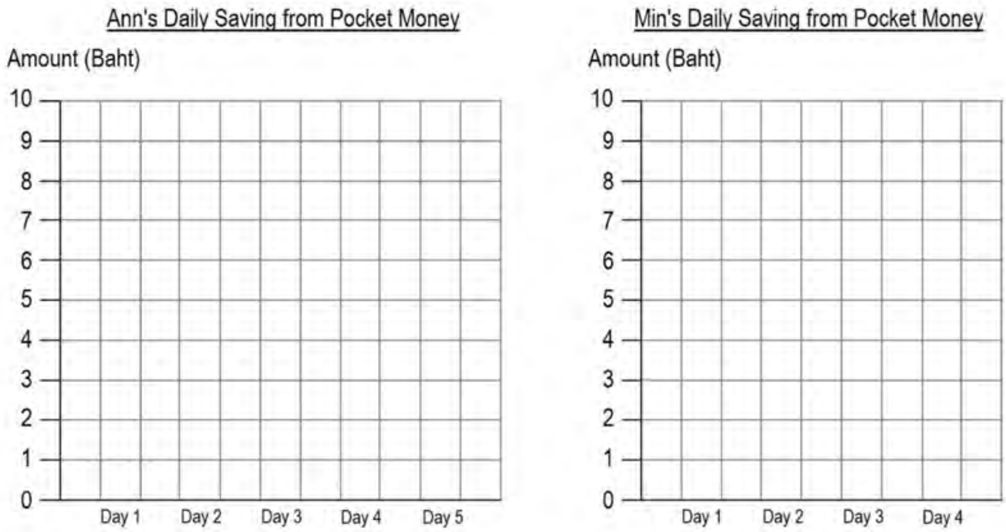


Diagram 3

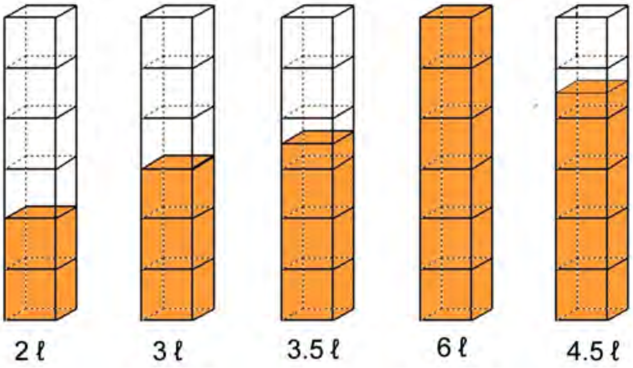


Diagram 4

iv. There is some orange juice in five containers as shown in Diagram 4.

- What difficulties may your students face if they are to use Jason's method to find the average amount of orange juice?

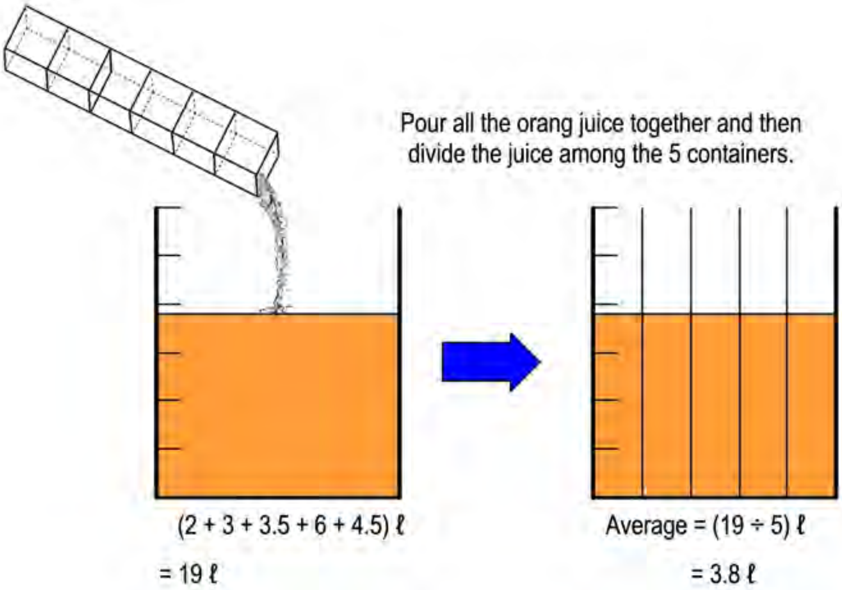
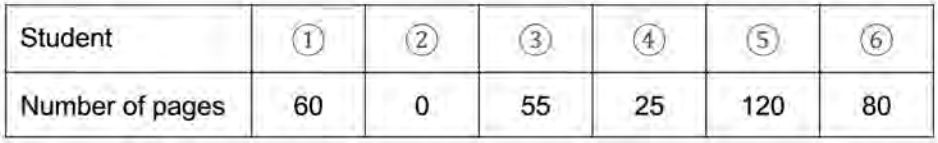


Diagram 5

- Diagram 5 shows Jason's idea for finding the average amount of orange juice.
  - ❖ What formula for finding the average can you derive from Jason's method?
  - ❖ What are the strengths and weaknesses of Jason's method compared to Cindy's method?

v. The same number which is averaged from some numbers is also called the mean of the original numbers. Table 2 shows the number of pages of the storybook read by 6 students in a week.

Table 2
Number of Pages of Story Book Read



Student	①	②	③	④	⑤	⑥
Number of pages	60	0	55	25	120	80

- When asked to calculate the mean number of pages read by the six students, Jason and John answered 68 and 56.7 pages, respectively.
  - ❖ Why are there different answers given as the mean value?
  - ❖ What is the correct value of the mean for the 6 students?
- How many pages of storybook must student ② read so that the mean will be 70 pages?

Task 2: Average as Population Density

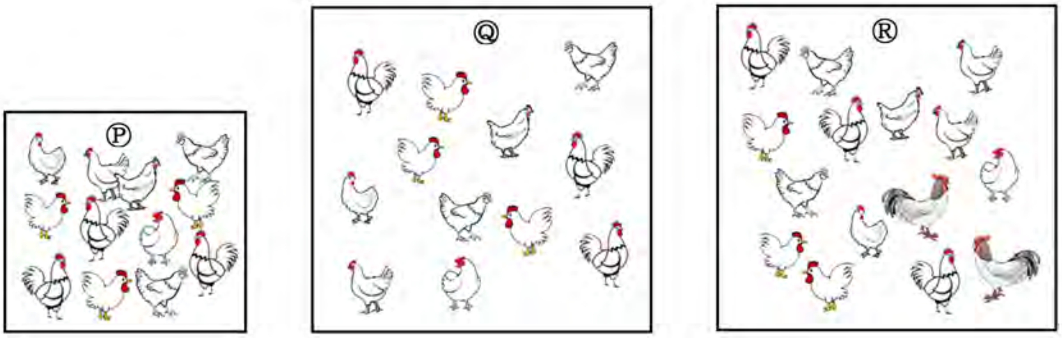


Diagram 6

i. Diagram 6 shows some chickens in enclosures Ⓟ, Ⓠ and Ⓡ. Enclosure Ⓟ, Ⓠ and Ⓡ has an area of 4 m^2, 9 m^2 and 9 m^2, respectively.

- Between Ⓟ and Ⓠ, which one is more crowded? Explain your reasoning.
- Between Ⓠ and Ⓡ, which one is more crowded? Explain your reasoning.
- Between Ⓟ and Ⓡ, which one is more crowded? Explain your reasoning.

ii. Crowdedness of the chicken is affected by two measures, the number of chickens and the area of the enclosure.

- How many chickens are there per 1 m^2 for each enclosure? Show your calculations.
- The number of chickens per 1 m^2 expresses the mean of crowdedness. Fill in the blank for the following two statements.
  - ❖ If the value of the mean of crowdedness is higher, the level of crowdedness is ____.
  - ❖ If the value of the level of crowdedness is lower, the mean of crowdedness is ____.
- Based on your calculations, which enclosure is the most crowded?

iii. Table 3 shows the population and area of 3 capital cities of ASEAN countries.

Table 3
Population and Land Area of Three ASEAN Capital Cities

City	Population (people)	Area (km^2)
Kuala Lumpur	1 982 112	243
Singapore	5 938 796	700
Bangkok	10 720 000	1 568

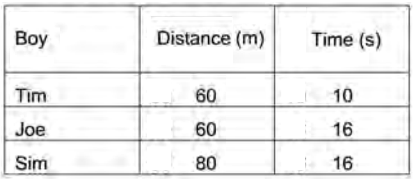
- Population density is the number of people per square kilometre.
  - ❖ Find the population density of the three cities, respectively.
- How is population density related to the level of the crowdedness of a city?
- Based on the population densities, which city is the most crowded?

iv. Explain how average is related to population density as a measurement per unit quantity.

Task 3: Average as Speed

Three boys, Joe, Tim and Sim were running in a field. Table 4 shows the distance covered in metres (m) and the time taken in seconds (s) by the three boys.

Table 4
Distance Run and Time Taken



Boy	Distance (m)	Time (s)
Tim	60	10
Joe	60	16
Sim	80	16

i. Compare the distance covered and the time taken by the three boys.

- Who ran faster between Tim and Joe? Explain your reason.
- Who ran faster between Joe and Sim? Explain your reason.
- Why is it more difficult to compare who ran faster between Tim and Sim?

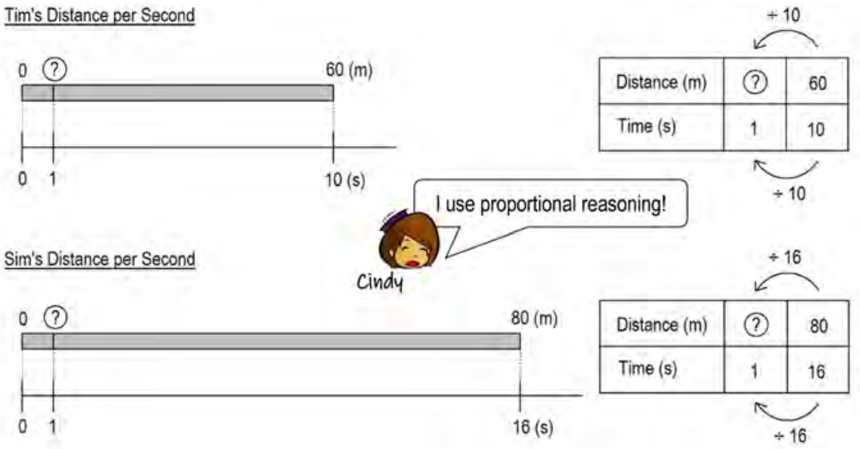


Diagram 7

ii. Cindy calculated the average distance per second for Tim and Sim to make a comparison as shown in Diagram 7.

- Explain how proportional reasoning is used by Cindy.
- Find the missing value ?.
- Who ran faster between Tim and Sim?

iii. Speed is an example of measurement per unit.

- How is speed expressed in this case?
- What is the measurement unit for speed in this case?
- Give some other examples of measurement units for speed?

Task 4: Applying Measurement Per Unit Quantity

Finding Average Speed

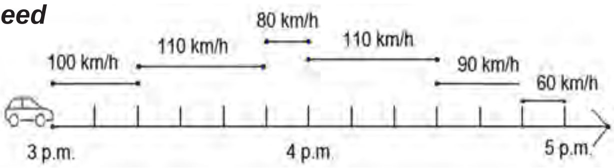


Diagram 8

i. Diagram 8 shows the speed of a car that changes with situations of traffic on the road. The number line shows the speed of the car travelling on a road for 2 hours.

- What is the average speed of the car? Show how you calculate.
- Why is the average speed necessary to be used in telling the speed of a travelling car?

ii. Town P is 150 km away from town Q. The highway connecting the two towns has a speed limit of 110 km/h.

- What is the shortest possible time for a car driver to travel from P to Q?
- Do you think the driver can reach Town Q as shown by your calculation? Why or why not?

Rate and Average

i. Ricky, Andy and Danny cycle to school every day. Table 5 shows the distance from home, time taken and cycling speed of the three students on a schooling day.

Table 5
Distance, Time and Cycling Speed of Three Students

Students	Distance from home	Time	Speed
Ricky	1.8 km	6 minutes	**? m/s**
Andy	1.2 km	**? minutes**	4 m/s
Danny	**? km**	12 minutes	2.5 m/s

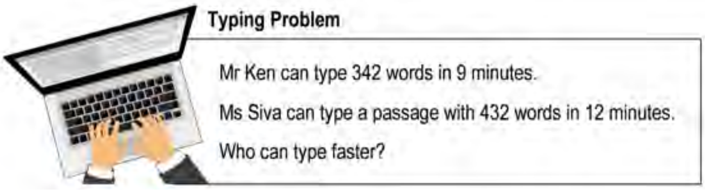
- Find the missing value for each question mark '?'.

ii. Rate can be expressed as a quantity measured per unit of another quantity.

- Explain the meaning of each of the following examples of rate.
  - ❖ A speed of 90 km/h.
  - ❖ Water flows from a pipe at 2.5 litres per second.
  - ❖ A tree grows at 58 cm per year.

iii. How is the rate related to the average? Illustrate your answer with a suitable example.

Problem Solving with Measurement Per Unit Quantity



Typing Problem

Mr Ken can type 342 words in 9 minutes.

Ms Siva can type a passage with 432 words in 12 minutes.

Who can type faster?

Diagram 9

i. Ms Kana gave a problem to her students as shown in Diagram 9.

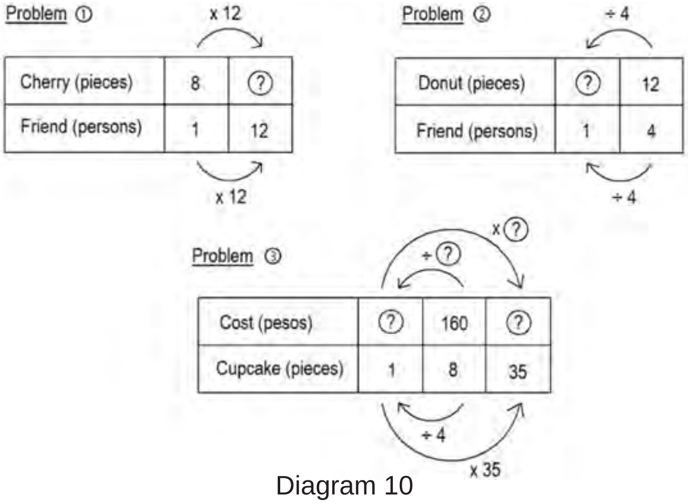
- Explain how the idea of a number of words per unit of time can be used to solve the problem.
- Find the rate of typing for Mr Ken and Ms Siva, respectively.

ii. Aida is preparing food and drink for her birthday party. Consider the following problems she needs to solve in the preparation.

Problem ①: Each friend is allocated 8 cherries. How many cherries are needed for 12 friends?

Problem ②: She prepares 12 pieces of doughnuts for 4 special friends coming to the party. How many doughnuts will each special friend get?

Problem ③: The cost of 8 cupcakes is 160 Philippines pesos. How much will she need to buy 35 cupcakes?



Problem ①

Cherry (pieces)	8	?
Friend (persons)	1	12

Problem ②

Donut (pieces)	?	12
Friend (persons)	1	4

Problem ③

Cost (pesos)	?	160	?
Cupcake (pieces)	1	8	35

Diagram 10

- These are problems related to measurement per unit quantity. Aida works out the solutions for the problems using the Rule of Three as shown in Diagram 10.
  - ❖ Find the missing value for each ?.
- Sequence the tasks to show the development of ideas on measurement per unit quantity.

Topic 8: Investigating the Area of a Circle

Standards 8.1:

Areas of a circle are discussed through the relationship between the radius and the circumference

i. Investigate the relationship between the diameter of a circle and its circumference using the idea of proportion
ii. Investigate the area of a circle by transforming it into a triangle or parallelogram and find the formula of the circle
iii. Estimate the area of inscribed and circumscribed shapes based on a known formula of area
iv. Enjoy estimating the area of irregular shapes with fluency in life

Sample Tasks for Understanding the Standards

Task1: Relationship Between Diameter and Circumference of Circular Objects

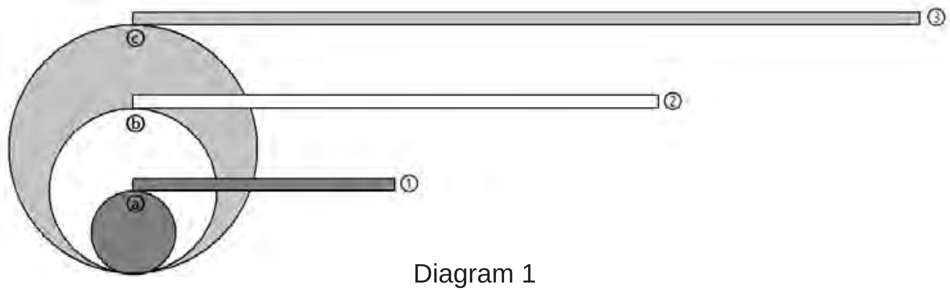


Diagram 1

i. Jason drew three circles, ⓐ,ⓑ, ⓒ on cardboards with diameters 10 cm, 20 cm and 30 cm respectively as shown in Diagram 1. Then, he rolled the circles one complete rotation and investigate how far each of them advanced as shown by tapes ①, ②, ③ in the diagram.

- What do you think the distance rolled is related to?

- If Jason cut out another circle with a diameter of 40 cm, estimate the distance rolled for this circle.

- Use cardboard to draw and cut out the four circles. Roll each circle one complete rotation to find the distance advanced and complete Table 1.

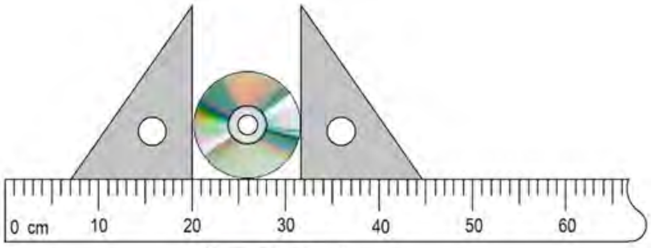
Table 1
Distance Advanced for One Complete Rotation

	ⓐ	ⓑ	ⓒ	
Diameter (cm)	10	20	30	40
Circumference (cm)				

- How does the circumference of a circle change when its diameter increases 2 times, 3 times and 4 times?

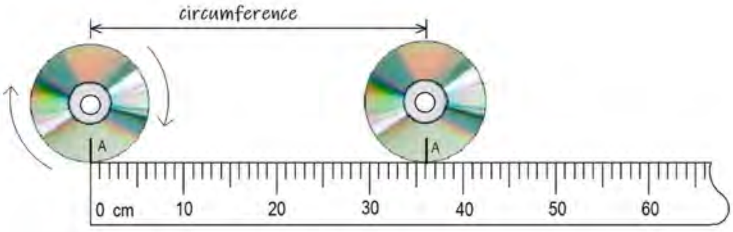
Diagram 2

ii. Jason tried to investigate further the relationship between the circumference and diameter of a circle by taking some measurements of some circular objects as shown in Diagram 2. He measured the circumference and diameter of each object using the methods shown in diagrams 3 and 4. The measurements are shown in Table 2.



Measure the diameter.

Diagram 3



Measure the circumference.
Roll the round object one complete rotation and measure how far it moves.

Diagram 4

Table 2
Circumference and Diameter of Round Objects

Objects	Circumference (cm)	Diameter (cm)	$\frac{\text{Circumference}}{\text{Diameter}}$
Compact Disk	36.1	11.5	
Clock	53.4	16.9	
Plate	72.5	23.1	

- Calculate the ratio $\frac{\text{Circumference}}{\text{Diameter}}$ for each circular object.

  (Give your answer to the nearest hundredth by rounding the thousandth.)

- Approximately, how many times is the diameter to the circumference?

- Based on your calculation, what conclusion can you make on the relationship between the circumference and diameter of a circle?

Note: The ratio $\frac{\text{Circumference}}{\text{Diameter}}$ is the same value for all circles regardless of the circles' sizes. This ratio for a circle is named *pi*, represented by the symbol π. It is a number that continues infinitely like 3.14159 … and we usually take 3.14 as its approximate value in the calculation.

iii. Jason concluded the following formula for any circle from his investigation.

Circumference = Diameter x 3.14

- Based on this formula, explain what it means by "the circumference of a circle is directly proportional to its diameter".

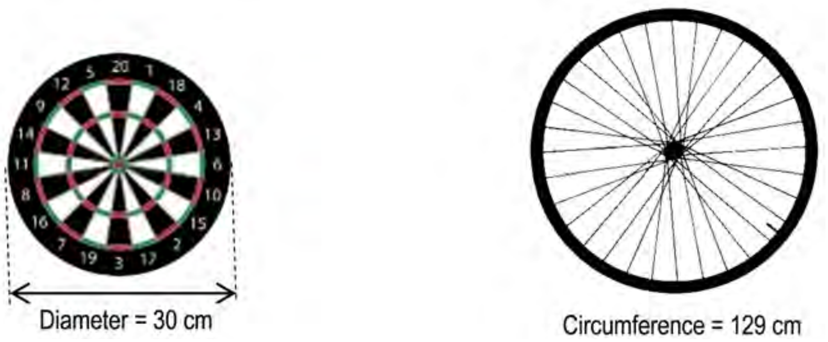


Diagram 5

iv. Diagram 5 shows another two circular objects, a dart board and a bicycle rim.

- Calculate the circumference of the dart board.
- Calculate the radius of the bicycle rim.

Task 2: Area of a Circle

Formula for the Area of a Circle

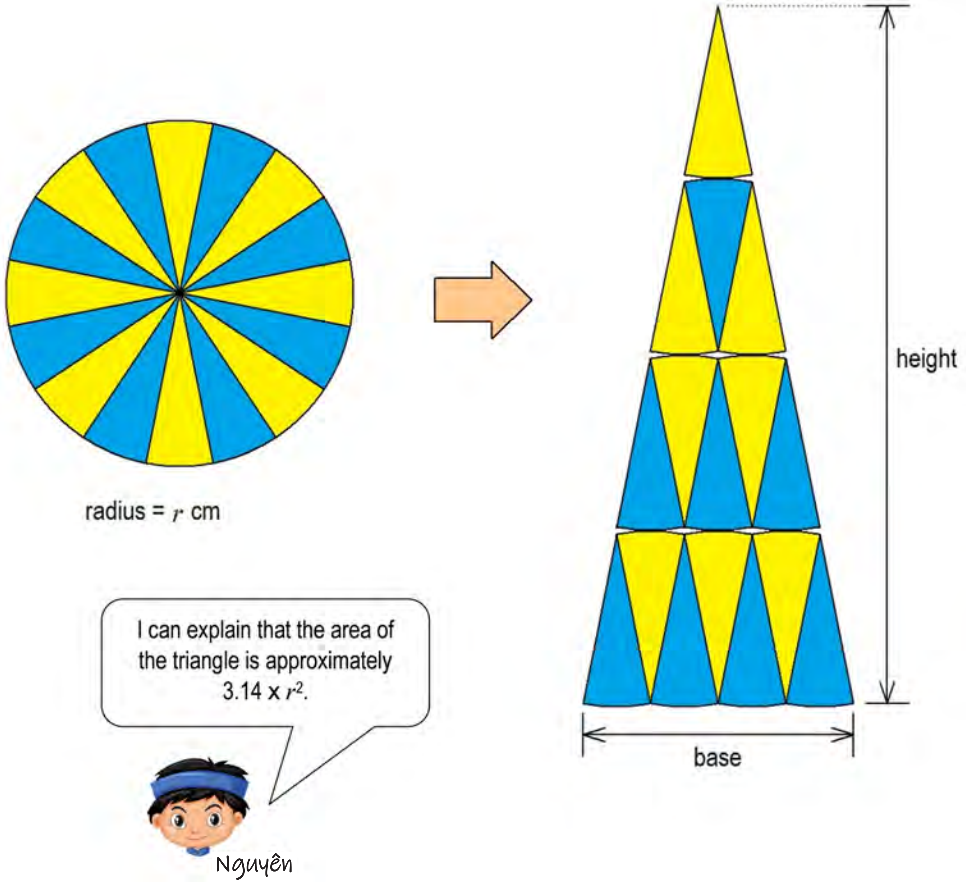


Diagram 6

i. Diagram 6 shows a circle divided into 16 equal parts. A student, Nguyên cut out and rearrange the parts into a triangle as shown in the diagram.

- If the radius of the circle is r cm, approximately how many cm will
  - The height of the triangle be?
  - The base of the triangle be?

    (Express your answer in terms of r.)
- Do you agree with Nguyên's formula for finding the area of the triangle?
  - Justify your decision.
  - How is the formula for finding the area of the circle related to this formula?
  - If the radius of the circle is 10 cm, find its area in cm².

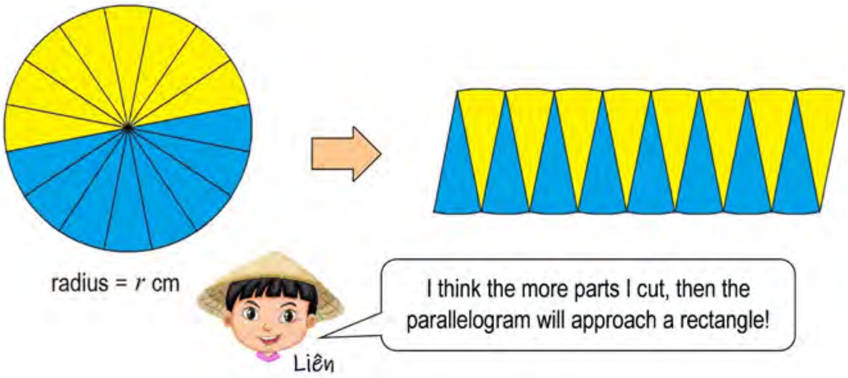


Diagram 7

ii. Another student, Liên, cut out and tried to rearrange the 16 parts into a rectangle. However, the new shape looks more like a parallelogram than a rectangle as shown in Diagram 7.

- What does Liên mean by the parallelogram will approach a rectangle?
- Do you agree with her idea? Justify your decision.

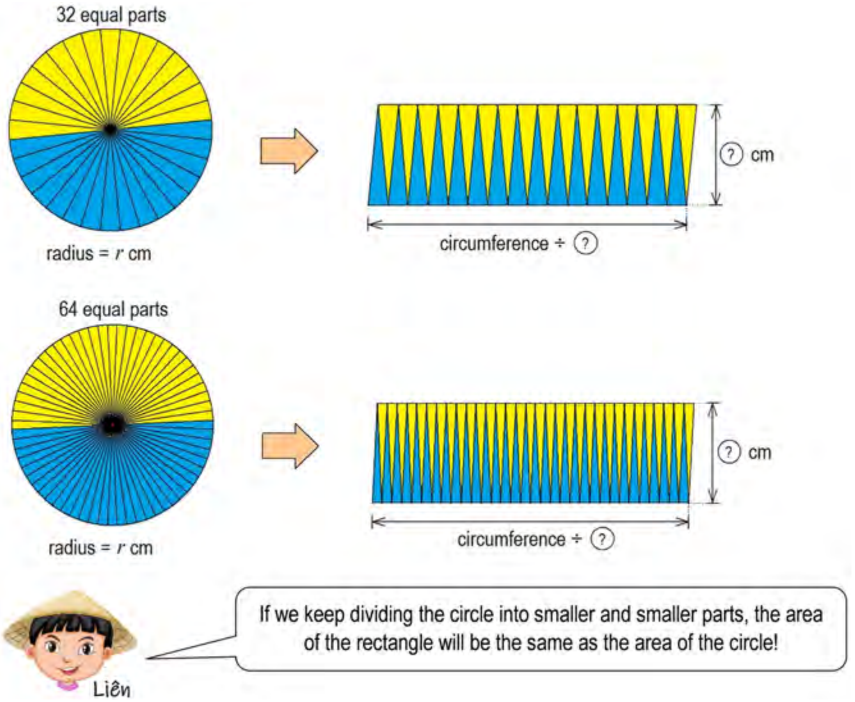


Diagram 8

iii. Liên tried to test her idea by cutting another two circles into 32 and 64 parts respectively. as shown in Diagram 8.

- Find the missing value for each ? in Diagram 8.
- Do you agree with Liên's conclusion the area of the rectangle will approach the area of the circle eventually? Why or why not?

iv. Based on Liên's conclusion, find the formula for the area of a circle with a radius of r cm.

- Find the area of the circle if its radius is 10 cm.

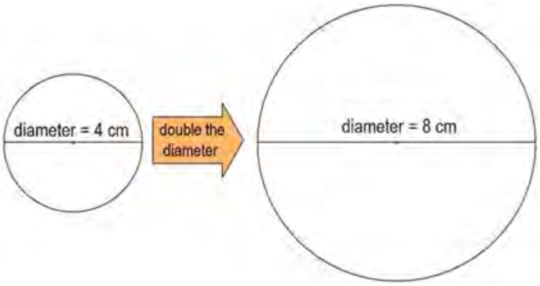


Diagram 9

v. Diagram 9 shows two circles. The diameter of the big circle is double the diameter of the small circle.

- What will happen to the circumference and the area of a circle when its diameter is double? Justify your answer with an appropriate calculation.
    - ❖ What if the diameter is tripled?

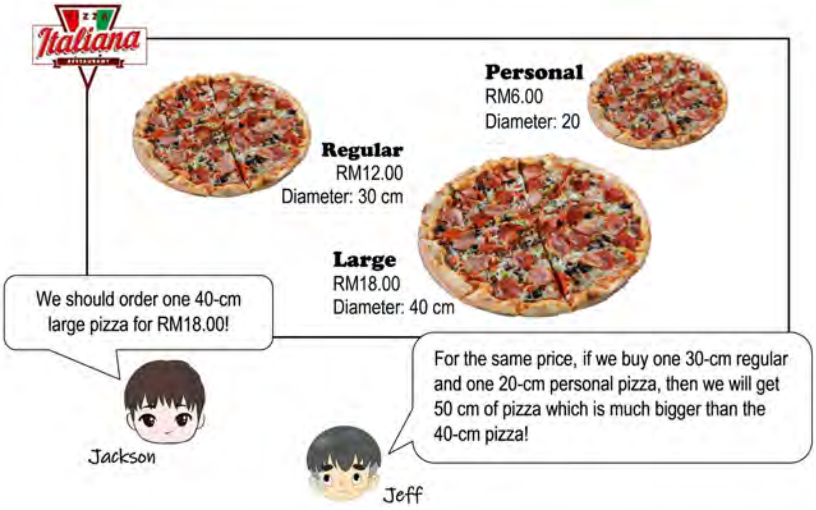


Diagram 10

vi. A restaurant in Kuala Lumpur sells pizza in three different sizes as shown in Diagram 10. Two brothers, Jackson and Jeff are given RM20 to spend in the restaurant for their lunch. Upon seeing the menu, the brothers have contradicting ideas on what pizza to order as shown in the diagram.

- Between the brothers, who has a misconception about the size of the pizza?
    - ❖ What is the misconception?
- What should the brothers order? Why?
- How would you help the brother correct the misconception?

Task 3: Finding Areas of Inscribed and Circumscribed Shapes

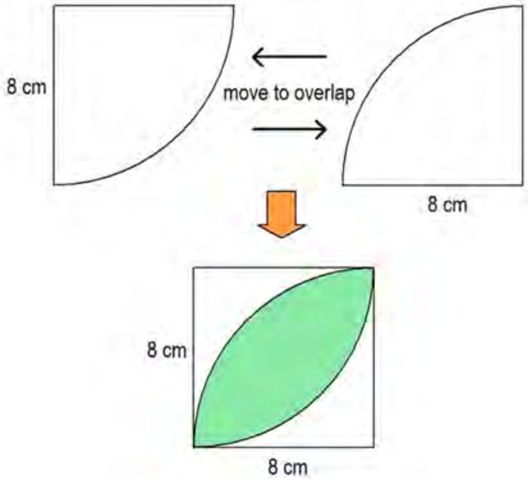


Diagram 11

i. Diagram 11 shows two quarter-circles overlapping to form a coloured region in a square.

- Find the area of the coloured region.
- Find the area of the uncoloured region.
- What are the subtasks that the students need to learn to solve the problem?
- How would you sequence the subtasks to facilitate your students' effort in solving the problem by themselves?

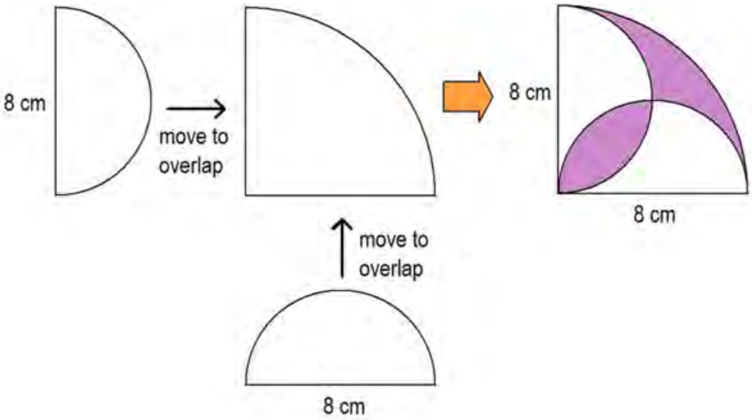


Diagram 12

ii. Diagram 12 shows two semi-circles overlap with a quarter circle to form a coloured region.

- Find the area of the coloured region.
- What are the subtasks that students need to learn to solve the problem?
- Why is task sequence important in developing students' ability to solve the problem?

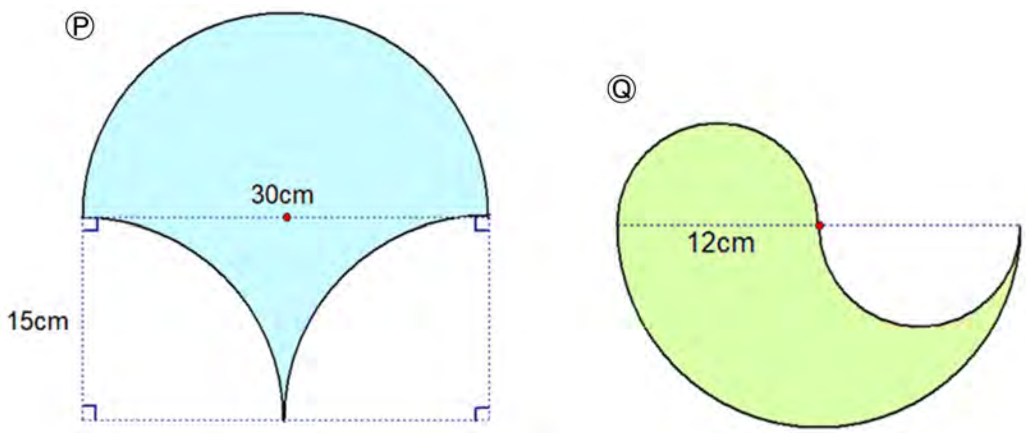
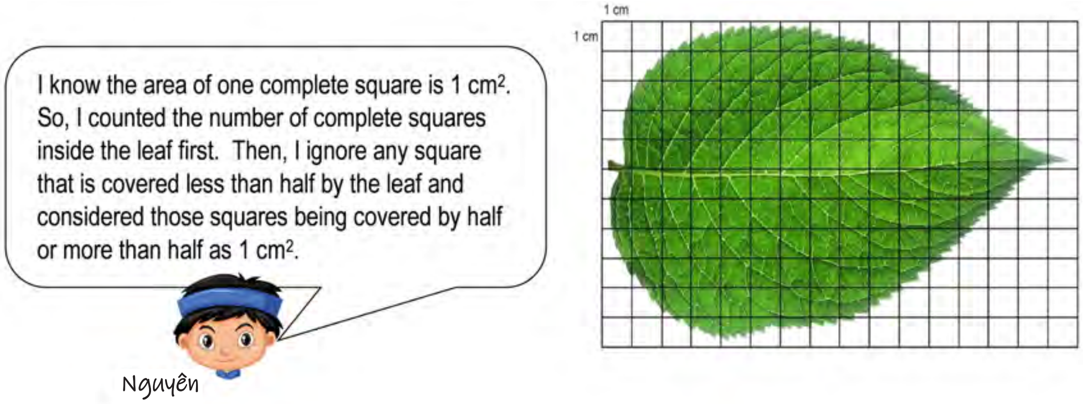


Diagram 13

iii. Find the area of the figure Ⓟ and Ⓠ in Diagram 13.

Task 4: Estimating Areas of Irregular Shapes

Nguyên's Method



Liên's Method

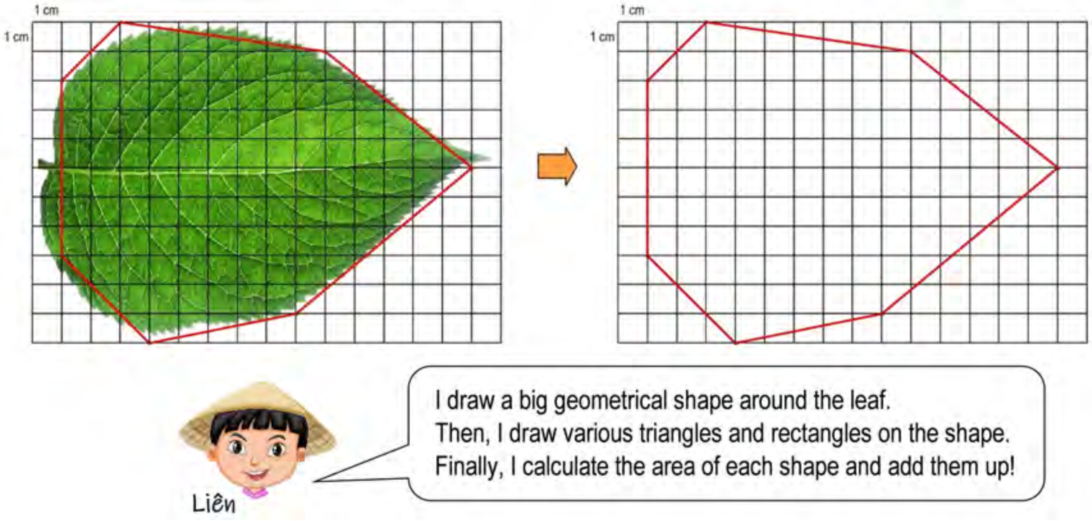


Diagram 14

i. Diagram 14 shows two students' methods to estimate the area of a leaf. Each square is 1 cm x 1 cm.

- Use Nguyên's method to estimate the area of the leaf.
- Use Liên's method to estimate the area of the leaf.

ii. Which method do you think is easier for your students? Justify your decision.

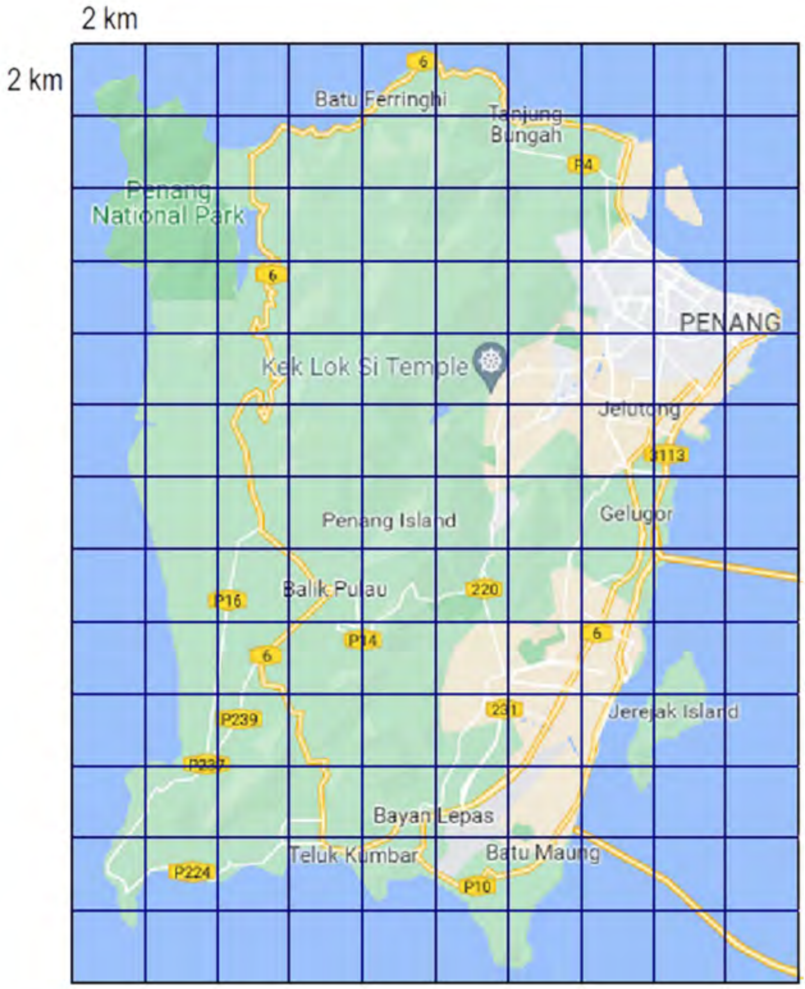


Diagram 15

iii. Diagram 15 shows the map of Penang Island. The measurement of each square is 2 km x 2 km.

- What is the area of a square, in square km.
- Estimate the area of Penang Island using Nguyên's and Liên's methods, respectively.
- Check your answers with information on Penang Island from the Internet.
  - ❖ Which method gives you a closer estimate of the actual area?

Note:

The area is a 2-dimensional measure. Notice that the area of a trapezium is $(a + b)h$ x ½; the area of a triangle is ah x ½; and the area of a circle is pi x r^2. Thus, each of these area formulas can be seen as a representation of proportionality with a constant.

Topic 9: Exchanging Local Currency with Currency in the ASEAN Community

Standards 9.1:

Exchanging local currency in the ASEAN community with the idea of rate

i. Extend the use of ratio for currency exchange (rate of exchange)
ii. Apply the four operations for money in appropriate notation in life
iii. Appreciate the value of money

Sample Tasks for Understanding the Standards

Task1: Ratio for Currency Exchange in ASEAN Community

The exchange rate is the price of one country's currency in terms of another currency. Table 1 shows the exchange rates between the Malaysian Ringgit (MYR) and currencies in the ASEAN community and U.S. Dollar (USD).

Table 1
The Exchange Rates of ASEAN Countries and the USA

Foreign Currency Units		Trading date: 21 June 2022 Time: 1700 [1 Malaysian ringgit =	Trading date: 21 June 2022 Time: 1700 [1 USD =
Currency	code	Exchange rate	Exchange rate
Brunei Dollar	BND	0.315	
Cambodian Riel	KHR	907.505	
Indonesian Rupiah	IDR	3365.700	
Philippine Peso	PHP	12.266	
Singapore Dollar	SGD	0.315	
Thai Baht	THB	8.010	
Vietnamese Dong	VND	5274.150	
Myanmar Kyat	MMK	416.042	
Lao Kip	LAK	3358.800	
U.S. Dollar	USD	0.227	

Based on OANDA's Currency Converter

i. The rate for Brunei Dollar (BND) is 0.315.
   - What does it mean?
   - How many MYR can exchange for 1 BND?

ii. The rate for Singapore Dollar (SGD) is also 0.315.

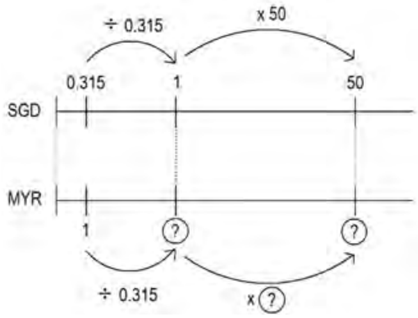
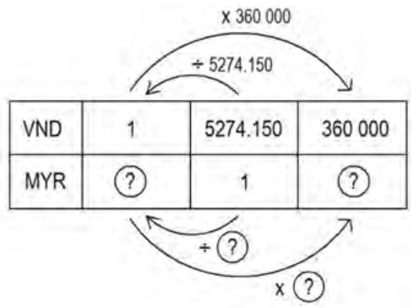


Diagram 1

Diagram 1 shows a pair of proportional number lines used by a student to exchange 50 SGD for MYR.

- Find the missing number in each ?.
- How many MYR can 50 SGD change for?



VND	1	5274.150	360 000
MYR	?	1	?

Diagram 2

iii. A store in Đồng Xuân market, Hanoi is selling crispy lotus seeds at 360 000 VND per kg. The rate for Vietnamese Dong (VND) is 5274.150. Diagram 2 shows a Rule of Three tables to change 360 000 VND to MYR.

- Find the missing number in each ?.
- How many MYR will 1 kg of crispy lotus seeds cost?

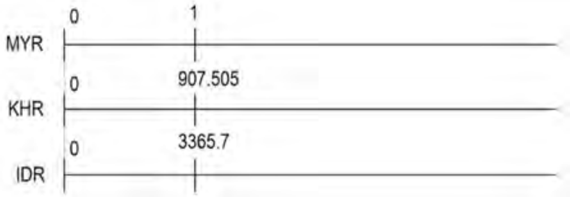


Diagram 3

iv. Diagram 3 shows three proportional number lines representing currency exchange rates for three currencies.

- Explain how you can use diagram 3 to exchange 100 000 Indonesian Rupiah for Combodia Riel?

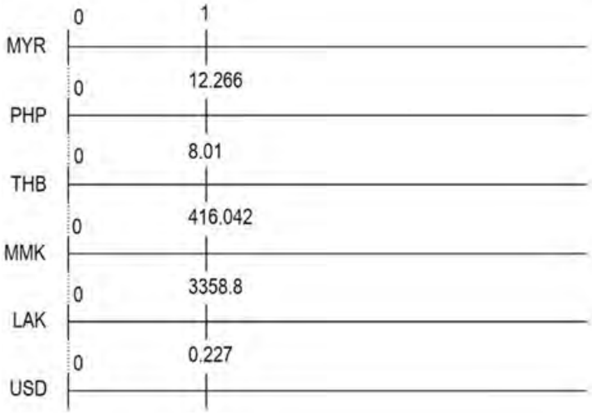


Diagram 4

v. Diagram 4 shows six proportional number lines representing currency exchange rates for six countries.

- Use the number lines to exchange 500 THB for
  - ❖ Philippines Peso, PHP
  - ❖ Lao Kip, LAK
  - ❖ Indonesian Rupiah, IDR
- Use the number lines to exchange 20 USD for
  - ❖ Myanmar Kyat, MMK
  - ❖ Thai Baht, THB
  - ❖ Malaysian Ringgit, MYR

vi. US dollar is a common based currency for exchange.

- How many MYR can be exchanged for 1 USD?
- Explain how the exchange rate is calculated.
- Calculate the exchange rates of other currencies to 1 USD and complete Table 1.

vii. Mr Joseph from the Philippines accompanied 10 grade-6 students to visit Borobudur in Jogjakarta. The entrance ticket for each foreign visitor is 25 USD.

- Based on the rates in Table 1, how much is the price of each ticket in Philippines Peso, PHP?
- What is the total amount for 11 tickets in PHP?
- How much is the total amount of 11 tickets when converted to Indonesian Rupiah, IDR?

viii. Based on the rates in Table 1, arrange the following currencies from the highest value to the lowest.

- 100 Indonesian rupiah, 100 Viatnamese Dong, 100 Thai Bhat, and 100 Peso
- Explain how you make the comparisons.

Task 2: Four Operations for Money

Diagram 5

i. Madam Aishah took her three children to Jollibee for a meal while in Cebu, Philippines. She ordered 3 sets of meal C1 and 1 set of meal C3 as shown in Diagram 5. The children also requested individual Mini Sundae Twirl.

- What is the total amount she has to pay in the Philippines Peso? Show the procedure of calculation.

- Aishah was not sure if the charges for the food were expensive. What would you suggest she do in order to find out if the food prices were reasonable?
- The following day, Madam Aishah ordered 2 sets of C4 with Chocolate Sunday Twirl for her two children, 1 set of C5 with Cookies'n Cream Sundae Twirl for another child, 1 set of C6 with a Vanilla Cone Twirl for herself.
    - ❖ What is the average cost of food for each person, in the Philippines Peso?

Malaysian Ringgit	Indonesian Rupiah	Thai baht	Philippines Peso	Cambodina Riel	Singapore Dollar	Viaetnamese Dong
RM1=	3365Rp	฿8	₱12	907 ៛	S$0.31	5274 Dong
 RM1	 2 900 Rp	 ฿15	 ₱9	 4 650 ៛	 S$2	 8 000 Dong

Diagram 6

ii. Diagram 6 shows the exchange rates of the Malaysian Ringgit to six other ASEAN countries. It also shows the price of seven items in the respective country.

- What item can you buy in other countries with one Malaysian Ringgit?
- How much Malaysian Ringgit do you need to buy a bottle of Coca-Cola in Thailand?
- Which is the most expensive item listed in the diagram?
- How much does an apple cost in your country?
    - ❖ Is it cheaper than the apple in Cambodia?
    - ❖ Why do you think apples cost so much in Cambodia?
- An ice-cream cone costs S$ 2 in Singapore.
    - ❖ How much does that cost when the value is exchanged to your local currency?
    - ❖ If you are at home, will you pay such a price for a piece of such ice cream? Why or why not?
    - ❖ What if you are travelling in Singapore?

Diagram 7

iii. Diagram 7 shows the prices of two best-seller sets of KFC fried chicken in Malaysia.

- Find out the prices of similar sets of KFC fried chicken in your country.
- Find out the current exchange rate of your country’s currency to the Malaysian Ringgit.
    - Is KFC selling the same sets at the same prices in your country?
    - Do you think it is justifiable for the price difference if any?

iv. Suggest Two ways to make students aware of the values of money in different currencies.

Topic 10: Extending the Relation of Time and Use of Calendar in Life

Standards 10.1:

Extending the relation of time and use of calendar in life

i. Convert time in 12-hour system with abbreviation a.m. and p.m. to 24-hour system and vice versa
ii. Investigate the numbers in calendar to relate days, weeks, months and year using the idea of number patterns
iii. Appreciate the significance of various calendars in life

Sample Tasks for Understanding the Standards

Task1: Telling Time in 24-hour System

Time Printed on Air Ticket

Flight 1 Fri, 01 Jul 2022 1 hours 5 minutes

19: 05 Penang (PEN)

20:10 Kuala Lumpur (KUL)
KUA2

1 hours 5 minutes

Diagram 1

i. Diagram 1 shows the departure and arrival time of a flight from Penang to Kuala Lumpur.

- When is the flight, in the morning or in the afternoon?
- What is the departure time? Write the time with abbreviation a.m. or p.m.
- What is the arrival time? Write the time with abbreviation a.m. or p.m.

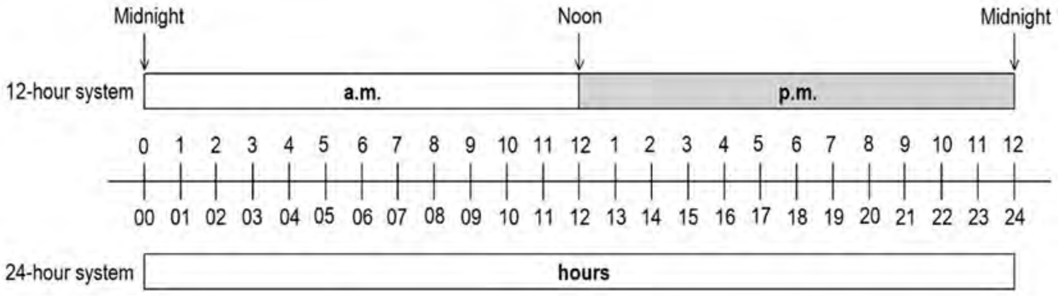


Diagram 2

ii. Diagram 2 shows a chart connecting the 24-hour system and the 12-hour system in telling time.

- Using the chart as a guide, convert the following time to the 12-hour system. Write the time using the abbreviation a.m. or p.m.
  - ❖ 14:55 hours
  - ❖ 09:10 hours
  - ❖ 18:30 hours

- Using the chart as a guide, convert the following time to the 24-hour system.
    - ❖ 5:00 a.m.
    - ❖ 8:15 a.m.
    - ❖ 4:35 p.m.

iii. What are the advantages of using the 24-hour system?

Task 2: Exploring Calendar Numbers

Number Patterns in Calendar

December 2022

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Diagram 3

i. Diagram 3 shows the calendar for December 2022. There are 7 columns of numbers in the calender, namely the Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday columns.

- What pattern do you observe between the numbers in each column?
- Why does the pattern exist?

December 2022

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Diagram 4

ii. Diagram 4 shows a left-to-right diagonal of numbers in the same calendar.

- What pattern do you observe between these numbers?
- Does the same pattern exist for other left-to right diagonals too?
- Why does the pattern exist?

December 2022

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Diagram 5

iii. Diagram 5 shows a right-to-left column of numbers in the same calendar.

- What pattern do you observe between these numbers?
- Does the same pattern exist for other right-to-left diagorals too?
- Why does the pattern exist?

Relationship of Numbers in Calendars

December 2022

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Diagram 6

i. Diagram 6 shows three consecutive numbers from a column in the same calendar.

- What relationship do you observe between these numbers?
- Is the relationship always true for any other three consecutive column numbers such as 14, 21 and 28?
- Let the expressions $(a - 7)$, a, and $(a + 7)$ represent any three consecutive column numbers.
  - ❖ Prove the relationship using these expressions.

December 2022

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Diagram 7

ii. Diagram 7 shows 1 Dec on the 2022 calendar.

- 1 Dec is a Thursday.
  - ❖ What day is 7 days later?
  - ❖ What day is 10 days later?
  - ❖ What day is 14 days later?
  - ❖ What day is 20 days later?
  - ❖ What pattern do you observe?
  - ❖ What calculation can you do to solve similar problems?
- Solve the following problems without refering to the calendar.
  - ❖ 4 Dec is a Sunday. What day is 21 days later?
  - ❖ 20 Dec is a Tuesday. What day is 25 days later?
  - ❖ 26 Dec is a Monday. What day is 10 January 2023?
  - ❖ Check all your answer with a calendar.

December 2022

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Diagram 8

iii. Diagram 8 shows a '2-by-2 square' of four numbers in the same calendar.

- What relationship do you observe between these numbers?
- Is the relationship true for other '2-by-2 squares'?
- Why does the relationship exist?
- Clare claims that the sum of these four numbers is (4 x 5) + 16 as shown in the diagram.
    - ❖ Verify Clare's claim.
    - ❖ Is the claim true for any other '2-by-2 square' of numbers?
    - ❖ Why is the claim true?

December 2022

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Diagram 9

iv. Diagram 9 shows a '3-by-3 square' of nine numbers in the same calendar and a claim on the sum of these numbers made by Jeff.

- What does Jeff mean?
- Verify Jeff's claim.
- Is the claim also true for any other '3-by-3 squares' of numbers in the calendar?
- Why is the claim true?

v. The Gregorian calendar that is universally used now is a solar calendar.

- What does it mean by solar calendar?
- Name other calendars that you know.
    - ❖ Compare the calendars. Explain the similarities and differences among the calendars.

vi. How will the tasks in this section inculcate your students' appreciation towards the values of mathematics?

vii. A calendar is used to track passage of time. How will our life be affected if there is no calendar?

Topic 11: Converting Quantities in Various System of Units

Standards 11.1:

Converting measurement quantities on international and non-international system with idea of base-10

- i. Convert measurement system of metre and kilogram with prefixes deci-, centi-, and mili-, and with deca-, hecto-, and kilo-
- ii. Convert measurement system of litre with cubic centimetre
- iii. Convert measurement system of area using are (a) and hectare (ha) with square metre
- iv. Convert measurement of local quantities with standard quantities
- v. Understand the unit system with power, such as metre, square metre and cubic metre

Sample Tasks for Understanding the Standards

Task1: Quantity and Unit of Measurement

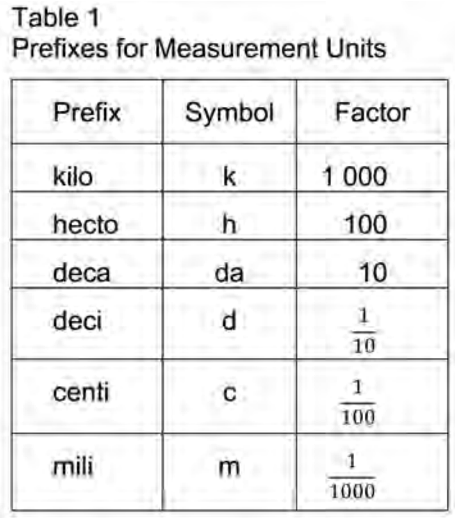


Table 1
Prefixes for Measurement Units

Prefix	Symbol	Factor
kilo	k	1 000
hecto	h	100
deca	da	10
deci	d	$\frac{1}{10}$
centi	c	$\frac{1}{100}$
mili	m	$\frac{1}{1000}$

i. Any quantity can be measured using a basic unit of measurement such as metre (m), gram (g), and litre (ℓ). Centi, kilo, and mili are common prefixes used to modify the magnitude of a unit. Table 1 shows some examples of prefixes.

- Write the measurements in the respective basic unit.

1 kilometre = ________ m

1 kilogram = ________ g

1 hectometre = ________ m

1 decametre = ________ m

1 decigram = ________ g

1 centimetre = ________m

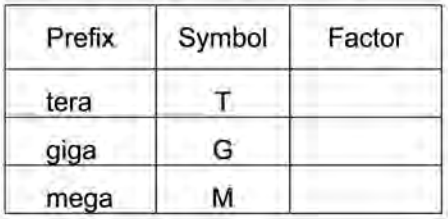
1 centilitre = ________ ℓ

1 mililitre = ________ ℓ,

ii. Arrange each of the following sets of measurements from least to the largest.

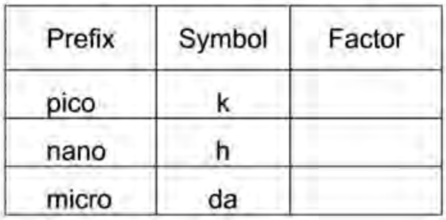
- 3 dm, 563 cm, 750 mm
- 2.7 kg, 42.5 hg, 499.6 cg

Table 2
Prefixes for Much Larger Units



Prefix	Symbol	Factor
tera	T	
giga	G	
mega	M	

Table 3
Prefixes for Much Smaller Units



Prefix	Symbol	Factor
pico	k	
nano	h	
micro	da	

iii. Kilo, hecto, and deca are examples of prefixes used to make the units larger, whereas deci, centi, and mili are prefixes used to make the units smaller. Tables 2 and 3 are more examples of prefixes.

- Fill in the missing factors in the tables.

Task 2: Conversion Between Units of Length

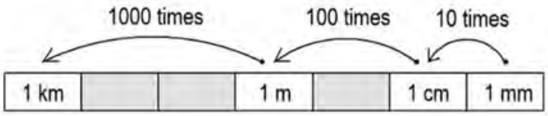


Diagram 1

i. Diagram 1 shows the relationships among units of length.

- With the help of Diagram 1, complete the following conversion of units for length.
  - ❖ 10 x 1 mm = ________ cm
  - ❖ 100 x 1 cm = ________ m
  - ❖ 1000 x 1 m = ________ km
  - ❖ ____ x 1 mm = 1 m
  - ❖ ____ x 1 mm = 1 km
  - ❖ ____ x 1 cm = 1 km

km			m		cm	mm
5	0	0	0	0	0	
		1	2	0	0	0

Diagram 2

ii. Diagram 2 shows a chart used to convert between units of length based on the relationships shown in Diagram 1. Two examples of conversions as shown in the chart are 5 km = 500 000 cm and 12 m = 12 000 mm.

- Use the chart to convert
  - ❖ 2 km to mm
  - ❖ 235 m to cm
  - ❖ 37 cm to mm
  - ❖ 5.2 m to cm
- Use the chart to convert
  - ❖ 7900 cm to m
  - ❖ 860 mm to m
  - ❖ 2375 cm to km

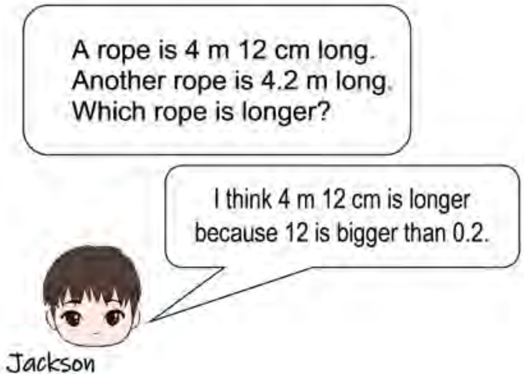


Diagram 3

iii. Diagram 3 shows a problem involving comparing the length of two ropes.

Jackson make a mistake as shown in the diagram.

- What are the possible sources of the mistake?
- How to help Jackson correct the mistake?

Task 3: Conversion Between Units of Mass

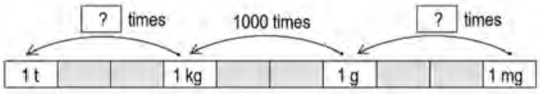


Diagram 4

i. Ton (t), kilogram (kg), gram (g) and milligram (mg) are units of mass and their relationships are shown in Diagram 4.

- Fill in the missing factor in each $\boxed{?}$.
- With the help of Diagram 4, complete the following conversion of units for mass.
  - ❖ 3 000 000 g = ________ t
  - ❖ 2 000 000 mg = ________ kg
- What next conversion tasks will you give to your students? Explain your reasons.

	▨	▨		▨	▨	g	▨	▨	

Diagram 5

ii. Diagram 5 shows an incomplete chart similar to Diagram 2 for conversion of units involving mass.

- Complete the chart.
- Use the chart to convert
  - ❖ 2.8 kg to mg
  - ❖ 6.37 t to kg
  - ❖ 692 g to mg
- Use the chart to convert
  - ❖ 4500 kg to t
  - ❖ 43860 mg to g
  - ❖ 120 g to kg

Diagram 6

iii. Diagram 6 shows four fruit baskets. The mass of each basket is written on a card next to it.

- Arrange the fruit baskets from lightest to the heaviest.

Task 4: Conversion Between Units of Area

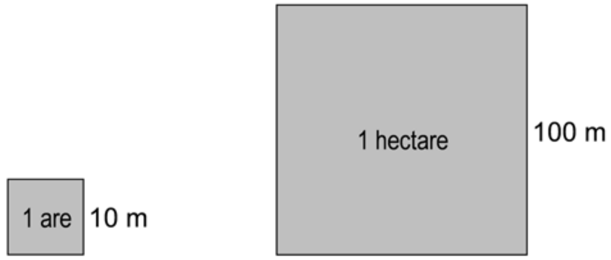


Diagram 7

i. Hectare and 'are' are two units in Metric System commonly used to measure land area. The size of 1 are (a) and 1 hectare (ha) is an area equal to a square that is 10 m and 100 m on each side respectively, as shown in Diagram 7.

- How many m^2 is 1 are?
- How many m^2 is 1 hectare?
- How many are is 1 hectare?
- How many km^2 is 1 hectare?

ii. 1 ha 9 a and 1.9 ha, which is larger?

iii. Arrange the following land areas from least to the largest.

4.2 ha	78.05 ha	3 889 m^2
63 500 m^2	238 a	356.7 a

Penang Island	○	○	45 cm^2
A Badminton Court	○	○	293 km^2
A Name Card	○	○	81.74 m^2
A School Compound	○	○	2.1 ha
An oil palm plantation	○	○	24 a

Diagram 8

iv. Diagram 8 shows some measurements of area.

- Match each area with its appropriate measurement.

Task 5: Units of Volume

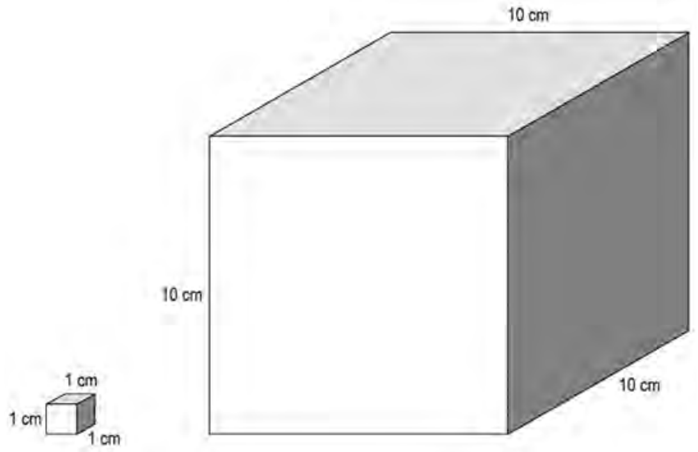


Diagram 9

i. Diagram 9 shows one large and one small cube, respectively. The small cubes can fill 1 cubic centimetre (cm³), also known as 1 mililitre (mℓ) of water and the large cube can fill 1 litre (ℓ) of water.

- How many small cubes are there in 1 big cube?
- How many cm³ are there in 1 ℓ ?

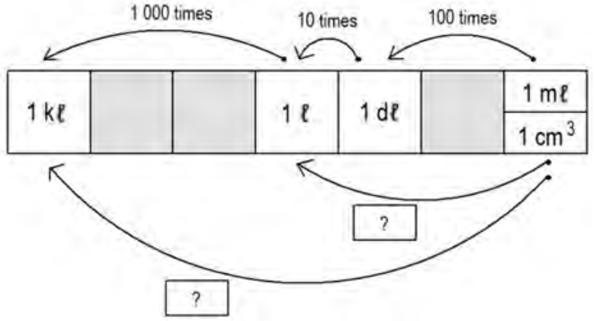


Diagram 10

ii. Diagram 10 shows the relationships among units for measuring volume of liquid.

- Fill in the missing factor in each ☐.
- With the help of Diagram 10, complete the following conversion of units for volume of liquid.

❖ 1 mℓ = __________ x 1 cm³

❖ 1 dℓ = __________ x 1 cm³

❖ 1 ℓ = __________ x 1 dℓ

❖ 1 kℓ = __________ x 1 cm³

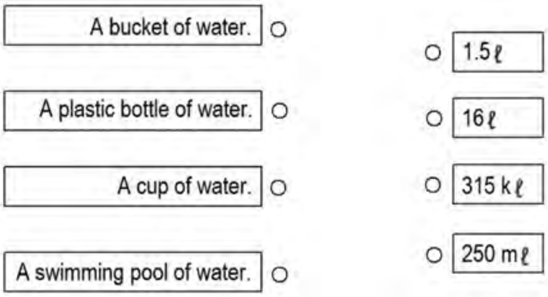
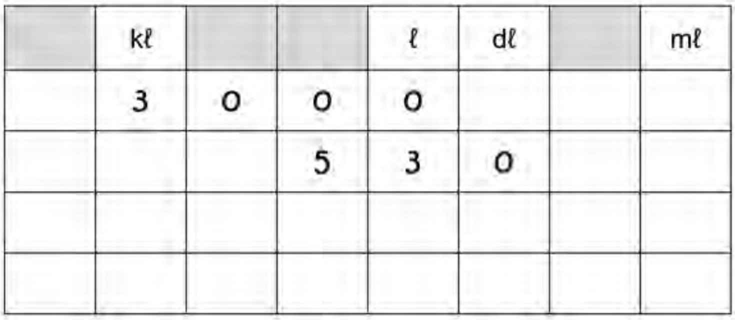


Diagram 11

iii. Diagram 11 shows some measurements of water capacity.

- Match each water capacity with its appropriate measurement of volume.



	kℓ			ℓ	dℓ		mℓ
	3	0	0	0			
			5	3	0		

Diagram 12

iv. Diagram 12 shows a chart used to convert between units of volume. Two examples of conversions as shown in the chart are 3 kℓ = 3000 ℓ and 53 ℓ = 530 dℓ.

- Use the chart to convert
  - ❖ 2 kℓ to cm^3
  - ❖ 18 dℓ to mℓ
  - ❖ 6.5 kℓ to dℓ
  - ❖ 2.6 ℓ to mℓ

v. 3 kℓ 600 dℓ and 3.6 kℓ, which is larger?

vi. Arrange the following volume of liquid from least to the largest.

7 ℓ 20 mℓ

7.2 ℓ

720 dℓ

720 cm^3

1 kℓ 2 dℓ

Topic 12: Showing Relationships Using a Venn Diagram

Standards 12.1:

Using a Venn diagram to show relationships of numbers and figures for making a clear logical deduction

i. Sort objects by their defining characteristics
ii. Show relationships of squares, rectangles, rhombuses, parallelograms, trapezium and quadrilaterals by using a Venn diagram
iii. Show the relationship between numbers
iv. Critique ambiguous reasoning by using a Venn diagram for making clear a definition

Sample Tasks for Understanding the Standards

Task1: Who am I?

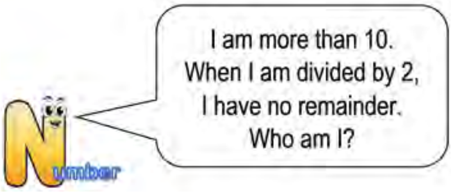


Diagram 1

i. Diagram 1 shows a number telling about its characteristics.

- Can the number be 8? Why or why not?
- Can the number be 17? Why or why not?
- What can the number be?

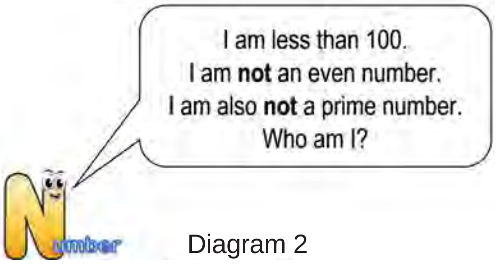


Diagram 2

ii. Diagram 2 shows another number.

- Can the number be 2? Why or why not?
- Can the number be 28? Why or why not?
- Can the number be 29? Why or why not?
- Can the number be 115? Why or why not?
- What can the number be?

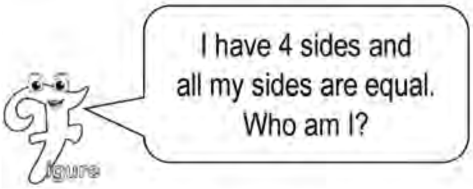


Diagram 3

iii. Diagram 3 shows a space figure telling about its characteristics.

- Can the figure be a triangle? Why or why not?
- What can the figure be?

Diagram 4

iv. Diagram 4 shows another space figure.

- What can the figure be?

Task 2: Relationships among Quadrilaterals

i. Quadrilaterals can be classified into (a) trapezium, (b) parallelogram, (c) rhombus, (d) rectangle, and (e) square.

- State the properties of each of the quadrilaterals.

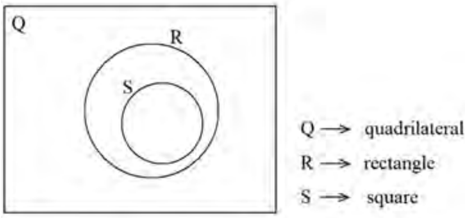


Diagram 5

ii. Diagram 5 is a Venn diagram showing the relationships among quadrilaterals, rectangles and squares.

- Decide whether each of the following statements is true or false.
  - ❖ All squares are quadrilaterals.
  - ❖ All rectangles are square.
  - ❖ Some quadrilaterals are squares.
  - ❖ Some squares are quadrilaterals.
  - ❖ A square is also a rectangle.
  - ❖ A quadrilateral is also a rectangle.
- Justify each of your decisions.

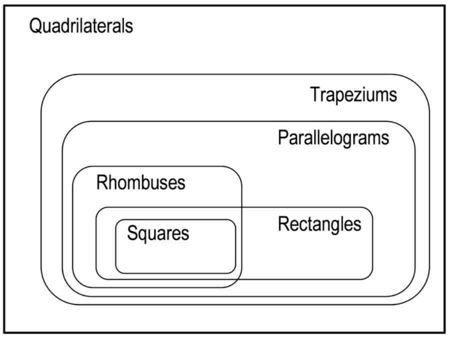


Diagram 6

iii. Diagram 6 shows a Venn diagram illustrating the relationships among quadrilaterals.

- A rectangle is also a trapezium. Why?
- A square is also a rhombus. Why?

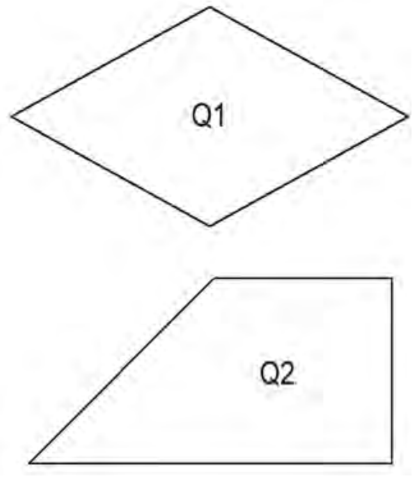


Diagram 7

iv. Diagram 7 shows two quadrilaterals, Q1 and Q2. In Q1, the lengths of all the sides are equal; whereas two of the sides of Q2 are parallel.

- Decide whether each of the statements about the quadrilaterals is true or false.
    - ❖ Q1 is a parallelogram.
    - ❖ Q1 is a trapezium.
    - ❖ Q2 is a trapezium.
    - ❖ Q2 is a parallelogram.
- Justify each of your decisions.

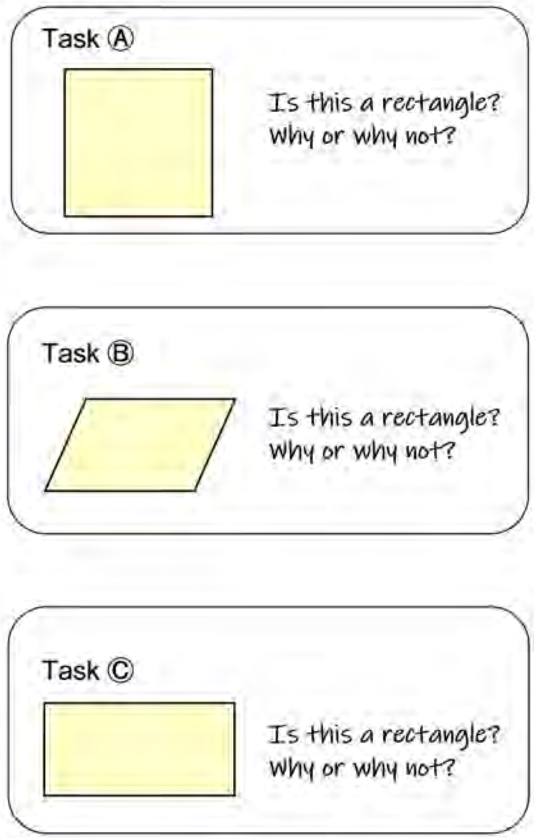


Diagram 8

v. Diagram 8 shows three tasks to help students understand the relationships between a rectangle, a square and a parallelogram.

- How will you order the sequence of the tasks?
- Justify your decision.
- What responses will you anticipate getting from your students for each of the tasks?
- Based on the anticipated responses, what additional specific questions will you ask to support your students' mathematical thinking in each task?
- What next task will you give to your students? Explain your reasons.

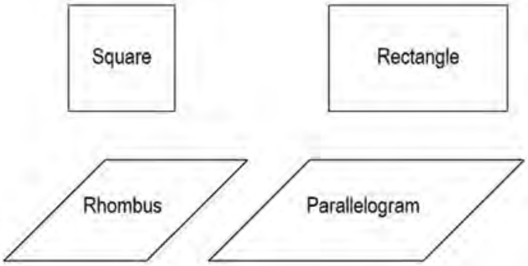


Diagram 9

vi. Diagram 9 shows a square, a rectangle, a rhombus and a parallelogram.

- Complete each of the following statements.
  - ❖ A square is also a rhombus, but a rhombus may not be a square. So, a rhombus is a square if __________.
  - ❖ A rectangle is also a parallelogram, but a parallelogram may not be a rectangle. So, a parallelogram is a rectangle if _________________.

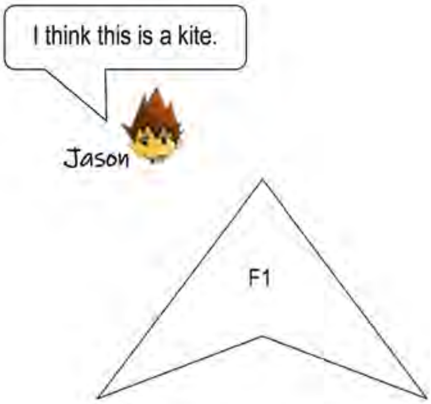


Diagram 10

vii. Diagram 10 shows a plane figure F1 with two pairs of equal adjacent sides. Jason said F1 is a kite.

- Is F1 a kite? Why or why not?

Task 3: Relationships among Numbers

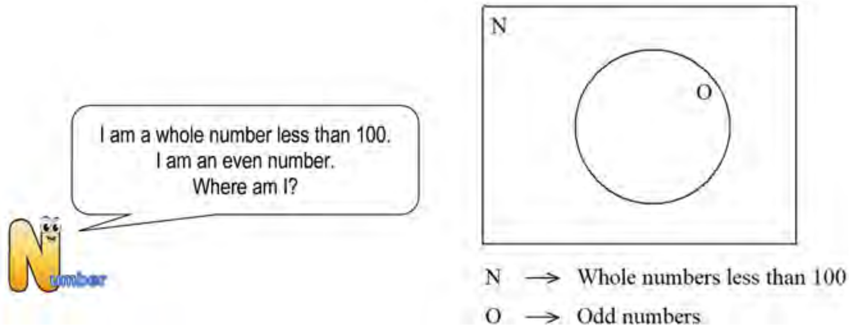


Diagram 11

i. Diagram 11 shows a number describing itself and a Venn diagram.

- Can the number be 188? Why or why not?
- Shade on the Venn diagram, the region that represents the number.

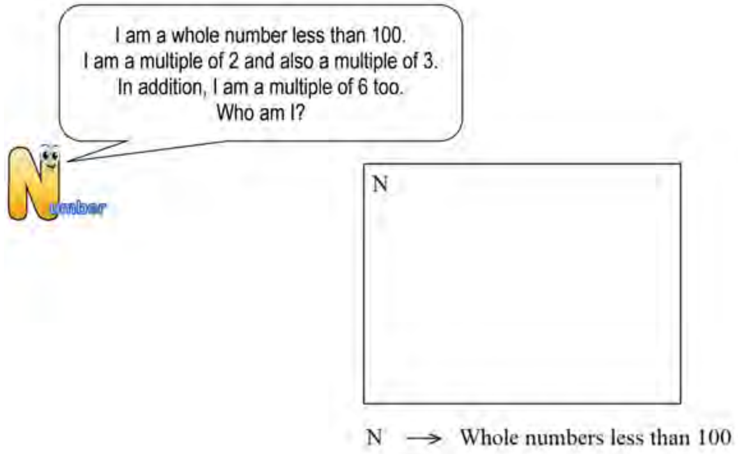
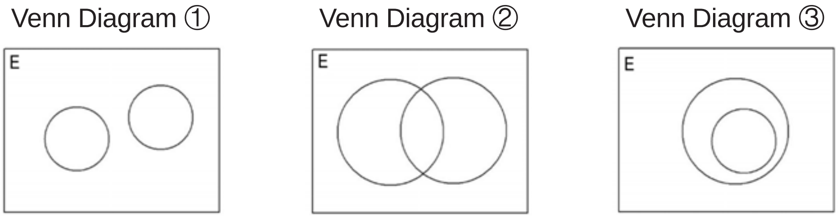


Diagram 12

ii. Diagram 12 shows another number describing its characteristics, and a Venn diagram represents whole numbers less than 100.

- Draw on the Venn diagram to express the relationship among multiples of 2, multiple of 3, and multiples of 6.
- Shade on the Venn diagram, the region that represents whole numbers less than 100 which are multiples of 6.
- Find a number that is a multiple of 2, but not a multiple of 6.
  - ❖ Shade on the Venn diagram, the region for the number.
- Can you find a number that is a multiple of 6, but not a multiple of 3? Explain your reasoning.



Venn Diagram ①　Venn Diagram ②　Venn Diagram ③

Diagram 13

- The relationship of even numbers to multiples of 4.
- The relationship of multiples of 4 to multiples of 5.
- The relationship of odd numbers to multiple of 6.

iii. Diagram 13 shows three Venn diagrams and three relationships of numbers. Set E has whole numbers between 100 and 200.

- Match the Venn diagrams with the relationships.
- Explain each of the relationships using specific examples.

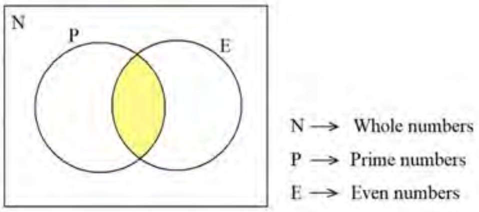


Diagram 14

iv. The Venn diagram in Diagram 14 shows the relationship of prime numbers to even numbers.

- Give an example of a number that is in the shaded region.
- Shade the region of odd numbers that are also prime numbers.
- Shade the region of even numbers that are not prime numbers.

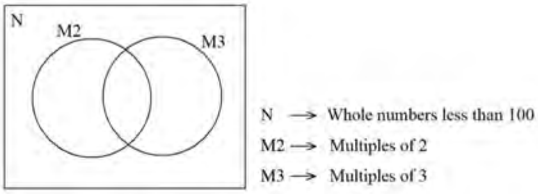


Diagram 15

v. The Venn diagram in Diagram 15 shows the relationship of multiples of 2 to multiples of 3 which are less than 100.

- How many numbers are multiples of 2?

- How many numbers are multiples of 3?
- How many numbers are multiples of 2 and multiples of 3?
    - Shade this region on the Venn diagram.
    - What is the largest common multiple of 2 and 3?
- How many numbers are multiple of 2 or multiples of 3?
    - Shade this region on the Venn diagram.
    - Give an example of such a number.
- How many numbers are multiple of 2, but not multiples of 3?
    - Shade this region on the Venn diagram.
    - Give an example of such a number.
- How many numbers are neither multiple of 2 nor multiple of 3?
    - Shade this region on the Venn diagram.
    - Give an example of such a number.

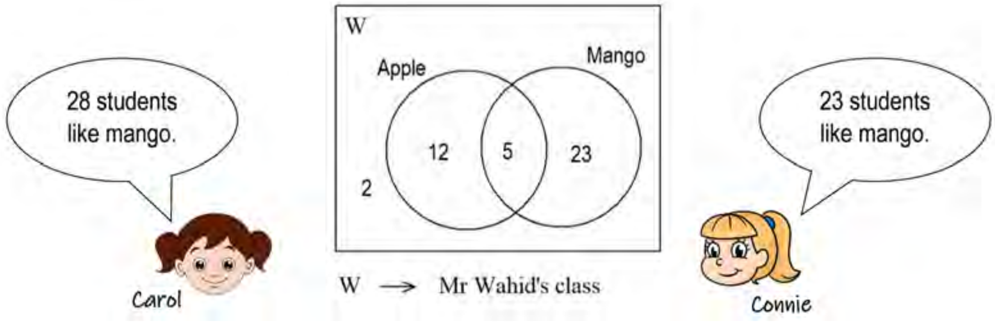


Diagram 16

vi. The Venn diagram in Diagram 16 shows the number of students who like apples and/or mangoes in Mr Wahid's class. Two students, Carol and Connie have different opinions on the number of students who like mangoes.

- How will you use this contradicting opinions in helping your students learn Venn diagram?
- How many students are there altogether in Mr Wahid's class?

CHAPTER 4

Plane Figures and Space Figures

Topic 1: Exploring Figures with their Components in the Plane

Standards 1.1:

Exploring figures with their components in the plane and using their properties

i. Examine parallel lines and perpendicular lines by drawing with instruments
ii. Examine quadrilaterals using parallel and perpendicular lines, and identify parallelogram, rhombus, and trapezium by discussion
iii. Find properties of figures through tessellations such as a triangle where the sum of the angles is 180 degrees, a straight angle
iv. Extend figures to polygons, and expand them to circles by knowing and using their properties

Sample Tasks for Understanding the Standards

Task 1: Perpendicular Lines

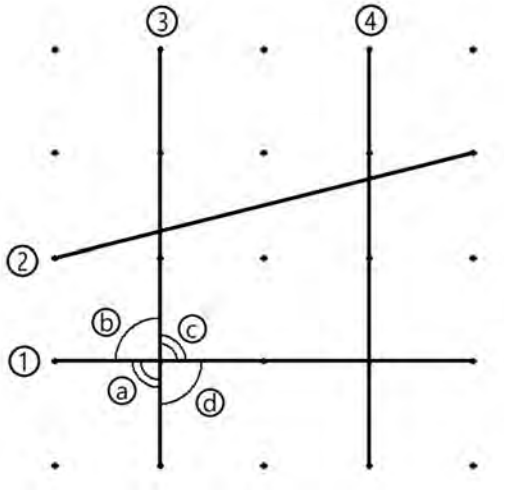


Diagram 1

i. Diagram 1 shows a quadrilateral formed by four lines drawn on a dot paper.
   - Line① and Line ③ are perpendicular.
     - ➢ What instrument can be used to measure an angle?
     - ➢ Measure angles ⓐ,ⓑ,ⓒ, and ⓓ respectively.
   - Is line ② perpendicular to line ③?
     - ❖ Explain your way of finding the answer.
     - ❖ Confirm your answer by paper folding.
     - ❖ Why is paper folding an error-free method to confirm your answer?
   - Which other line is also perpendicular to line ①?
   - Draw another line on Diagram 1 that is perpendicular to line ③.

ii. What are perpendicular lines?

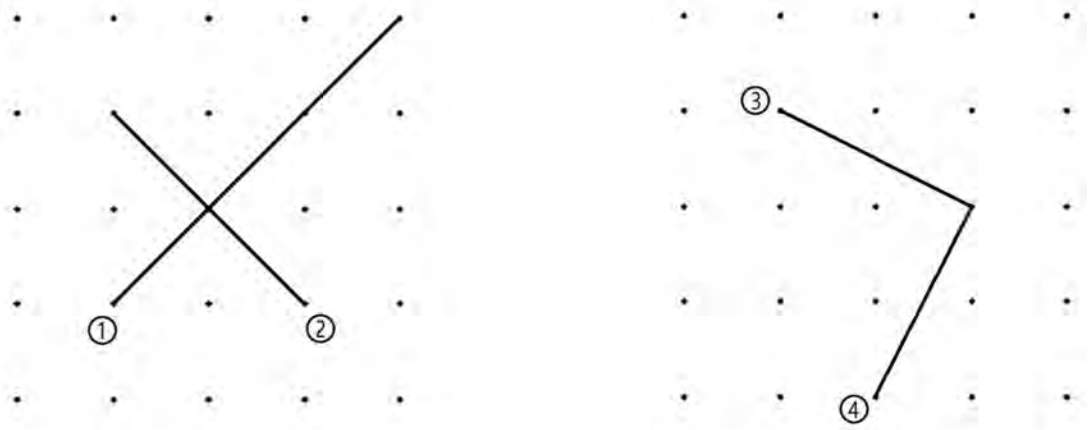


Diagram 2

iii. Diagram 2 shows two pairs of lines.

- Which pairs of lines are perpendicular?
- Explain your way to find the answer.
  - ❖ Explain how the properties of a square can be used to confirm your answer.
- Think of other ways to find the answer.

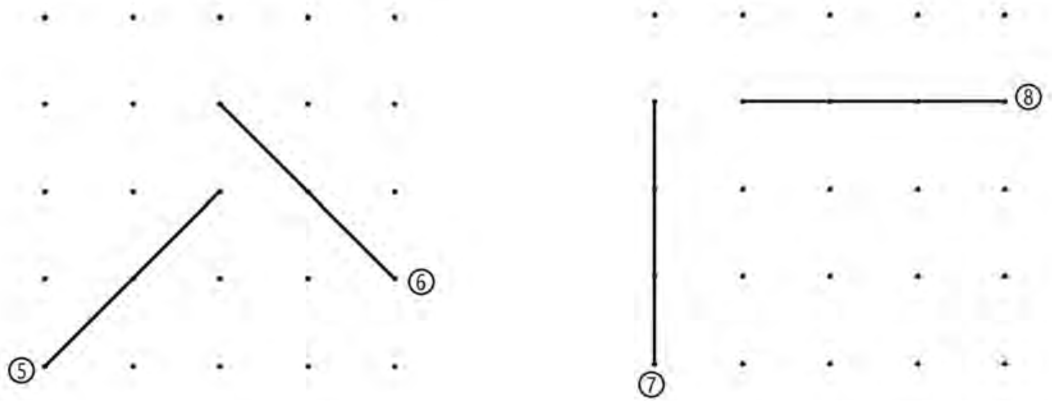


Diagram 3

iv. Diagram 3 shows another two pairs of lines.

- Which pairs of lines are perpendicular?
- Explain your ways of finding the answers.
  - ❖ Explain how the properties of a square and parallel lines can be used to confirm your answers.

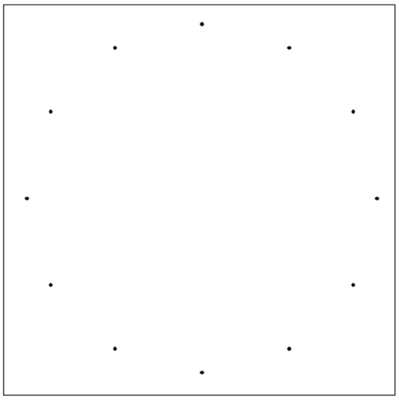


Diagram 4

v. Diagram 4 shows a clock geoboard.

- Design a task to help your students learn perpendicular lines using a clock geoboard with rubber bands.

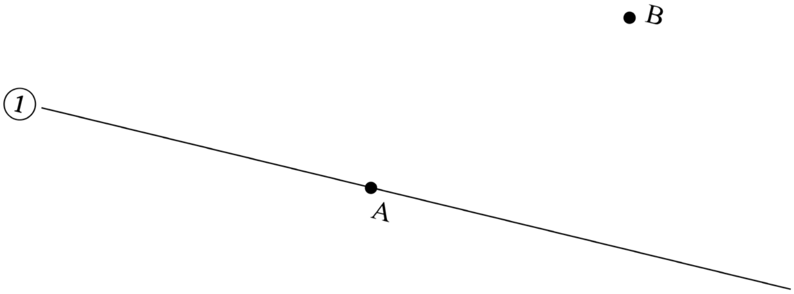


Diagram 5

vi. Diagram 5 shows line ① and two points, A and B. Use a ruler and a protractor or a set square to help you with the following subtasks.

- Draw a line such that it passes through point A and is perpendicular to line ①.
- Draw another line such that it passes through point B and is perpendicular to line ①.

vii. Get a piece of A4 paper. Fold the paper to make two perpendicular lines.

- How many folds do you need to make?
- Must each fold always divide the paper into two equal parts? Explain your answer.
- What are the possible different reasons that your students may suggest to explain why the two folded lines are perpendicular?

Task 2: Parallel Lines

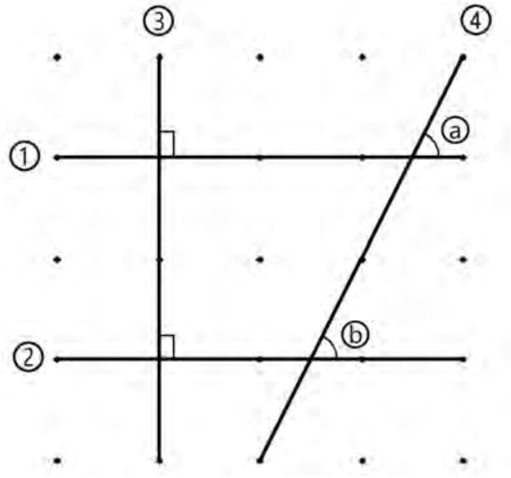


Diagram 6

i. Diagram 6 shows a quadrilateral formed by four lines drawn on a dot paper.

- Line ③ is perpendicular to both lines ① and ②.
  - ❖ What is the value of the angle of intersection between line③ and lines ① and ②, respectively?
- Line ④ is another line crosses both lines ① and ②.
  - ❖ Measure the values of angles ⓐ and ⓑ.
  - ❖ What do you find out about these two angles?
- Lines ① and ② are said to be a pair of parallel lines.
  - ❖ Based on what you have found so far, explain the meaning of parallel lines.

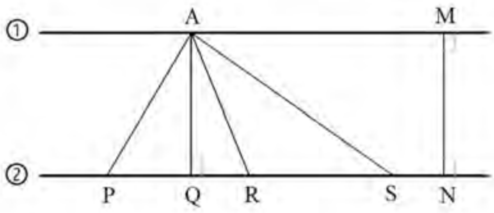


Diagram 7

ii. Diagram 7 shows two parallel lines, ① and ②. Five other line segments, AP, AQ, AR, AS and MN intersect with lines ① and ②.

- Arrange the distance, AP, AQ, AR, and AS in ascending order.
- How do you determine the order?
- Compare the distance between AQ and MN.
  - ❖ What can you conclude from your comparison?
- Will lines ① and ② meet if they are extended at both ends? Why or why not?

iii. Two students, Jose and Pedro were asked to draw a parallel line to line ①. They used different methods as shown in Diagram 8.

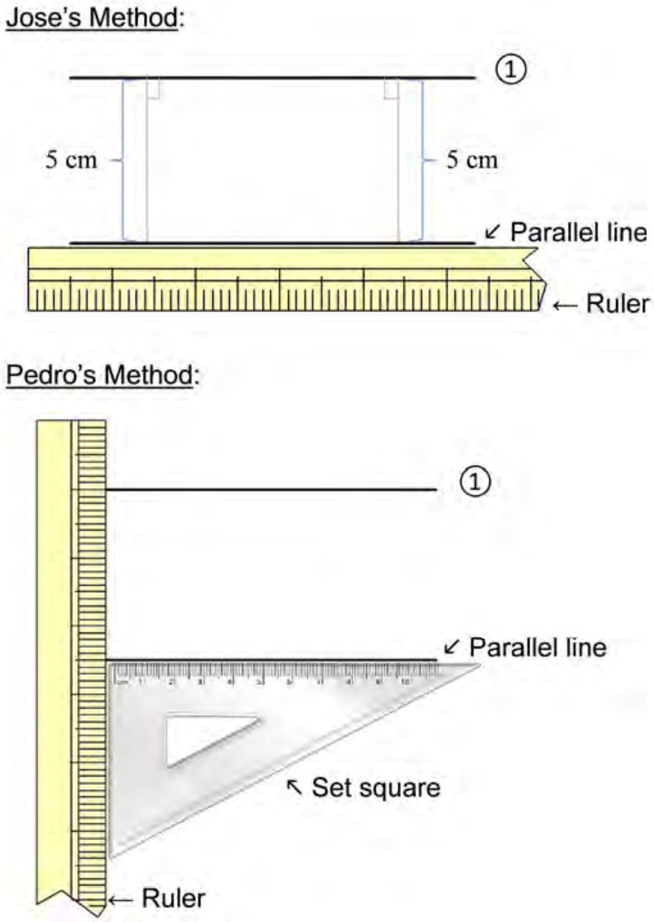


Diagram 8

- Explain the two students' methods and the reason why both methods are appropriate.

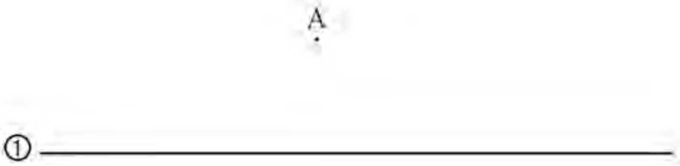


Diagram 9

iv. Diagram 9 shows line ① and point A.

- Draw a line which passes through point A and is parallel to line ①.
- Draw another two lines that are parallel to line ① and 3 cm apart.

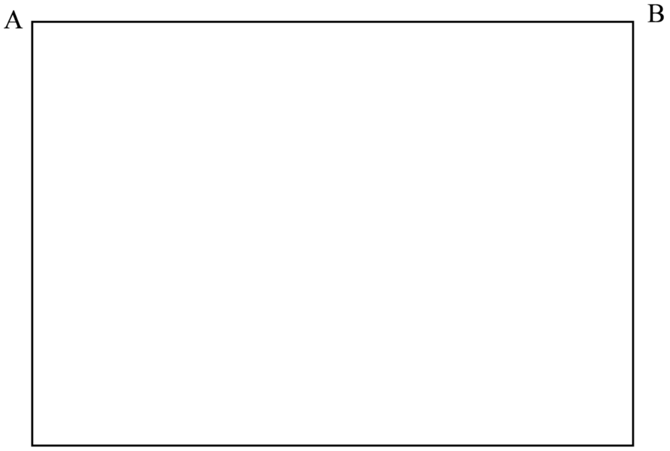


Diagram 10

v. Diagram 10 shows a sheet of rectangular paper with side AB.

- Using a scaled ruler **only**, draw a parallel line 2 cm away from side AB on Diagram 10.

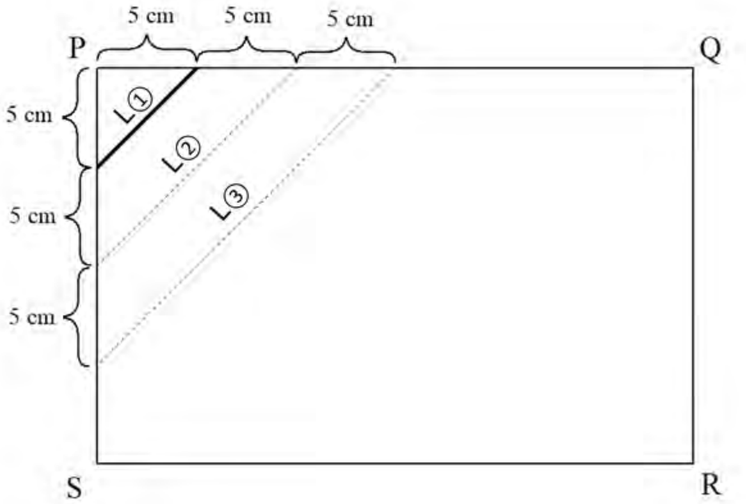


Diagram 11

vi. Diagram 11 shows another sheet of A4 paper, PQRS. Three line segments, L① , L② , and L③ are drawn as shown in the diagram.

- Are the three line segments, L① , L② , and L③ parallel lines? Why or why not?
- Using a scaled ruler only, draw another parallel line to L① , L② , and L③.

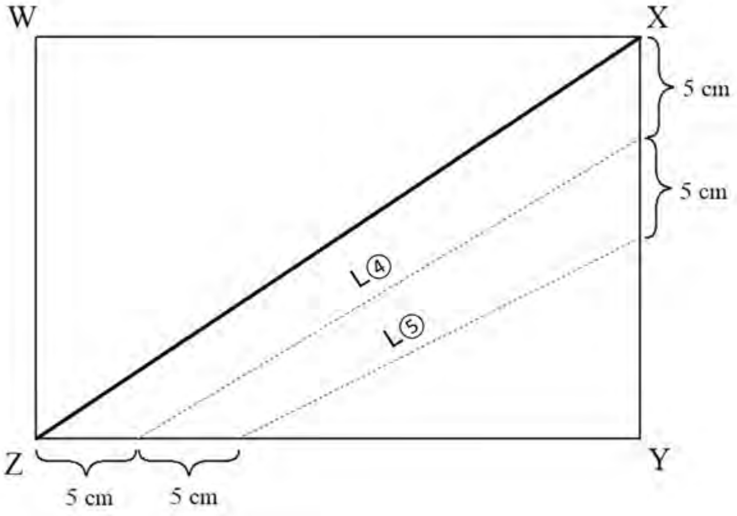


Diagram 12

vii. Diagram 12 shows a third sheet of A4 paper, WXYZ. Taking the diagonal ZX as a reference, two line segments, L④ and L⑤, are drawn as shown in the diagram.

- Are the two line segments, L④ and L⑤ parallel to diagonal ZX? Why or why not?
- Using a scaled ruler only, explain how you could fold the A4 paper to construct a parallel line segment that is 5 cm away from diagonal ZX.

Task 3: Perpendicular and Parallel Lines Surrounding Us

Stairs

Kite

Door

Diagram 13

i. Diagram 13 shows some objects surrounding us.

- Identify pairs of perpendicular lines and pairs of parallel lines in each of these objects.

Diagram 14

ii. Diagram 14 shows the drawing of a pair of parallel railway lines.

- Supposedly, parallel lines never intersect but why do the two railway lines draw to meet at one endpoint?

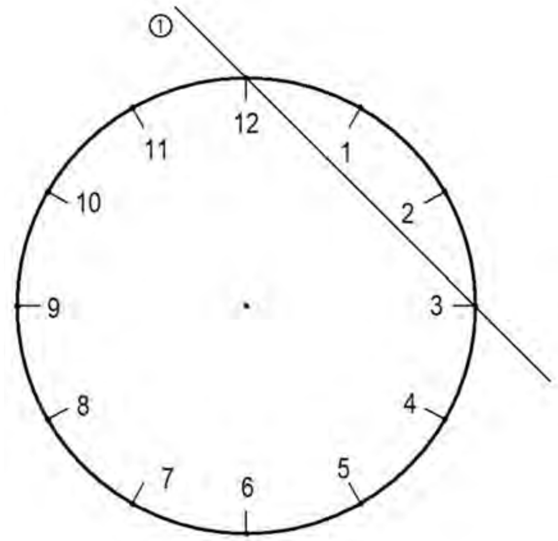


Diagram 15

iii. Diagram 15 shows a clock face with line ① joining 3 to 12.

- Using a ruler and pencil only, draw
  - ❖ a line joining any two numbers on the clock face that is perpendicular to line ①.
  - ❖ A line joining any two numbers on the clock face that is parallel to line ①.
- Other than measuring the angles of intersection, explain how you know that the lines are perpendicular or parallel.

Task 4: Different Types of Quadrilaterals

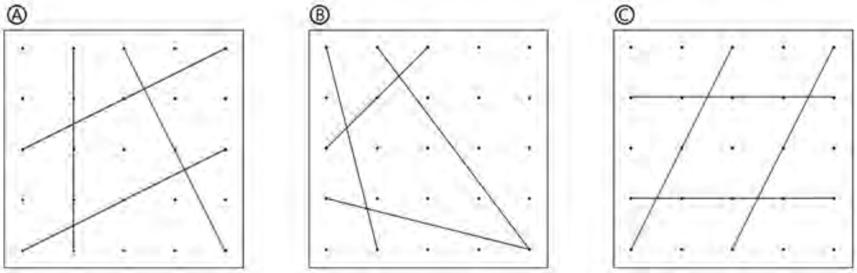


Diagram 16

i. A quadrilateral is a closed figure with 4 straight sides. Diagram 16 shows three types of quadrilaterals formed by joining the dots with four lines.

- Name each of the quadrilaterals.
- How many pairs of parallel lines can you find in each of the quadrilaterals?
- Draw a Venn diagram to show the relationships that exist among these quadrilaterals.
- Explain each relationship.

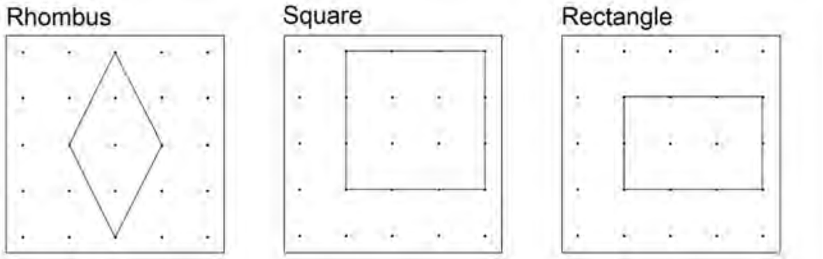


Diagram 17

ii. Diagram 17 shows another three types of quadrilaterals.

- Explain why each of these quadrilaterals is also a parallelogram.
- Explain how these three quadrilaterals fit into your Venn diagram in subtask (i).


Bridge Table Rack

Diagram 18

iii. Diagram 18 shows some trapezium found in our surroundings.

- Look for more examples of the trapezium in your home surrounding.
- Look for examples of parallelograms and rhombuses in your home surrounding.

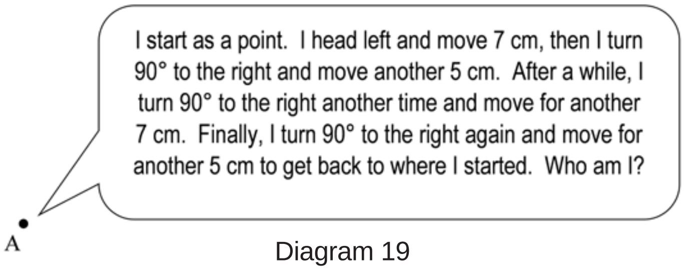


Diagram 19

iv. Diagram 19 shows a point, A. What figure will A become after making the movet?

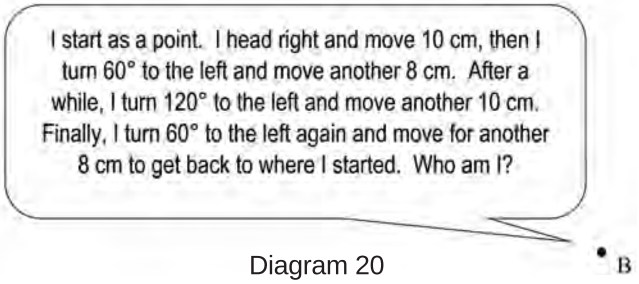


Diagram 20

v. Diagram 20 shows another point, B. What figure will B become after making the move?

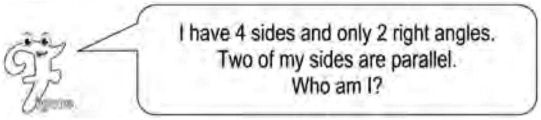


Diagram 21

vi. Diagram 21 shows a figure telling about its properties. What can the figure be?

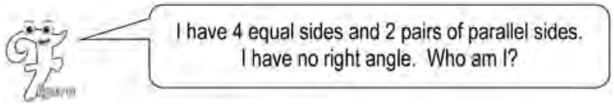


Diagram 22

vii. Diagram 22 shows another figure.

- Can the figure be a square? Why or why not?
- What can the figure be?

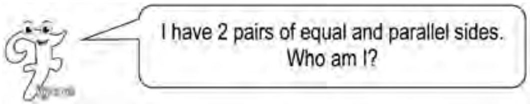


Diagram 23

viii. Diagram 23 shows a third figure.

- Can the figure be a rectangle? Why or why not?
- Can the figure be a square? Why or why not?
- What else can the figure be?

Task 5: Sum of Interior Angles of Polygons

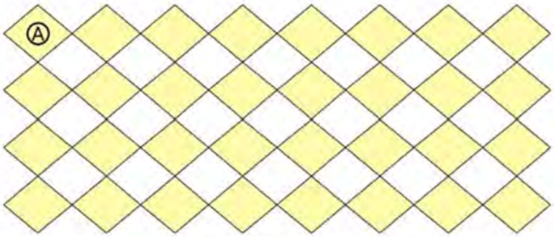


Diagram 24

i. Diagram 24 shows a tessellation pattern formed by a rhombus Ⓐ.

- Explain what is a tessellation pattern.

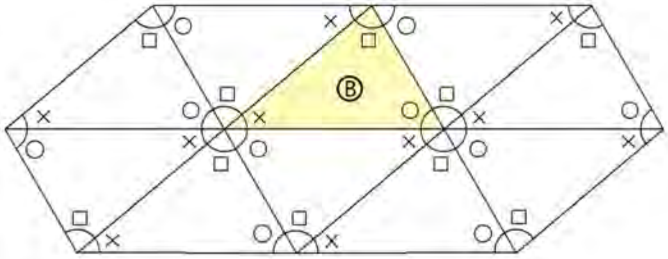


Diagram 25

ii. Diagram 25 shows a tessellation pattern formed by a triangle Ⓑ, with interior angles ∡X, ∡O, and ∡□. Cut out and use triangle Ⓑ in Material Sheet 1 at the end of this topic to make the tessellation pattern.

- Based on the tessellation pattern, explain why the sum of the three interior angles of the triangle Ⓑ is 180°.
- Other than using the tessellation pattern, how will you prove that the sum of three interior angles of any triangle is 180°?

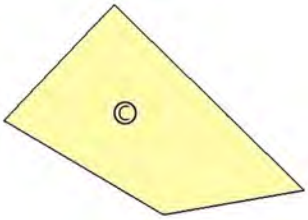


Diagram 26

iii. Diagram 26 shows quadrilateral Ⓒ.

- Cut out and use quadrilateral Ⓒ in Material Sheet 2 at the end of this topic to create a tessellation pattern.
- Use the tessellation pattern to find the sum of all interior angles of a quadrilateral Ⓒ.
- Explain your reasoning for finding the sum of the interior angles.

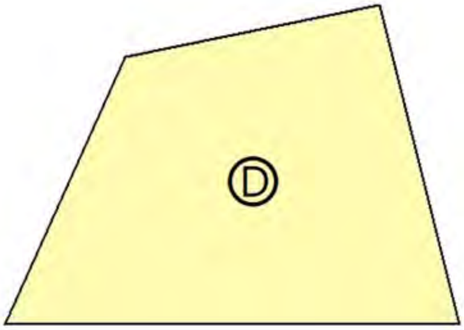
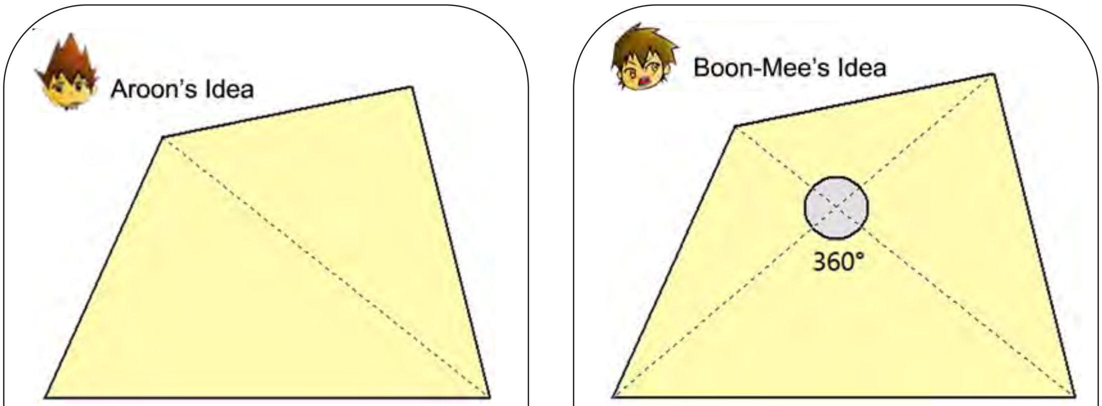


Diagram 27

iv. Diagram 27 shows a quadrilateral Ⓓ. Without forming a tessellation pattern, two students, Aroon and Boon-Mee, used two different ideas to find the sum of all interior angles of the quadrilateral Ⓓ as shown in Diagram 28.

- Explain the two ideas.



Divide the quadrilateral into two triangles by drawing a diagonal.

So, the sum of the four interior angles of the quadrilateral is 2 times 180°.

Divide the quadrilateral into four triangles by drawing two diagonals.

So, the sum of the four interior angles of the quadrilateral is 4 times 180°, subtract the extra 360°.

Diagram 28

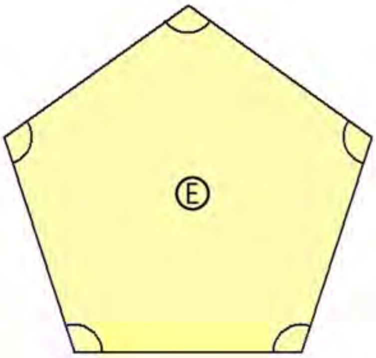
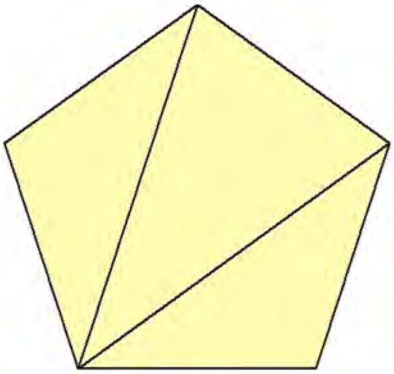


Diagram 29

v. Diagram 29 shows a regular pentagon Ⓔ with five interior angles. Aroon and Boon-Mee used two different methods to find the sum of all interior angles of the pentagon Ⓔ as shown in Diagram 30.

- Explain the reasonings for the two methods.
- Explain why the pentagon Ⓔ cannot form a tesselation pattern.
- Explain how this task can nurture your students' appreciation towards the beautifulness of mathematical ideas.

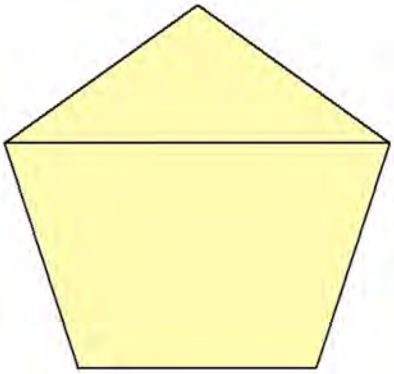
Aroon's Method



Divide the pentagon into three triangles by drawing two diagonals.

So, the sum of the five interior angles of the pentagon is 3 times 180°.

Boon-Mee's Method



Divide the pentagon into one triangle and one quadrilateral by drawing one diagonal.

So, the sum of the five interior angles of the pentagon is 180° adds up with 360°.

Diagram 30

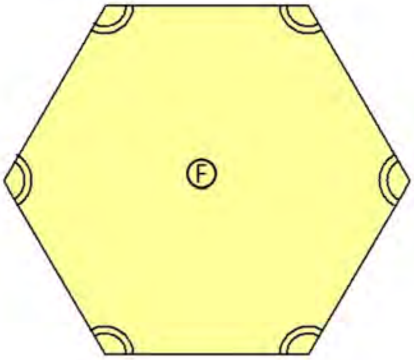


Diagram 31

vi. Diagram 31 shows a regular hexagon Ⓕ, with 6 interior angles.

- Find the size of each interior angle.
- Explain your reasoning for finding the interior angle.
- Can the hexagon Ⓕ be used to form a tessellation pattern? Explain your reasons.

vii. Diagram 32 shows four tasks involving interior angles of polygons.

- Which task will you choose to be the first task for your students? Explain your reasons.
- Explain how the knowledge learnt in Task ① can help to simplify the way to solve Task ④?
- How will you sequence the four tasks for your students?
- Explain the reasons for your sequence of tasks.
- How can sequencing the tasks encourages your students to learn mathematics by and for themselves?

Task ①:

Prove that the sum of interior angles of any heptagon is 900°.

Task ②:

Prove that the sum of interior angles of any triangle is 180°.

Task ③:

Prove that the sum of interior angles of any quadrilateral is 360°.

Task ④:

Prove that the sum of interior angles of any octagon is 1080°.

Diagram 32

viii. Diagram 33 shows a concave quadrilateral CDEF.

- Find the sum of the interior angles.

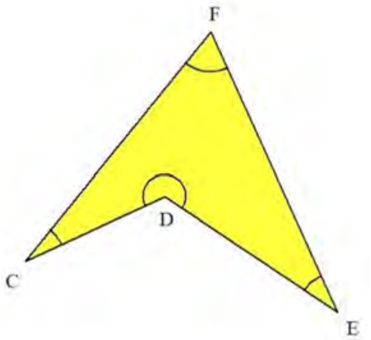


Diagram 33

ix. Diagram 34 shows a concave pentagon PQRST.

- Find the sum of the interior angles.

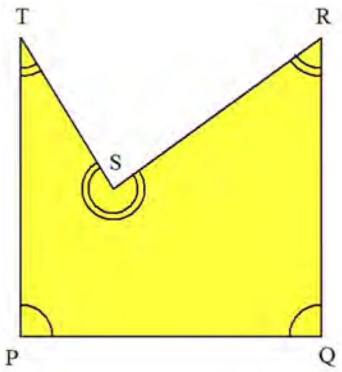


Diagram 34

Task 6: Circle

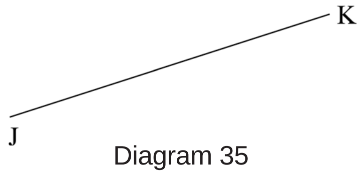


Diagram 35

i. Diagram 35 shows a line JK.

- Explain how a compass together with a ruler can be used to measure the length of JK.

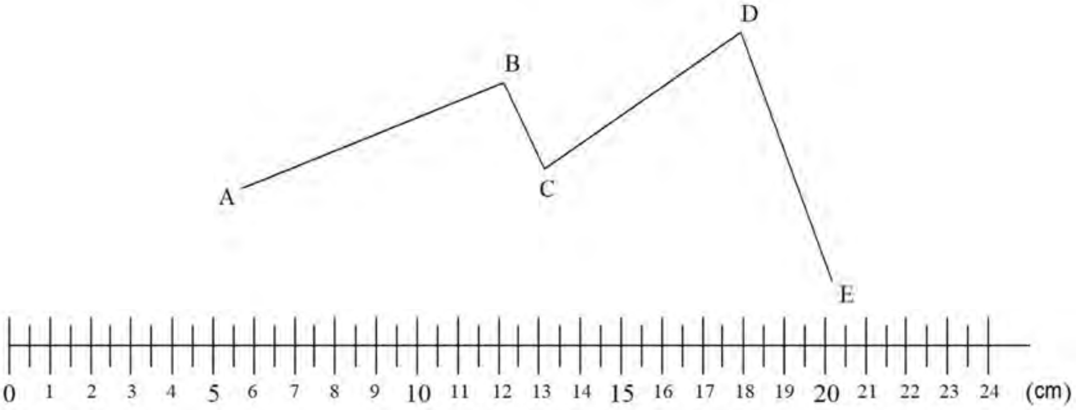


Diagram 36

ii. Diagram 36 shows a zigzag road connecting five towns A, B, C, D, and E on a map and a number line.

- Explain how a compass can be used to mark the total distance from A to E, going through B, C and D, on the number line.
- With the help of a compass, mark the total distance AE on the number line.

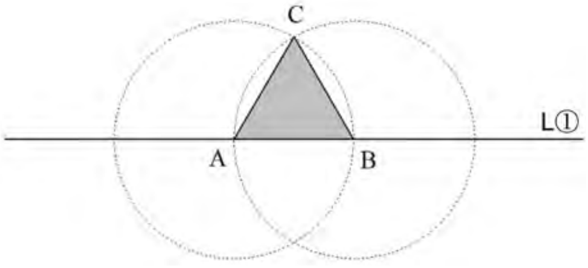


Diagram 37

iii. A triangle ABC is constructed by drawing two circles, with centres at A and B on line L① as shown in Diagram 37.

- Explain why triangle ABC is an equilateral triangle.
- Using a compass and a ruler only, reproduce triangle ABC, with AB = 5 cm.

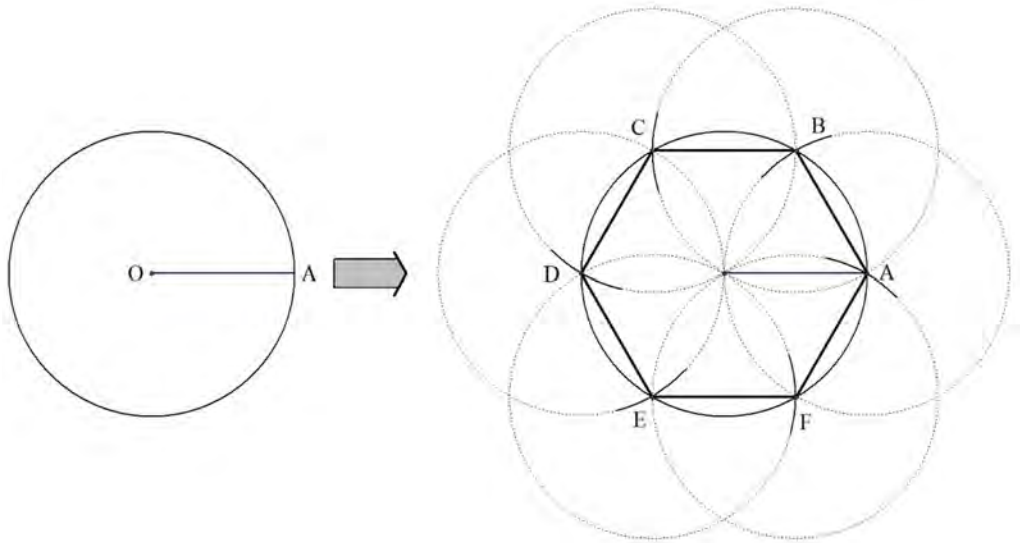


Diagram 38

iv. Diagram 38 shows a circle with centre O and radius OA. Six other identical circles are then constructed with centres on the circumference of the circle as shown in the diagram. A regular hexagon is constructed by joining the points A, B, C, D, E and F.

- Explain why the six sides of the hexagon are of the same length.
- Using a compass and ruler only, reproduce hexagon ABCDEF, with AB = 5 cm.

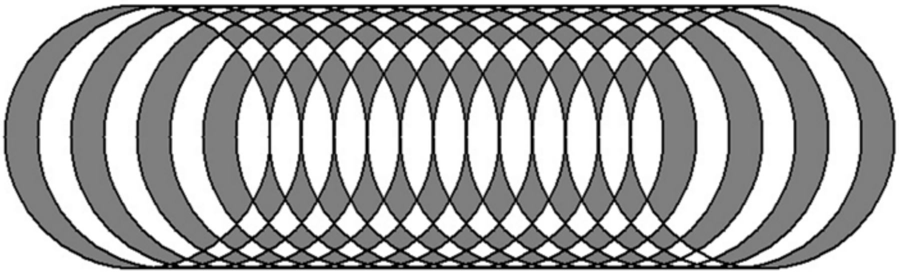


Diagram 39

v. Diagram 39 shows a design created by drawing 13 identical circles along a straight line.

- Using a compass and ruler only, reproduce the design.
- What property of a circle is used in creating the design? Explain your answer.

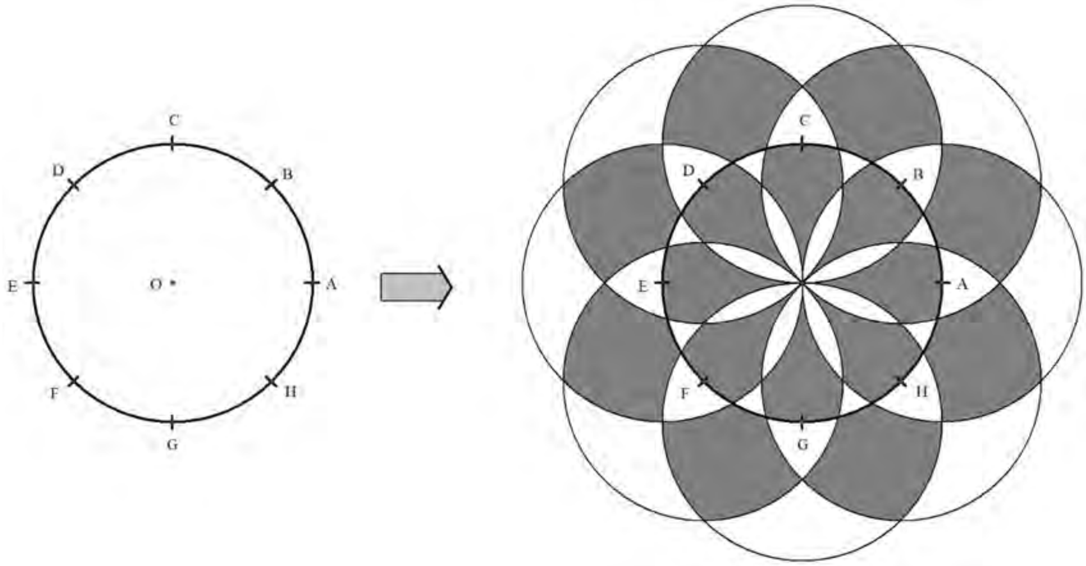


Diagram 40

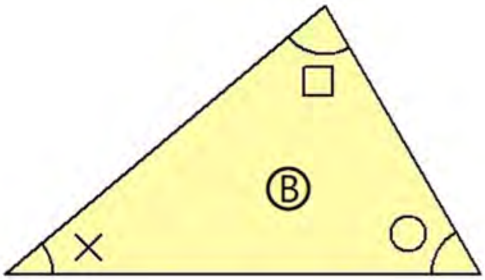
vi. Diagram 40 shows a circle with centre O and eight other points, A, B, C, D, E, F, G and H, on its circumference. The design is created by constructing eight other identical circles with each of the eight points as the centre for each circle.

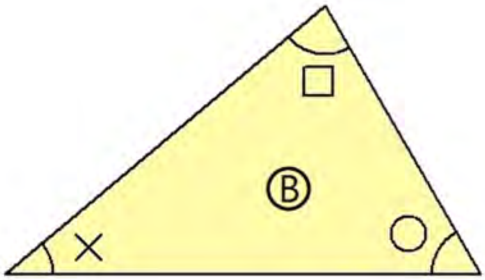
- Using a compass and ruler only, reproduce the design.
- What mathematical ideas are embedded in this task?
- In what ways can this task help to develop an appreciation towards circles?

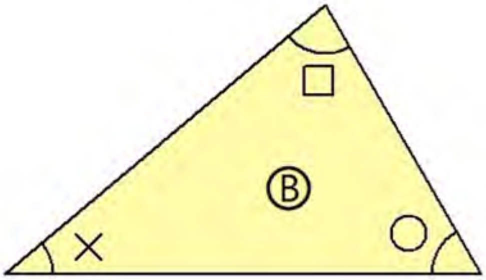
vii. With the help of a compass, create your artistic design using circles.

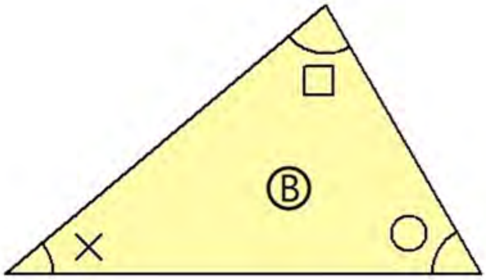
viii. What mathematical ideas in future grades can be extended from Task 6? Explain your answer with examples.

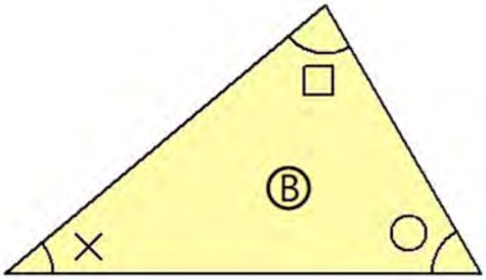
Material Sheet 1: Tessellation of Triangles

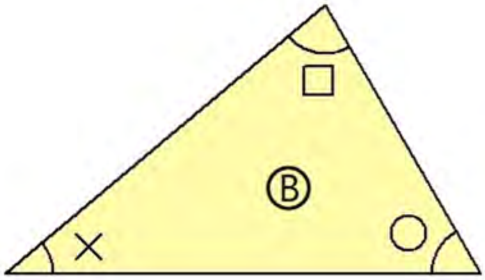


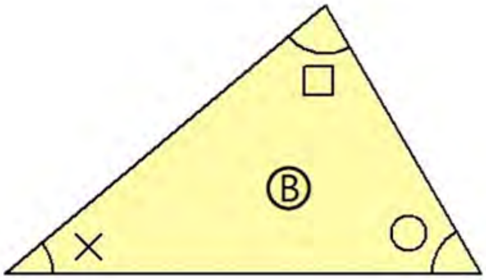


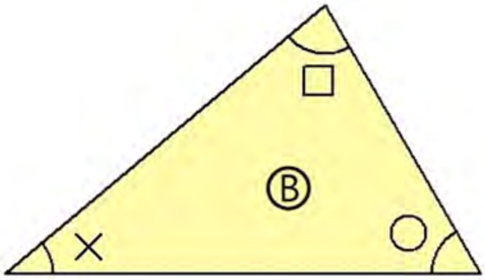


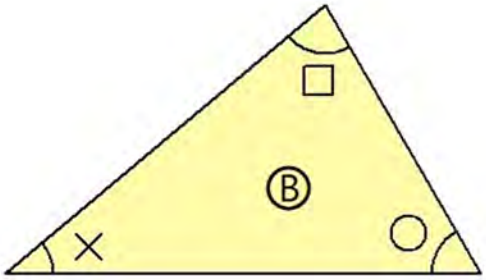


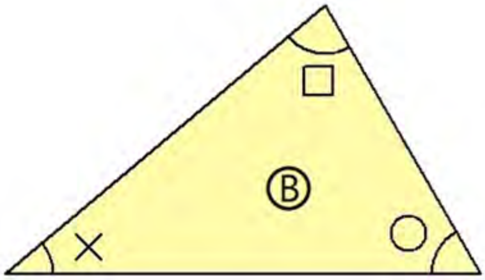




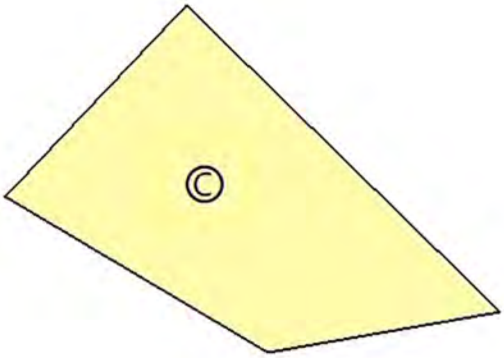


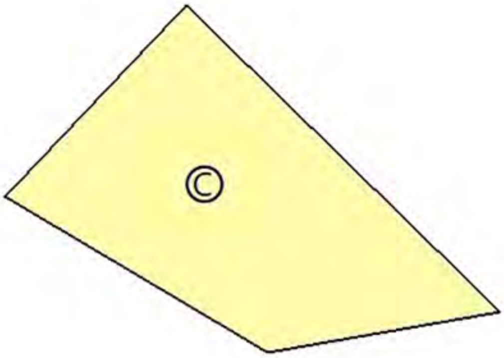


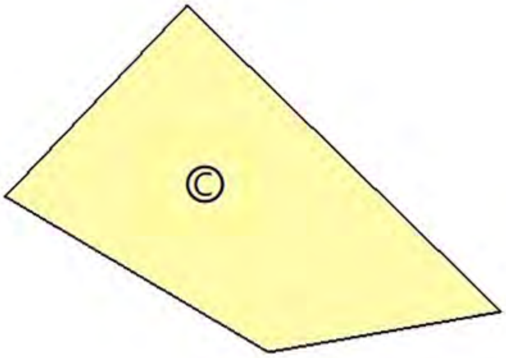


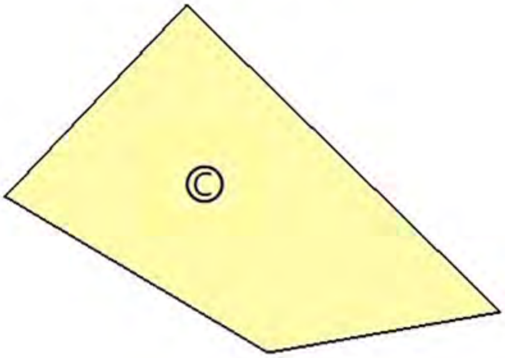


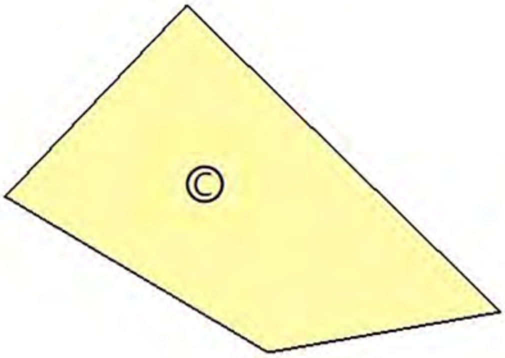
Material Sheet 2: Tessellation of Quadrilaterals

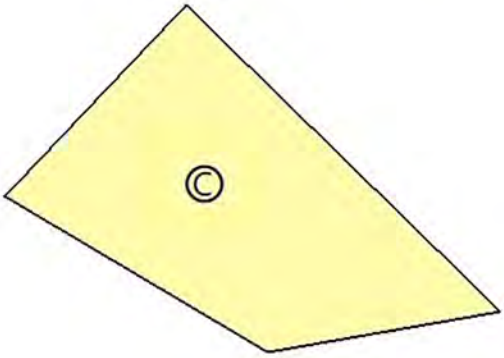


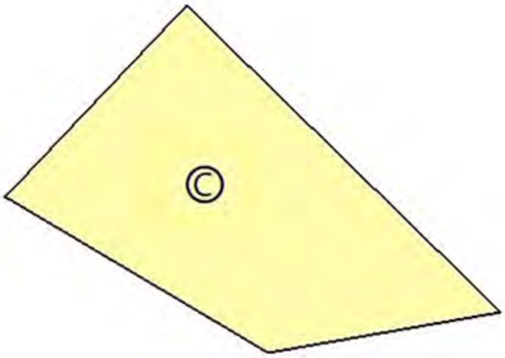


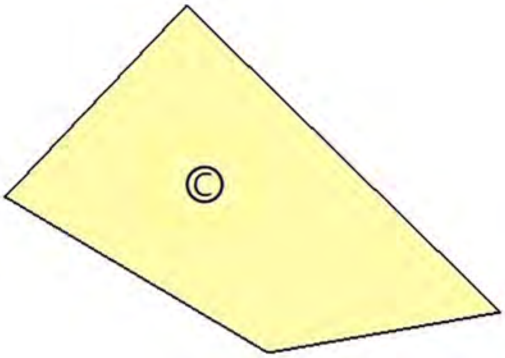












Topic 2: Exploring Space Figures with Their Components in Relation to the Plane

Standards 2.1:

Exploring rectangular prisms and cubes with their components
i. Identify the relationship among faces, edges and vertices for drawing sketch
ii. Explore the nets of a rectangular prism and find the corresponding position between components
iii. Explore the perpendicularity and parallelism between the faces of a rectangular prism
iv. Explain positions in rectangular prisms with the idea of 3 dimensions

Sample Tasks for Understanding the Standards

Task 1: Rectangular Prism and Cube

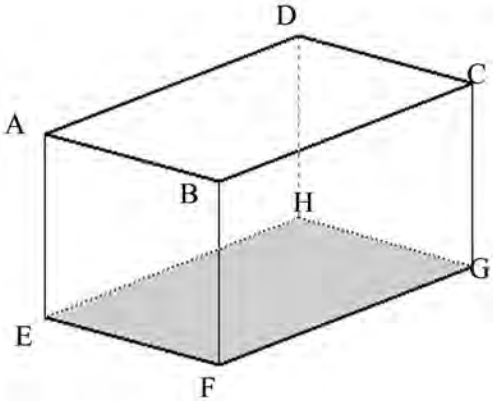


Diagram 1

i. Diagram 1 shows a rectangular prism ABCDEFGH, with a shaded base EFGH.

- Determine all pairs of parallel faces.
- Which faces are perpendicular to
  - ❖ face EFGH?
  - ❖ face ABFE?
- Which edges are perpendicular to
  - ❖ face EFGH?
  - ❖ face ADHE?
- Which edges are perpendicular to edge BC?
- Which edges are parallel to edge BC?

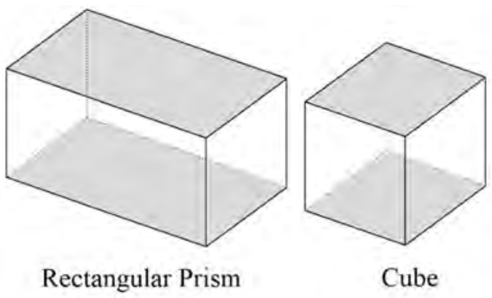


Diagram 2

i. Diagram 2 shows a cube and a rectangular prism each with two shaded parallel faces.

- What is the same and what is different between these two solid figures?
- Find the number of vertices (V), edges (E), and faces (F) for each solid figure.
- For each of the solid figures, let □ be the number of edges of each of the shaded base shapes. Find the relationship between
  - ❖ V and □
  - ❖ E and □
  - ❖ F and □
- Find the relationship between V, E, and F.

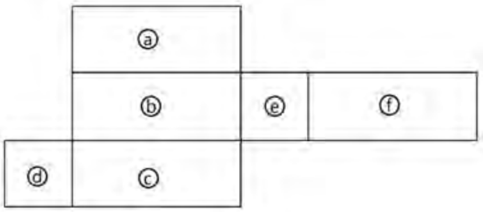


Diagram 3

ii. Diagram 3 shows a net with six faces of a rectangular prism.

- What shape is the base of this prism? Explain your answer.
- Determine all pairs of parallel faces of this prism.
- Determine all faces that are perpendicular to the face ⓕ.

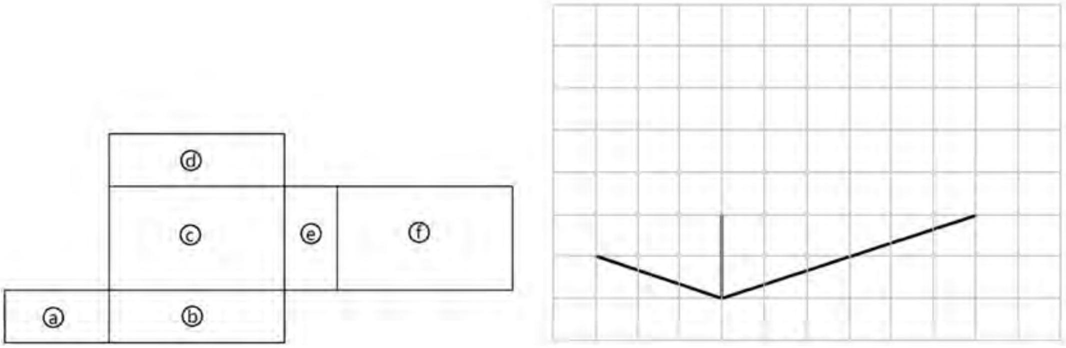


Diagram 4

iii. Diagram 4 shows a net and an incomplete sketch of a rectangular prism.

- Complete the sketch of the prism.
- Determine all pairs of opposite faces of the prism.

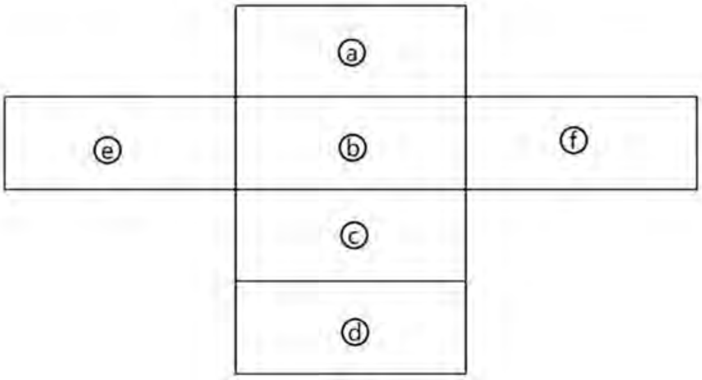


Diagram 5

iv. Diagram 5 shows an arrangement of six congruent rectangles.

- Explain why this arrangement will not form a rectangular prism.
- Explain how you will modify the arrangement to make a net for a rectangular prism.

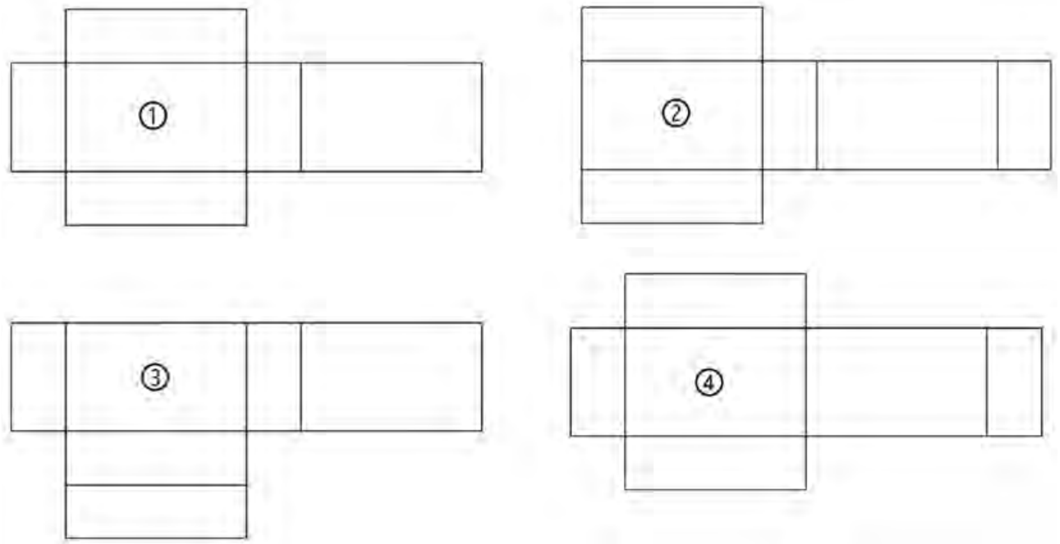


Diagram 6

v. Diagram 6 shows four arrangements of six faces to make a rectangular prism.

- Determine, which of these arrangements are not nets of the rectangular prism.
- Explain any clue that you use to identify nets of a rectangular prism.

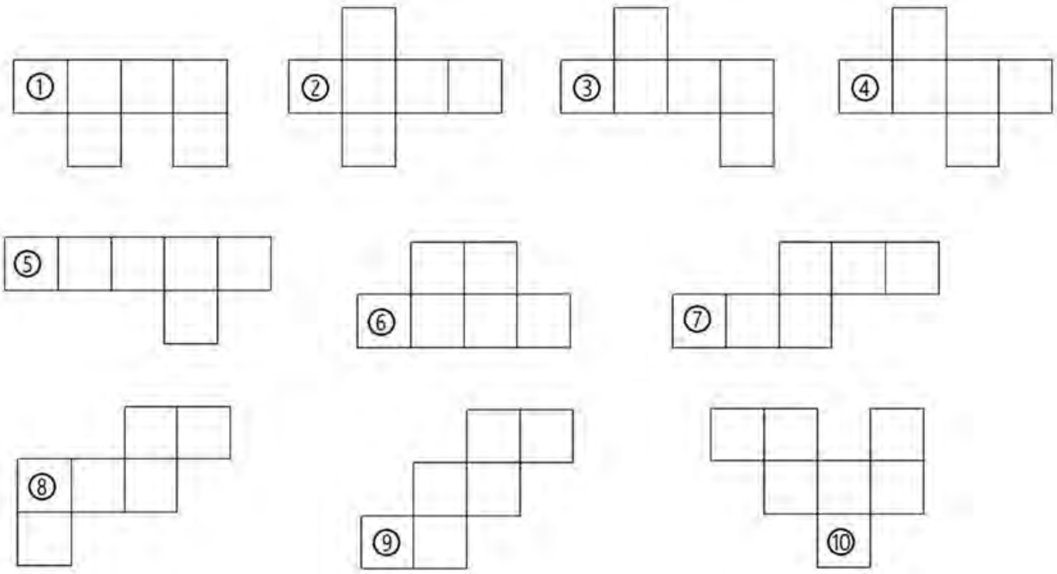


Diagram 7

vi. Diagram 7 shows ten arrangements of squares.

- Determine which of these arrangements are nets of a cube.
- Explain any clue that you use to determine the nets of a cube.

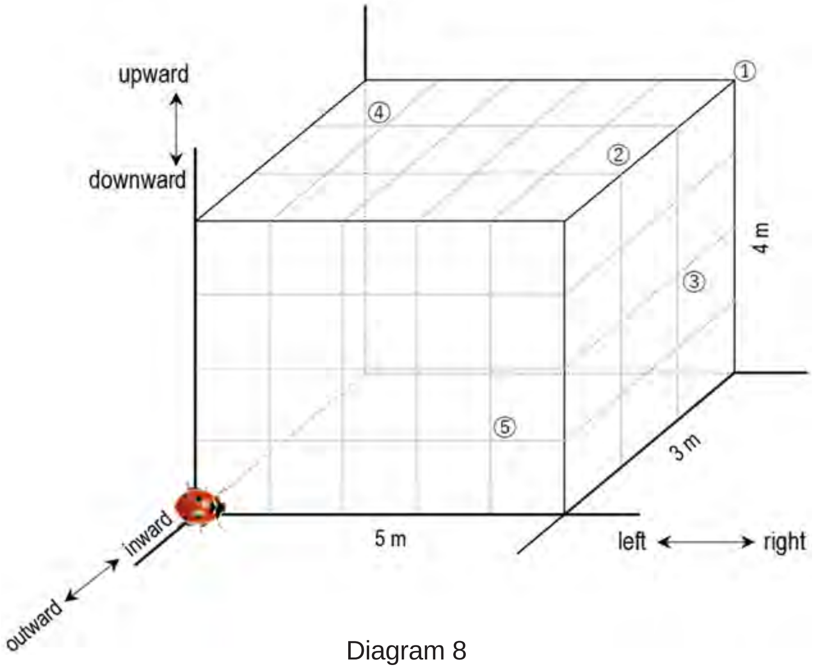


Diagram 8

vii. Diagram 8 shows a giant rectangular prism with a length 5 m, width 3m, and height 4 m. A beetle is resting at one of its vertices as shown and this beetle can crawl around on the faces of the prism. However, it only knows three directions of movement, which are left-right, inward-outward, and upward-downward as shown in the diagram.

- The beetle crawls 5 m to the right, turns and crawls 3 m inward, and finally turns again to crawl 4 m upward.
    - ❖ At what location is the beetle now?
- Describe the movement of the beetle from its original position to
    - ❖ location ②.
    - ❖ location ③.
    - ❖ location ④.
- Describe the movement of the beetle to crawl from location ④ to location ⑤.
- Sometime, the beetle will fly to move beyond the prism faces. From location ②, the beetle flies 2 m upward, then 2 m to the right and stop in the air.
    - ❖ From the location in the air, describe the movement of the beetle to location ③.
- From location ③, the beetle flies 1 m to the right, then 2 m outward, and finally 2 m downward.
    - ❖ At what location is the beetle now?

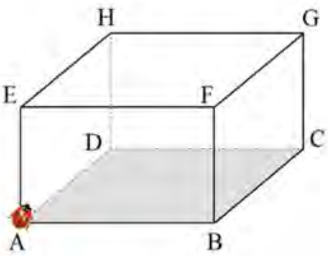


Diagram 9

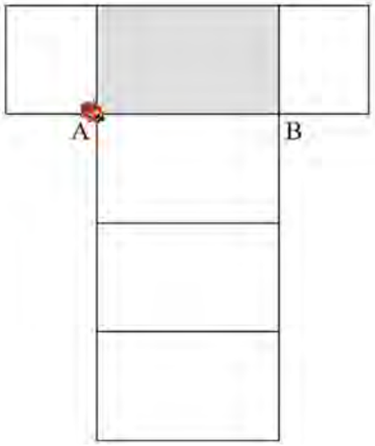


Diagram 10

viii. Diagram 9 shows a rectangular prism with a beetle resting at vertex A, whereas Diagram 10 shows a net of the rectangular prism when it is opened up.

- Mark and label on Diagram 10, vertices C, D, E, F, G, and H.
- Explain why these vertices appear twice in the net.
- Explain why vertices A and B appear once only in the net.
- The beetle is crawling from vertex A to vertex G. A student claimed that the shortest route for the beetle to crawl from vertex A to vertex G is to crawl along edge AE, then along EG in a straight line.
  - ❖ Is the student's claim valid?
  - ❖ Explain how you will use the net in Diagram 10 to help the student visualise the shortest route.

Standards 2.2:

Extending rectangular prism to other solids such as prisms and cylinders

i. Extend the number of relationships among faces, edges and vertices for drawing sketch

ii. Explore nets of prisms and cylinders, and find the corresponding position between components

iii. Distinguish prism and cylinder by the relationship of their faces

Sample Tasks for Understanding the Standards

Task 1: Prisms and Cylinders

What is a prism?

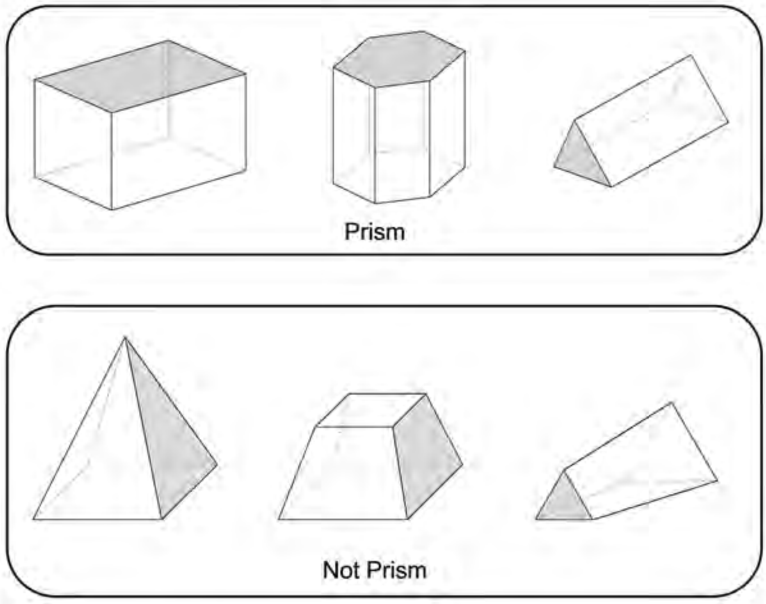


Diagram 1

i. Diagram 1 shows some examples and non-examples of prisms.

- What is a prism?

ii. Diagram 2 shows a cylinder.

- Why is a cylinder not a prism?

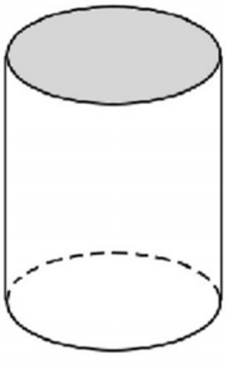


Diagram 2

Diagram 3

iii. Diagram 3 shows a file holder.

- Is the file holder a prism? Why or why not?

Diagram 4

iv. Diagram 4 shows a structure from Egypt.

- Is the structure a prism? Why or why not?

Diagram 5

v. Diagram 5 shows a traditional house in Malaysia. Study the structure of the house.

- Identify and sketch any prism in the structure of the house.

Relationship Among Faces, Edges and Vertices of a Prism

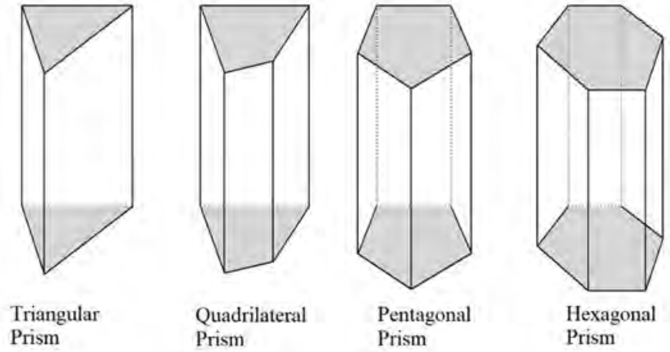


Diagram 6

i. Diagram 6 shows four prisms, each with two shaded parallel faces, known as the bases of the prism.

- What can you say about the shape and sizes of the two parallel faces for each prism?
- What is the shape of all the other faces that are not shaded?
- Which faces are perpendicular to the shaded bases?

ii. For each prism, determine and fill in Table 1, its number of vertices (V), edges (E) and faces (F). Then, determine the relationship among V, E and F.

Table 1
Number of Vertices, Edges and Faces

	Triangular Prism	Quadrilateral Prism	Pentagonal Prism	Hexagonal Prism
Shape of bases				
Number of edges of one base (n)				
Number of vertices (V) of the prism				
Number edges (E) of the prism				
Number of faces (F) of the prism				

- Determine and write a mathematics sentence to explain the relationship between
  - V and n
  - E and n
  - F and n
- Determine and write a mathematics sentence to explain the relationship between V, E and F.
- If your students have difficulty in dealing with the use of algebraic symbols such as n, V, E and F, how will you present this task to them?

Task 2: Nets of Prism and Cylinder

Net of Prism

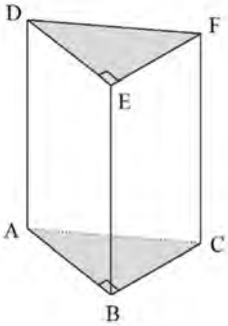


Diagram 7

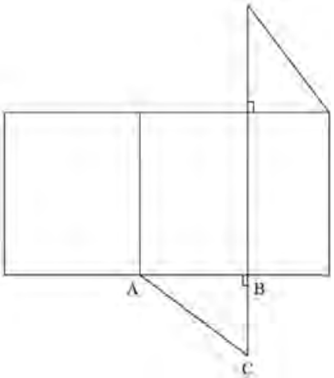


Diagram 8

i. Diagram 7 shows a prism with a right-angled triangle base. The prism is cut out to form a net as shown in Diagram 8.

  - Mark and label the vertices D, E, and F on the net in Diagram 8.
  - Identify and label the edge AD, BE, and CF on the net in Diagram 8.
  - Sketch another net for this triangular prism.

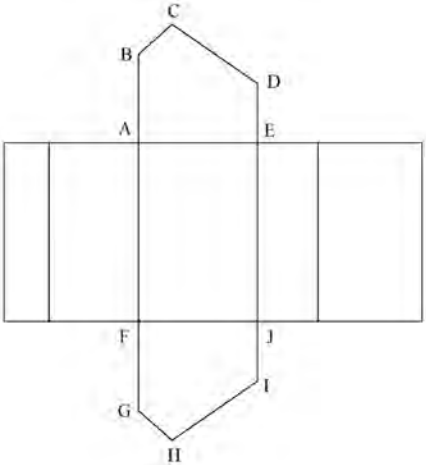


Diagram 9

ii. Diagram 9 shows a net of a prism.

  - Based on the net, sketch the prism and name the prism.
  - How many pairs of parallel faces are there in the prism? Shade the parallel pairs of faces with different colours on the net in Diagram 9.
  - How many faces are perpendicular to face ABCDE? Shade the perpendicular faces with another colour on the net in Diagram 9.

Net of Cylinder

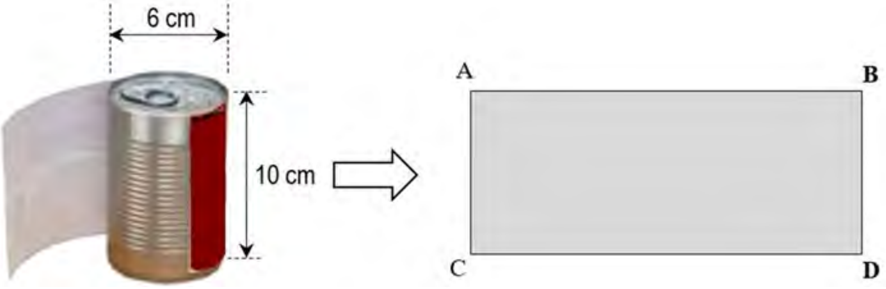


Diagram 10

i. A face of a cylinder is cut out and flattened to be a rectangle ABCD as shown in Diagram 10.

- Find the length of
  - ❖ AB
  - ❖ AC

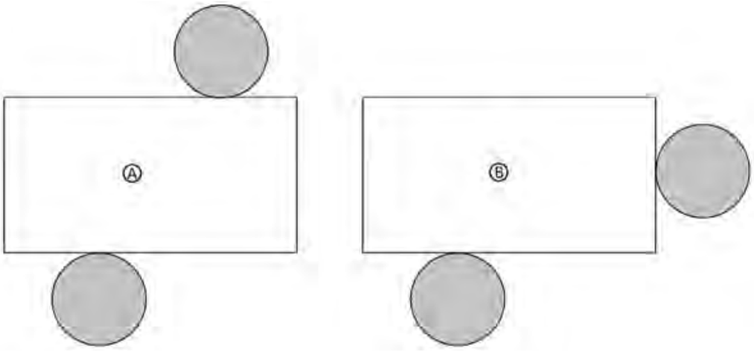


Diagram 11

ii. Diagram 11 shows two arrangements of circles and rectangles, Ⓐ and Ⓑ.

- Which of these arrangements is or is not a net for a cylinder? Why or why not?

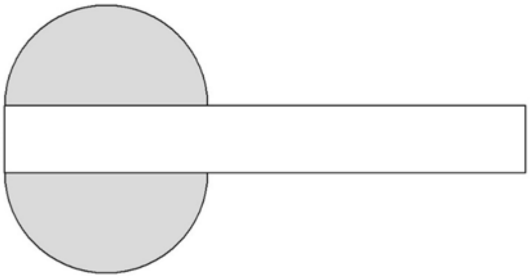


Diagram 12

iii. Diagram 12 shows the net of a solid with two semi-circular faces.

- Sketch the solid.

Task 3: Who am I?

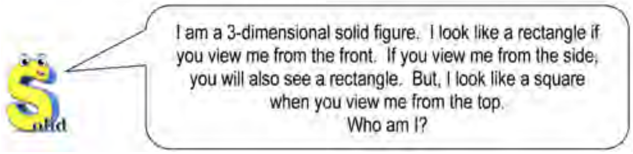


Diagram 13

i. Diagram 13 shows a solid figure talking about how it looks from different directions.

- Can the solid figure be a prism? Why or why not?
- What can the solid figure be?

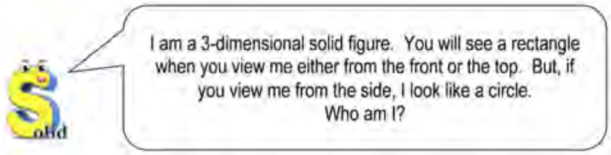


Diagram 14

ii. Diagram 14 shows another solid figure talking about how it looks from different directions.

- Can the solid figure be a prism? Why or why not?
- What can the solid figure be?

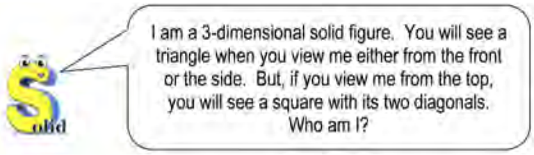


Diagram 15

iii. Diagram 15 shows a third solid figure talking about how it looks from different directions.

- Can the solid figure be a prism? Why or why not?
- What can the solid figure be?

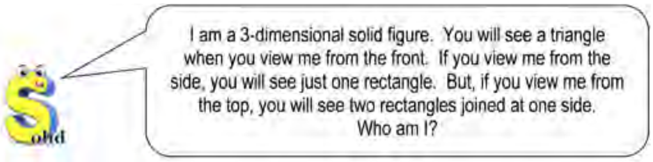


Diagram 16

iv. Diagram 16 shows a fourth solid figure talking about its look from different directions.

- Can the solid figure be a prism? Why or why not?
- What can the solid figure be?

Topic 3: Exploring Figures with Congruence, Symmetry and Enlargement

Standards 3.1:

Exploring the properties of congruence

i. Explore properties of figures which fit when overlapped and identify conditions of congruency with corresponding points and sides
ii. Draw congruent figures using minimum conditions and confirm by measuring angles and sides
iii. Appreciate the usefulness of congruent figures by tessellation

Sample Tasks for Understanding the Standards

Task 1: Congruent Figures

Congruent Figures

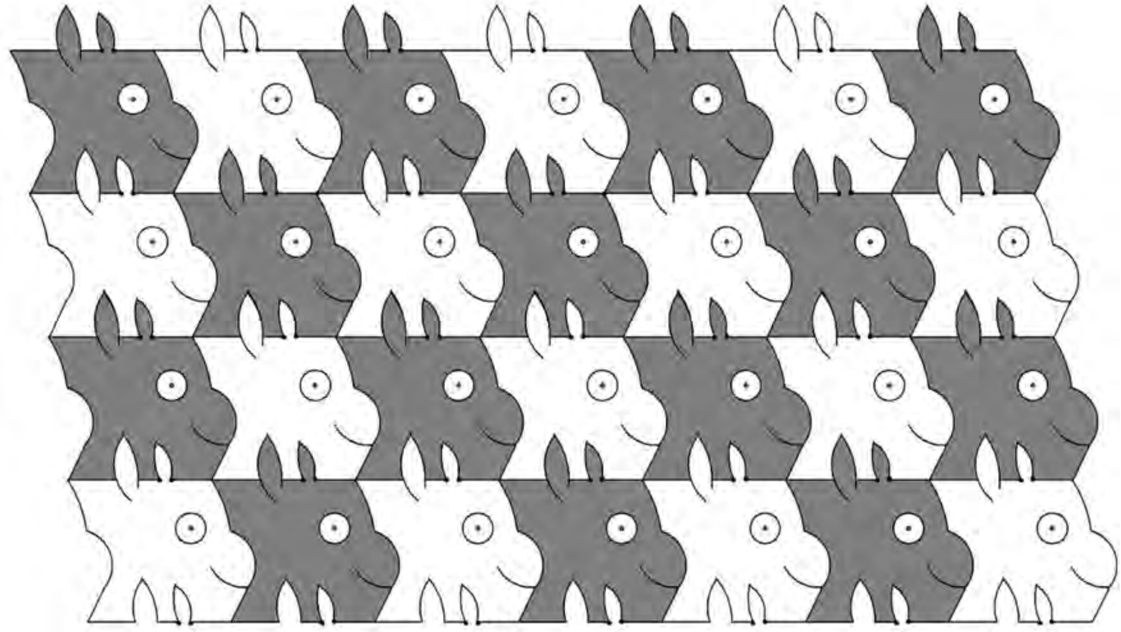


Diagram 1

i. Diagram 1 shows a tessellation of dark and white bear heads. The bear heads are congruent with one another. Examine the shape and size of each bear's head.

- Explain the meaning of congruent figures.
- Explain how the tessellation of figures can be used to develop the concept of congruency.
- Explain how the tessellation of congruent figures can be used to nurture appreciation towards the beautifulness of mathematics.

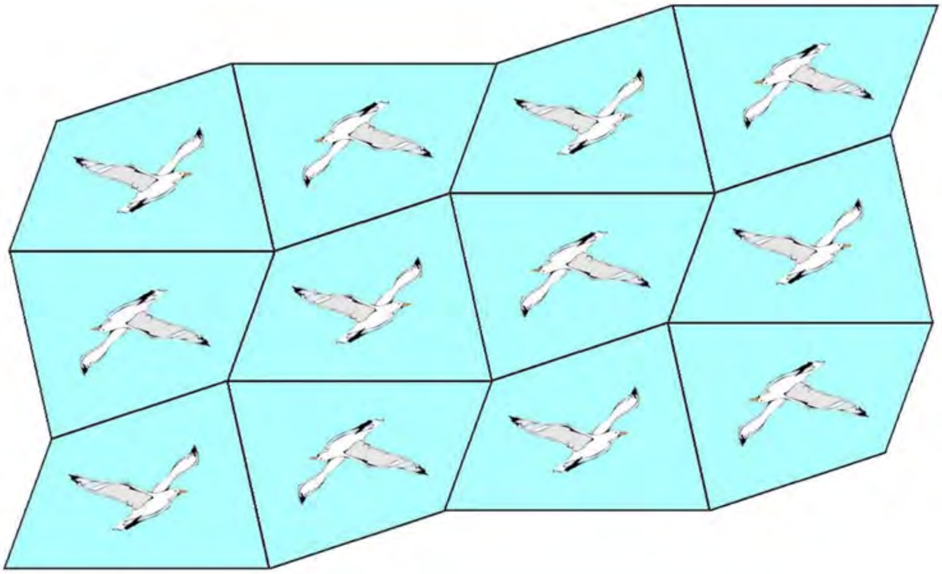


Diagram 2

ii. Diagram 2 shows a tessellation of congruent quadrilaterals.

- What is the relationship between the corresponding angles of the quadrilaterals?
- What is the relationship between the corresponding sides of the quadrilaterals?

Congruent Triangles

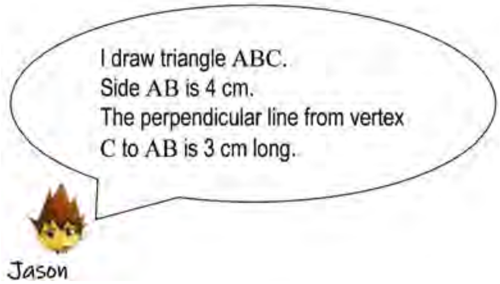


Diagram 3

i. Jason drew a triangle and he told his friend about his triangle. Diagram 3 shows the information he gave to his friends. From the information, his friend was trying to draw a triangle.

   - Based on the information given, is it possible for Jason's friend to be certain of drawing another triangle exactly the same as triangle ABC? Why or why not?

   - What is the minimum information needed for Jason's friend to be certain of drawing the exact triangle ABC?

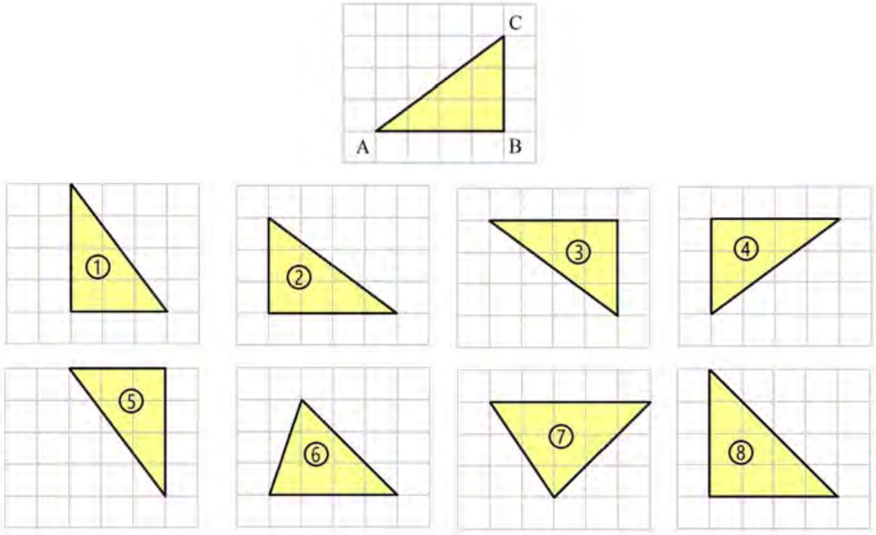


Diagram 4

ii. Diagram 4 shows a triangle ABC and eight other triangles.

   - Determine which of the eight triangles fit by overlapping with triangle ABC.

   - What can you conclude about the congruency of these triangles with triangle ABC?

iii. A student claimed that: "Congruent triangles are triangles that have the same size and shape."

   - Is the student's claim valid? Why or why not?

iv. Determine if each of the following statements is true or false. Explain your reasoning.

   - Two triangles are congruent if they have the same three sides.

   - Two triangles are congruent if they have the same three angles.

v. Two sides and a corresponding angle of triangle ABC are given as shown in Diagram 5. Jason used a protractor and compass to draw the triangle as shown in Diagram 6.

- How many different triangles can be drawn to fulfil the given condition? Explain your answer.

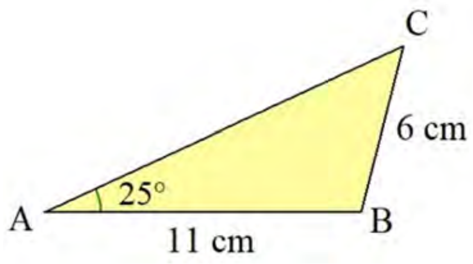


Diagram 5

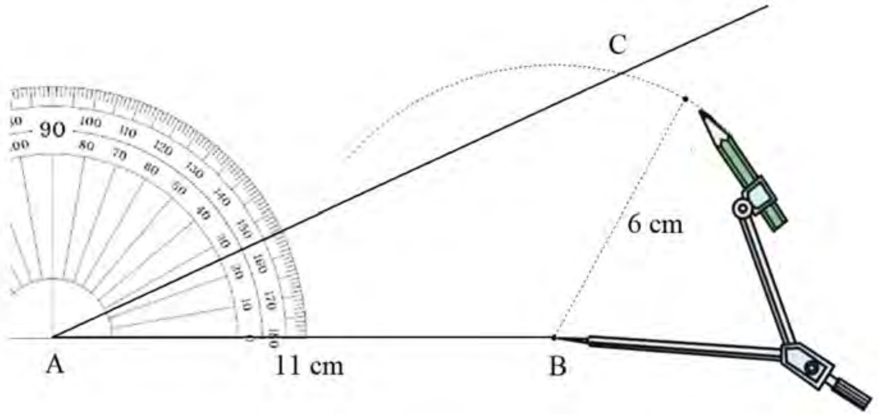


Diagram 6

vi. Two sides and the included angle of triangle DEF are given as shown in Diagram 7. Diagram 8 shows part of the process of drawing the triangle.

- How many different triangles can be drawn? Explain your answer.

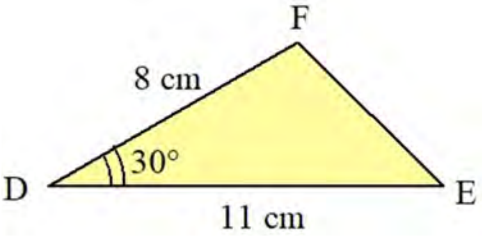


Diagram 7

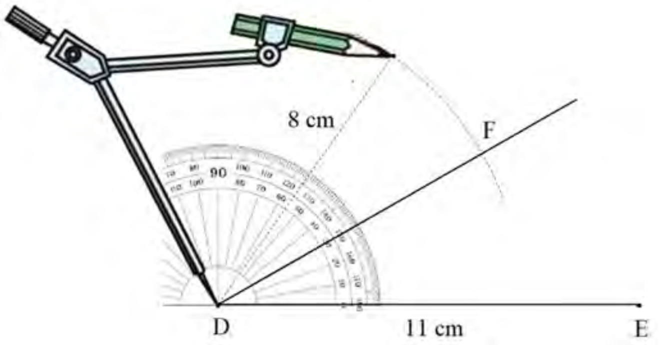


Diagram 8

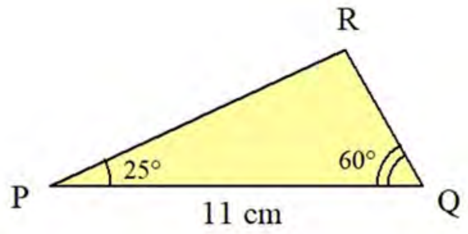


Diagram 9

vii. Two angles and the included side of triangle PQR are given as shown in Diagram 9.

- Use a ruler, a protractor and a compass to draw the triangle.
- How many different triangles can be drawn? Explain your answer.

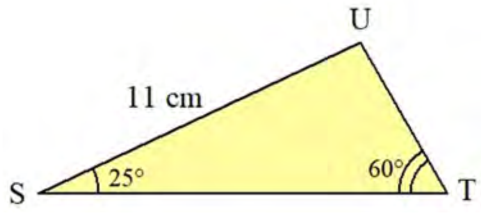


Diagram 10

viii. Two angles and a corresponding side of triangle STU are given as shown in Diagram 10.

- Use a ruler, a protractor and a compass to draw the triangle.
- How many different triangles can be drawn? Explain your answer.

Standards 3.2:

Exploring the properties of symmetry.

i. Explore the properties of figures which reflect and identify conditions of symmetry with line and its correspondence
ii. Draw symmetrical figures using conditions in an appropriate location
iii. Appreciate the usefulness of symmetry in designs

Sample Tasks for Understanding the Standards

Task 1: Line Symmetrical Designs

Diagram 1

i. Diagram 1 shows three ethnic designs from ASEAN countries. Duplicate and cut out each of the designs. Fold each of the cut-out designs into two congruent parts that fit on top of each other. The folded line is a line of symmetry.

- What is a line of symmetry?
- Which design has more than one line of symmetry? Explain your answer.

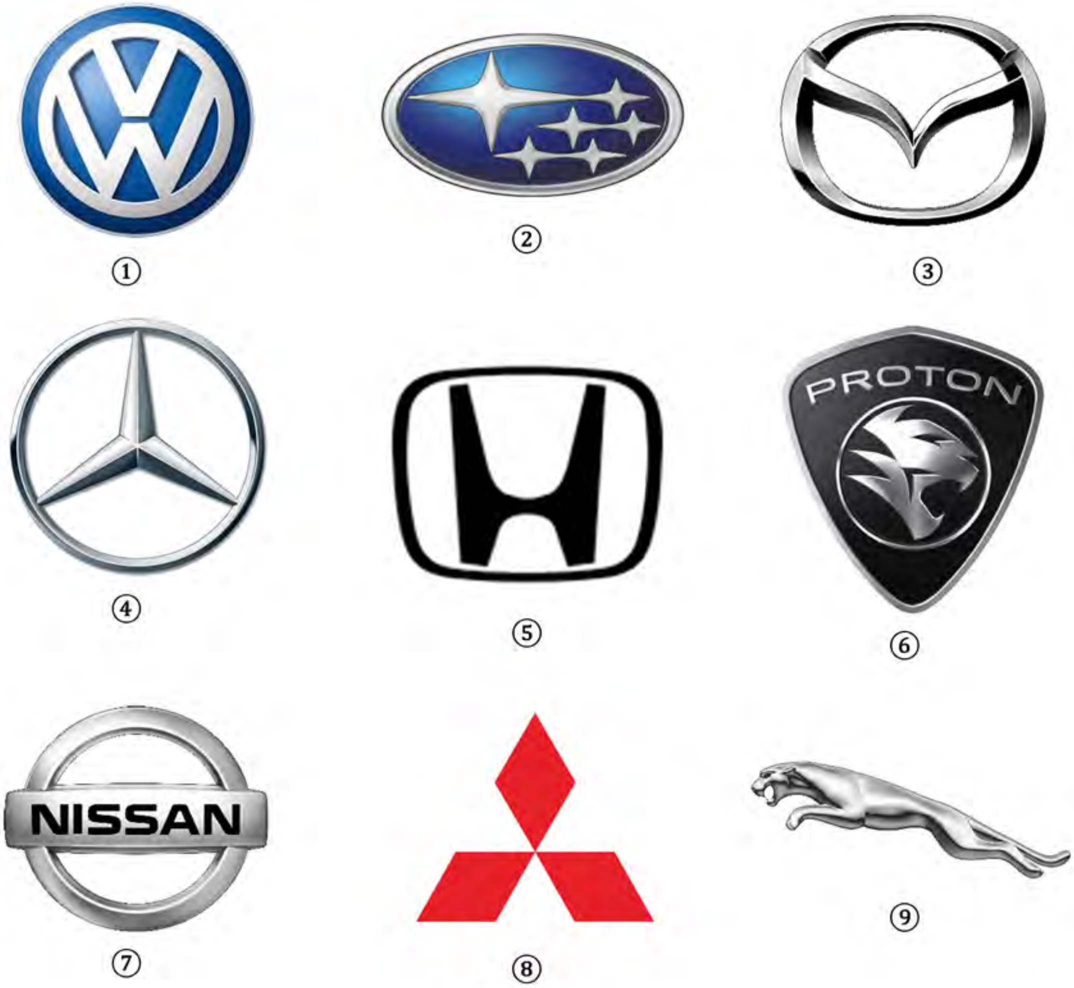


Diagram 2

ii. Line symmetry is a common motif in many designs. Diagram 2 shows nine logos of some automobile companies.

- Determine if each of the logos has any line of symmetry. If yes, draw all the lines of symmetries. If not, redesign the logo so that it will have at least one line of symmetry.

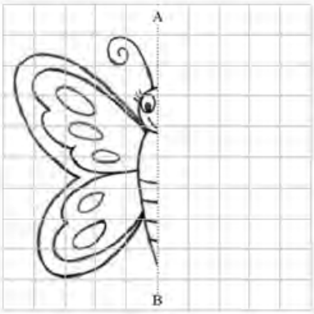


Diagram 3

iii. Diagram 3 shows an incomplete drawing of a butterfly. Given that AB is the line of symmetry of the butterfly.

- Complete the drawing.

iv. Explain how the tasks in this section will nurture your students' appreciation towards the beautifulness and usefulness of line symmetry.

Task 2: Line Symmetry and Paper Cutting

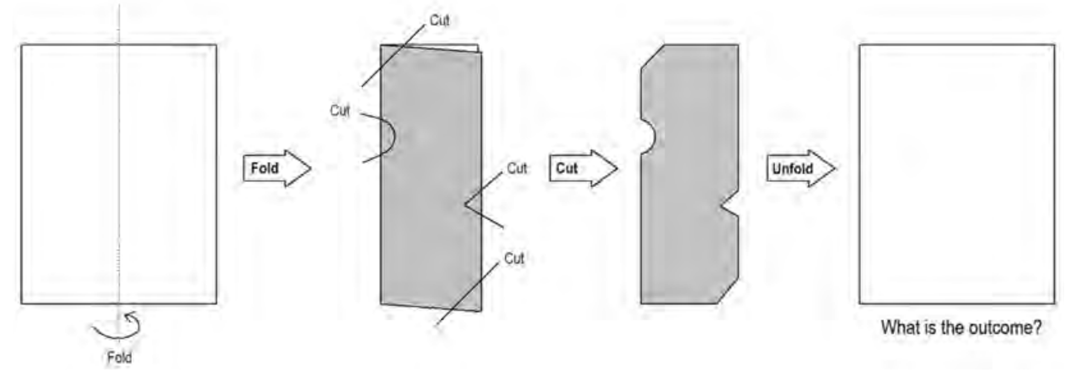


Diagram 4

i. Diagram 4 shows a piece of A4 paper being folded into halves. Then, several parts were cut off from the folded paper as shown.

- Sketch on Diagram 4, the outcome when the paper is unfolded.
    - ❖ How many lines of symmetry are there?
    - ❖ Mark the line of symmetry.
- Reproduce the outcome by folding and cutting a piece of A4 paper to check your answer.

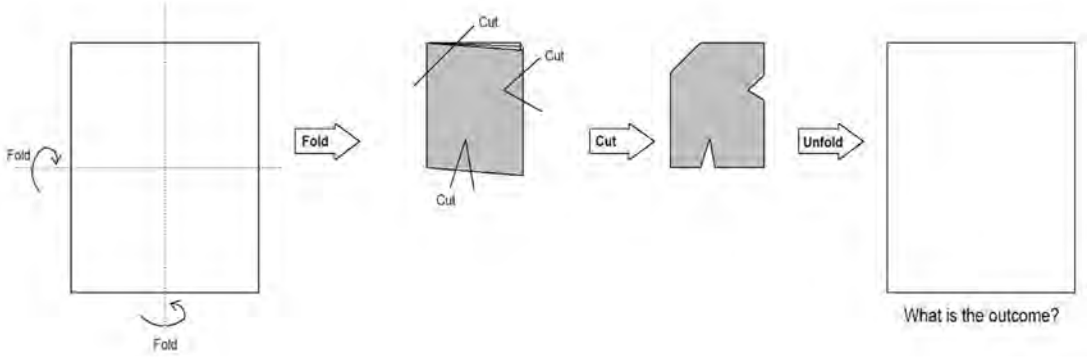


Diagram 5

ii. Diagram 5 shows another piece of A4 paper being folded into quarters. Similarly, several parts were cut off as shown.

- Sketch on Diagram 5, the outcome when the paper is unfolded.
    - ❖ How many lines of symmetries are there?
    - ❖ Mark the lines of symmetries.
- Reproduce the outcome by folding and cutting to check your answer.

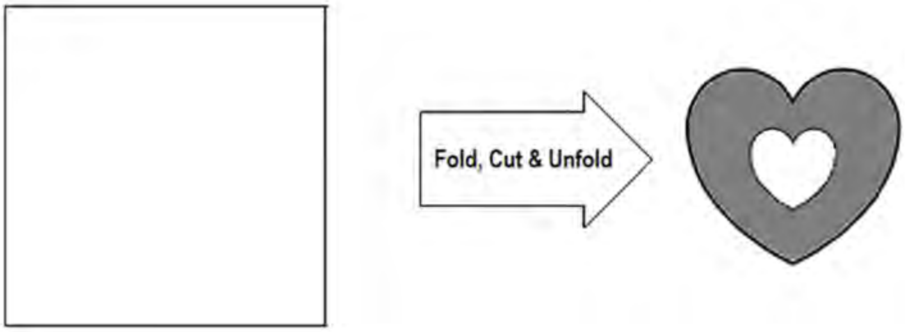


Diagram 6

iii. Diagram 6 shows a “double hearts” design created by folding and cutting a piece of square paper.

- Explain the folding and cutting process to produce the “double hearts”.
- How many lines of symmetry does the “double hearts” have?

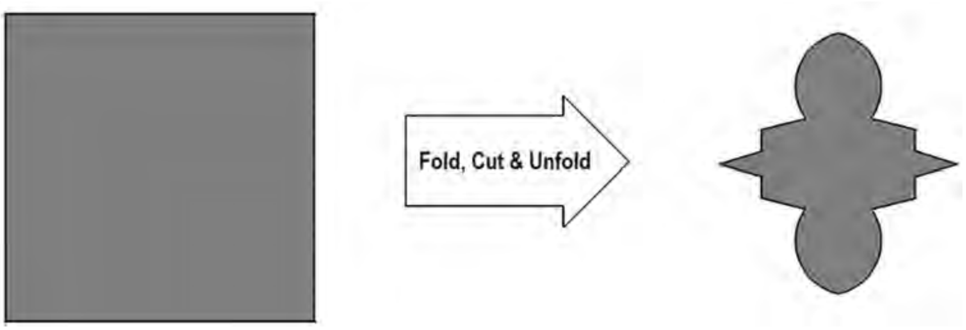


Diagram 7

iv. Diagram 7 shows an artistic design created from a piece of square paper.

- Explain how the square paper can be folded **twice** and cut to produce the design.
- How many lines of symmetry does the design have?
- Reproduce the design from a piece of square paper.

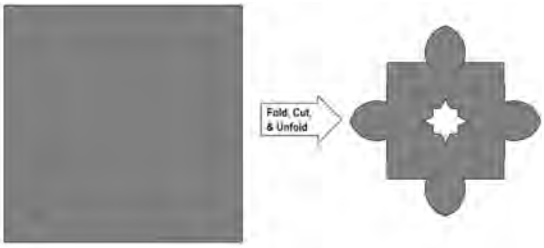


Diagram 8

v. Diagram 8 shows an artistic design created from another piece of square paper.

- Explain how the square paper can be folded three times and cut to produce the design.
- How many lines of symmetry does the design have?
- Reproduce the design from a piece of square paper.

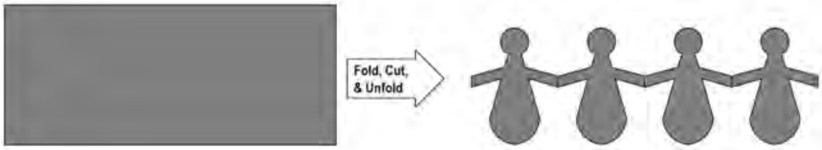


Diagram 9

vi. Diagram 9 shows a piece of rectangular paper, folded three times, and cut to create a chain of dolls.

- Explain how the chain of dolls can be created.
- How many lines of symmetry does the chain of dolls have?
- Reproduce the chain of dolls.

vii. Using a piece of paper, create your artistic design by folding and cutting the paper.

- Explain how you fold and cut the paper in creating the design.

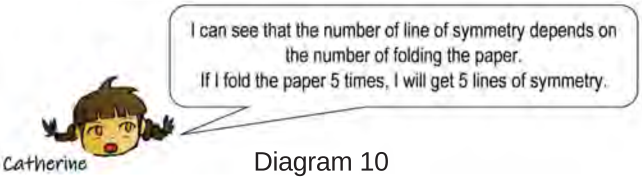


Diagram 10

viii. After making some symmetrical designs by folding and cutting paper, Catherine, as in Diagram 10, makes a conjecture about the relationship between the number of lines of symmetry and the number of folding.

- Is Catherine’s conjecture always, sometimes, or never true?
- Explain your answer.

Task 3: Properties of Figures with Line Symmetry

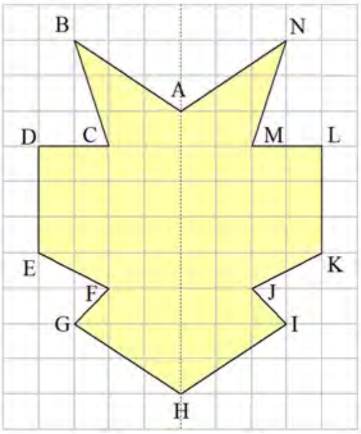


Diagram 11

i. In Diagram 11, when the shape is folded along the line of symmetry AH, the two parts of the shape fit exactly on top of each other.

- B and N is a pair of corresponding points. Draw a line segment from B to N.
  - ❖ What is the angle of intersection between the line segment and AH?
- Identify and state all other pairs of corresponding points. Repeat drawing a line segment joining each pair of corresponding points.
  - ❖ What is the angle of intersection between each line segment and AH?
  - ❖ What conclusion can you make about these angles of intersection?
  - ❖ What is the relationship between the perpendicular distance from each point of a corresponding pair to AH?
- BA and AN are a pair of corresponding sides.
  - ❖ Compare the lengths of BA and AN. What can you conclude about these two corresponding sides?
- Identify and state all other pairs of corresponding sides.
  - ❖ What is the relationship between the lengths of each pair of sides?
- ∠ABC and ∠AMN are a pair of corresponding angles.
  - ❖ Compare the sizes of the two corresponding angles. What can you conclude about these two corresponding angles?
- Identify and state all other pairs of angles.
  - ❖ What is the relationship between the sizes of each pair of angles?

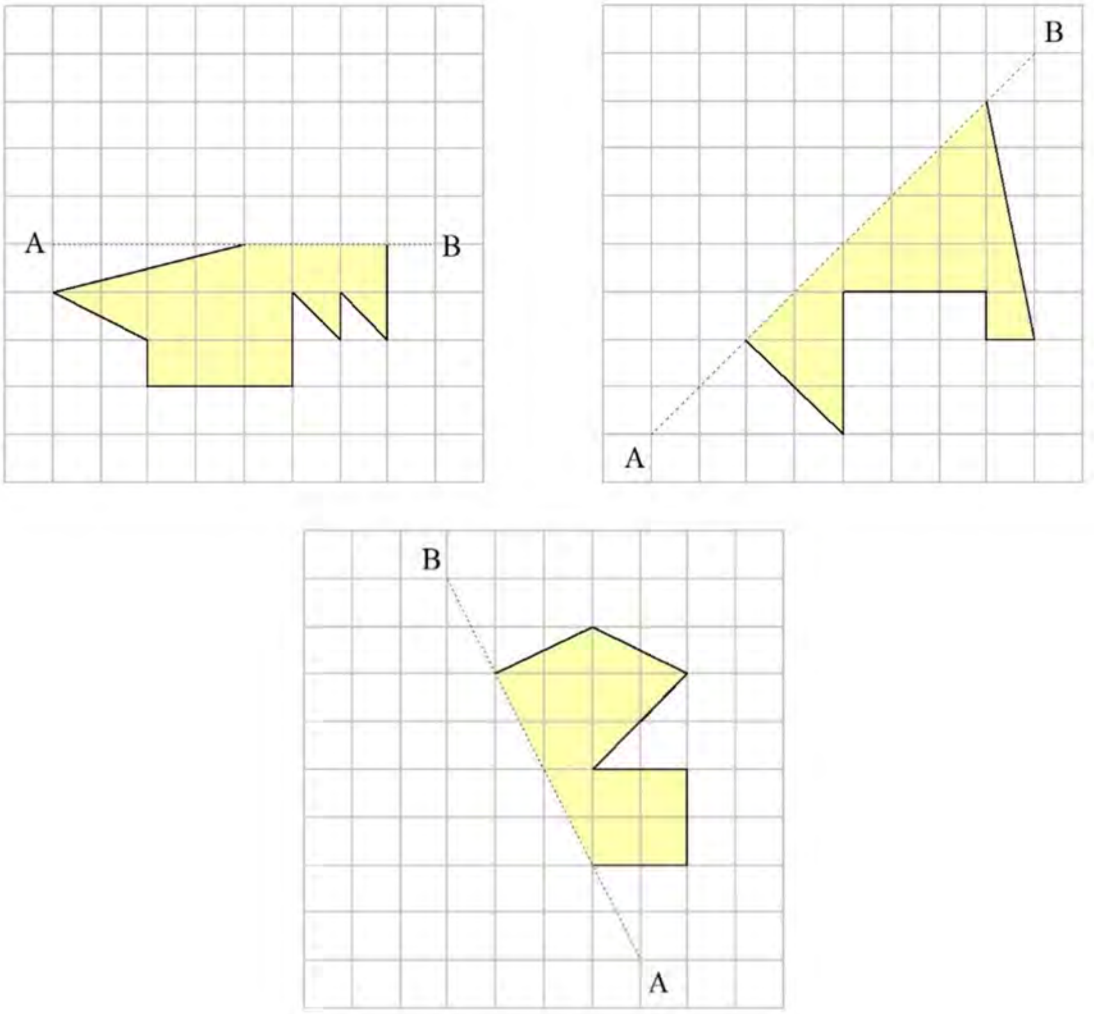
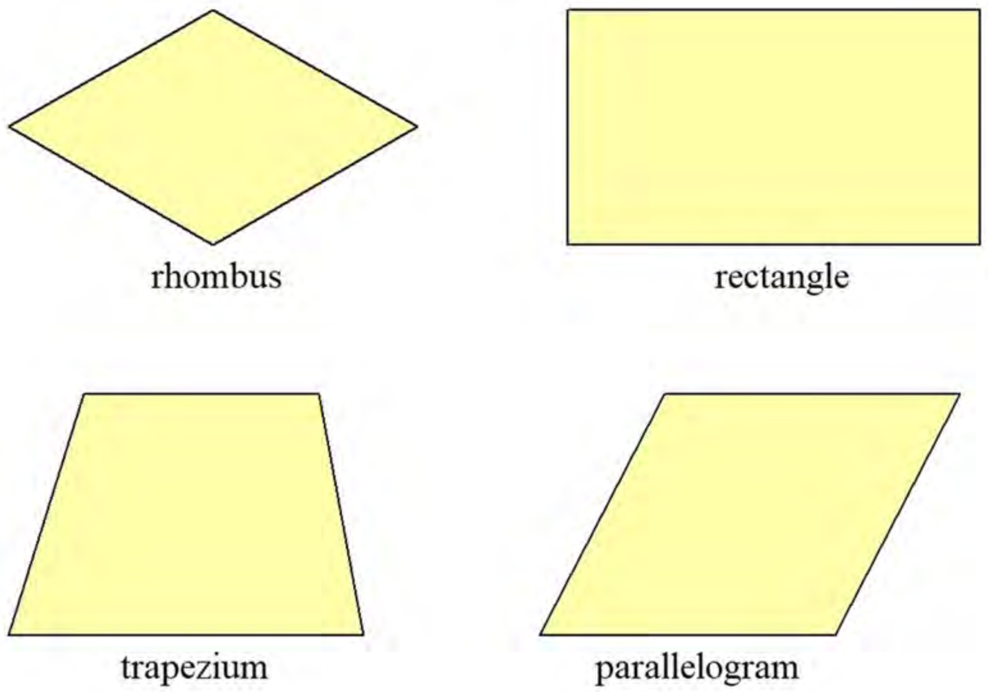


Diagram 12

ii. Diagram 12 shows incomplete drawings of three shapes. Given that AB is the line of symmetry.

- Complete the drawings.
- What different ways can be used by your students to complete these tasks?
- Many students may have difficulties completing these tasks. What different ways will you suggest to support these students' learning?

Task 3: Polygons and Line Symmetry



rhombus

rectangle

trapezium

parallelogram

Diagram 13

i. Diagram 13 shows four quadrilaterals.

- Which quadrilaterals have line symmetry?
- How many lines of symmetry does each have?
- Duplicate and cut out each quadrilateral. Fold the quadrilaterals to verify your answers.

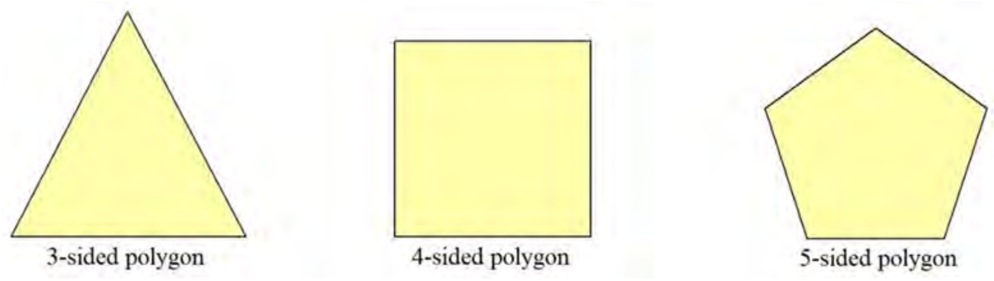


Diagram 14

ii. A regular polygon has all its sides with equal length. Diagram 14 shows six regular polygons with a different number of sides.

- A 3-sided regular polygon is known as an equilateral triangle, whereas a 4-sided polygon with all equal sides as shown is known as a square.
    - ❖ What is the name of the 5-sided regular polygon in Diagram 14?
- How many lines of symmetry does each of the regular polygons have?
- Complete Table 1.

Table 1
Number of Lines of Symmetry

Number of sides	3	4	5
Number of lines of symmetry			

- ❖ Make a conjecture about the relationship between the number of sides and the lines of symmetry of a regular polygon.
- ❖ Verify your conjecture using the 6-sided, 7-sided, and 8-sided polygons in Material Sheet 3 at the end of this topic.

- How many lines of symmetry does a 10-sided regular polygon have?
    - ❖ Cut out and fold the 10-sided polygon in Material Sheet 3 to verify your answer.
- What mathematical reasoning can this task develop among your students?
  Explain your answer.

Standards 3.3:

Exploring the Properties of Enlargement

i. Explore properties of figures in finding the centre of enlargement in simple cases such as rectangle

ii. Draw an enlargement of a rectangle using the ratio (multiplication of the value of the ratio)

iii. Appreciate the usefulness of enlargement in the interpretation of a map

Sample Tasks for Understanding the Standards

Task 1: Changing the Size of a Shape

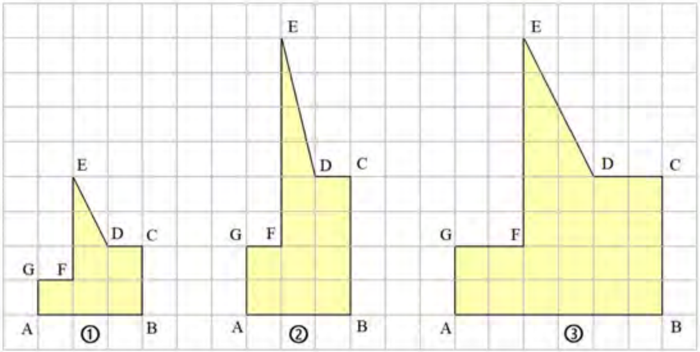
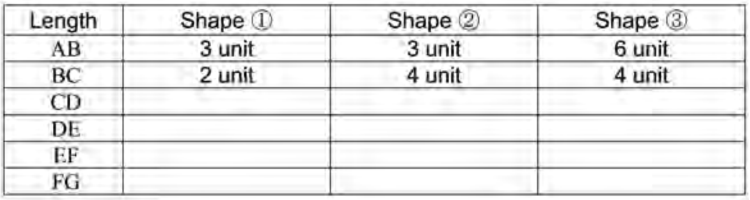


Diagram 1

i. Diagram 1 shows three shapes, ①, ②, and ③. Shape ② is a different shape from ①, but shape ③ is considered the same shape as ①. Table 1 recorded the length of each side of the three shapes, whereas Table 2 shows the ratio of each side of shape ② and shape ③ to shape ①.

- Complete Table 1

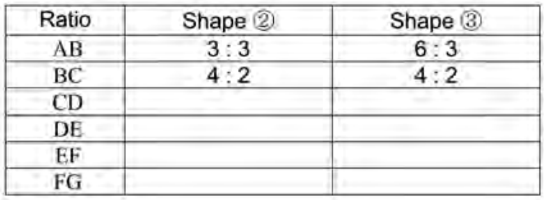
Table 1
Length of Sides



Length	Shape ①	Shape ②	Shape ③
AB	3 unit	3 unit	6 unit
BC	2 unit	4 unit	4 unit
CD			
DE			
EF			
FG			

- Complete Table 2

Table 2
Ratio of Side Compare to Shape ①



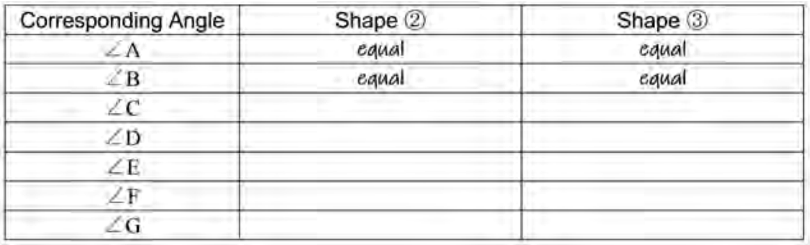
Ratio	Shape ②	Shape ③
AB	3 : 3	6 : 3
BC	4 : 2	4 : 2
CD		
DE		
EF		
FG		

ii. There are seven interior angles in each of the shapes. These angles are ∠A, ∠B, ∠C, ∠D, ∠E, ∠F, and ∠G.

- Duplicate and cut out shapes ①, ② and ③. Compare the corresponding angles of shapes ② and ③, with shape ①, then complete Table 3.

Table 3

Comparison Between Corresponding Angles of Shapes ② and ③ with Shape ①



Corresponding Angle	Shape ②	Shape ③
∠A	equal	equal
∠B	equal	equal
∠C		
∠D		
∠E		
∠F		
∠G		

iii. Why is shape ③ considered same as shape ①, but shape ② is not?

iv. Shape ③ is an enlarged shape of ①, whereas shape ① is a reduced shape of ③.

- Conclude the relationships between corresponding sides and corresponding angles between ① and ③.

v. What is the ratio of the area of shape ③ to shape ①?

- Make a conjecture about the ratio of the side and the ratio of the area of an enlarged or reduced shape.
- Test your conjecture with shapes ④ and ⑤ in Diagram 2, as well as shapes ⑥ and ⑦ in Diagram 3.

Diagram 2

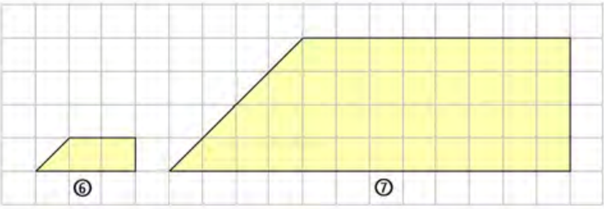


Diagram 3

Task 2: Drawing Enlarged or Reduced Shapes Using Sides and Angles

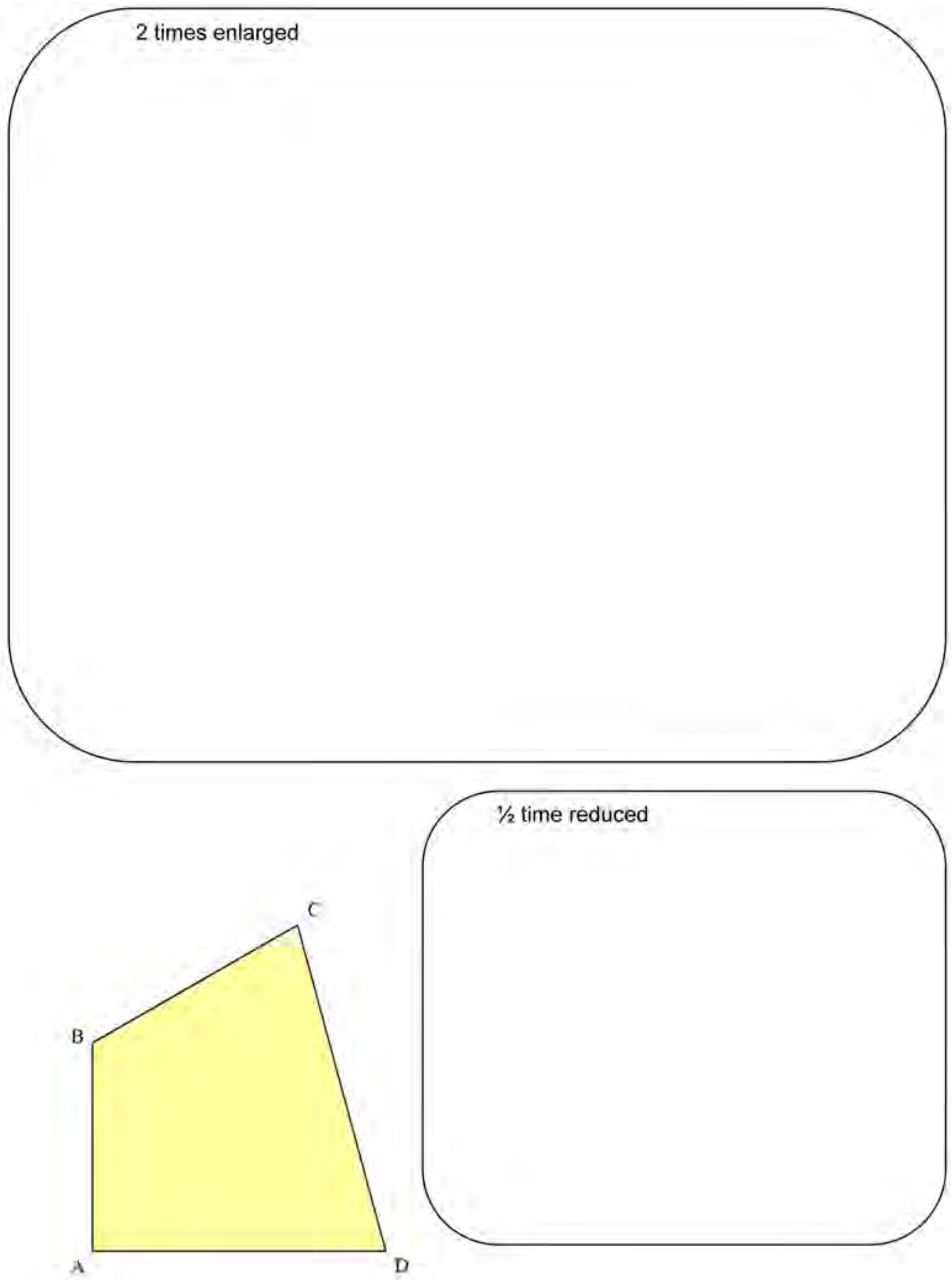


Diagram 4

i. Diagram 4 shows a quadrilateral ABCD.

- With the help of a ruler, compass and protractor, draw a 2 times enlarged drawing and a ½ time reduced drawing of ABCD in the space provided in Diagram 4.

Task 3: Drawing Enlarged or Reduced Shapes Using a Centre of Enlargement

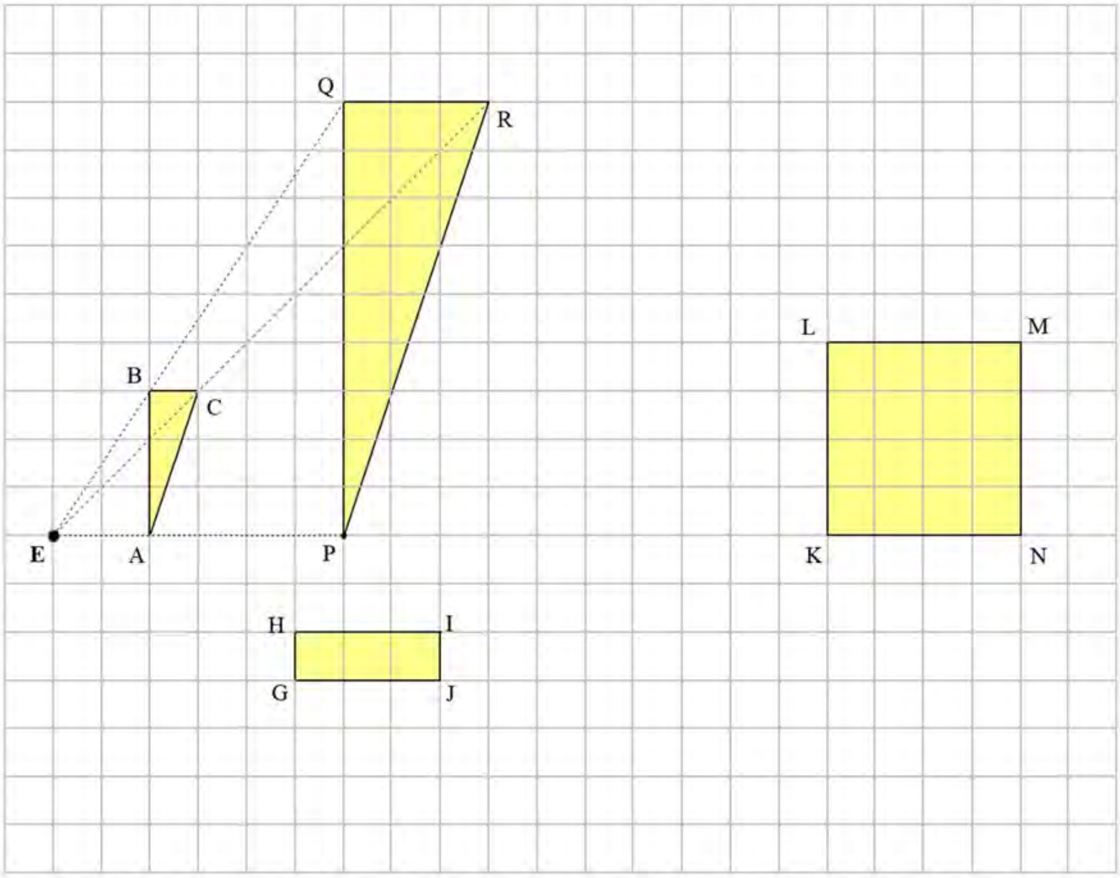


Diagram 5

i. In Diagram 5, triangle ABC is enlarged 2 times from centre E to become triangle PQR.

- Study the distance from the centre of enlargement, E, to points A, B, C, P, Q, and R.
    - ❖ What can you conclude about the ratios EP : EA, EQ : EB, and ER : EC?
- Rectangle GHIJ is also enlarged 2 times from centre E to become rectangle STUV.
    - ❖ Draw a rectangle STUV on Diagram 5.
- Square KLMN is reduced ½ time from centre E to become square WXYZ.
    - ❖ Draw square WXYZ on Diagram 5.

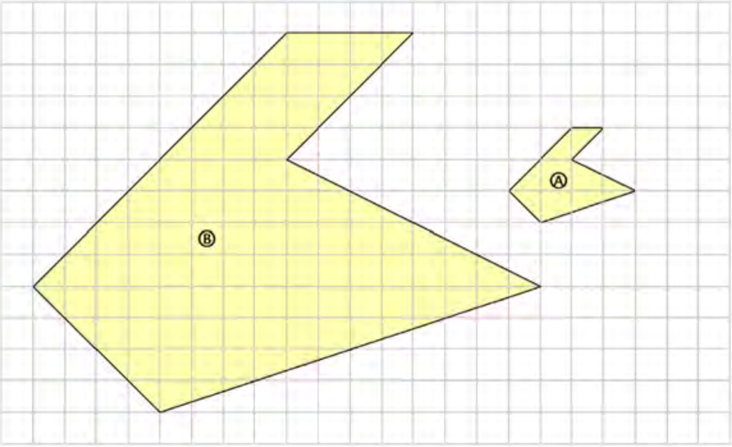
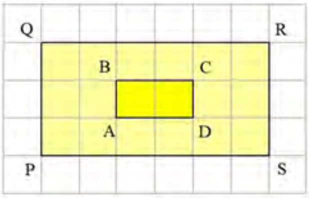


Diagram 6

ii. In diagram 6, shape Ⓐ is enlarged to become shape Ⓑ.

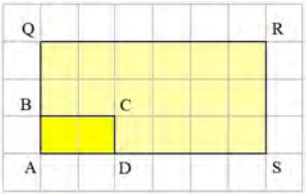
- Locate and mark the centre of enlargement on Diagram 6.
- How many times is shape Ⓑ larger than shape Ⓐ?

Task ①



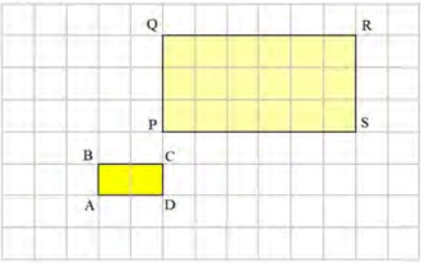
Rectangle ABCD is enlarged to become rectangle PQRS. Determine the centre of enlargement.

Task ②



Rectangle ABCD is enlarged to become rectangle AQRS. Determine the centre of enlargement.

Task ③



Rectangle ABCD is enlarged to become rectangle PQRS. Determine the centre of enlargement.

Diagram 7

iii. Diagram 7 shows three tasks to find centres of enlargement.

- How will you order the sequence of tasks for your students?
- Explain your answer.

Task 3: Map Reading

Diagram 8

i. Diagram 8 shows the drawing of a house with a reduced scale of 1 : 50.

- What does the reduced scale of 1 : 50 mean?
- The length of the kitchen is 6 cm on the reduced drawing. What is the actual length of the kitchen in m?
- The actual width of the living room is 5.2 m. How long is it in cm on the reduced drawing?

Diagram 9

ii. Diagram 9 shows two popular street art paintings by the Lithuanian artist, Ernest Zacharevic, and another popular tourist site on Penang Island. Diagram 10 shows the locations of these sites on a map of Georgetown.

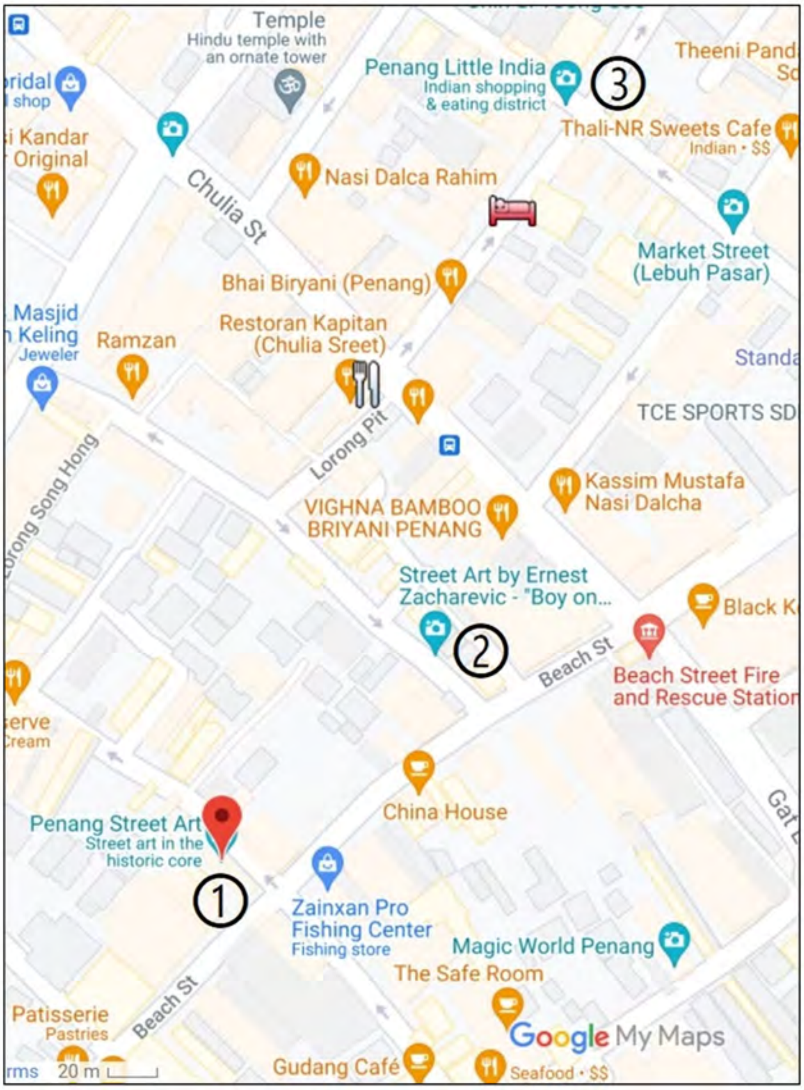
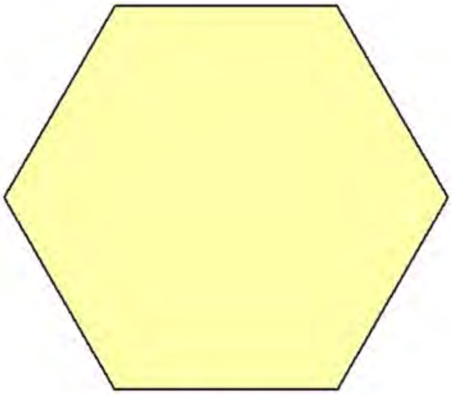


Diagram 10

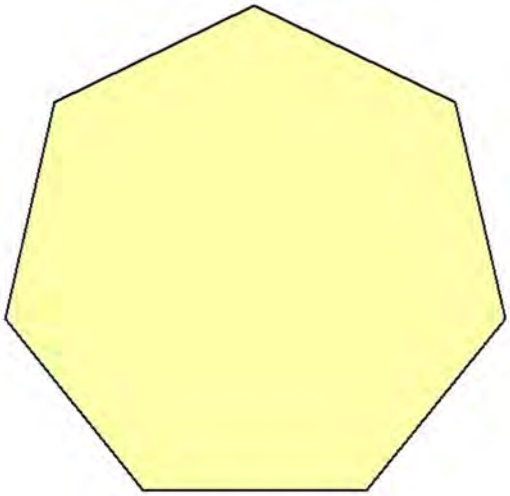
iii. A map is a drawing that represents an area of land or sea based on a reduced scale, which states the ratio that represents how much is it reduced from the real length. The scale of the map in Diagram 10 is 1 : 2500 as indicated in the map.

- How far is it from site ① to site ② along Beach Street, in the actual distance?
- A tourist starts walking from site ① to site ② along Beach Street at 3 km per hour. How many minutes will the walk take?
- Another tourist wants to go from site ② to site ③.
- What are the possible routes?
- Which route is the shortest?

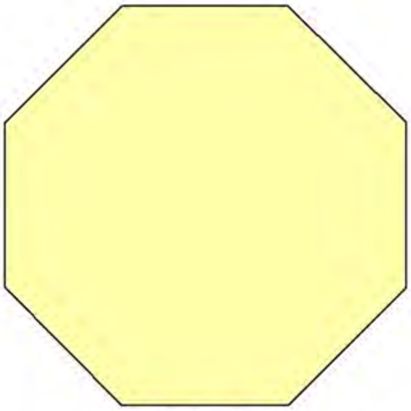
Material Sheet 3: n-sided Polygons



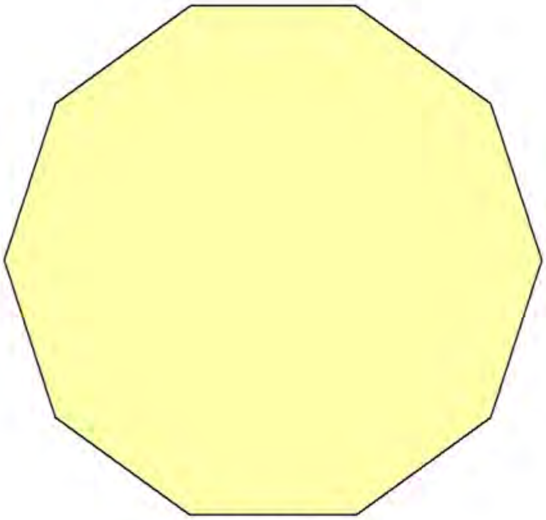
6-sided polygon



7-sided polygon



8-sided polygon



10-sided polygon

CHAPTER 5

Data Handling and Graphs

Topic 1: Arranging Tables for Data Representations

Standards 1.1:

Collecting and arranging data

i. Explore how to collect multi-category data based on a situation
ii. Explore how to arrange and read multi-category data on appropriate tables
iii. Appreciate the use of multi-category tables in situation

Sample Tasks for Understanding the Standards

Task 1: Collecting Multi-Category Data on a Situation

Table 1
Record of Injuries

Grade	Locations	Types of Injuries
4	Playground	Cut
5	Corridor	Bruise
5	Corridor	Bruise
1	Classroom	Scratch
3	Gymnasium	Scratch
3	Playground	Scratch
6	Gymnasium	Fracture
5	Classroom	Cut
4	Playground	Scratch
5	Gymnasium	Scratch
3	Gymnasium	Bruise
6	Playground	Sprain
6	Corridor	Sprain
2	Gymnasium	Scratch
1	Classroom	Bruise
5	Classroom	Cut
5	Gymnasium	Scratch
3	Stairs	Bruise
4	Gymnasium	Scratch
2	Playground	Bruise
6	Classroom	Scratch
4	Corridor	Bruise
4	Playground	Scratch
2	Gymnasium	Sprain
2	Gymnasium	Sprain
3	Playground	Scratch
3	Gymnasium	Sprain

Table 1 shows a record of injuries in a school over a period of one month.

Table 2 and Table 3 show two sets of data collected from this record.

Table 2
Locations of Injuries

Locations	Numbers of Students
Playground	
Corridor	
Classroom	
Gymnasium	
Stairs	
Total	

Table 3
Types of Injuries

Types	Numbers of Students
Cut	
Bruise	
Scratch	
Fracture	
Sprain	
Total	

i. Complete Table 2 and Table 3.

ii. Based on the tables, answer the following questions:

- Where did injuries happen most frequently?
- Which type of injury happened most frequently?
- What can you say about the types of injuries that happened at the gymnasium?
- What can you say about the locations of Cut?
- Which of these questions cannot be answered using information from Table 2 and Table 3? Explain your answers.

iii. In order to answer the unanswered questions, a student rearranges the data into Table 4.

Table 4
Locations and Types of Injuries

Type / Location	Cut	Bruise	Scratch	Fracture	Sprain	Total
Playground						
Corridor						
Classroom						
Gymnasium						
Stairs						
Total						

Complete Table 4.

- In what way is Table 4 different from Tables 2 and 3?
- Answer the unanswered questions in (ii).
- What can you conclude about the types and locations of injuries based on information from Table 4?

iv. In what way is Table 4 more useful than Tables 2 and 3? How will you instil appreciation towards the usefulness of Table 4 among your students?

v. Build two other tables to analyse the (a) types of injuries, and (b) locations, related to grade levels, respectively.

- How will you analyse the information derived from each table?
- What relationship can you find between the grade levels and the types of injuries based on your analysis?
- What relationship can you find between the grade levels and the locations of injuries?
- How shall you rearrange the data in Table 4 to see at a glance, any trend in
  - ❖ the type of injury?
  - ❖ the location of injury?

Topic 2: Drawing and Reading Graphs for Analysing Data

Standards 2.1:

Drawing and reading line graphs for knowing the visualised pattern as the basis for tendency of change

- i. Introduce line graphs based on appropriate situations such as rainfall, temperature and others
- ii. Distinguish line graph from bar graph for observation such as increase, decrease, and no-change
- iii. Introduce the graph of proportion using the idea of a line graph and read the gradient by a constant ratio
- iv. Appreciate the line graph in various situations

Sample Tasks for Understanding the Standards

Task 1: Line Graphs

i. Table 1 shows the average monthly temperature, measured in ºC, for the periods 1961-1990 and 1991-2020 in Myanmar.

Table 1
Average Monthly Temperature (ºC) for 1961-1990 and 1991-2020 in Myanmar

Month	Temperature (ºC)	
	1961-1990	1991-2020
1	18.1	18.7
2	19.8	20.4
3	22.8	23.2
4	25.5	25.8
5	25.8	26.0
6	25.2	25.5
7	24.9	25.2
8	24.9	25.2
9	24.9	25.2
10	24.1	24.5
11	21.5	22.0
12	18.7	19.4

Retrieved from:

World Bank Group, Climate Change Knowledge Portal for Development Practitioners and Policy Makers

https://climateknowledgeportal.worldbank.org/country/myanmar-burma

and

https://climateknowledgeportal.worldbank.org/country/myanmar-burma/climate-data-historical

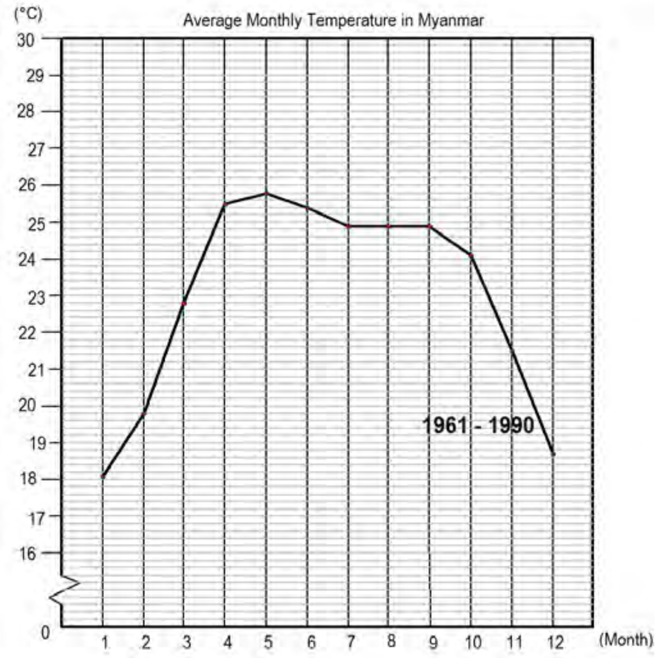


Diagram 1

Diagram 1 shows a line graph plotted for the period 1961-1990.

- Based on the line graph, describe the changes in temperatures from month to month.
- In which month is the temperature highest?
- In which month is the temperature lowest?
- Which part of the graph shows
    - ❖ Increase in temperature?
    - ❖ Decrease in temperature?
    - ❖ No change in temperature?
- Between which two consecutive months does the temperature change most?
- Are the slopes of the graph for Jan-Feb, Feb-Mac, and Mac-Apr the same? Justify your answer.
- Are the slopes of the graph for Sept-Oct and Nov-Dec the same? Justify your answer.
- What can you conclude about the change in temperature from
    - ❖ January to April?
    - ❖ October to December?

- Plot on Diagram 1, another line graph for the period 1991-2020
    - ❖ Compare the changes in temperatures between 1961-1990 and 1991-2020.
    - ❖ What trend in the changes in temperatures do you see?
- Look for the average monthly temperature in your country from the World Bank Climate Change Knowledge Portal. Plot the relevant line graphs to compare the changes in temperatures for different periods of time.
    - ❖ What can you conclude on the issue of global warming in your country?
    - ❖ How will you use line graphs to instil awareness and appreciation towards sustainable development among your students?

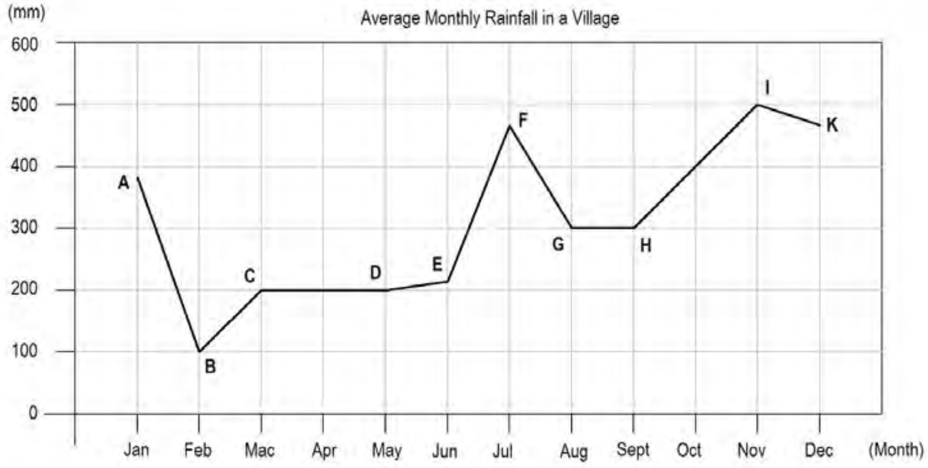


Diagram 2

ii. Diagram 2 shows a sketch of a line graph of average monthly rainfall (mm) over a period of 12 months in a village in an ASEAN country.

- Identify and explain which part of the line graph shows
    - ❖ slight increase in monthly rainfall;
    - ❖ significant increase in monthly rainfall;
    - ❖ slight decrease in monthly rainfall;
    - ❖ significant decrease in monthly rainfall;
    - ❖ no change in monthly rainfall, for that part of the time.
- What can you say about the change in value for parts
    - ❖ CD and GH,
    - ❖ BC and HI.

- Look for the average monthly rainfall data for your region.
    - ❖ Plot the relevant line graphs to analyse the data.
    - ❖ What conclusion can you make on the change in rainfall for your region?

Task 2: Graph of Proportion

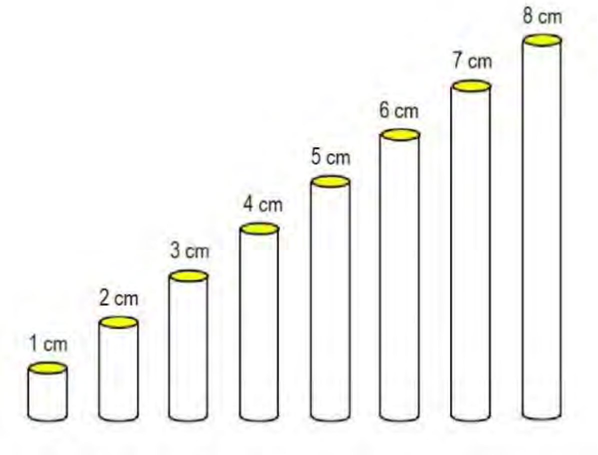


Diagram 3

Diagram 3 shows eight wires with different lengths, in cm. Each of the wires was weighed, in g, and the result is shown in Table 2.

Table 2
Length of a Wire and Weight

Length (cm)	1	2	3	4	5	6	7	8
Weight (g)	3	6	9	12	15	18	21	24

i. Study Table 2.

- What pattern do you see in the changes in weight and length?
- Find the values of the quotient, (weight) ÷ (length) for each pair of the values.
- Write a mathematical sentence to represent the relationship between the weight and length of the wire.

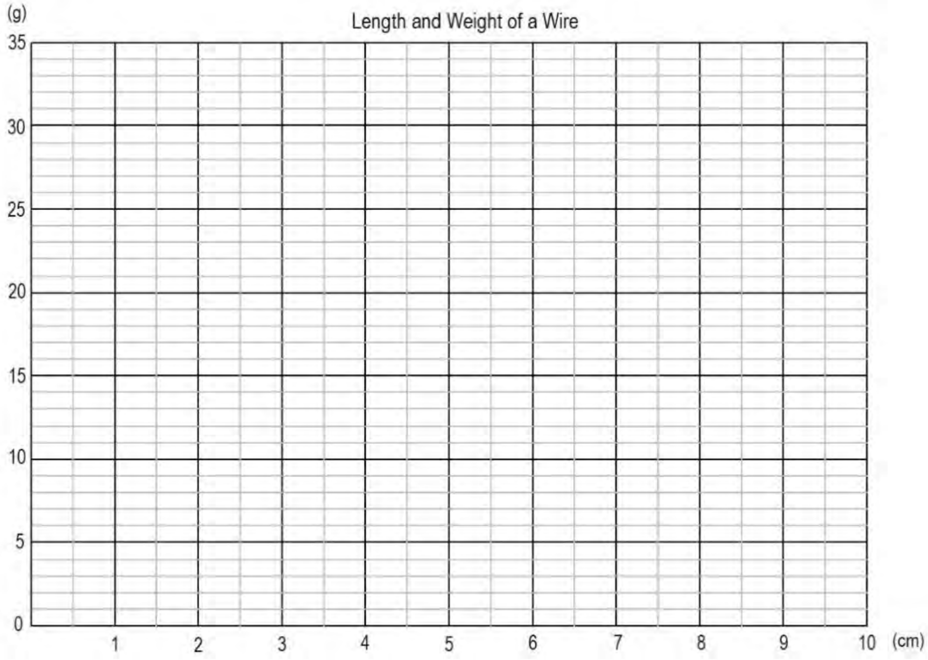


Diagram 4

ii. Draw a graph on Diagram 4 to show this set of data.

- What is the difference between this graph and the line graph in Task 1?
- Why is there a difference?

iii. Use the graph to find the weight of

- a length of 4.5 cm wire,
- a length of 9 cm wire.
- Justify your method.

iv. Another wire weighs 20 g.

- Use the graph to find the length of the wire.
- Justify your method.

Standards 2.2:

Drawing and reading band graph and pie chart for representing ratio in a whole
i. Explore how to scale a band or circle for representing the ratio or percent
ii. Use the band graph and pie chart for comparison of different groups
iii. Appreciate the band graph and pie chart in a situation

Sample Tasks for Understanding the Standards

Task 1: Comparing Ratio or Percent with Band Graph

Expressing Ratio as Decimal in a Band Graph

Table 1
Number of Passengers

	Bus ①	Bus ②
Number of passengers	15	28
Number of seats	21	42

i. Table 1 shows the number of passengers and seats of two buses on a trip to Kuala Lumpur.

- Which bus is more crowded?
- What mathematical ideas do you use to determine crowdedness? Explain your answer.

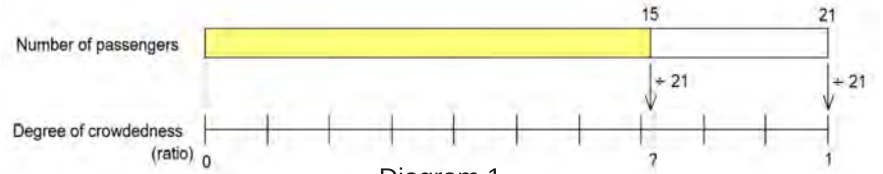


Diagram 1

ii. A student uses ratio to determine the degree of crowdedness. Diagram 1 shows a band graph drawn by the student to solve the problem involving Bus ①.

- What does a ratio of 1 mean in terms of the crowdedness of Bus ①?
- What is the degree of the crowdedness of Bus ①?
- Draw another band graph to determine the degree of the crowdedness of Bus ②.
- Can the degree of crowdedness be more than 1? Explain your answer.

Table 2
Number of Vehicles

Vehicle	Bus	Van	Car
Number	6	8	12

iii. There are 6 buses, 8 vans and 12 cars in a parking lot as shown in Table 2. Diagram 2 shows a band graph involving the ratios of these buses, vans and cars.

- What is the ratio of the number of buses to the number of vans?
- What is the ratio of the number of cars to the number of vans?

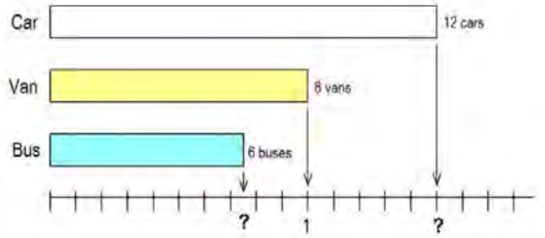


Diagram 2

iv. Draw another band graph to find the ratio of the number of vans to the number of cars.

Expressing Ratio as Percentage in a Band Graph

i. There are 40 boys in a class of 50 students. Diagram 3 shows a band graph converting the ratio of boys to percentages.

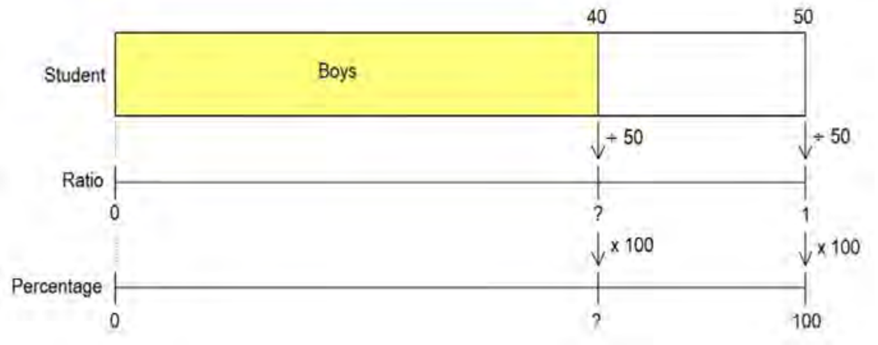


Diagram 3

- Find the ratio of the boys to the total number of students in decimal numbers and percentages.
- Why do we multiply the ratio with 100 to change it into a percentage?

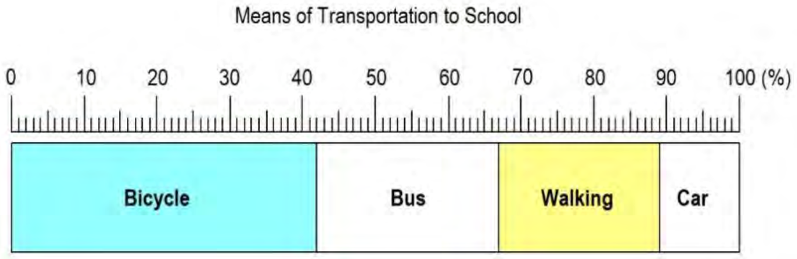


Diagram 4

ii. Diagram 4 shows a band graph comparing the means of transportation to the school of a group of students by percentage.

- What is the percentage of students cycled to school compared to the total number of students?
- What is the percentage of bus, walking and car compared to the total number of students, respectively?
- Which means of transportation has the least number of students?
- There are 150 students in the group. How many students walked to the school?
- What is the advantage of using a band graph to show the percentages of each means of transportation?

iii. Table 3 and Table 4 show the favourite ice cream flavours of two groups of children.

Table 3
Favourite Ice Cream Flavours of First Graders

Flavour	Number of Children	Percentage (%)
Chocolate	12	
Strawberry	5	
Vanilla	3	
Mocha	3	
Others	2	
Total	25	

Table 4
Favourite Ice Cream Flavours of Sixth Graders

Flavour	Number of Children	Percentage (%)
Chocolate	9	
Strawberry	10	
Vanilla	5	
Mocha	2	
Others	6	
Total	32	

- Find each ratio to the total and round it to the nearest hundredth.
- Convert each ratio to a percentage for each flavour and fill up the tables.
- Draw a band graph for each grade on Diagram 5.

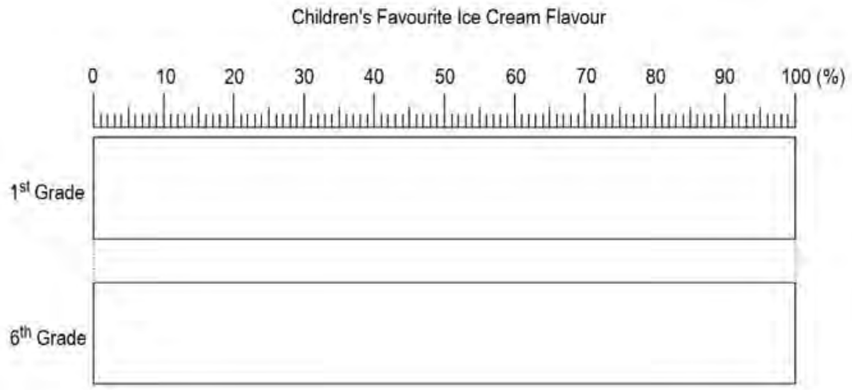


Diagram 5

- Discuss your finding based on the two band graphs.

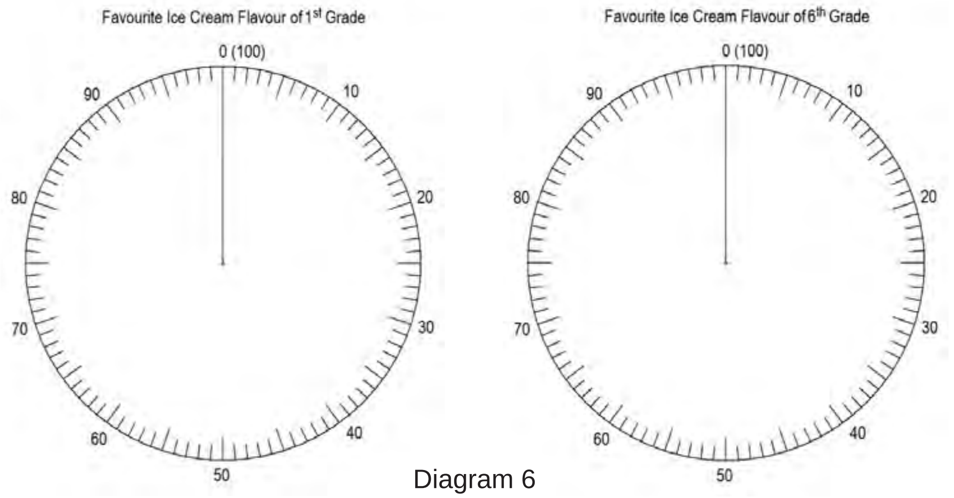


Diagram 6

iv. Draw a pie chart for each grade on Diagram 6 to show the favourite ice cream flavours of the two groups of children.

v. Band graph or pie chart, which of these graphs is easier to compare the favourite ice cream flavours of the two groups of children? Justify your choice.

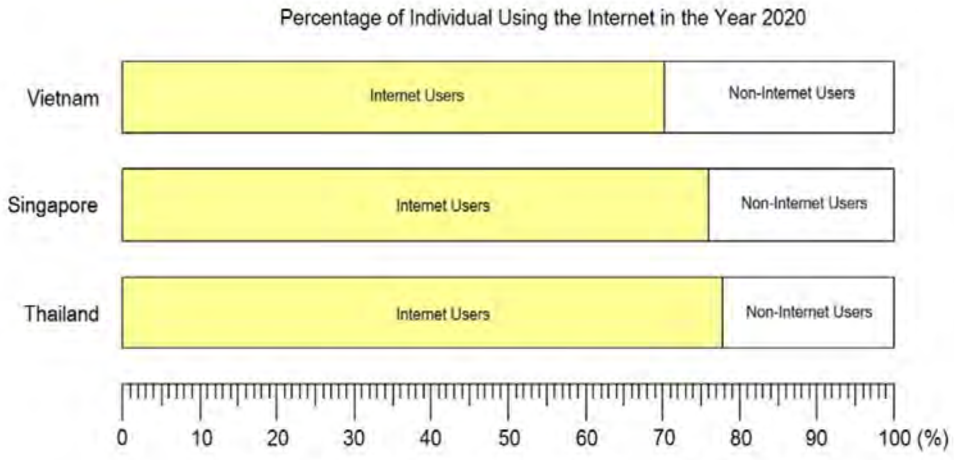


Diagram 7

Data retrieved from The World Bank DataBank portal
https://databank.worldbank.org/source/world-development-indicators

vi. The band graphs in Diagram 7 show the percentage of the population using the Internet in the year 2020 for three SEAMEO countries.

- A student claimed that more individuals were using the Internet in Thailand than in Singapore. Is the claim valid? Explain your reasons.
- Another student claimed that more individuals were using the Internet in Singapore than in Vietnam. Is the claim valid? Explain your reasons.

vii. Table 5 shows the daily total calorie intake per person in Vietnam and Japan for the year 2011.

Table 5
Daily Calories Intake per Person by Country

Country	Vietnam	Japan
Calories Per Person	2704	2717

Retrieved from What the world eats? National Geography Magazine website. https://www.nationalgeographic.com/what-the-world-eats/

The band graphs in Diagram 8 show the percentage of daily calorie intake from different types of food for the countries.

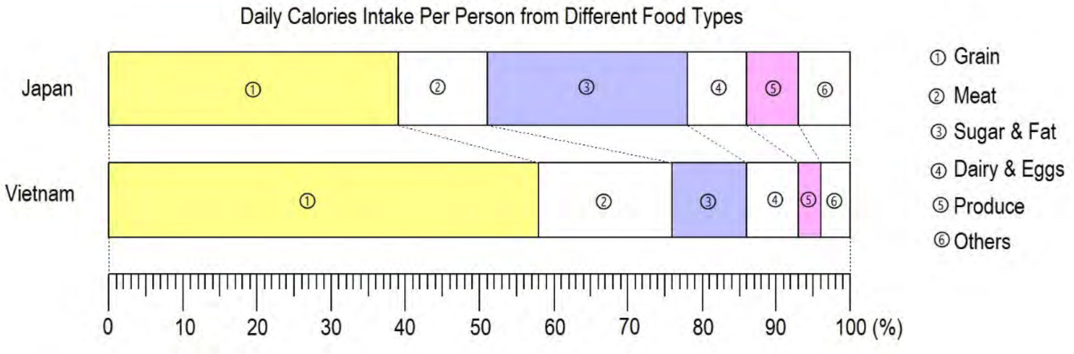


Diagram 8

Ariff wrote down the following conclusions from the graphs.

① The Vietnamese get the least percentage of calories from produce.
② The Vietnamese get more calories from grains than the Japanese.
③ The Vietnamese and the Japanese get the same amount of calories from dairy and eggs.

- Decide if each of the conclusions is true or false.
- Justify your decision.

viii. Table 6 shows the daily total calorie intake per person of Vietnamese 50 years apart.

Table 6
Vietnamese Daily Food Intake for 1961 and 2011

Year	1961	2011
Daily Food Intake (Calories Per Person)	1907	2704

Retrieved from What the world eats? National Geography Magazine website.
https://www.nationalgeographic.com/what-the-world-eats/

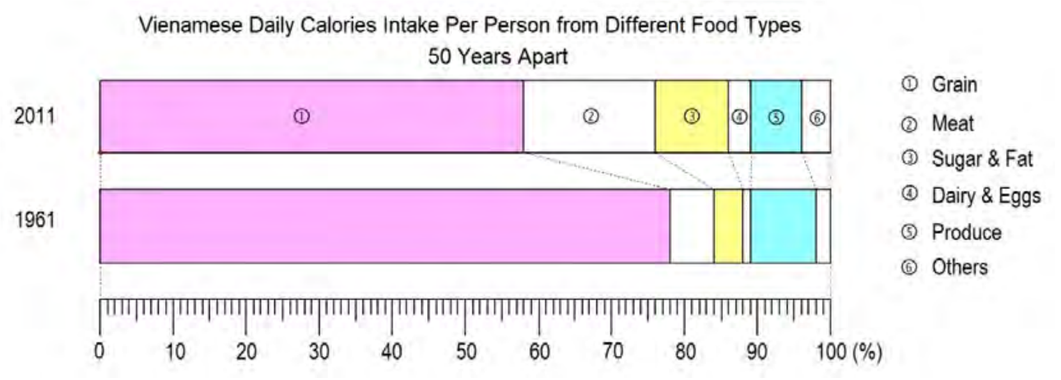


Diagram 9

The band graphs in Diagram 9 show the percentage of calories intake per person from different food types.

- Based on the band graphs, discuss the change in the Vietnamese diet pattern after 50 years.
- How can this task be used to promote a healthy lifestyle among your students?

Standards 2.3:

Reading histogram for analysing frequency distribution
i. Draw a simple histogram from the frequency table on situations
ii. Read various histograms for analysing data distribution
iii. Use averages to compare different groups in the same situation with histograms

Sample Tasks for Understanding the Standards

Task 1: Average Scores

187, 199, 182, 184, 191, 204,
194, 196, 187, 201, 185, 196 (cm)

Diagram 1

The numbers in Diagram 1 show the height (in centimetres) of 12 members of a basketball team. Two students, Baskoro and Satria, calculated the average height of this team using two different methods as shown in Diagram 2.

Baskoro's Method

(187 + 199 + 182 + 184 + 191 + 204 + 194 + 196 + 187 + 201 + 185 + 196) ÷ 12

= 192.2 cm

Satria's Method

180 + (7 + 19 + 2 + 4 + 11 + 24 + 14 + 16 + 7 + 21 + 5 + 16) ÷ 12

= 180 + 12.2

= 192.2 cm

Diagram 2

i. Why do both methods give the same answer?

ii. Which method do you think will be easier for your students? Explain your reasons.

Task 2: Analysing Data Distribution

i. The data in Table 3 shows the heights, arranged from shortest to the tallest, of a group of 12-year-old children.

Table 3
Height of 12-Year-Old Children (cm)

Child	Height (cm)
C1	110
C2	111
C3	111
C4	112
C5	112
C6	112
C7	112
C8	113
C9	115
C10	117
C11	118
C12	119
C13	119
C14	121
C15	121
C16	121

Child	Height (cm)
C17	122
C18	122
C19	123
C20	125
C21	125
C22	125
C23	125
C24	126
C25	126
C26	126
C27	126
C28	126
C29	126
C30	127
C31	127
C32	127

Child	Height (cm)
C33	127
C34	127
C35	127
C36	128
C37	128
C38	128
C39	129
C40	129
C41	132
C42	133
C43	134
C44	135
C45	135
C46	136
C47	141

- Find the average height of these children.

ii. Table 4 shows the data separated into class intervals of 5 cm.

Table 4
Distribution of Heights

Class	Height (cm)	Number of Children
①	Greater or Equal Less Than 110 – 115	8
②	115 – 120	5
③	120 – 125	
④	125 – 130	
⑤	130 – 135	
⑥	135 – 140	
⑦	140 – 145	

- In which class interval does the height of 120 cm belong?
- Fill in the number of children in each blank space in Table 4.
- Draw a histogram to show the distribution of children’s heights in Diagram 3.

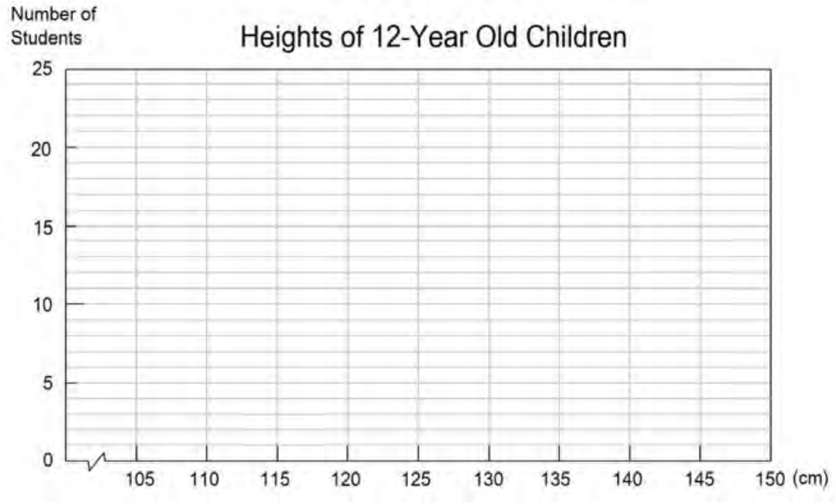


Diagram 3

- Based on the histogram, answer the following questions.

  ⓐ Which class interval has the most number of children?
  ⓑ Which class interval does the average height that you calculated in (i) belong to?
  ⓒ What is the height in the centre (24th place) and which class interval does this height belong to?

- Based on your answers in ⓐ, ⓑ, and ⓒ, which one explains the data well? Justify your choice.

Task 3: Comparing Two Groups of Data

i. Table 1 shows a record of mathematics test scores of two groups of students, 20 boys and 18 girls.

Table 1
Mathematics Test Score of Two Groups

Group	Score									
Boy	62	82	64	85	71	66	82	65	69	71
	71	63	75	63	81	56	58	91	68	73
Girl	80	66	69	73	78	77	68	82	75	77
	74	70	69	46	64	70	74	77	-	-

- What are the highest and lowest scores for each group?

- What is the average score for each group?

ii. Baskoro and Satria, compare the test scores of the two groups using different measures and came to different conclusions as follows:

Baskoro's Reasoning

"Both the highest and lowest scores of the boys are larger than the highest and lowest scores of the girls, respectively. Therefore, the boys performed better than the girls in the test."

Satria's Reasoning

"The average score of the girls is larger than the average score of the boys. Therefore, the girls performed better than the boys on the test.

- Whose reasoning do you support? Explain your reasons.

iii. The test scores in Table 1 are separated by class intervals of 5 marks as shown in Table 2.

Table 2
Distribution of Test Scores

Class	Score	Number of Students	
		Boy Group	Girl Group
①	Greater or Equal Less Than 45 – 50	-	
②	50 – 55	-	
③	55 – 60	3	
④	60 – 65	5	
⑤	65 – 70	3	
⑥	70 – 75	2	
⑦	75 – 80	-	
⑧	80 – 85	5	
⑨	85 – 90	1	
⑩	90 – 95	1	

- Fill in the number of students for the girl group in Table 2.
- Compare the test scores of the boys and girls.
  - ❖ Which group has more students with test scores greater or equal to 80 marks?
  - ❖ Which group has more students with test scores less than 70 marks?
  - ❖ Which group has more students with test scores greater or equal to 70 marks and less than 90 marks?

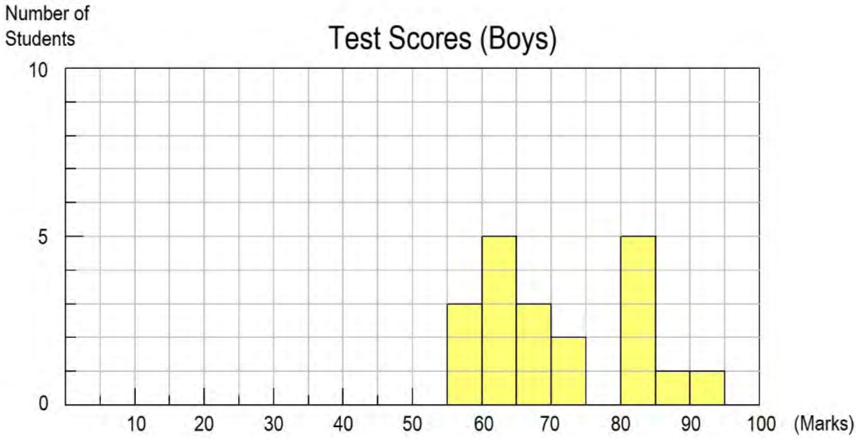


Diagram 4

iv. Diagram 4 shows a histogram drawn for the test scores of the boys.

- How many boys scored greater than or equal to 80 marks?
- How many percent of boys scored greater than or equal to 60 marks but less than 70 marks?

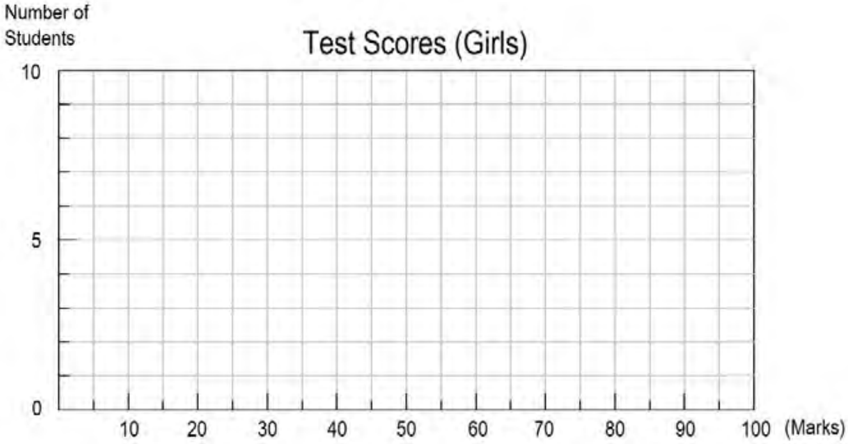


Diagram 5

v. Draw a histogram for the girls' test scores on diagram 5.

- Compare the shapes of the two histograms in Diagrams 4 and 5.
  - ❖ Discuss the distribution of the boys' and the girls' test scores.
- In which class interval, greater than or equal to and less than, do most students belong to each group?
- How many percent of students scored less than 60 marks in each group?
- How many percent of students scored greater or equal to 75 marks in each group?

Topic 3: Using Graphs in PPDAC Cycle

Standards:

Identifying appropriate graphs for problem solving in data handling using the PPDAC cycle

i. Analyse a problem situation and discuss the expected outcomes before collecting data to clarify the purpose of a survey
ii. Plan the survey for the intended purpose
iii. Collect the data based on the purpose of the survey
iv. Use appropriate graphical representation which is most suitable for the purpose
v. Appreciate the use of graphs for making the conclusion

Sample Tasks for Understanding the Standards

Task 1: Choosing the Appropriate Graph to Handle Data

Diagram 1

Diagram 1 shows an ice cream vending machine. The Entrepreneur Club of a school intends to set up such vending machines to generate income for the club. A committee was formed to discuss and make decisions for setting up the machine. A decision made by the committee was although there are 12 flavours of ice cream, 6 flavours only will be chosen to fill a machine. As such, a problem faced by the committee is:

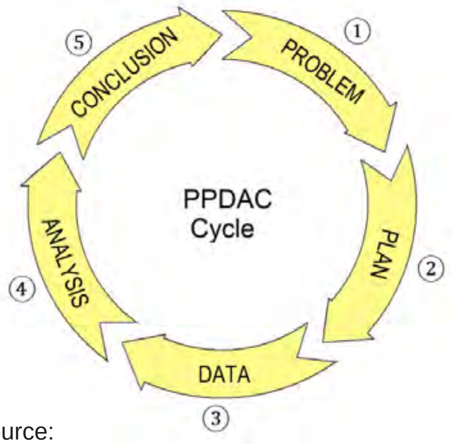
“What 6 flavours are to be chosen?”

i. The following questions concerning each flavour of ice cream were raised during the committee meeting.

① What is the unit price of each flavour?
② What is the number of students who like each flavour?
③ What is the colour of each flavour?

- Which question is directly relevant to the problem?
- What data will help the club choose the 6 flavours?
- How can the data be collected?

ii. Diagram 2 shows the five phases of the Problem Plan Data Analysis Conclusion (PPDAC) cycle for data handling and a brief description of the five phases.



Ⓐ
- Identify the right problem to solve
- Think of possible ways to solve the problem

Ⓑ
- Construct tables & graphs
- Calculate summary statistics
- Analyse data

Ⓒ
- Identify the data needed to solve the problem
- Think of ways to collect the data

Ⓓ
- Interpret result
- Explain what it means
- Ask further questions

Ⓔ
- Collect the data
- Clean the data

Source:
1. MathsNZ Students https://students.mathsnz.com/9.10/pdfs/1.pdf
2. Data Education in Schools https://dataschools.education/

Diagram 2

- Match each description with its corresponding phase.
- Explain briefly, what the Entrepreneur Club committee needs to do in each of the phases.

iii. The 12 ice cream flavours available are Vanilla, Mint, Chocolate, Orange, Mango, Coffee, Mocha, Raspberry, Neapolitan, Coconut, Pecan, and Strawberry. The committee collected the data on favourite flavours from 80 students and 20 teachers as shown in Diagram 3.

Vanilla	Mocha	Mocha	Raspberry	Chocolate	Vanilla	Vanilla	Neapolitan	Mango	Strawberry
Mint	Vanilla	Raspberry	Chocolate	Vanilla	Raspberry	Vanilla	Mango	Strawberry	Pecan
Chocolate	Vanilla	Chocolate	Mango	Raspberry	Chocolate	Chocolate	Mocha	Vanilla	Neapolitan
Vanilla	Strawberry	Chocolate	Vanilla	Vanilla	Mocha	Coconut	Mango	Mango	Coconut
Mint	Coffee	Neapolitan	Mint	Pecan	Vanilla	Mocha	Strawberry	Vanilla	Mango
Strawberry	Vanilla	Mango	Mint	Chocolate	Chocolate	Mango	Strawberry	Strawberry	Mocha
Mango	Chocolate	Vanilla	Strawberry	Mocha	Orange	Raspberry	Vanilla	Orange	Coconut
Vanilla	Mint	Coconut	Chocolate	Raspberry	Vanilla	Chocolate	Mocha	Mango	Strawberry
Chocolate	Mango	Mango	Vanilla	Vanilla	Raspberry	Coffee	Orange	Coconut	Chocolate
Coffee	Coffee	Chocolate	Coffee	Strawberry	Chocolate	Vanilla	Chocolate	Strawberry	Mocha

Diagram 3

- Build a table to show the number of people who like each flavour.
- What graphical representation will you use to analyse the data? Explain your choice.
- Analyse the data using your choice of graphical representation.
  - ❖ What 6 flavours will you choose on behalf of the committee? Explain your reasons.

iv. Data handling using the PPDAC cycle is often used in making decisions in problem situations. As an example, Penang Island is a popular tourist destination in Malaysia. Its attractions include hawker food, street art, heritage buildings, clan jetties, and sand beaches. Diagram 4 shows some of these attractions.

Diagram 4

Mr Zamri, an owner of a tour agency in Penang Island, intends to plan tour packages which include visits to popular tourist spots, meals, accommodation, and transportation.

- List down some data-relevant questions that Mr Zamri should ask to help him plan the tour packages?
- Choose one question from your list.
  - ❖ Explain what data should Mr Zamri collect to help him answer the question.
  - ❖ Explain briefly, how the data can be collected.
- Collect some data related to your chosen question from your class of students.

- Organise and represent your collected data using appropriate tables and graphs.
    - ❖ Analyse the data.
- Make some recommendations to Mr Zamri based on your analysis of data.

v. Identify a contextual problem situation in your country.

- Plan a project that engages the PPDAC cycle to solve the problem.
- Implement the cycle.
- Present your findings.

vi. To what extent can this task nurture appreciation toward the usefulness of data handling among your students?

Topic 4: Applying Data Handling for Sustainable Living

Standards:

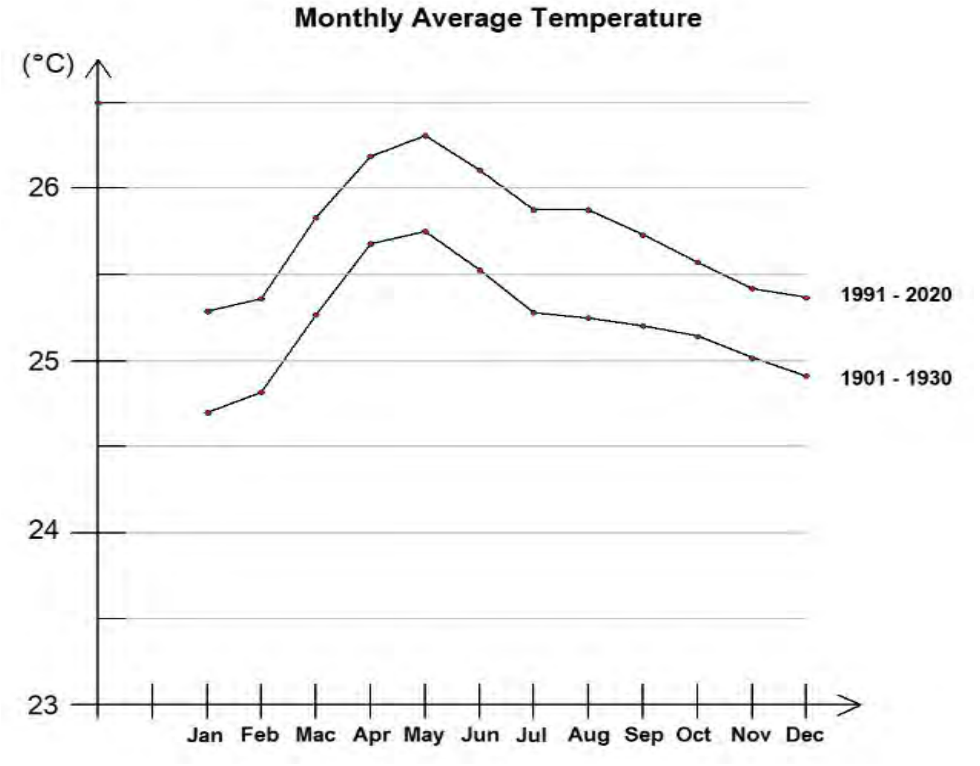
Applying data handling for sustainable development and appreciating the power of data handling for predicting the future.

i. Read data related to sustainable development on SDGs and adopting positive views for the betterment of the society
ii. Understand the idea of probability as a ratio and percentage in reading the data for situations related to sustainable development
iii. Experience implementing a project of reasonable size in data handling to achieve sustainable development and appreciate the power of data handling

Sample Tasks for Understanding the Standards

Task 1: Read Data Related to Sustainability

Climate Change



Data retrieved from Climate Change Knowledge Portal
https://climateknowledgeportal.worldbank.org/country/malaysia/climate-data-historical

Diagram 1

i. The graph in Diagram 1 shows the monthly average temperature of Malaysia over two periods of time.

  - Discuss the effect of global warming on the climate of Malaysia over the years.

ii. Search for the climate-change data of your country from the Climate Change Knowledge Portal or your country's meteorological department.

- Draw suitable graphs to represent the climate change data of your country.
- Compare the global warming effects between your country and Malaysia.

Land Use Change

Table 2 shows some information on the forest land area (in square km) in Cambodia and Vietnam 30 years apart.

Table 2
Forest Land Area in Square Kilometres by Years

Year	Cambodia (square km)	Vietnam (square km)
1990	110 048	93 760
2020	80 684	146 431

Data retrieved from The World Bank Data Bank portal
https://databank.worldbank.org/source/world-development-indicators

The band graphs in Diagrams 2 and 3 show the percentage of forest and non-forest land by area in Cambodia and Vietnam respectively, for the years 1990 and 2020.

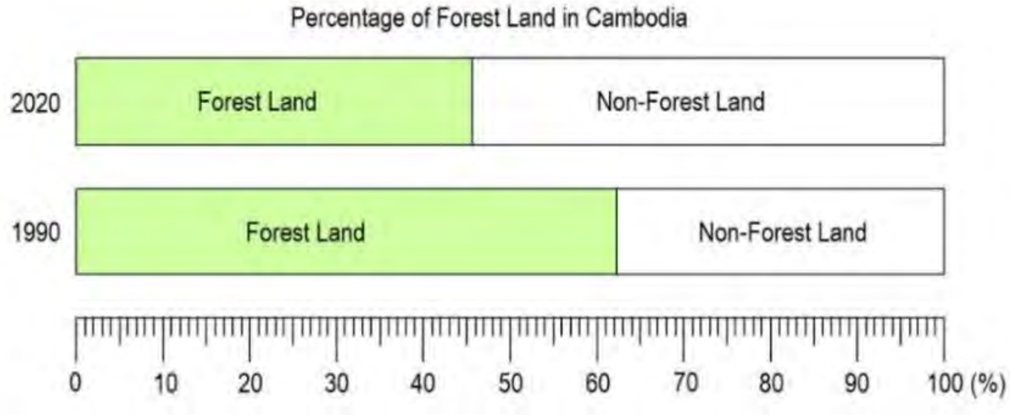


Diagram 2

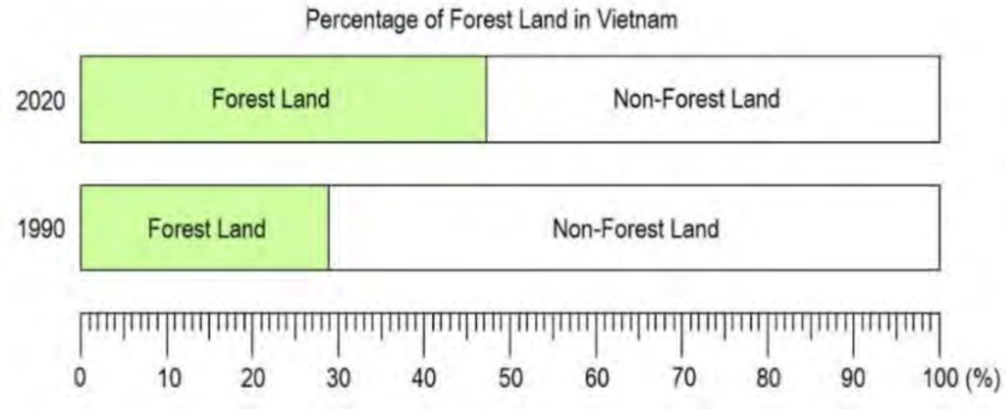


Diagram 3

i. Discuss the changes in forest land area in the two countries based on the band graphs.

ii. Approximately, what is the percentage of decrease in forest land in Cambodia from the year 1990 to 2020?

- What may be the causes for the decrease in forest land?
- How does the decrease in forest land affect the well-being of Cambodians?

iii. Approximately, what is the percentage of increase in forest land in Vietnam from the year 1990 to 2020?

- How is it possible that the percentage of forest land increases after 30 years?

iv. How will you use this task to raise awareness of the importance of sustainable development?

Carbon Dioxide Emission

Table 3
Global Annual CO_2 Emission from Burning of Fossil Fuels and Cement Production

Year	Annual CO_2 Emission (Billion tonnes)
1900	1.95
1910	3.03
1920	3.51
1930	3.92
1940	4.85
1950	6.00
1960	9.39
1970	14.90
1980	19.49
1990	22.75
2000	25.23
2010	33.34
2020	34.81

Data retrieved from Our World in Data website.
https://ourworldindata.org/co2-emissions

Table 3 shows the data on global annual carbon dioxide emissions from the burning of fossil fuels for energy and cement production.

i. Draw a line graph to show the change in carbon dioxide emissions from 1900 to 2020.

ii. Describe the trend in the annual emission of CO_2 in the world.

iii. Predict what may happen to the emission in 2030.

iv. How does carbon dioxide emission affect the environment?

Data Handling and Sustainable Living

i. In what ways is data handling important in promoting sustainable living?

ii. How will you use the tasks in this topic to create awareness of the importance of sustainable development among your students?

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Malaysian Primary Mathematics Curriculum:
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http://bpk.moe.gov.my/index.php/terbitan-bpk/kurikulum-sekolah-menengah/category/319-kurikulum-menengah

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Thailand Mathematics Curriculum: Years 1 – 12
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Singapore Mathematics Curriculum: Primary 1 – 6
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Singapore Mathematics Curriculum: Secondary One to Four
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APPENDIX A

Framework for CCRLS in Mathematics

Nature of Mathematics

Mathematics has been recognised as a necessary literacy for citizenship and not only for living economically but also to establish a society with fruitful arguments and creations for better living. It has been taught as a basic language for all academic subjects using visual and logical-symbolic representations. In this information society, mathematics has increased its role in establishing 21st-century skills through reviewing mathematics as the science of patterns for future prediction and designing with big data which produces innovation not only for technology advancement but also for business models.

Mathematics is an essential subject to establish common reasoning for the sustainable development of society through viable arguments in understanding each other and developing critical reasoning as the habit of mind. Mathematics should be learned as the basis for all subjects. For clarifying the framework in CCRLS on mathematics and by knowing the role of mathematics education, the humanistic and philosophical natures of mathematics are confirmed as follows:

The humanistic nature of mathematics is explained by the attitudes of competitiveness and understanding of others by challenging mathematicians such as Blaise Pascal, Rene Descartes, Isaac Newton and Gottfried Wilhelm Leibniz. For example, if you read the letter from Pascal to Pierre de Fermat, you recognise the competitive attitude of Blaise Pascal toward Fermat's intelligence and seek a way to be understood the excellence of his finding on Pascal's Triangles. If we read Pascal's Pensées, we recognise how Pascal denied Descartes's geometry using algebra from the aspect of ancient Greece geometry. On the other hand, Descartes tried to overcome the difficulties of ancient geometry through algebra. If you read the letter from Descartes to Elisabeth, you recognise how Descartes appreciated and felt happy the Royal Highness Elisabeth used his ideas of algebra in geometry. Despite being a princess, Elisabeth had been continuously learning mathematics in her life.

There were discussions on who developed calculus between Britain and the Continent. In that context, Johann Bernoulli, a continental mathematician, posed a question in the journal about the Brachistochrone problem, the locus of the point on the circumference of the circle when it rotates on the line. No one replied and Bernoulli extended the deadline for the answer and asked Newton to reply. Newton answered it within a day. Finally, six contributions of the appropriate answer including Newton and other Continental mathematicians were accepted. All those stories show that mathematics embraces the humanistic nature of proficiency for competitiveness and understanding others for sharing ideas.

The philosophical nature of mathematics can be explained from ontological and epistemological perspectives. From the ontological perspective, mathematics can be seen as a subject for universal understanding and a common scientific language. Plato and Aristotle are usually compared on this perspective. Plato believes that the existence of the world of "idea" and mathematics existed in the world of "idea" in Platonism. In this context, mathematical creation is usually explained by the word "discover" which means taking out the cover from which it has already existed. At the moment of discovery, the reasonable, harmony and beautifulness of a mathematical system is usually felt. Aristotle tried to explain about reaching an idea from the "material" to the "form". This explains that abstract mathematics can be understood with concrete materials using terms such as "modelling", "instruments", "aembodiment", "metaphor" and "change representation". From both ontological perspectives, mathematics can be understood and acquired by anyone and if acquired, it serves as a common scientific language which is used to express any subject. Once representing the ideas using the shared common language, the world can possibly be perceived in the same view autonomously.

From the epistemological perspective, mathematics can be developed through processes which are necessary to acquire mathematical values and ways of thinking. From this perspective, idealism and materialism are compared. In the context of Hegel, a member of German idealism, Imre Lakatos explained the development of mathematics through proof and refutation by using counter-examples. In another word, beyond the contradiction is the nature of the mathematical activity and it provides the opportunity to think mathematically for overcoming. In this context, mathematics is not fixed but an expandable system that can be restructured through a process of dialectic in constructing viable arguments. Plato also used dialectic for reaching ideas with examples of mathematics. The origin of dialectic is known as the origin of indirect proof. In education today, dialectic is a part of critical thinking for creation. Parallel perspectives for mathematical developments are given by George Polya and Hans Freudenthal. For the discovery of mathematics, Polya explained mathematical problem-solving processes with mathematical ideas and mathematical ways of thinking in general. Freudenthal enhanced the activity to reorganise mathematics with the term mathematisation on the principle of reinvention.

Genetic epistemologist Jean Piaget established his theory for operations based on various theories, including the discussion of Freudenthal and explained the mathematical development of operations by the term reflective abstraction. Reflection is also a necessary activity for mathematisation by Freudenthal. On materialism, under Vygotskyian perspective, intermediate tools such as language become the basis for reasoning in the mind. Under his theory, the high-quality mathematical thinking can be developed depending on high-quality communication in mathematics classrooms. Dialectical-critical discussion should be enhanced in mathematics class. From both the epistemological perspectives, mathematics can be developed through the processes of communication, problem-solving and mathematisation which include the reorganisation of mathematics. Those processes are necessary to acquire mathematical values and ways of thinking through reflection.

Aims of Mathematics in CCRLS

The aims of mathematics in CCRLS for developing basic human characters, creative human capital, and well-qualified citizens in ASEAN for a harmonious society are as follows:

- Develop mathematical values, attitudes and habits of mind for human character,
- Develop mathematical thinking and be able to engage in appropriate processes,
- Acquire proficiency in mathematics contents and apply mathematics in appropriate situations.

Framework for CCRLS in Mathematics as shown in Figure 3 is developed based on the three components with discussions of the humanistic and philosophical nature of mathematics. This framework also depicts the concrete ideas of mathematics learning of the above aims.

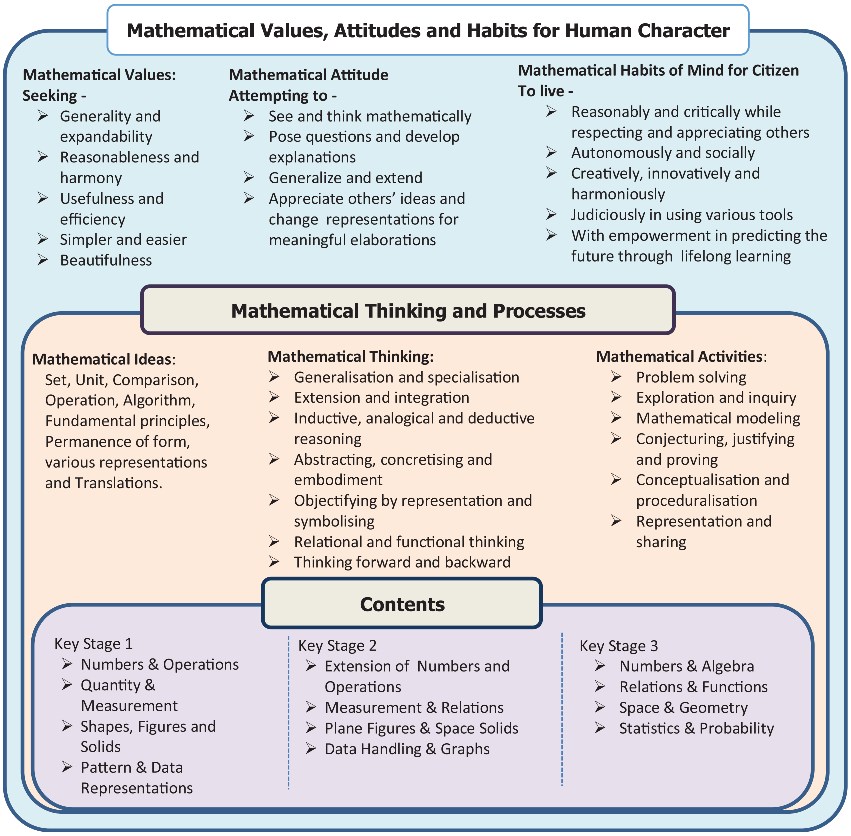

Figure 3. CCRLS Framework for Mathematics and Aims of Mathematics Learning: Old edition

Mathematical Values, Attitude and Habits for Human Character

For cultivating basic human characters, values, attitudes and habits of mind are essentials to be developed through mathematics. Values are basis for setting objectives and making decisions for future directions. Attitudes are mindsets for attempting to pursue undertakings. Habits of mind are necessary for soft skills to live harmoniously in society. Mathematical values, mathematical attitudes and habits of mind are simultaneously developed and inculcated through learning the content knowledge.

Essential examples of values, attitudes and habits of mind are given in Figure 3. On mathematical values, generalisable and expandable ideas are usually recognised as strong ideas. Explaining why proving is necessary for mathematics is a way of seeking reasonableness. Harmony and beautifulness are described not only in relation to mathematical arts but also in the science of patterns and systems of mathematics. Usefulness and simplicity are used in the selection of mathematical ideas and procedures.

On mathematical attitudes, “seeing and thinking mathematically” means attempting to use the mathematics learned for seeing and thinking about objects. Posing questions and providing explanations such as the “why” and the “when” are the ordinary sequence for thinking mathematically. Changing representation to other ways such as modelling can overcome running out of ideas in problem-solving. The mindset for trying to understand others is the basis to explain one’s own ideas that are understandable by the rest with appreciation. Producing a concept with a definition operationally is a manner of mathematics.

On mathematical habits of mind, for citizens to live, mathematical attitudes and values are necessary for reasoning critically and reasonably. Appreciating and respecting other ideas is also necessary. Mathematics is developed independently for those who appreciate life creatively, innovatively and harmoniously. Seeking an easier and more effective manner of selecting appropriate tools is necessary. Mathematics is a subject to challenge and experience competitiveness, appreciation with others, and develop the mindset for lifelong learning, personal development and social mobility.

Mathematical Thinking and Processes

For developing creative human capital, mathematical ideas, mathematical ways of thinking and mathematical activities are essential. Mathematical ideas are process skills involving mathematical concepts. Mathematical thinking is a mathematical way of reasoning in general which does not depend on specific concepts. Mathematical activities are various types of activities such as problem-solving, exploration and inquiry. Mathematical processes which include these components are necessary skills to use mathematics in our life, such as innovation in this society (e.g., Internet of Things (IoT)). In the context of education, competency referring to mathematical processes is the basis for STEM and STEAM education as well as a basis for social science and economic education.

Mathematical ideas serve as the basis of content knowledge related to promoting and developing mathematical thinking. Some key ideas of mathematics are used as a special process. The fundamental ideas of set and unit lead to a more hierarchical and simple structural relationship. The ability to compare, operate, and perform algorithms of related functions enables efficient ways of learning mathematics and solving problems in learners’ life with mathematics.

In the case of a set, it is a mathematical idea related to conditions and elements. It is related to activity in grouping and distinguishing with other groups by conditions. For example, 3 red flowers and 4 white flowers become 7 flowers, if we change the condition of the set by not considering the colours. “A” and non-A” is a simple manner to distinguish sets with logical reasoning. For categorising, we use intervals such as $x > 0$, $x < 0$, and $x = 0$. This situation can be seen in the hyperbolic graph where $y=1/x$.

In the case of a unit, it is a mathematical idea that is related to a process to produce and apply the unit with operations. In some cases, trying to find the common denominator is the way to find the unit of two given quantities. A tentative unit such as arbitrary units can be set and applied locally whereas standard units are used globally. The combination of different quantities produces new measurement quantities such as distance with respect to time produced speed. A square unit such as square centimetres is a unit of area.

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[1] STEM refers to Science, Technology, Engineering and Mathematics. STEAM refers to Science, Technology, Engineering, Arts and Mathematics or Applied Mathematics.

Mathematical thinking is well discussed by George Polya. Inductive, analogical and deductive reasoning are major logical reasoning at school. However, deductive reasoning is enhanced in relation to formal logic and inductive and analogical reasoning are not well recognised. Polya enlightens the importance of those reasoning in mathematics. On the process of mathematisation by Hans Freudenthal, objectifying of the method is necessary. David Toll mentioned it by the term thinkable concept on the process of conceptual development. Polya mentioned thinking forward and backwards in relation to ancient Greek term analysis.

Mathematical activities are ways to represent a mathematical process. The problem-solving process was analysed by Polya. He influenced problem-solving with various strategies. Technology enhances the activities of conjecturing and visualising inquiries. Conceptualisation is done based on procedures such as the procedure 3+3+3+3=12, which becomes the basis for 4x3. The proceduralisation of multiplication is done through developing the multiplication table, the idea of distribution and memorising.

Content

For cultivating well-qualified citizens, content knowledge of mathematics is essential. The content of mathematics is usually divided by the set of mathematics. However, developing human characters and creative human capital should be developed through mathematical processes. Values, attitudes and habits of mind are driving forces for engagement in mathematical processes. Thus, without involving human character formations with mathematical process skills, content knowledge of mathematics cannot be realised. The content is divided into three stages in CCRLS and every stage has four strands. Between the stages, the names of the strands are directly connected and those on the standard level are well connected too. The names of strands for every key stage are as follows:

Key Stages	Strands
Key Stage 1	Numbers and Operations
	Quantity and Measurement
	Shapes, Figures and Solids
	Pattern and Data Representations
Key Stage 2	Extension of Numbers and Operations
	Measurement and Relations
	Plane Figures & Space Figures
	Data Handling and Graphs
Key Stage 3	Numbers and Algebra
	Relations and Functions
	Space and Geometry
	Statistics and Probability

In every stage, four content strands are mutually related. Between the key stages, all strands in different key stages are mutually related. The same content strand names are not used to indicate development and reorganisation beyond each stage. For example, “Numbers and Operations” in Key Stage 1, “Extension of Numbers and Operation” in Key Stage 2, and “Numbers and Algebra” in Key Stage 3 are well connected. These names of the content strands show the extension and integration of contents. For example, even and odd numbers can be taught at any stage with different definitions. At Key Stage 1, even numbers can be introduced as “counting by two” which does not include zero. In Key Stage 2, it can be re-defined by a number divisible by two. Finally, in Key Stage 3, it can be re-defined as a multiple of two in integers which includes zero. Although we use the same name as even numbers, they are conceptually different. The definition in Key Stage 1 is based on counting, Key Stage 2 is based on division and Key Stage 3 is based on algebraic notation. Expressing such theoretical differences requires names of strands for content to be distinguished. In

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[2] *Strands used to explain mutual relation of content (Jeremy Kilpatrick, Jane Swafford, Bradford Findell. “Adding it up”, National Academies Press. 2001). The term domain is sometimes used for compartmentalization through categorisation of content.*

[3] *In algebraic notation of numbers, addition and multiplication are major operations. Subtraction can be represented by addition of negative numbers and division can be represented by reciprocal or multiplicative inverse property.*

the case of measurement, there is no strand name of measurement in Key Stage 3. Key Stage 1 relates to the quantity and setting of the units. In Key Stage 2, it extends to non-additive quantity beyond dimension. In Key Stage 3, the idea of unit and measurement is embedded in every strand. For example, the square root in the Numbers and Algebra strand is an irrational number which means unmeasurable, the Pythagorean Theorem in Space and Geometry strand is used for measuring, proportional function in the Relations and Functions strand is used for counting the number of nails by weight, and in Statistics and Probability strand, new measurement units are expressed such as quartile for boxplot.

Context to Link the Three Components

Three components in Figure 3 should be embedded in every key stage as standards for the content of teaching. The "Mathematical values, attitudes, habits for human character" component and the "Mathematical thinking and processes" component cannot exist without the "Content" component. The first two components can be taught through teaching with the content. For teaching those three components at the same time, context is introduced as shown in Figure 4.

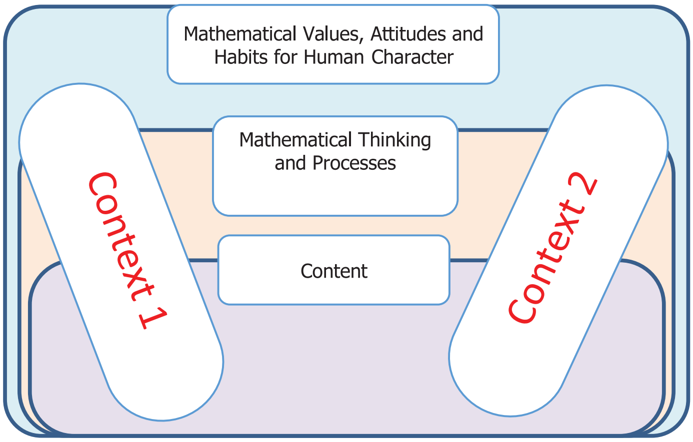

Figure 4. Interlinking of the three components with the context

On a given context, three components are well connected. For this reason, classroom activities for developing competencies should be designed to link all of them. The following contexts are samples:

- Explore a problem with curiosity in a situation and attempt to formulate mathematical problems
- Apply the mathematics learned, listen to other's ideas and appreciate the usefulness, power and beauty of mathematics
- Enjoy classroom communications on mathematical ideas in solving problems with patience and developing perseverance
- Feel the excitement of "Eureka" with enthusiasm for the solutions and explanation of unknown problems
- Think about ways of explanation using understandable representations such as language, symbols, diagrams and notation of mathematics
- Discuss the differences in seeing situations before and after learning mathematics
- Explain, understand others and conclude mathematical ideas
- Explore ideas through inductive and deductive reasoning when solving problems to foster mathematical curiosity
- Explore ideas with examples and counter-examples
- Feel confident in using mathematics to analyse and solve contextual problems both in school and in real-life situations
- Promote knowledge, skills and attitudes necessary to pursue further learning in mathematics
- Enhance communication skills with the language of mathematics
- Promote abstract, logical, critical and metacognitive thinking to assess one's own and other's work
- Foster critical reasoning for appreciating other's perspectives
- Promote critical appreciation of the use of information and communication technology in mathematics
- Appreciate the universality of mathematics and its multicultural and historical perspectives

Those contexts are chosen for illustrating the interwoven links of the two components with contents. It looks like methods of teaching, however, all three components are the subjects of teaching in the contexts.

Extracted and edited from:

SEAMEO Basic Education Standards (SEA-BES): Common Core Regional Learning Standards (CCRLS) in Mathematics and Science. SEAMEO RECSAM. (Mangao, Ahmad, & Isoda, 2017, p. 2-11)

APPENDIX B

Terminologies Explained

Mathematical Thinking and Processes

Higher order thinking is the terminology for curriculum but it is not specified in mathematics. Here, it is explained generally as acceptable terms from the perspective of mathematics in education. On the Mathematical thinking and processes, the following terms are the sample which can be seen on SEA-BES: CCRLS for making clear descriptions of the objective of teaching. If you can use these terminologies for writing the objectives of teaching, you would be able to consider how you teach them in the process. Mathematical Thinking can be explained through Mathematical Ideas, Mathematical Ways of Thinking, and Mathematical Attitude. Mathematical attitude is a component of the Value, Attitude and Human Character Formation in Appendix A. Mathematical Ideas, Mathematical Ways of Thinking can be developed through the reflection of the process and Value and Attitude can be developed through appreciation.

Mathematical Ideas

Even through every mathematics content embedded some ideas, there are essential mathematical ideas which are used in various occasions. Mathematical ideas are not exclusive but functions as complementary. The followings are samples of essential mathematical ideas.

Terminology	Explanation
Set	A set is a collection of elements based on certain conditions. When the condition of the set changes, result of reasoning related to the set may change too. Sets are compared by one-to-one correspondence. Basically, the idea of set is reflected through activities that require us to think about membership (elements and conditions) of a set. In addition, activities involving subsets, cardinality and power are extended ideas of set. Number of elements called cardinal or cardinal number (or set number). Ordinal number does not imply number of elements. Other ideas include operations of sets such as union, intersection, complement, the ordered pair/combination of elements such as Cartesian products and dimension mapping. A number system is a set with structures that has the structures of equality, order (greater, less than), and operations, which is developed and extended throughout the curriculum from natural number to complex number: the set of complex numbers does not have the structure of order. In Mathematics, a set is the bases to set algebraic structure and the bases for logic such as universal proposition and existential proposition.
Unit	Unit is necessary for counting, measurement, number line, operations and transformation. It is represented as “denomination’ for discrete quantity, such as 1 “apple’ for situations involving counting, or continuous quantity, such as 1 gram for situations involving measurement. Mathematically, unit is used to indicate a number by mapping it with the quantity in a situation. In a situation, it can be fixed based on the context of comparison, which can either be direct or indirect comparison. In this context, a remainder or a difference from a comparison can be used for fixing a new arbitrary unit for measurement which is a fraction of the original unit. This process of determining a new unit is the application of Euclidean algorithm for finding the greatest common divisor. For the base-10 place value number system, every column is defined by the units such as ones, tens, hundreds and so on. However, in other place value number system such as the binary system, every column is defined by the units such as

	ones, twos, fours and so on. Therefore, in a place value number system, the unit is not always a multiple of the power of ten. In addition, various other number systems are made up of different units. For the calendar system, the lunar calendar is based on 30 (29.5) days, while the solar calendar is based on 365 (365.25) days. On the other hand, the imperial and U.S. customary measurements include units in the base-12 and base-16 systems. In the ancient Chinese and Japanese systems, there were units in the base-4, thus also included the base-16. On the other hand, the units used in different currency systems are dependent on differing culture and countries. However, many countries had lost the unit of 1/100 on their currency systems, which originated from "per centos" that means percent. Even though the based-10 place value system is used to represent the value of money, many currency systems are using the units for 2, 5, and 25 in their denominations instead. Unit for a new quantity can be derived from ratio of different quantities. For example, the unit for speed (km/h) is the ratio of distance (km) over time (h) which cannot be added directly. A car moves at 30km/h and then moves at 20km/h does not mean the car moves at 50km/h. Identity element for multiplication is one but additive identity is zero. Identity for multiplication is the base for multiplicative and proportional reasoning. Inverse element for multiplication is defined by using one.
Comparison	Comparison of concrete objects can be done directly or indirectly without measurement unit. As mentioned at the Unit, direct comparison can be used to fix a new unit of measurement, whereas indirect comparison can be used to promote logic for transitivity which includes syllogism. Comparison of multiple denominated numbers with different unit quantities on the same magnitude such as 5.2 m and 5 m 12 cm can be done if they are represented by a single denominated number by the unified unit quantity, such as 520 cm and 512 cm. Furthermore, comparison of expressions which has the same answers on the same operation such as 2+4, 3+3 and 4+2 can be used to find rules and patterns. For example, 2+4 = 4+2 can show commutative rule for addition whereas for 2+4 = 3+3 can be used to show pattern when 1 is added to 2 and subtracted from 4, the sum is still the same. Comparison of fractions is an activity to find the unit fraction. For comparison of fractions such ad $\frac{1}{2}$ and $\frac{1}{3}$, we have to find the unit fraction $\frac{1}{6}$ which can measure $\frac{1}{2}$ and $\frac{1}{3}$. $\frac{1}{6}$ is the common denominator for $\frac{1}{2}$ and $\frac{1}{3}$. The algorithm to find the unit fraction as the common denominator is called 'reduction of fraction'. On numbers, the relationship of two numbers can be equal, greater or less. The number set up to real number is a total/linear order set, thus up to real number set, two numbers can be compared. However, complex number as an extension of real number cannot be compared directly because it has two dimensions. On the number line as real number, the size of number (distance) is defined by the difference, 1 = 2-1 = 3-2 = 4-3 =.... Here the difference is the value of subtraction as binary operation and can be seen as the equivalence class. On the idea of equivalence class, the value of operations can be compared. On the plane such as complex plane, even though the number is not simply ordered, the size of

	number (distance) is defined by Pythagorean theorem. By using this definition, I1I= I($\frac{\sqrt{2}}{2}$)(1+ i)I = IiI = I($\frac{\sqrt{2}}{2}$)(1- i)I=... the theorem produce the distance on the plane and the distance can be compared. Explained at the Unit, on measurement, the magnitude is given by defining the unit of magnitude. One of the ways to produce the unit magnitude is a direct comparison which provides the difference and Euclidean Algorithm produce the unit of measurement as the greatest common devisor. Another way to produce the unit is by using the ratio and so on. Such a newly produced magnitude usually lost the linearity. In Physics, ‘db’ is the size of volume which is produced by the common logarithm of sound pressure. ‘db’ is fitting well for human impression of the size of sounds on its linearity. It is known as Weber-Fechner’s law that human senses are proportional to the logarithm of stimulus. On science, logarithmic scale is used for Semi-log graph and Log-log graph for demonstrating the linearity even it is an exponential phenomenon. Logarithm produce the scale to illustrate multiplicative phenomena as an additive phenomenon. In any mathematical investigation, particularly in the mathematics classroom, problem solving approach, comparisons of various ideas, representations and solutions are key activities for discussion and appreciation. This comparison is a nature of mathematical activity.
Operations	Addition, Subtraction, multiplication and division are four basic arithmetic operations. Mathematically, these are binary operations involving any two numbers with symbols of operations, +, –, × and ÷. Polynomial expressions can be seen as a combination of binary operations. Mental arithmetic may be used in column method with the base-10 place value system. An operation is not just a rule but can be demonstrated by using various representations. For example, an operation can be represented by the manipulation of concrete objects as well as expressions. However, the manipulating process is not the same as the operation process because it cannot be recorded without using a diagram. From Key Stage 3 up to the field theory at the university levels, arithmetic operations are expressed as addition and multiplication. On the axiom of vector space, negative vector is represented by using the minus symbol. Mathematically, operations and set are used for to establish mathematical structure.
Algorithm	Algorithm is a set of sequential activity at the special situation to produce a particular solution to a task. The column method is based on the base-10 place value system. When using this method, 200 + 300 is done by just 2 + 3 on the hundreds place value. This algorithm is adapted for mental calculation. Representation of column method is not universal like an expression as algebraic representation. Algorithm for the column method is fixed as a formal form based on culture, however it can be created from the manipulation of the base ten blocks. A formula also functions like an algorithm. It can be applied without understanding its meaning. However, it cannot be created without understanding and recognising its underlying structure or meaning. If we understand the structure or meaning such as ratio and proportionality, it is not necessary to memorise it. In Informatics, algorithm is usually defined by programing language through analysing procedure and setting several functions by the program are necessary.

Fundamental Principles	Fundamental principles are the rules which is related with mathematical structures and forms in general. Commutativity, Associativity and Distributivity are three fundamental principles for arithmetic operations. Commutativity does not work on subtraction and division. On the discussion of Distributivity, if *a*, *b*, and *c*, are positive numbers, then the expressions $a(b + c)$, $(b + c)a$, $a(b – c)$, and $(b – c)a$ are different. However, if *a*, *b*, *c* is both positive and negative numbers, then the four expressions can be seen as the same. There are also other fundamental principles for arithmetic operations at the elementary level such as the followings: $\begin{array}{ccccc} 1 & + & 9 & = & 10 \\ \downarrow{+1} & & \downarrow{-1} & & \\ 2 & + & 8 & = & 10 \end{array}$ $\begin{array}{ccccc} 2 & \times & 3 & = & 6 \\ \downarrow{\times 10} & & & & \downarrow{\times 10} \\ 20 & \times & 3 & = & 60 \end{array}$ $\begin{array}{ccccc} 8.1 & \div & 9 & = & 0.9 \\ \downarrow{\times 10} & & & & \uparrow{\div 10} \\ 81 & \div & 9 & = & 9 \end{array}$ $\begin{array}{ccccc} 8.1 & \div & 9 & = & 0.9 \\ & & \downarrow{\times 10} & & \downarrow{\div 10} \\ 8.1 & \div & 90 & = & 0.09 \end{array}$ Principles can be identified through comparison. They are necessary for the explanation of algorithms and thinking about how to calculate by using models and other representations. On the extension of numbers and operations, principles are used for the discussion of the permanence of form (see the permanence of form). Not only algebraic participles, there are other forms that can be extended. In geometry, the extendable nature of a line changes its functions in curriculum. For example, shape is extended to figure; edge, which may include the inner part of a shape, is extended to side, which may not include the inner part of a figure. Then, the side is extended to a line which enable the discussion on the possibility of escribed circles. In addition, parallel lines are necessary to derive the area formula for triangles with various heights.
Permanence of Form	Permanence of the equivalence of form, Hankel's Principle, is known as Commutativity, Associativity, and Distributivity for algebra for the field theory. Permanence of form had appeared in history of mathematics in 16th century and functioned to shift from arithmetic algebra to symbolic algebra. On Peacock's Permanence of Form, it is not only the limited three rules like Hankel's, but it is applied to algebraic symbolic form for overcoming arithmetic algebra which had limited the positive number and 0. In Education, the form is not a limited expression but includes the patterns and the permanence of form can be used in various occasions. Especially, it is used for the extension of numbers and operations from elementary level to secondary level education like the followings: $\begin{array}{ccccc} (+3) & + & (+2) & = & +5 \\ & & & & \downarrow{-1} \\ (+3) & + & (+1) & = & +4 \\ & & & & \downarrow{-1} \\ (+3) & + & 0 & = & +3 \\ & & & & \downarrow \\ (+3) & + & (-1) & = & ? \end{array}$ $\begin{array}{ccccc} (+3) & - & (+2) & = & (+1) \\ & & \downarrow{+1} & & \\ (+3) & - & (+1) & = & (+2) \\ & & \downarrow{+1} & & \\ (+3) & - & 0 & = & (+3) \\ & & \downarrow & & \\ (+3) & - & (-1) & = & ? \end{array}$

	The '?' are unknown, not yet learned. However, people could imagine the '?' by analogical reasoning with the idea of the permanence of the patterns. Here, the permanence of patterns is used as hypothesis and it makes possible to apply it to the unknown cases. Permanence of form is used for initiation of number in Key Stage 1 and later throughout all other key stages. For examples, it is used to explain the necessity of zero (0) and the sum of any number with zero (0) as at Key Stage 1.
Various Representations and Translations	Every specified representation provides some meanings on its essential nature of representation which can be produced by specified symbols and operations. Different representations have the different nature, use different symbols and operations. Every representation of specific mathematical idea has the limitation of interpretation on its nature, thus, thinking by using only one specified representation provides the limitation of reasoning and understanding. If one type of representation is translated to another type of representation, then the representation of idea can be interpreted in other ways. If the idea of specified representation with certain embedded nature is translated into various representations, a rich and comprehensive meaning and use will be produced. However, for making the translation meaningful, it is necessary to know the way of translations which consists with the correspondences between symbols and operations on different representations. For example, proportional number lines only functions for teachers and students who know well how to represent the proportionality on the tape diagrams and number lines, and so on. If they know what it is, they can use it for explanation to produce the expression. Comprehensive learning of mathematics by using various representations and translations are necessary however students have to learn how to represent it in other representations and translate, at first, as well as between expressions and situations.
Pattern, Recursion and Invariant	Mathematics is the science of patterns, in other words, the science of finding invariants. A pattern means the invariant, something with no change in repetition. In the case of numbers, it is usually related to natural number sequences and something constant. For example, sequence patterns such as the arithmetic sequence have the same difference and the geometric sequence has the same ratio. In a table, if the first difference is constant it is an arithmetic sequence, but if the second sequence is constant it is a quadratic sequence. The table also shows some functional relationships between x and y. The pattern is usually found on the table when given pairs of numbers are lined in an appropriate order. Lining ordered pairs under natural numbers appropriately is necessary to find the pattern. Recursion on natural number sequence is usually represented by the recurrence formula. In mathematics, it is used for complete induction. In the programming, recursion means just repetition in a limited set on the natural number for the recurrence formula and it focuses on the part of complete induction if p(k) is true then p(k+1) is true. In the case of a figure, a pattern is usually found on the tessellation of the congruence triangle. It is an appropriate repletion of the same triangle. If we tessellate it, we can find the invariant properties of angles in relation to parallel lines. On the tessellated design as a whole, we can also find translation, rotation and symmetry. We can also find the enlarged figure which shows the similarity of the figures. Mandelbrot set on infinite geometry is the recursion of figures with the natural number sequence. On the programming, we just realize finite cases on the screen.

	An invariant is a stronger word in mathematics if we compared it with the usage pattern just meant repetition in informatics such as programming because it is necessary to be proved on the mathematics system. Dynamic Geometry Software (DGS) provides ways to find something invariant which should be proved. It is normal, nothing strange, that two lines meet at one point if it is not parallel. However, it is very special, and strange, if three lines meet at one point because three lines usually produce a triangle. Thus, the constructions of circumcenter, incenter and centroid on any triangles are very much amazing and should be proven. Graphing Tools (GT) for functions are the tools to represent invariant properties in Algebra and Analysis. On $f(x)=ax^2+bx+c$, if it is drawn by GT with parameters a, b, c, and fix b and c as any constant, and changes a real number, the graph of f(x) produces the family of functions on the screen. Any graph of the f(x) family intersects with the y-axis on (0, c) and $y=bx+c$ is the tangent line for any f(x). To say these findings as invariant in mathematics, we should prove it on the chosen mathematical system. Various proofs are possible if we change the theory in mathematics.

Mathematical Ways of Thinking

Mathematical ways of thinking support the student's thinking process by and for themselves.

Generalisation and Specialisation	Generalisation is to consider the general under the given conditions of situations through extending set or reducing conditions. Any task in mathematics textbooks is usually explained by using some examples which are known as special cases, however, it is usually preferred to discuss the general ideas. Given conditions are usually unclear in textbooks, however if students are asked to consider other cases such as by saying 'for example', the conditions become clear because we may need to consider the condition and set for producing other example. In the upper grades, variable, domain, range, parameter, discreet, continuous, zero, finite and infinite become the terms of set to consider the conditions for general. Specialisation is to consider the thinkable or more simple example on situations and necessary to find the hidden conditions. Considering general with thinkable example is called generalisable-special example. In mathematics, general theory is usually stronger than local theory. It is one of the objectives for generalisation and specialisation. In mathematical inquiry, the process is usually on going from special cases to a general case to establish a stronger theory. Thus, so many cases, task sequence of every unit in textbooks is progressed from special cases to general for generating simple procedures, exceptionally the tasks for exercise and training the procedures which usually progress from general to special because students already learned the general. For students engaging in generalisation and specialisation by and for themselves, students have to produce examples. Thus, developing students who say 'for example' by and for themselves is a minimum requirement for teaching mathematics.

Extension and Integration	Extension means extend the structure beyond the known set on the process of the enlargement of set. On this meaning, extension can be seen as a generalization. The product of multiplication is usually increased if both the factors are natural numbers. However, if we extend it into fractions and decimals, there are cases where the products decease. On the extension of structure beyond set, learned knowledge produce the misconception which is explained by the over generalisation of learned knowledge. In mathematics curriculum, we cannot initiate numbers from fractions and decimals instead of natural numbers. Thus, producing misconception is an inevitable nature of mathematics curriculum and its learning. Even through it is a source of difficulty for students to learn mathematics, it is the most necessary opportunity to think mathematically and to justify the permanent ideas to be extended. After experiencing the extension with misconceptions, if students learn what ideas can be extended, it is a moment of integration. Extension and Integration is a mathematical process for mathematisation to reorganise mathematics. It implicates that the school mathematics curriculum is a kind of nets which connect local theory of mathematics as knot even though their strings/paths include various inconsistencies with contradictions. It is a long-term principle for task sequence to establish relearning on the spiral curriculum In order to develop the way of thinking for extension and integration, teachers need efforts to clarify the repetition of both, the same patterns for extensions and the reflections after every extension in classroom. For example, there are repetitions for the extensions of numbers from Key Stage 1 to 3. On every extension of numbers, there are discussions of the existence of number with quantity, the comparison with equality, greater and lesser, and constructing operations of numbers with the permanence of form. Through the reflection of every repetition, students are able to learn what should be done on the extension of numbers.
Inductive, Analogical and Deductive Reasoning	These three reasoning are general ways of reasoning as for the components of logical reasoning in any subject in our life, however mathematics is the best subjects in schools to teach them. **Inductive reasoning** is the reasoning to generalise the limited number of cases to the whole set or situations. Three considerable cases or more will be the minimum to be considered inductive reasoning. To promote inductive reasoning, teachers usually provide the table for finding the patterns, however it is not the way to develop the inductive reasoning. To promote inductive reasoning, students need to consider various possible parameters in a situation and choose two or further parameters by fixing other parameters. Then, consider the relationship in the situation for knowing the cause and effect, and subsequently, set the ways to check the cases by well ordering of the cause and get the effect, follow by recording the data in a table. To promote inductive reasoning, teachers usually provide the table first for finding the patterns, however it is not the way to develop the inductive reasoning. It is the concern of students because to develop inductive reasoning, students have to learn the ordering of natural numbers is vital because it is the cause of effect and beautifulness of patters that is originated from the order of natural number. Table is a tool for finding pattern but students cannot produce their inductive reasoning by and for themselves as long as it is given by teachers. Students have to learn the way of ordering for finding the pattern, inductively because it is the manner of experiment in Science and Engineering.

	Analogical reasoning is the reasoning to apply known ideas to the unknown set or situations when we recognise the similarity with the known set or situations. Depending on the context it is called abduction. Most of the reasoning to find ways of solution for unknown-problem solving by using what we already knew is analogical reasoning. In analogical reasoning, it is to recognising similarity between the unknown-problem and the known problems. Even though the rule of translations between different representations are not well established, analogical reasoning may function as a metaphor for understanding. Many teachers explain operations by using diagrams. It appears meaningful to provide the hint for solving but most of students cannot use the hint by and for themselves because students do not recognise the similarity as in their own analogy. To develop analogical reasoning, the most necessary way is to develop the habit to use what students learned before by and for themselves. Providing assisting tasks before posing the unknown problem is also used as a strategy to find similarity. **Deductive reasoning** is the reasoning with components of already approved notions and given by using 'if… then' and logics for propositions such as transitivity rule. 'If not' also functions for proof by contradiction as well as counter examples. In cases where the rules of translations are well established, the translations of various representations still function under their limitations too. Various methods for proving such as a complete induction is also done by deductive reasoning. For finding the ways of explaining and proving, inductive and analogical reasoning are necessary, and analytical reasoning, thinking backward from conclusion to the given, is also used but these reasoning do not allow to write at the formal proof by deductive reasoning. Arithmetic and algebraic operation can be seen as automatised deductive reasoning. Most of students do it just by recognising the structure of expression intuitively without explaining why. For clarifying the reason, teachers are necessary to ask why? For developing the three reasoning, knowing objectives of reasoning are necessary. Inductive reasoning is applied to find general hypothesises. Analogical reasoning is applied to challenge unknown problem solving. Deductive reasoning is applied to explain or proof in general on local system.
Abstracting, Concretising and Embodiment	Abstracting and concretising are changing perspectives relatively by changing representations such as expressions. Abstracting is usually done for making clear a structure. Concretising is usually done for making ideas meaningful by concrete objects. For numerical expressions, manipulative and diagrams function as concrete. For algebraic expression, numerical expressions function as concrete objects. For both examples, abstract representations and concrete representations do not correspond one-to-one on translation because concrete representation usually have some limitations but concrete representations function as metaphors of abstract ideas. Embodiment functions in both abstracting and concretising. When abstract ideas can be concretised, it implicates those abstract ideas are embedded with some specified concrete ideas. When concrete ideas can be abstracted, it implicates concrete ideas are embedded into abstract ideas. Both the embodiments function for understanding ideas using metaphors but their translations are usually limited only for corresponding contents. In the case abstract and concrete objects are well translated on both directions, both of them function as mathematical representation. Otherwise, embodiment is a useful word for explaining the metaphor on the given context in process to establish mathematical representations in general.

Objectifying operations for symbolising and establishing new operations on mathematisation	A mathematical representation can be characterised by its symbols and operations with specified purpose and context. In the process of mathematisation, lower-level operational matters are usually objectified for new symbolising and its operations. Until Key Stage 2, numbers do not mean the positive and the negative number. The number in red on financial matter is large if the number is 'large'. The number in black on financial matter is large if the number is 'large'. Here, the meaning of 'large' are defined at the opposite number rays, thus cannot be compared the numbers in red and black easily. At Key Stage 3, as for integration, we have to alternate new symbols and operations. We represent the red number by the negative symbol '-' and the black number by the positive symbol '+' and integrate the one direction for comparison into one dimensional number line. Here, on Key Stage 3, 'larger' for comparison (operational matter on number rays) on lower level become the object of higher level to produce the comparison (operational matter on number line) for new number symbols with positive and negative. It is the process of mathematisation by objectifying the operational matter to establish new symbols and operations. The process of abstracting from concrete can be seen as the process of mathematisation.
Relational and Functional Thinking	Relational and functional thinking are ways of thinking that can be represented by relation and function if we need describing them by mathematical notation. It has been used as major terminologies to explain mathematical thinking at the Klein Movement, 100 years ago. Relation in pure mathematics is a basic axiom. In relation to functional thinking, it is known as the ordered pairs between two sets. Mapping is a relationship and ordering is also a relationship and it supports keeping the structure or not. Equivalence class is a relationship in which axiom is defined by reflective, symmetric and transitive properties. Function is a binary relation between two sets that associates every element of the first set to exactly one element to the second set. The second set is usually numbers in mathematics and both sets are numbers in education. Relational thinking is the thinking which can be represented by various representations such as graphs. It is the activity for students who do not know such representations, and teachers have to teach such representations on the necessity of students to engage in their activity: It is a welcome for students to create their necessary informal representation such as graph and diagram by themselves. Historically, characteristics of functional thinking was described by Hamley (1934). He defined functional thinking by four elements: Class, Order, Variable and Correspondence, Class is a set which has possibility to include equivalence in operations such as 5 can be seen as the value of 1+4, 2+3, 3+2, 4+1. Order is discussed within a set and between sets. Variables are domain and range which are not the same as the origin set and the destination set. Correspondence is discussed between domain and range. Functional thinking is useful to predict and control a situation. In elementary level, proportionality and operations are used for functional thinking on this objective. For example, proportional number line is a representation of functional thinking. For knowing the properties of each function, tables and graphs are useful. Teachers usually provide the tables and the graph papers from the beginning of experiment for finding patterns and properties of function. However, this is not the way to develop functional thinking because teachers take over the opportunities from

	students to consider and fix the sets, orders, variables, correspondences and so on by themselves. Rate of change is used to check if a function has linearity. In the case if it is constant, it is a liner function. If not, otherwise. The limit of the rate of change for finding the tangent is the definition of differentiation. To show pattern beyond order, semi-log graph and logarithm graph are in Physics and Engineering. Correlation in statistics is a relation but not necessary functions as a causal relation.
Thinking Forward and Backward	Thinking forward and backward are the terminologies of Polya. In Pappus of Alexandria (4th Century A. D.), thinking forward corresponds to synthesis and thinking backward corresponds to analysis. Synthesis is the deductive reasoning (proving) from the given and known whereas analysis is the reasoning from the conclusion for finding the possible ways of reasoning from the given and known. From Ancient Greece, analysis is a method of heuristics which was the ways to find adjoin lines on construction problem, valance for area and volume problem. Descartes used unknown *x* for algebraic problem. Leibniz used unknown limit *x* of the function for calculus. These are hypothetic-heuristic reasoning beginning from the conclusion, such as if the construction is achieved, if the valance is kept, if the unknown *x* is given, and if the limit of *x* existed. Since analysis began from hypothesis, without proving from the given, people used to believe that analysis produced tautology which is not allowed to write in the system. On the other hands, modern mathematics system itself begins from the axiom as presupposition. Thus, if the ways of analytic reasoning become a part of presupposition, it is allowed in a written form. On this reason, the unknown *x* is able to be written in Algebra and the limit of *x* is able to be written in Calculus. However, in school mathematics, before such reformations of mathematics, analysis, a method of heuristics, do not allow to be written in a part of proof. It is a reason why some mathematics textbooks look very difficult to understand because they have to be written in the form of deductive reasoning for the construction of the system from axiom which does not include heuristics and ways of findings. On this consequence, there are old fashioned textbooks which just oriented to exercise the procedures as rules without explaining why. Current school textbooks, which orients to write the problem solving process with various solutions as well as misconception by using what already learned, include the heuristics such as thinking backward and so on. In the standards of Key Stage 1 and 2, addition and subtraction are inverse operation, and multiplication and division are inverse operation as for verification of answers on operations. Such ideas are reformulated at the algebra in Key Stage 3. In the case where 'If your saying (conclusion) is true, it produces contradiction which we already knew' is known as dialectic in communication and it was formalised as the proof by contradiction in mathematics. It is also the way of analysis for thinking backward but accepted as the way of proving by counter example and Reductio ad absurdum. In mathematical communication, thinking backward is a part. Without the preparation of lesson plan which includes thinking backward, teachers cannot realise the classroom communication including misconception because the counter example is the component for the proof by contradiction. Though the communication of both objective and way of reasoning such as thinking backwards by using what students already learned, we are able to develop students' mathematical thinking.

Mathematical Activities

Mathematical Activities usually explain the teaching and learning process but also embedded the mathematical ideas, thinking, values and so on. As for style of teaching approach, they are usually enhanced however the style of teaching itself does not make clear them.

Problem Solving	Pure mathematicians inquire problems that are never solved yet and they develop new theorems for solving their problems. It produces a part of system. Such authentic activity is the model of problem solving in education because it usually embedded rich ideas, ways of thinking and values in mathematics. As mathematicians usually pose problems for themselves, the activity includes problem posing and reflection which is necessary for establishing new theories. In education, there are two major approaches for embedding them in learning. The first one is setting the time, unit or project for the problem solving. Here, solving problem itself is an objective for students. It focused on heuristics: it is usually observed unexpectedly and to plan it is inevitably not easy. On this difficulty, the problem solving tasks are usually provided in two types. The first type focuses on mathematical modelling from the real world. The second type focuses on open ended tasks for students because it provides the opportunity for various solutions. The second one is called the problem solving approach, tries to teach content through problem solving in classes. This case is only possible if teachers prepare the task sequence that enabling students to challenge the unknown-task by using what students already learned (ZPD). In problem solving approach, the tasks given by teachers are planned for students where they are able to learn the content, mathematical ideas and ways of thinking. For teachers, solving the tasks itself is not the objective of their classes but students reveal the objectives of teaching by teachers through recognising problematic as an unknown and finding the way of solutions. For students to be able to learn by and for themselves, it is necessary to plan the class with the preparation of future learning as well as by using learned knowledge. Textbooks such as the Japanese textbooks equipped task sequence for this purpose. In such textbooks, heuristics is not an accidental matter but a purposeful matter because every task in the textbook will be solved by utilising the already learned representation and so on. It is called a ‘guided discovery’ under ZPD because it expects well-learned students on the learning trajectory and never expects the genius students on producing unknown ideas. For observers, the way of teaching by the problem solving approach in a class cannot be distinguished with the open-ended approach. The open approach is not necessary to prepare the task sequence because it is characterised by an independent open-ended task. If teachers set the open-ended task independently, it is the first approach. If teachers set the task sequence of open ended tasks for learning mathematics itself, it is the problem solving approach, the second one. Both of the approaches were known as Japanese innovation in textbooks since 1934 for elementary level and since 1943 for secondary level which was done by Prof. Shimada Shigeru and others. Prof. Nobuhiko Nohoda was known as he theorized the open approach with open-ended task and task sequence. Currently, Japanese textbooks until the middle school level equipped the task sequence for problem solving approach. Even though problem solving in education resemble activities of authentic mathematicians, it is actually not the same because problems of mathematicians are usually unsolvable beyond years and decades, while tasks in classroom can be explained by teachers who posed the problems. When teachers refer problem solving in education, it includes various objectives such as developing mathematical ideas, thinking, values and attitudes. These terminologies are used

	in education to develop students who learn mathematics by and for themselves while mathematicians only use some of them. On teacher education, if teachers only learned the content of mathematics, they may lack the opportunity to learn the necessary terminology. If teachers do not know it, the higher order thinking becomes a black box which cannot be explained.
Exploration and Inquiry	Exploration has been enhanced in finding hypothesis by using technology such as Dynamic Geometry Software and Graphing Software. These software provide the environment for students to explore invariant in diversity, easily. Exploration on the environment support to produce hypothesis. Inquiry which includes exploration orients to the justification and proving through reflections. Both exploration and inquiry enhance questioning by students. Thus, the process and finding will be depending on students' questioning sequence, not likely on problem solving by the task which teachers are able to design before the class.
Mathematical Modelling, Programming and Mathematisation	Mathematical modelling is a necessary way to solve real world problems in the world. Mathematical model is hypothetically set by using mathematics to represent the situation of the problem. Mathematical answers based on the model are confirmed in the situation through interpretation. Modelling enhanced various possibility to apply various representations in mathematics. In education, modelling has been enhanced for the problem solving after the new math movement which recognised school mathematics with set and structure. In this era of Artificial Intelligence (AI) and Big Data, modelling is done through programming with algorithms and it is also a part of mathematics. Before mathematical modelling, mathematics had functioned as the metaphor, theory and language for the nature. In Ancient Greece, music and astronomy were exactly theorised under geometric representations. Today, mathematics itself has various theories with algebraic representations, thus, modelling means hypothetical approach for the real world by using universal mathematical language. Mathematisation has two usages: The first usage is for science and engineering in the establishment of mathematical model and produce new mathematics theories based on the model. In physics, mathematical problem solving of nature has produced the various theorisation in mathematics for solving problems in general. The second usage is used in education as the principle of mathematics curriculum sequence which enhances reorganisation of mathematical experiences. It includes the process of extension and integration by using the prior learned knowledge. From the perspective of mathematical problem solving, second usage aimed to develop or reinvent mathematics by students through the solving mathematics problems with support of teachers, textbook and curriculum. For mathematisation, setting appropriate task sequence has the key role to develop or revise known mathematical ideas, concepts and procedure.

Conjecturing, Justifying and Proving	Conjecturing, justifying and proving have been on the context of proof and refutation. In education, students conjecture hypothesis with reasoning on exemplar. Conjecture is conceived through generalisation and justifying with appropriate conditions. Proving includes not only the formal proof in a local system but also the various ways of explanations. Counter example is a way of refutation. Counter example is meaningful because it sets off the reasoning from which if the conclusion is true. Proving is the activity to reach proof. On this meaning, probing is not the proof itself. Collecting and explaining with examples can be also included.
Conceptualisation and Proceduralisation	Conceptual knowledge is the knowledge to explain the meaning and why, and used for the conscious reasoning, and procedural knowledge is the skilful knowledge used for unconscious-automatised reasoning. A unit of mathematics textbook usually begins from the initiations of new conceptual knowledge by using learned procedural-conceptual knowledge which is called conceptualisation. After the initiations, new conceptual knowledge formulates the new procedural knowledge for convenience and the exercises produce proficiency of procedure which is called proceduralisation. In mathematics, the proposition format 'if…, then…' is the basic format to represent knowledge, however in school mathematics, it is impossible to make clear the proposition from the beginning because 'if' part can be clarified later. For example, 'number' changes the meaning several times in school curriculum. In multiplication, products become large if it is […] number. Until numbers are extended to decimal, […] part cannot be learned. On this problematic, it is normal that students meet the difficulty in their learning and challenge to produce appropriate knowledge if they do have a chance to over-generalise knowledge for knowing […] part Misconception (overgeneralized conception) is usually appear in the process of learning. It is a nature of mathematics curriculum. On this reason for producing exact knowledge in mathematics, conceptualisation and proceduralisation is a journey that continues recursively in mathematics learning to change the view of mathematics. It is the opportunity to learn necessary mathematical ideas and ways of thinking, and to develop value and attitude. The process of mathematisation also can be seen from the perspective of conceptualisation and proceduralisation on the context of number and algebra.
Representations and Sharing	A mathematical representation consists of symbols, operations, and objective (context). Solving equations in algebra can be seen as specific ordered elements of every equation which has the same answer. Order of equations shows the context that it is the process of solving. Operation of equations between an equation to another equation can be explained by the property of equality. Each equation is a symbolic sentence. If operations of representation are missing, it is not a mathematical representation even through it has some artistic images such as diagrams. When students draw diagrams, it is sharable if the rule of the drawing (operation) are shared. In a classroom, students produce their own images in diagrams. It is helpful to encourage for their explanation by themselves but every explanation is independent until knowing the hidden ideas as in the comparison. Other students cannot re-present it until the ways of drawing (operation) is shared. Thus, comparison of various representations is a part of the process to recognise ideas for producing symbol and operations, and translations as in mathematical representations. In mathematics, different representations use different symbols and operations. If there is a rule for correspondence between symbols and operations on different

	representations, it can be translated and produce rich meanings. It is the way to produce a mathematics system. In mathematics, representation system can be defined universally, which is the product of convention by mathematicians. For teachers, it looks far for students' activity in the classroom, however it is the opportunity for students to reinvent the representation and its system through considering the why and how. For example, to produce metre as the measurement quantity includes such activities: There are historical episodes on why and how 'm' was defined by Condorcet and others in the middle of the French Revolution. On the area of Engineering, Informatic and Science, applied mathematicians usually try to produce new measurements based on their necessity of research and development to conceptualise the idea mathematically and operationally. Setting the measurable quantity is a part of mathematical modelling for real-world problem solving because if it is measurable, we can apply known mathematics.

APPENDIX C

Mathematical Values

Values indicate the direction that we seek. Thus, they set the direction of thinking. On values, in mathematics, generalisable and expandable ideas are usually recognised as strong ideas. Proving is usually necessary in mathematics for seeking reasonableness. Harmony and beautifulness are described not only in relation to mathematical arts but also in the science of patterns and the basic structure of the system of mathematics. Usefulness and simplicity are used in the selection of mathematical ideas and procedures. Here, each term does not mean an independent category but is related to a feeling such as ‘Simple is beautiful in Mathematics.’

Mathematical Values seeking for:

Generality and Expandability

School mathematics seeks to establish general theory as well as University mathematics. For example, in the case of multiplication, the following learning process is going through seeking general operation with extension. At the initiation stage of multiplication, if students acquired the multiplication table from Row 1 to Row 9, they are released from accumulation (repeated addition) and began to learn multiplication as the binary operation. However, without extension of numbers for multiplication such as Row 10, Row 11, and so on, they still have to do repeated addition. Shall they have to memorise all of them? It is impossible! If they acquire column multiplication, they can multiply any digits with their acquired knowledge of the multiplication table. The column multiplication can extend to decimals if they are able to manage the position of the decimal point on the base ten place value notation. On these extensions and generalizations of multiplication, the accumulation form functioned to learn multiplication as repeated addition at the initiation, however, in the next stage, the column form under the place value system functioned to alternate the accumulation to the column multiplication as a binary operation. These are the process to seek generality and expandability in the case of multiplication.

For learning the value to seek generality and expandability, teachers are necessary to begin from the un-general and before-expansion cases, even though teachers knew the general and the final extended forms. As long as teachers try to teach their general knowledge, students will never have the opportunity to learn these values. A good teacher can design the process of generalisation and expansion and seeks to teach the values through reflection and appreciation. If students appreciate, they can learn the values and seek them by and for themselves.

Reasonableness and Harmony

Mathematics is reasonable. Good teachers usually ask why for developing students who will explain the reason by and for themselves by using what students already learned. Here, a logical-reasonable explanation for using learned knowledge is not always deductive, but more analogical and inductive is related to numbers at the primary level. Significance and objective are also considered as the reason why. For teaching reasonableness, students may have to feel something strange or good first and explain it by using the learned knowledge or something sharable. An unreasonable situation is necessary for creating the opportunity to learn the reason itself. This meant good teachers know what is unreasonable for students with a comparison of what students already learned and plan the process to recognise the unreasonable and to reach appropriate reasons by using what already learned. In the problem-solving approach, the unknown problem itself is set by such a teacher to provide students with the feeling of unknown or unreasonableness. However, on the given appropriate task sequence designed by good teachers, students are able to use what they already learned for the unknown problem. Thus, on the well-designed task sequence, teachers are possible to ask students why.

Mathematics is a harmonious/harmonised subject: how do you explain it? In Ancient Greece, harmony is a name of the subject of mathematics represented by the Pythagorean music scale developed by the ratio, 1 to 2, 2 to 3 and 3 to 4. Buildings were also constructed by the special ratio in Ancient Greece. Some ratios were used to represent beautifulness (Eros) in the Era.

In the case of music, we have to consider the overtone for the development of scale and code. The height of scale (sound) is developed by a kind of multiple however dividing real strings (code) itself for the development of the musical instruments themselves are division as the multiple of reciprocal. It was the origin of music as a subject of education in Europe. In current mathematics, harmony has very much limited use such as the harmonic series as for the extension of overtone. On another story, the current music scale itself is defined as equal temperaments which are the products of multiple to produce a harmonised code in orchestration and it is not the ratios on the Pythagorean scale itself because it produces special growl in a special case.

Here, as for mathematics education, harmony as a seeking value is metaphorically used such as the harmony produced by the current music orchestration even if it is not the usage of current mathematics. Firstly, at the moment, mathematical notions are proved on a system, it is recognised that already existed from the beginning in a system: It is called pre-supposed/established harmony likely the providence such as the god already planned before. Up to the final moment to be proved in a system, the notions do not have a clear position, however at the proved moment, it is a part of the system based on mathematical structure. Even though they are not proved in one system (theory), they can be used hypothetically as true. Even, each of them is proved on the different systems (theories), not one system, and used at the same time as long as it works. However, it looks something strange until re-setting the harmonised position in a system like the history of negative and imaginary numbers.

In music, different scale/temperament system produces different sounds however the music looks the same. For example, Current Buch players use the instruments under equal temperaments even though they used different cords. We are listening to and playing different Bach music however we still recognise it as Bach. Metaphorically, it implies that we recognise mathematical notions on the different systems and use them harmoniously through proven under different systems/theories, even though they should be proved in a system, later. Indeed, the school mathematics curriculum is the possible sequence of local theories which includes several contradictions among them, however, we are possible to accept them harmoniously. In this metaphorical usage, a teacher needs to prepare the class based on various mathematical systems/theories which students already learned.

Secondly, with more metaphorical usage in a mathematics classroom, each student looks at a player to produce mathematical notions harmoniously with collaboration. Current orchestration is possible through each player and conductor’s contributions and collaborations. Each player is a necessary component to produce a kind of resonance of code. Metaphorically, it is a kind of verbal and non-verbal communication in the mathematics classroom and each student’s idea has the role to produce harmonised reasoning as a whole and to be appreciated the likely resonance of code. Even though there is a student who does not talk verbally, he or she is contributing by non-verbal manners such as attitude and eye line and so on. A well-harmonised lesson can be seen through such attitudes and so on.

Usefulness and Efficiency

Mathematics usually alternates the meanings which are necessary for reasoning to the procedure which automates and compresses the reasoning. If this alternation is recognised and done by students, they are able to learn the efficiency of the procedure. If teachers only teach the final procedure from the beginning without any meaning, students lost the opportunity to learn efficiently. Even though teachers take over the opportunity from students, teachers are able to teach the usefulness of the application problem after they teach the procedure. In the previous example of multiplication, accumulation is necessary at the beginning but if we memorise the multiplication table at once and used it for column multiplication it is unnecessary to consider the accumulation. What good things you learned about column multiplication can be explained if they learned the accumulation before column multiplication. What are the good things you learned about positive and negative numbers? These questions are firstly answered by usefulness and efficiency even though there are other values. For answering these questions, students might have the opportunity to compare before and after.

Simpler and Easier

These values are similar to the value of usefulness and efficiency, however, simpler is usually used for the selection of procedure and setting the mathematical form. In the development of the procedure, we chose simpler forms and so on. For example, numbers are read from the largest place value. Thus, children usually try to add from the largest place at the beginning. However, on the column addition, if it has the carrying, they have to rewrite the number several times in the process. If they decide to add from the lowest place, it becomes simple. For explaining the procedure, we use a diagram such as a proportional number line for making easier to produce the expression and interpret the meaning. If students can draw a diagram for an explanation of the meaning, it must be easier to understand.

Beautifulness

In mathematics teaching, beautifulness uses several occasions with various values such as reasoning is simple and thus beautiful. If we find structure/pattern/invariant, it is beautiful. If we make a line with equal signs in the operation of the equation easier to see, then it is beautiful. For students to feel the beautifulness, teachers need to set students to feel the ugly/not beautiful situation and change it to feel beautiful. For example, it is just a heap of coins before the arrangement of the same coins. However, if the coins are arranged in vertical bars as in ordering natural numbers, we can alternate the numbers to the height. and if the coins are regrouped by the height of 10 coins, we can easily count by 10. Comparisons before and after are necessary to discuss how beautiful is the order.

Beautifulness is also discussed in relation to specific mathematical ideas. In number and operation, the answer of operation should be a number, then $1 \div 3 = \frac{1}{3}$, thus $\frac{1}{3}$ is a number even if it is $0.\dot{3}$. $\frac{1}{3}$ looks more beautiful than $0.\dot{3}$. In algebra, the general form of the quadratic function is $y = ax^2 + bx + c$ and the standard form of the quadratic function is y=$a(x-\alpha)^2+\beta$. If we do not know how these two forms are beautiful, we cannot operate a quadratic function. In a figure, symmetry is usually found in the tessellation of a figure. Symmetry is the word to represent the beautifulness of geometry and algebra. On the sequence, the recursion on the numbered sequence is beautiful because it is a representation of invariant pattern.

APPENDIX D

Strand: Mathematical Process – Humanity for Key Stage 2

As a follow-up of Key Stage 1, activities are designed to enable an appreciation of knowledge and skills learned and the ways of learning such as applying knowledge of number sense to solve daily problems. Mathematical processes such as communication and reasoning are used to provide explanations for mathematical problems and modelling. The ability to connect and reason mathematical ideas would trigger excitement among learners. Discussions of misconceptions are usually enjoyable and challenging. Mathematics learning usually begins from situations at Key Stage 1. In Key Stage 2, the development of mathematics is possible through discussions for the extension of the forms. Appreciation of ideas and representations learned become part of the enjoyable activities. Through the consistent use of representations such as diagrams, the application of learning becomes meaningful.

Standards:

Enjoying problem-solving through various questioning and conjecturing for extension of operations into decimals and fractions with proportionality and new quantities such as area and volume
Enjoying measuring through settings and using the area and volume in situations
Using ratio and rate in situations
Using number lines, tables, and area diagrams for representing operations and relationships in situations
Establishing the idea of proportion to integrate various relations with the consistency of representations
Enjoying tiling with various figures and blocks
Producing valuable explanations based on established knowledge, shareable representations and examples
Performing activities of grouping and enjoy representing with Venn diagram
Experiencing PPDAC (Problem-Plan-Data-Analysis-Conclusion) cycle and modelling cycle in simple projects in life
Preparing sustainable life with number sense and mathematical representations
Utilising ICT tools as well as notebooks and other technological tools
Promoting creative and global citizenship for sustainable development of community using mathematics

Enjoying problem-solving through various questioning and conjecturing for extension of operations into decimals and fractions with proportionality and new quantities such as area and volume[1]

i. Pose questions to develop a division algorithm in vertical form using multiplication and subtraction
ii. Pose questions to develop multiplication and division of decimal numbers using the idea of proportionality with tables and number lines
iii. Pose questions to develop multiplication and division of fractions using the idea of proportionality with tables, area diagrams and number lines
iv. Pose questions to extend multiplication and division algorithm in vertical form to decimal numbers and discuss decimal points
v. Pose questions to use decimals and fractions in situations
vi. Pose questions to use area and volume in life
vii. Pose questions to use ratio and rate in life
viii. Pose conjectures based on ideas learned such as when multiplying, the answer becomes larger

Enjoying measuring through settings and using the area and volume in situations

i. Compare directly and indirectly areas and volumes
ii. Set tentative units from difference for measuring area and volume[2]
iii. Give the formula for the area and volume for counting units
iv. Use measurement for communication in daily life

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[1] *This is connected to the three strands, Extension of Numbers and Operations, Measurement and Relations, and Plane Figures and Space Figures.*

[2] *Euclidean algorithm is a method of finding the largest common divisor of two numbers.*

Using ratio and rate in situations[3]

i. Understand division as partitive (between different quantities) and quotative (between the same quantity) in situations
ii. Develop the idea of ratio and rate utilising the idea of average and per unit with tables and number lines
iii. Communicate using the idea of population density and velocity in life

Using number lines, tables, and area diagrams for representing operations and relations in situations[4]

i. Represent proportionality on number lines with the idea of multiplication tables
ii. Use number lines, tables, and area diagrams for explaining operations and relations of proportionality in situations

Establishing the idea of proportion to integrate various relations with the consistency of representations[5]

i. Use the idea of proportion as the relation of various quantities in life
ii. Identify through the idea of proportion using tables, letters, and graphs
iii. Adopt the idea of proportion to angles, arcs and areas of circles
iv. Adopt the idea of proportion to area and volume
v. Adopt the idea of proportion to enlargement
vi. Use ratio for data handling such as percent and understand the difficulties to extend it to proportion

Enjoying tiling with various figures and blocks[6]

i. Appreciate producing parallel lines with a tessellation of figures
ii. Explain the properties of figures in tessellations by reflections, rotations and translations
iii. Develop nets from solids and explain the properties of solids by each of the component figures
iv. Use the idea of tiling for calculating the area and volume

Producing valuable explanations based on established knowledge, shareable representations and examples

i. Establish the habit of explanation by referring to prior learning and ask questions using terms such as why, how, what, if and if not, and reply using examples and 'for example' in a discussion
ii. Assessing the appropriateness of explanations using representations such as generality, simplicity and clarity
iii. Use other's ideas to produce a better understanding
iv. Use inductive reasoning for extending formulae

Performing activities of grouping and enjoying representing with Venn diagram

i. Use the idea of the Venn diagram for social study
ii. Understand classifications based on characteristics and represent them by using Venn diagrams

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[3] *Ratio and proportion bridge multiplication and division in situation of two quantities with reference to Extension of Numbers and Operations and Measurement and Relations.*

[4] *This is a bridge to the Extension of Numbers and Operations and Measurement and Relations.*

[5] *Bridge to the three strands, Measurement and Relations, Plane Figures and Space Figures and Data Handling and Graphs.*

[6] *Connected to the two strands, Measurement and Relations, and Plane Figures and Space Figures.*

Experiencing PPDAC (Problem-Plan-Data-Analysis-Conclusion) cycle and modelling cycle in simple projects in life

i. Understand the problem of context
ii. Plan appropriate strategies to solve the problem
iii. Gather data and analyse using appropriate methods and tools
iv. Draw a conclusion with justification based on data analysis

Preparing sustainable life with number sense and mathematical representations[7]

v. Use minimum and sequential use of resources in situations
vi. Use data with number sense such as order of quantity and percentage for the discussion of matters related to sustainable development
vii. Estimate the efficient use of resources in situations
viii. Maximise the use of resources through an appropriate arrangement in a space such as a room
ix. Understand "equally likely" of resources in situations

Utilising ICT tools as well as notebooks and other technological tools

i. Use internet data for the discussion of matters related to sustainable development
ii. Distinguish appropriate or inappropriate qualitative and quantitative data for using ICT
iii. Use calculators for organising data such as average
iv. Use calculators for operations in necessary context
v. Use projectors for sharing ideas as well as board writing
vi. Enjoy using notebooks to exchange learning experiences with each other such as in mathematics journal writing
vii. Use protractors, triangular compasses, straight edges, and clinometers for drawing and measuring
viii. Use the idea of proportionality to use mechanisms such as rotating once and moving twice (wheels, gears)
ix. Use various tools for conjecturing and justifying

Promoting creative and global citizenship for sustainable development of community using mathematics

i. Utilise notebooks, journal books and appropriate ICT tools to record and find good ideas and share them with others
ii. Prepare and present ideas using posters and projectors to promote good practices in the community
iii. Listen to other's ideas and ask questions for better designs
iv. Utilise information, properties, models and visible representations as the basis for reasoning
v. Utilise practical arts, home economics and outdoor studies to investigate local issues for improving welfare of life

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[7] *It is related to Numbers and Operations, Pattern and Data Representations and Quantity and Measurement all under Key Stage 1.*

APPENDIX E

The four strands on content learning standards and the strand on Mathematical Process-Humanity of Key Stage one

Key Stage 1

Key Stage 1 (KS1) serves as the foundation of knowledge covering the basic facts and skills developed through simple hands-on activities, manipulation of concrete objects, pictorial and symbolic representations. This stage focuses on arousing interest, enjoyment and curiosity in the subject through exploration of patterns, characterisation, identification and describing shapes, performing the four fundamental operations, identifying its algorithm, and understanding basic mathematical concepts and skills experienced in daily life. Calculations of quantities will also be established to carefully and wilfully understand the attribution of objects used to make direct and indirect comparisons.

Strand: Numbers and Operations

The number is introduced with situations, concrete objects, pictorial, symbolic representations and extended based on knowledge and skills learned. Ways of counting and distributions are extended to addition, subtraction, multiplication and division. The base ten number system is the key to extending the numbers and operations for standard algorithms in vertical form. Also, various procedures of calculations and algorithms are focused on. Models and diagrams are used for extensions instead of concrete materials themselves. Number sense will be developed through the establishment of fluency in calculations with connection to situations and models. Fractions and decimals are introduced with manipulatives.

Topics:

Introducing Numbers up to 120
Introducing Addition and Subtraction
Utilising Addition and Subtraction
Extending Numbers with Based Ten System up to 1 000 000 Gradually
Producing Vertical Forms for Addition and Subtraction and Acquiring Fluency of Standard Algorithms
Introducing Multiplication and Produce Multiplication Algorithm
Introducing Division and Extending it to Remainder
Introducing Fractions and Extending to Addition and Subtraction of Similar Fractions
Introducing Decimals and Extending to Addition and Subtraction

Introducing Numbers up to 120

Enjoying counting orally and manipulatively with number names, without symbolic numerals

i. Develop fluency in the order of number names and use them based on situations[1]
ii. Set the initial object for counting, the direction of counting and recognise the last object with one-to-one correspondence
iii. Distinguish the original situation with concrete objects and the representation of counting chips, blocks or marbles

Understanding and using the cardinality and ordinality of numbers with objects and numerals[2] through activities of grouping, corresponding and ordering, and developing number sense[3] and appreciating its beautifulness

i. Group objects for counting with conditions such as cups, flowers, and rabbits in situations and introduce numerals

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[1] *The denomination such as 3 cups, 2 cups, and one cup is described at strand on the Quantity and Measurement*

[2] *Inclusive in reading and writing of numerals*

[3] *Units for counting that describe the Quantity and Measurement*

ii. Obtain fluency in counting concrete objects and understand counting on, and recognise the necessity of zero
iii. Compare different sets by one-to-one correspondence and recognize larger, smaller or equal with appreciation in drawing paths between objects.
iv. Compose and decompose numbers for strengthening the number sense[4] and cardinality
v. Understand the difference between ordinal and cardinal numbers and use them appropriately in situations and challenge the mixed sequence.
vi. Acquire number sense[5] and appreciate the beautifulness of ordering numerals with and without concrete objects

Introducing the base-ten system with groupings of 10 and extending numbers up to 120[6].

i. Extend numbers to more than 10 with base-ten manipulative representing numbers in ones and tens, and appreciate the base-ten numeration system.
ii. Extend the order sequence of numbers to more than 10 in relation to the size of ones and tens and compare numbers using numerals in every place value.
iii. Introduce number lines to represent the order of numbers and for comparison starting from zero and counting by ones, twos, fives, and tens.
iv. Enjoy various ways of the distribution of objects with counting such as playing cards, and explain it and enhance the number sense
v. Draw a diagram for representing the size of the number with base-ten blocks such as a flat (square) for a unit of hundred and a rectangular bar for a unit of ten

Introducing Addition and Subtraction

Understanding situations for addition up to 10 and obtaining fluency of using addition in situations

i. Introduce situations (together, combine, and increase) for addition and explain it orally with manipulative to define addition for operation
ii. Develop fluency in addition expressions using a composition of numbers for easier calculation with number sense for the composition of numbers
iii. Apply addition with fluency in learners' life

Extending addition to more than 10 and obtaining fluency of using addition in situations

i. Extend addition situations and think about how to answer using the idea of making 10 with decomposition and composition of numbers
ii. Explain the idea of addition with place value using base-ten blocks
iii. Develop fluency in addition expressions to more than 10 for easier calculation
iv. Apply addition fluency in learners' life.

Understanding situations for subtraction up to 10 and obtaining fluency of using subtraction in situations

i. Introduce subtraction situations (remaining, complement, and difference) and explain orally with manipulative to define subtraction for operation[7]
ii. Develop fluency in subtraction expressions using the decomposition of numbers for easier calculation
iii. Apply subtraction fluency in learners' life

Extending subtraction to more than 10 and obtaining fluency in` using subtraction in situations

i. Extend subtraction with situations and think about how to answer using the idea of 10 with addition and subtraction of numbers (composition and decomposition of numbers)
ii. Explain the idea of subtraction in place value using base ten blocks

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[4] *Relationship of composing and decomposing numbers become the preparation for addition and subtraction for inverse operation*
[5] *Number pattern is discussed under Pattern and Data Representations*
[6] *For discussing the difference of hundred twenty is not twelve ten in English*
[7] *Distinguish minuend and subtrahend*

iii. Develop fluency in subtraction expressions to more than 10 for easier calculation
iv. Apply subtraction fluency in learners' daily life

Utilising Addition and Subtraction

Utilising addition and subtraction in various situations and understanding their relationships

i. Understand the difference between addition and subtraction situations with tape diagrams
ii. Explain subtraction as an inverse of addition situations with tape diagrams
iii. Understand addition with three numbers, subtraction with three numbers and combination of addition and subtraction situations
iv. Apply addition and subtraction in various situations such as in ordering numbers

Extending Numbers with Base-Ten System Up to 1 000 000 Gradually

Extending numbers using base-ten system up to 1 000

i. Experience counting of 1 000 by using various units and appreciate the necessity of the base ten system
ii. Extend the order of numbers to more than 1 000 in relation to the size of ones, tens and hundreds
iii. Use a partial number line to compare the size of numbers through a translation of the size of every digit with the appropriate scale
iv. Represent appropriate diagram to show the size of numbers without counting such as three of hundreds mean 30 of tens and visualise the relative size of numbers
v. Represent larger or smaller numbers by the symbol of inequality

Extending numbers using a base-ten system up to 10 000

i. Visualise the 10 000 by using thousand, hundred, ten and one as units
ii. Extend the order sequence of numbers to more than 10 000 in relation to the size of ones, tens, hundreds and thousands
iii. Use a number line with an appropriate scale to the show size of numbers and the relative size of numbers while focusing on the scale

Extending numbers using base-ten system up to 1 000 000[8]

i. Extend numbers up to1 000 000 and learn the representation of the place value for grouping every 3-digit numeral system up to a million
ii. Write large numbers using grouping of a 3-digit numeral system[9] such as thousand as a unit and compare numbers in relation to it
iii. Develop number sense such as larger and smaller based on comparison of place values through visualisation of the relative size of numbers

Producing Vertical Form Addition and Subtraction[10] and Acquiring Fluency in Standard Algorithms

Thinking about the easier ways for addition and subtraction and producing vertical form algorithms

i. Think about easier ways of addition or subtraction situations and use models with base-ten blocks meaningfully for representing the base-ten system

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[8] *One million is too big for counting and is introduced only for learning the three-digit system*

[9] *3-digit numeral system such as 123 times thousand equals the same way of reading plus thousand. In the case of Chinese, the four-digit numeral system is used.*

[10] *Understanding the relationship between addition and subtraction is discussed under Pattern and Data Representations*

ii. Produce and elaborate efficient ways and identify the standard algorithms[11] in relation to the base-ten system with appreciation
iii. Explain the algorithms of borrowing and carrying with regrouping of base-ten models
iv. Acquire fluency in addition and subtraction algorithms

Acquiring fluency in standard algorithms for addition and subtraction and extending up to 4-digit numbers

i. Extend the vertical form addition and subtraction through the extension of numbers and appreciate the explanation using the base-ten block model
ii. Develop fluency in every extension up to 3-digit numbers and simple case for 4-digit numbers

Developing number sense[12] for estimation[13] and using a calculator judiciously for addition and subtraction

i. Develop number sense for mental arithmetic with estimation for addition or subtraction of numbers
ii. Identify necessary situations to use calculators judiciously in real life.
iii. Appreciate the use of calculators in the case of large numbers for finding the total and the difference

Introducing Multiplication and Produce Multiplication Algorithm

Introducing multiplication and mastering multiplication table

i. Understand the meaning of multiplication[14] situations with models using the idea of addition and distinguish from the common addition to find the total number
ii. Produce a multiplication table in the case of counting by 2 and 5 with array diagrams, pictures or block models and extend it until 9 and 1 with an appreciation of patterns[15]
iii. Develop a sense for multiplication through mental calculation with fluency
iv. Use multiplication in daily life, differentiating multiplication in various situations with the understanding that any number can be a unit for counting in multiplication

Producing multiplication in vertical form and obtaining fluency

i. Think about easier ways of multiplication in the case of numbers greater than 10 using array diagrams and block models
ii. Develop multiplication in vertical form using multiplication table, array, model, and base-ten system with appreciation
iii. Extend the multiplication algorithm to 3-digit times 2-digit numbers
iv. Obtain fluency in the standard algorithm for multiplication
v. Use estimation with the multiplication of tens or hundreds in life
vi. Compare the multiplication expressions which is larger, smaller or equivalent
vii. Appreciate the use of calculators sensibly in life in the case of large numbers

Introducing Division and Extending It to Remainder

Introducing division with two different situations and finding the answers by multiplication

i. Understand division with quotative and partitive division for distribution situations
ii. Think about how to find the answer to division situations by distribution using diagrams, repeated subtractions and multiplication
iii. Obtain fluency to identify answers of division through the inverse operation of multiplication
iv. Appreciate the use of multiplication table for acquiring mental division

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[11] *Various algorithms are possible and there is no one specific form because depending on the country, the vertical form itself is not the same. Here, the standard algorithm means the selected appropriate form.*

[12] *Money system is discussed under Measurement and Relations*

[13] *Rounding numbers are treated in key stage 2 under Measurement and Relations*

[14] *Meaning of area is described in Measurement and Relations*

[15] *Multiplication row of 1 is not a repeated addition*

Extending division into the case of remainders and using division for distribution in daily situations

i. Extend division situations with remainders and understand division as a repeated subtraction with remainders.
ii. Obtain fluency in the division and apply it in daily situations
iii. Understand simple cases of the division algorithm

Introducing Fractions and Extending to Addition and Subtraction of Similar Fractions

Introducing simple fractions such as halves, quarters and so on using paper folding and drawing diagrams

i. Introduce simple fractions using paper folding and drawing diagrams in the context of part-whole relationship
ii. Use "*a half of*" and *"a quarter of*" in a daily context such as half a slice of bread
iii. Count a quarter for representing one quarter, two quarters, three quarters, and four quarters
iv. Compare and explain simple fractions in the case where the whole is the same

Extending fractions using tape diagram and number line to one, and think about how to add or subtract similar fractions for producing a simple algorithm

i. Extend fractions to more than one unit quantity for representing the remaining parts (unit fraction) such as measuring the length of tape, recognising the remaining parts as a unit measure of length, and understanding proper and improper fractions
ii. Appreciate fractions with quantities in two ways; firstly, the whole is a unit of quantity and secondly, based on the number of unit fraction
iii. Compare fractions in the case where the whole is the same and explain it with a tape diagram or number line, and develop fraction number sense such as with quantities and so on
iv. Think about how to add or subtract similar fractions with a tape diagram or number line and produce a simple algorithm with fluency

Introducing Decimals and Extending to Addition and Subtraction

Introducing decimals to tenths, and extending addition and subtraction into decimals

i. Introduce simple decimals to tenths by remaining part such as using a tape diagram with appreciation
ii. Compare the size of decimal numbers on a number line with the idea of place value
iii. Extend addition and subtraction of decimals utilising the place value system in the vertical form up to tenths
iv. Think about appropriate place value for applying addition and subtraction in life

Strand: Quantity and Measurement

Attributes of objects are used to make direct and in-direct comparisons., The non-standard and standard units are also used for comparison. Counting activities denominate units of quantities such as cups for volumes, arm-length and hand-spans for length. Standard units such as metre, centimetre, kilogram and litre are introduced. Time and durations which are not the base ten system are introduced. Money is not a complete model for a base-ten system. The concept of conservation of quantities will be established through the calculation of quantities. The sense of quantity is developed through the appropriate selection of measurement tools.

Topics:

Comparing Size, Directly and Indirectly, Using Appropriate Attributes and Non-Standard Units
Introducing Quantity of Length and Expand It to Distance
Introducing Quantity of Mass for its Measurement and Operation
Introducing Quantity of Liquid Capacity for its Measurement and Operation
Introducing Time and Duration and its Operation
Introducing Money as Quantity

Comparing Size, Directly and Indirectly, Using Appropriate Attributes and Non-Standard Units

Comparing and describing quantity using appropriate expression

i. Compare two objects directly by attributes instead of stating in length and amount of water such as longer or shorter and less or more
ii. Compare two objects indirectly using non-standard units to appreciate the unification of units
iii. Use appropriate denomination[16] of quantity (such as number of cups) for counting and appreciate the usage of units for quantity in a suitable context

Introducing Quantity of Length and Expanding to Distance

Introducing centimetre for length and extending to millimetres and metre

i. Compare the length of different objects and introduce centimetre with a calibrated tape[17] of one centimetre
ii. Demonstrate equivalent length with addition and subtraction such as part-part whole
iii. Extend centimetre to millimetre to represent remaining parts with ideas of equally dividing and the idea of making tens
iv. Extend centimetre to metre to measure using a metre stick
v. Estimate the length of objects and select appropriate tools or measuring units for measurement with fluency
vi. Convert mixed and common units of length for comparison[18]
vii. Convert mixed and common units of length when adding or subtracting in acquiring the sense for quantity

Introducing distance for the extension of length

i. Introduce kilometres to measure distance travelled using various tools and appreciate the experiences of measuring skills
ii. Distinguish the distance travelled and the distance between two places on the map
iii. Compare mixed units of length with an appropriate scale on a number line

Introducing Quantity of Mass and Its Measurement and Operation

Introducing gram for mass and extending to kilogram and tons

i. Compare the mass of different objects directly using balance and introduce gram
ii. Demonstrate equivalent mass with addition and subtraction such as part-part whole
iii. Extend gram to kilogram, measure with a weighing scale
iv. Extend kilogram to metric ton through the relative measure (such as 25 children, each weighing 40 kilograms)
v. Estimate the mass of objects and select appropriate tools or measuring units for measurement with fluency
vi. Convert mixed and common units of mass for comparison
vii. Convert mixed and common units of mass for addition and subtraction in acquiring the sense of quantity

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[16] *Denomination is necessary for learning the group of counting. It also describes pattern and data representations and number and operation both of Key Stage 1*

[17] *The plane tape can be used for direct comparison and indirect comparison by marking. If the tape is scaled by a non-standard unit, we can use it for measurement. If the tape is scaled by one centimetre we can define the length of the centimetre*

[18] *Which one is longer, 2 m 3 cm or 203 mm?*

Introducing Quantity of Liquid[19] Capacity and Its Measurement and Operation

Introducing litre for capacity of liquid and extending to millilitre

i. Compare the amount of water in different containers and introduce litre with measuring cups of 1 litre
ii. Demonstrate equivalent capacity with addition and subtraction such as part-part whole
iii. Extend litre by decilitre/100-millilitre cup for representing remaining parts with ideas of equally dividing and making 10, and extend until millilitre
iv. Estimate the capacity of containers and select the appropriate measuring unit
v. Convert mixed and common units of capacity for comparison
vi. Convert mixed and common units of capacity for addition and subtraction in acquiring the sense of quantity

Introducing Time and Duration, and Its Operation

Introducing analogue time and extending to duration

i. Tell and write analogue time of the day corresponding with different activities in daily life such as morning, noon, afternoon, day and night.
ii. Show time by using a clock face with an hour hand and a minute hand
iii. Understand the relative movement of clock hands

Extending clock time to a duration of one day[20]

i. Introduce duration in hours and minutes based on the beginning time and end time of activities
ii. Express time and duration on a timeline, and understand duration as the difference between two distinguished times
iii. Addition and subtraction of duration and time
iv. Extend time and duration to seconds
v. Convert mixed and common units of duration for comparison
vi. Estimate the duration of time and select an appropriate measuring unit for measurement with fluency and appreciate the significance of time and duration in life
vii. Appreciate the difference in time depending on the area (time zone) and the seasons

Introducing Money as Quantity

Introducing money as quantity and use as the model of the base-ten system[21]

i. Introduce units of money using notes and coins and determine the correct amount of money
ii. Use counting by fives and so on for the base-10 system
iii. Appreciate the fluency in the calculation of money with all the four operations
iv. Appreciate number sense for the conversion and transaction of money in daily life

Strand: Shapes, Figures and Solids

Basic skills of exploring, identifying, characterising and describing shapes, figures and solids are learned based on their features. Activities such as paper folding enable the exploration of various features of shapes. Identification of similarities and differences in shapes and solids enables classification to be done for defining figures. Using appropriate materials and tools, relationships in drawing, building and comparing the 2D shapes and 3D objects are considered. Through these activities, the skills for using the knowledge of figures and solids will be developed. The compass is introduced to draw circles and mark scales of the same length.

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[19] *The density which explains the relationship between mass and liquid capacity is usually learned in science at a later stage. In the case of CCRLS Science, it starts in Key Stage 2 such as 1 cubic centimetre of water is equivalent to 1 gram.*

[20] *Calendar is possible in the keys stage 1 under Pattern and Data Representation*

[21] *Coins and notes are dependent on the country. Some countries use currency units of twenty and twenty-five which are in coins or notes. These forms are not appropriate for the model of the base-10 system*

Topics:

Exploring shapes of objects
Characterising shapes for figures and solids
Explaining positions and directions

Exploring Shapes of Objects

Exploring shapes of objects for finding their attributes

i. Roll, fold, stack, arrange, trace, cut, draw, and trace objects (blocks such as boxes, cans and so on) for knowing their attributes
ii. Use attributes of blocks for drawing the picture by tracing shapes on the paper and explain how to draw it with the shapes
iii. Create patterns of shapes (trees, rockets and so on) by using the attributes and recognise the characteristics of shapes[22]
iv. Appreciate functions of shapes of objects in learners' life
v. Appreciate the names of shapes in daily life by using one's mother tongue

Characterising the Shapes for Figures and Solids

Describing figures with characters of shapes

i. Use characteristics of shapes for understanding figures (quadrilateral, square, rectangle and triangle, right angle, same length)
ii. Introduce line and right angle with relation to activities such as paper folding and use it for describing figures with simple properties, such as a triangle has 3 lines
iii. Classify triangles by specific components, such as side, vertex and angle (right-angled triangle, equilateral, isosceles) and then know the properties of each classification
iv. Reorganising rectangular shapes and squared shapes as figures by using the right angle and length of sides

Describing solids with characteristics of shapes

i. Use the characteristics of shapes to understand solids such as boxes can be developed by six rectangular parts with simple properties
ii. Develop boxes with the properties
iii. Appreciate solids around daily life by considering the functions of the solids

Drawing a circle and recognising the sphere based on the circle

i. Think about how to draw a circle and find the centre and radius
ii. Draw a circle with an instrument such as a compass
iii. Enjoy drawing pictures using the function of circles such as Spirograph
iv. Find the largest circle of the sphere with a diameter and identify the sphere by its centre and radius
v. Appreciate circles and spheres in daily life such as manholes, and the difference between a soccer ball and a rugby ball

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[22] *Pattern of shapes is discussed in Key stage 1 under Pattern and Data Representations*

Explaining Positions and Directions

Exploring how to explain a position and direction

1. Identify simple positions and directions of an object accurately using various ways such as in my perspective, in your perspective in the classroom, and the left, right, front, back, west, east, north, south and with measurement
2. Draw the map around the classroom with consideration of locations
3. Design a game to appreciate the changing of positions and directions in a classroom

Strand: Pattern and Data Representations

Various types of patterns such as the number sequence and repetition of shapes are considered. The size of pictures can be represented by the number sequence. Tessellation of shapes and paper folding can be represented by the repetition of shapes. Exploration of patterns and features is also considered to represent the data structure in our life using pictographs and bar graphs. Patterns and features produce the meaning of data and represent mathematical information. Patterns are represented by diagrams and mathematical sentences which are also used for communication in identifying and classifying situations to produce meaningful interpretations.

Topics:

Using Patterns under the Number Sequence
Producing Harmony of Shapes using Patterns
Collecting Data and Represent the Structure

Using Patterns under the Number Sequence[23]

Arranging objects for beautiful patterns under the number sequence

1. Know the beautifulness of patterns in cases of arranging objects based on number sequence
2. Arrange objects according to number sequence to find simple patterns
3. Arrange expressions such as addition and subtraction to find simple patterns
4. Express the representation of patterns using placeholders (empty box)
5. Enjoy the arrangement of objects based on number sequence in daily life
6. Find patterns on number tables such as in calendars[24]

Producing harmony of shapes using patterns[25]

Arranging tiles of different or similar shapes in creating harmony

1. Know the beautifulness of patterns in cases of arranging the objects based on shapes, colours and sizes
2. Arrange objects according to shapes, colours and sizes to show patterns
3. Arrange boxes according to shapes, colours and sizes to create a structure
4. Arrange circles and spheres for designing
5. Enjoy the creation based on different shapes, colours and sizes in daily life

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[23] *Number sequence will be discussed in Key Stage 1 under Numbers and Operations*

[24] *Time and duration are discussed in key stage 1 under Quantity and Measurement*

[25] *Harmony of shapes will be discussed in Key Stage 1 under Shapes, Figures and Solids*

Collecting data and representing the structure

Collecting data through categorisation for getting information

i. Explore the purpose of why data is being collected
ii. Grouped data by creating similar attributes on the denomination[26] of categories and counting them (check mark and count)
iii. Think about what information is obtained from the tables with categories and how to use it

Organising the data collected and representation using pictograms for easy visualisation

i. Produce the table and pictograms from collected data under each category
ii. Interpretation of tables and pictograms as a simple conclusion about the data being presented.
iii. Appreciate pictograms through collecting data and adding data in daily activities in learners' life

Representing a data structure by using a bar graph to predict the future of communities

i. Understand how to draw bar graphs from a table using data categories and sort the graph for showing its structure
ii. Appreciate ways of presenting data such as using tables, pictograms and bar graphs with sorting for predicting their future communities
iii. Appreciate the use of data for making a decision

Strand: Mathematical Process - Humanity

Enjoyable mathematical activities are designed to bridge the standards in different strands. Exploration of various number sequences, skip counting, addition and subtraction operations help to develop a number sense that is essential to support explanations of contextual scenarios and mathematical ideas. Mathematical ways of posing questions in daily life are also necessary to learn at this stage. The ability to select simple, general and reasonable ideas enables effective future learning. The application of number sense provides a facility for preparing sustainable life. The use of ICT tools and other technological tools provides convenience in daily life. At the initial stage, concrete model manipulation is enjoyable, however, drawing a diagram is most necessary for explaining complicated situations by using simple representation.

Enjoying problem solving through various questioning for four operations in situations
Enjoying measuring through setting and using the units in various situations
Using blocks as models and its diagram for performing operations in base ten
Enjoying tiling with various shapes and colours
Explaining ideas using various and appropriate representations
Selecting simple, general and reasonable ideas which can apply to future learning
Preparing sustainable life with a number sense
Utilizing ICT tools such as calculators as well as other tools such as notebooks and other instruments such as clocks
Promoting creative and global citizenship for sustainable development of neighbourhood using mathematics

Enjoying problem solving through various questioning for four operations in situations[27]

i. In addition, pose questions for altogether and increase situations
ii. In subtraction, pose questions for remaining and differences in situations
iii. In multiplication, pose questions for the number of groups in situations
iv. In division, pose questions for partition and quotation in situations
v. Enjoy questioning by using a combination of operations in various situations
vi. In operations, pose questions to find easier ways of calculation
vii. Use posing questions for four operations on measurements in daily life

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[26] *Denomination will be learned in Key stage 1 under Quantity and Measurement*
[27] *It is related to Numbers and Operations and Quantity and Measurement both in Key Stage 1.*

Enjoying measuring through setting and using the units in various situations[28]

i. Compare directly and indirectly
ii. Set tentative units from difference for measuring
iii. Give appropriate names (denominations) for counting units
iv. Use measurement for communication in daily life
v. Use tables and diagrams for showing the data of measures

Using blocks as models and its diagram for performing operations in base ten[29]

i. Show increasing and decreasing patterns using blocks
ii. Show based ten system using blocks, the unit cube is 1, the bar stick is 10 and the flat block represents 100
iii. Explain the addition and subtraction algorithm in vertical form using a base-ten block model
iv. Explain multiplication table with grouped blocks
v. Explain division using equal distribution of blocks and repeated subtraction of blocks
vi. Use the number of blocks for measurement in daily life

Enjoying tiling with various shapes and colours[30]

i. Appreciate producing beautiful designs with various shapes and finding the pattern to explain it
ii. Reflect, rotate and translate to produce patterns
iii. Cut and paste various shapes and colours to form the box and ball such as developing the globe from a map

Explaining ideas using various and appropriate representations[31]

i. Explain four operations using pictures, diagrams, blocks and expressions for developing ideas
ii. Explain measurement using measuring tools, tape diagrams, containers and paper folding for sharing ideas
iii. Make a decision on how to explain the figures and the solids by using manipulative objects or diagrams or only verbal explanation
iv. Explain patterns using diagrams, numbers, tables and expressions with a blank box
v. Ask questions using terms such as why, how, what, if and if not, and reply using examples and ‘for example’ in discussions
vi. Change the representation and translate it appropriately into daily life

Selecting simple, general and reasonable ideas which can apply to future learning[32]

i Discuss the argument for the easier ways for addition and subtraction algorithms in vertical form
ii Extend the algorithm to large numbers for convenience and fluency
iii Use the pattern of increase in multiplication table for convenience
iv Use multiplication tables for finding the answers to division

Applying number sense[33] acquired in Key Stage 1 for preparing sustainable life[34]

i Use mathematics for the minimum and sequential use of resources in situations
ii Estimate for efficient use of resources in situations

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[28] *It is related to Quantity and Measurement and Pattern and Data Representations both in Key Stage 1.*

[29] *It is related to Pattern and Data Representations and Numbers and Operations both in Key Stage 1.*

[30] *It is related to Shapes, Figures and Solids and Pattern and Data Representations both in Key Stage 1.*

[31] *It is related to all strands in Key Stage 1.*

[32] *It is related to Numbers and Operations and Pattern and Data Representations both in Key Stage 1.*

[33] *It is related to Numbers and Operations, Pattern and Data Representations and Quantity and Measurement all in included in Key Stage 1.*

[34] *Sustainable development goals were crafted at the 70th Session of the United Nations General Assembly and indicated them as universal value in education.*

iii Maximize the use of resources through an appropriate arrangement in space
iv Understand equally likely of resources in situations

Utilising ICT tools such as calculators as well as other tools such as notebooks and other instruments such as clocks[35]

i Use calculators for poly addition in situations
ii Use mental calculations for estimations
iii Use a balance scale to produce equality and inequality
iv Use cups, tapes, stopwatch, and weighing scales for measuring distances and weights
v Use calculators to explain the process of calculation by solving backwards and understand the relationship of addition and subtraction, and multiplication and division.
vi Enjoy using notebooks to exchange learning with each other such as mathematics journal writing
vii Enjoy presentations with board writing
viii Use various tools for conjecturing and justifying

Promoting creative and global citizenship for sustainable development of neighbourhood using mathematics

i. Utilise notebooks and journal books to record and find good ideas and share them with others
ii. Prepare and present ideas using posters to promote good practices in the neighbourhood
iii. Listen to other’s ideas and ask questions for better creation
iv. Utilise information, properties and models as a basis for reasoning
v. Utilise practical arts and outdoor studies to investigate local issues for improving the welfare of life

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[35] *STEM education is enhanced. Mathematics is the major and base subject for STEM Education in Key Stage 1 hence, technological contents are included in Mathematics.*

APPENDIX F

The four strands of content learning standards and the strand on Mathematical Process-Humanity of Key Stage Three

KEY STAGE 3

Key Stage 3 (KS3) can be developed based on Key Stage 2. This stage focuses on numbers and algebra, relations and functions, space and geometry, and statistics and probability. Symbolic representations allow the dealing of abstract ideas and concepts that enhance critical and creative thinking through the application of knowledge. Understanding and using mathematical concepts and principles in this stage through discussions, dialogue, and arguments enable learners to participate in contemporary societal, economic, technological, political, environmental and mathematical issues. This stage is the basis for the creation of a better future with predictions. It bridges further mathematics learning in various job demands.

Strand: Numbers and Algebra

Numbers are extended to positive and negative numbers and square roots. Algebraic expressions are already introduced by the mathematical sentences and symbols at Key Stage 2. At Key Stage 3, algebra is operated by expressions and equations until the second degree. On the extension from numbers to symbolic algebra, various possible ways of calculations are explored until their appropriateness is established. Like and unlike terms are introduced in an algebraic sentence and in simplifying expressions. Properties of equations are introduced for finding simple equivalents and solving equations with fluency. Substitution, addition and subtraction of equations enable further operations of simultaneous equations. Expansion and factorisation enable further operations of the polynomials. Finally, quadratic equations can be solved using various operations.

Topics:

Extending Numbers to Positive and Negative Numbers
Utilising Letters for Algebraic Expressions and Equations
Algebraic Expressions, Monomials and Polynomials and Simultaneous Equations
Expansion and Factorisation of Polynomials
Extending Numbers with Square Roots
Solving Quadratic Equations

Extending Numbers to Positive and Negative Numbers

Extending numbers to positive and negative numbers and integrating four operations into addition and multiplication

i. Understand the necessity and significance of extending numbers to positive and negative numbers in relation to directed numbers with quantity
ii. Compare numbers which are greater or less than on the extended number line and use absolute value for the distance from zero
iii. Extend operations to positive and negative numbers and explain the reason
iv. Get efficiency on calculation in relation to the algebraic sum

Utilising Letters for Algebraic Expressions and Equations

Extending the utilisation of letters for a general representation of situations and finding ways to simplify algebraic expressions

i. Appreciate the utilisation of letters for a general representation of situations to see the expression as a process and value

ii. Find ways to simplify expressions using distributive law and figural explanations, establish the calculation with like and unlike letters
iii. Acquire fluency in simplifying an expression and appreciate it for representing the pattern of the situation

Thinking about a set of numbers in algebraic expression with letters as variables and representing them with equality and inequality

i. Recognise numbers as positive and negative numbers, and explain integers as a part of numbers
ii. Represent a set of numbers using variables with equality and inequality
iii. Translate given sets of numbers on the number lines using interval and inequality notations
iv. Appreciate redefining even and odd numbers using letters to represent different sets of variables

Thinking about how to solve simple linear equation

i. Review the answers to equations from the set of numbers and think backwards
ii. Know the properties of equations which keep the set of answers of the equation
iii. Appreciate the efficient use of properties of equations to solve linear equation
iv. Use equations based on life situations to develop fluency, solve equations, and interpret the solution

Algebraic Expressions, Monomials and Polynomials and Simultaneous Equations

Thinking about the calculations of monomials and polynomials for simple case[1]

i. Introduce terms, monomials and polynomials
ii. Introduce a number raised to the power of two as a square, and a number raised to the third power as a cube
iii. Get fluency in the calculation of polynomials such as combining like terms and the use of the four operations in simple cases

Thinking about how to solve simultaneous equations in the case of linear equations

i. Understand the meaning of solutions of linear equations and simultaneous linear equations as a pair of numbers
ii. Know the substitution and elimination methods of solving simultaneous linear equations
iii. Get fluency in selecting the methods from the form of the simultaneous linear equations
iv. Appreciate simultaneous linear equations in situations

Expansion and Factorisation of Polynomials

Acquisition to see the polynomials in the second degree with expansion and factorisation and use it

i. Use the distributive law to explain the formulae for expansion and explain them on diagrams
ii. Acquire proficiency in selecting and using the appropriate formulae
iii. Use the expansion formulae to factorise the second-degree expression and recognise both formulae with the inverse operation
iv. Solve the simple second-degree equations using the factorisation and apply it in life situations

Extending Numbers with Square Roots

Extending numbers with square roots and calculating the square roots algebraically
i. Define square root and discuss ways to estimate the nearest value of a square root by the Sandwich Theorem

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[1] *Simple case may vary from and depending on the countries based on the mapping of curriculum.*

ii. Understand that some square roots cannot be represented as fractions
iii. Compare square roots using a number line and understand that the order does not change but the differences between two consecutive square roots varied
iv. Think about multiplication and addition of square roots and understand the algebraic way of calculation which is similar to polynomial
v. Appreciate square roots in applying to situations in life[2]

Solving Quadratic Equations

Solving simple second-degree equations using factorisation and apply on the situation

i. Find the answers to simple second-degree quadratic equations by substitution and explore by completing the square, quadratic formula, and factorisation[3]
ii. Get fluency to select the appropriate ways for solving quadratic equations
iii. Apply quadratic equations in life situations

Strand: Relations and Functions

Relationships are represented by equations and a system of equations. Functional relations are treated amongst situations, tables, and equations of function are introduced based on patterns and relations with algebraic representation in Key Stage 2 and Key Stage 3. The solution of a simple equation is done by equivalence deduction based on algebra learnt earlier. Two variables in simultaneous equations as a simple system of equations are solved by substitution and additive-subtractive methods. Three representations, table, equation and graph, are used as methods to analyse the properties of every function. Proportion and inverse proportions are redefined with those representations mentioned. The proportional function is extended to line functions. The comparisons of inverse proportion and line functions are made clear by the property of linearity with a 'constant ratio of change'. The concept of proportion is extended to the function of $y = ax^2$. Ways of translations between table and equation, equation and graph, and graph and table are specific skills for every function with fluency.

Topics:

Extending Proportion and Inverse Proportion to Functions with Variables
Exploring Linear Function in Relation to Proportions
Exploring Simple Quadratic Function
Generalising Functions

Extending Proportion and Inverse Proportion to Functions with Variables

Extending proportion and inverse proportion to functions with variables on positive and negative numbers

i. Extend proportions to positive and negative numbers, using tables and equations on situations
ii. Plot a set of points as a graph for proportions defined in ordered pairs (x, y) in the coordinate plane using appropriate scales precisely[4]
iii. Introduce inverse proportion using tables, equations and graphs
iv. Introduce function as correspondences of two variables in situations
v. Explore the property of proportional function with a comparison of inverse proportional function
vi. Appreciate proportion and inverse proportion functions in life

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[2] *Pythagorean Theorem is discussed in Space and Geometry strand.*

[3] *The graph of quadratic equation will be treated in Relations and Functions strand.*

[4] *Utilising ICT is recommended in mathematical activities.*

Exploring Linear Function in Relation to Proportions

Exploring linear function in relation to proportion and inverse proportions

i. Identify linear functions based on situations represented by tables and compare them with proportional functions
ii. Explore properties of the linear function represented by tables, equations and graphs and compare it with direct and inverse proportional functions
iii. Acquire fluency to translate the rate of change of a linear function represented in the table, as a coefficient in an expression and gradient in a graph[5]
iv. Acquire fluency to translate y values of $x = 0$ in a table, constant in an expression, and *y-intercept* in a graph
v. Apply the graphs of linear functions to solve simultaneous equations
vi. Apply the linear function for data representation on situations to determine the best-fit line

Exploring Simple Quadratic Function

Exploring quadratic function $y = ax^2$ in relation to a linear function

i. Identify the quadratic function on situations using tables and comparing it with a linear function
ii. Explore properties of a quadratic function using tables, equations as well as graphs and compare it with a linear function
iii. Apply the quadratic function to situations in daily life and appreciate it

Generalising Functions

Generalising functions with various representations[6] of situations

i. Distinguish domain, range and intervals and is appropriately used for explaining the function
ii. Use various situations for generalising ideas of functions such as moving point A and moving point B with a time
iii. Compose a graph as a function of two or more graphs with different domains in a situation
iv. Introduce situations of step-functions[7] with a graph for generalisation of the idea of a function which cannot be represented by an equation

Strand: Space Geometry

Space and Geometry provide ways of reasoning for exploring properties in geometry and produce ways of argument to explain justifications of visual reasoning. The calculations of angles are not just simple calculation but also ways of using geometric propositions to justify answers by explaining why it is correct based on basic properties. By explaining the relationship of figures using transformation, the properties of congruency and describing similarity are identified and described. Finding the value of angles and building arguments for proving are means for developing the habit of reasoning in the properties of plane figures. The conditions of congruence and similarities, properties of circles, are also used to explain and prove the appropriateness of geometric conjectures in relation to triangles, quadrilaterals, and circles. Dynamic geometric software as well as a simple compass and ruler are used for conjecturing. It shows general ideas from consistency in variations. The counterexample is also found as a special case from variations.

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[5] *Family of linear functions are recommended to use ICT tool under mathematical activities strand.*

[6] *Utilising functions as model in daily life which is necessary for STEM education.*

[7] *Intervals are taught in the Key Stage 3 on Numbers and Algebra.*

Topics

Exploring Angles, Construction and Designs in Geometry
Exploring Space with its Components
Exploring Ways of Argument for Proving and Its Application in Geometry

Exploring Angles, Construction and Designs in Geometry

Exploring angles to explain simple properties on the plane geometry and doing the simple geometrical Construction[8]

i. Explain how to determine the value of angles using the geometrical properties of parallel lines, intersecting lines, and properties of figures
ii. Use a ruler and compass to construct a simple figure such as perpendicular lines and bisectors
iii. Appreciate the process of reasoning that utilises the properties of angles and their congruency in simple geometrical constructions

Exploring the relationship of figures using congruency and enlargement for designs

i. Explore the congruence of figures through reflection, rotation and translation and explain the congruency using a line of symmetry, point of symmetry and parallel lines
ii. Explore the similarity of figures with enlargement using points, ratios, and correspondences
iii. Enjoy using transformations in creating designs

Exploring Space with its Components

Exploring space by using the properties of planes, lines and their combinations to form solids

i. Explore the properties produced by planes, lines and their combinations, such as parallel lines produced by the intersection of parallel planes with another plane
ii. Produce solids by combining planes such as nets and motion such as rotation, reflection and translation
iii. Recognise the space of an object based on its properties and projection

Exploring Ways of Argument for Proving and its Application in Geometry[9]

Exploring properties of congruency and similarity on plane geometry

i. Explore ways of arguments using the congruence of two triangles and appreciate the logic of the argument in simple proving
ii. Explore ways of arguments using the similarity of two triangles based on ratio and angles and appreciate the logic of arguments in simple proving
iii. Explore the proof of the properties of circles such as inscribed angles, and intercepted arcs
iv. Appreciate proving through making the order of proven propositions to find new propositions

Exploring Pythagorean theorem in solving problems in plane geometry and spaces

i. Explore the proving of the Pythagorean theorem using a diagram and use it in solving problems involving plane figures

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[8] *Simple geometric construction is discussed by the ruler and compass with reasoning. Dynamic Geometric software usually draws entire circles. For knowing invariant dynamic geometric software is useful.*

[9] *Dynamic Geometry software is useful to find the invariant properties which is discussed in Mathematical Processes and Humanity strand in Key Stage 3.*

[10] *Pythagorean theorem is used for re-understanding the topic on square root under Key Stage 3 Numbers and Algebra.*
Mean, median and quartile are fixed depending on the data. However, mode changes depending on the class.

ii. Apply the Pythagorean theorem on the prism by viewing the figures through faces.
iii. Explore the situations for simple trigonometry using special angles in relation to the Pythagorean theorem
iv. Appreciate the use of the Pythagorean theorem in life[10]

Strand: Statistics and Probability

Data handling is extended to explore the dispersion of histogram with mean, median, mode and range. Exploratory data analysis (EDA) attempting to represent and visualise the structure from the given data using Information Communication and Technology (ICT) is enhanced. The histogram shows different dispersion if we change the class. Probability is introduced as a ratio with the law of large numbers. Sample space with the assumption of equal probability becomes the point of discussion. Logical analysis to understand whole possible cases such as a tree diagram is introduced for knowing the ways to represent logical reasoning. Histogram can be seen relatively and produce frequency distribution polygon. The difference between the sample and the population is discussed. Boxplots with quartiles are an extension of the median and the range is used for comparisons of distributions. Using skills of statistics and probability make problem solving in situations possible. Analysing and identifying the trends in situations for making decisions is necessary such as issues for sustainable living.

Topics:

Exploring Distribution with the Understanding of Variability
Exploring Probability with the Law of Large Numbers and Sample Space
Exploring Statistics with Sampling

Exploring Distribution with the Understanding of Variability

Exploring distribution with histograms, central tendency and variability

i. Use histograms with different class intervals to show a different distribution of the same set of data
ii. Identify alternative ways to show distribution such as dot plots, box-plot and frequency polygons
iii. Investigate central tendencies such as mean, median, mode[11] and their relationships in a distribution
iv. Investigate dispersion such as range and inter-quartile range in a distribution
v. Appreciate the analysis of variability through the finding of the hidden structure of distribution on situations using the measure of central tendency and dispersion

Exploring Probability with the Law of Large Numbers and Sample Space

Exploring probability with descriptive statistics, the law of large numbers and sample space

i. Experiment with tossing coins and dice to explore the distribution of the relative frequency and understand the law of large numbers
ii. Use the idea of equally likely outcomes to infer the value of the probability[12]
iii. Analyse sample space of situations represented by a table to determine the probability and use it for predicting the occurrence
iv. Use various representations such as tables, tree diagrams, histograms and frequency polygons for finding the probability
v. Analyse data related to issues on sustainable development and use probability to infer and predict future events

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[11] *Mean, median and quartile are fixed depending on the data. However, mode changes depending on the class.*

[12] *The probability here is called equiprobability when all possible cases are equally likely. In the upper grade level, probability will be redefined based on distributions.*

Exploring Statistics with Sampling

Exploring sampling with the understanding of randomness

i. Discuss the hidden hypothesis behind the sample and population
ii. Use randomness to explain sampling
iii. Analyse the data exploratory such as dividing the original into two for knowing better data representations and discuss appropriateness such as regrouping
iv. Appreciate data sampling in a situation with sustainable development

Strand: Mathematical Processes – Humanity

The critical argument in mathematics is enhanced through communication with others beyond Key Stage 2. These proposed challenging activities will promote metacognitive thinking at a different level of arguments to make sense of mathematics. Translating real-life activities into mathematical models and solving problems using appropriate strategies are emphasised in functional situations. The process of doing mathematical activities involves patience that develops perseverance in learners and takes responsibility for one's own learning. At this stage, the habitual practice of self-learning will eventually develop confidence, thus, the opportunity for challenges to extend mathematics and the ability to plan sequences of future learning is also enhanced.

Standards:

Enjoying problem solving through various questioning and conjecturing for extension of operations into algebra, space and geometry, relationship and functions, and statistics and probability
Enjoying measuring space using calculations with various formulae
Producing proof in geometry and algebra
Utilising tables, graphs and expressions in situations
Using diagrams for exploring possible and various cases logically
Exploring graphs of functions by rotation, by symmetry and by translation of proportional function
Understanding ways for extension of numbers
Designing sustainable life with mathematics
Utilising ICT tools as well as other technological tools
Promoting creative and global citizenship for sustainable development of society in mathematics

Enjoying problem solving through various questioning and conjecturing for extension of operations into algebra, space and geometry, relationship and functions, and statistics and probability[13]

i. Pose questions to extend numbers and operations into positive and negative numbers, algebraic operations, and further extension into polynomial operations, and numbers with square roots
ii. Pose questions to solve linear equations, simultaneous equations and simple quadratic equations
iii. Pose assumptions in geometry as objects of argument and proof
iv. Pose questions to transform three-dimensional objects into two-dimensional shapes and vice versa
v. Pose questions in relations and functions for knowing properties of different types of function
vi. Pose questions to exploratory of data handling for knowing the structure of distributions
vii. Pose questions that apply PPDAC in relation to statistical problem solving
viii. Pose questions in relation to "equally likely" events
ix. Pose assumptions to discuss the hypothesis based on the sample and population
x. Pose conjecturing such as if x increases and y decreases then it is an inverse proportion

Enjoying measuring space using calculations with various formulae[14]

i. Extend the number line to positive and negative numbers and compare the size of numbers with the idea of absolute value

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[13] *Connected to the three strands, namely Numbers and Algebra, Relations and Functions, and Space and Geometry.*
[14] *Connected to the three strands, namely Numbers and Algebra, Relations and Functions, and Space and Geometry.*

- ii. Derive the square root using unit squared paper through the idea of the area
- iii. Explain the expansion of polynomials using area diagrams
- iv. Use the projection of space figures to plane figures using the Pythagorean theorem
- v. Apply similarity and simple trigonometry for measurement
- vi. Use common factors to explain factorisation of the area of a rectangle based on the area of a square

Producing proof in geometry and algebra

- i. Have an assumption through exploration and produce propositions
- ii. Justify the proposition using examples and counter-examples to understanding
- iii. Rewrite propositions from sentences to mathematical expressions by using letters and diagrams
- iv. Search the ways of proving by thinking backwards from the conclusion and thinking forward from the given
- v. Show the proof and critique for the shareable and reasonableness
- vi. Deduce other propositions in the process of proving and after proving using what if and what if not
- vii. Adapt ways of proving to other similar propositions of proof
- viii. Explain the written proof in geometry and algebra by the known
- ix. Revise others' explanations meaningfully

Utilising tables, graphs and expressions in situations[15]

- i. Explore the properties of functions by using tables, graphs and expressions and establish the fluency of connections among them for interpreting functions in the context
- ii. Analyse the distribution of raw data by using tables, graphs and expressions in daily life

Using diagrams for exploring possible and various cases logically[16]

- i. Use number line with inequality to identify range and set
- ii. Use a circle to identify the relationship between the circumference and the central angle (acute, obtuse and right)
- iii. Use a rectangle and rotate a point on the side rectangle to draw the graph of the area
- iv. Use tree diagram for thinking about all possible cases sequentially

Exploring graph of functions by rotation, by symmetry and by translation of proportional function[17]

- i. Use the slope of a graph for the proportional function to rotate the graph or to determine the point of intersection
- ii. Explore to know the nature of two simultaneous equations by using translation
- iii. Use the y-axis, x-axis and as the line of symmetry to explore the proportional function
- iv. Explain the graph of the linear function by translation of the proportional function.

Understanding ways for extension of numbers[18]

- i. Extend the numbers based on the necessity of solving equations such as $x + 5 = 3$ and , and show examples for demonstrating the existence such as on the number lines, and understand it as set
- ii. Compare the size of numbers and identify how to explain the order of numbers and their equivalence
- iii. Extend operations for keeping the form[19] beyond the limitations of meaning[20]

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[15] *Connected to the strand on Relations and Functions.*

[16] *Connected to the two strands on Relations and Functions and Space and Geometry.*

[17] *Connected to the two strands on Numbers and Algebra and Relations and Functions.*

[18] *Connected to the strand on Numbers and Algebra.*

[19] *There are three meanings of form: (1) Permanence of form means "keep the pattern of operation" such as (-3)x(+2)=-6, (-3)x(+1)=-3, (-3)x0=0, and (-3)x(-1)=+3, and (-3)x(-2)=+6. Here, the product of the pattern increases by 3; (2) The form means "Principle of the permanence of equivalence form" which means to keep the law of commutativity, associativity and distributivity; and (3) The form means the axiom of field in Algebra. Normally, in education, we only treat (1) and(2).*

[20] *For the extension of numbers to positive and negative numbers, beyond the limitations of meaning such as subtract smaller number from larger number. For the extension of numbers to irrational number, beyond the limitation of meaning such as rational number is quotient (value of division).*

Designing models for sustainability using mathematics[21]

i. Discuss and utilise probabilities in life such as weather forecasting for planning
ii. Design cost-saving lifestyle models through comparison of data such as cost of electricity, water consumption, and survey
iii. Plan emergency evacuation such as heavy rain and landslide where the calculations on the amount of water in barrel per minute exceed the maximum standards
iv. Forecast the future with mathematics

Utilising ICT tools as well as other technological tools

i. Use dynamic geometry software for assumption, specialisation and generalisation
ii. Use a graphing tool for comparison of the graph and knowing the properties of a function
iii. Use data to analyse statistics with software
iv. Use internet data for the discussion of sustainable development
v. Use calculators for operations in the necessary context
vi. Use a projector for sharing ideas such as project surveys, reporting and presentation
vii. Use the idea of function to control a mechanism
viii. Use ICT tools for conjecturing and justifying to produce the object of proving.

Promoting creative and global citizenship for the sustainable development of society using mathematics

i. Utilise notebooks, journal books and appropriate ICT tools to wisely record and produce good ideas for sharing with others
ii. Prepare and present ideas using posters, projectors, pamphlets and social media to promote good practices in society
iii. Promote the beautifulness, reasonableness and simplicity of mathematics through contextual situations in the society
iv. Listen to other’s ideas and ask questions for better designs, craftsmanship and innovations
v. Utilise information, properties, models and visible representations as the basis for making intelligent decisions
vi. Utilise practical arts, home economics, financial mathematics and outdoor studies to investigate local issues for improving welfare of life

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[21] *Connected to the three strands, Numbers and Algebra, Relations and Functions and Space and Geometry.*